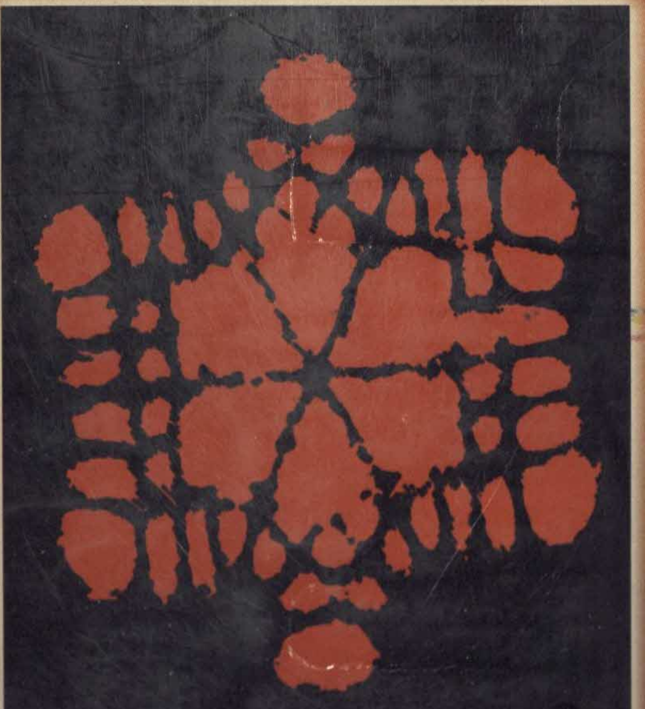
DR. MAHADEB DASKHAN

A TEXT BOOK OF

PHYSICS



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CONSTITUTION OF INDIA

PREAMBLE

WE, THE PEOPLE OF INDIA, having solemnly resolved to constitute India into a **SOVEREIGN SOCIALIST SECULAR DEMOCRATIC REPUBLIC** AND TO SECURE TO ALL CITIZENS :

JUSTICE, social, economic and political;

LIBERTY of thought, expression, belief, faith and worship;

EQUALITY of status and of opportunity : and to promote among them all

FRATERNITY assuring the dignity of the individual and the unity and integrity of the Nation :

IN OUR CONSTITUENT ASSEMBLY THIS TWENTY-SIXTH day of November, 1949, do **HEREBY ADOPT, ENACT AND GIVE TO OURSELVES THIS CONSTITUTION.**

FUNDAMENTAL RIGHTS UNDER THE CONSTITUTION OF INDIA

RIGHT TO EQUALITY

- The state shall not deny to any person equality before the low or equal protection of the law.
- The state shall not discriminate against any citizen on the ground only of religion, race, caste, sex, place or birth.
- There shall be equality of opportunity for all citizens in matters relating to employment or appointment to any office under the state.
- Untouchability in practice in any form is forbidden and punishable.

RIGHT TO FREEDOM

- To freedom of speech and expression.
- To assemble peaceably and without arms.
- To form associations or unions.
- To move freely throughout the territory of India.
- To reside and settled in any part of the territory of India.
- To practice and profess, or to carry on any occupation, trade and business.
- To provide for Right to Life and Personal Liberty, except according to procedure established by law.
- State shall provide free and compulsary education to all children of the age of 6—14 years in a manner to be determined by the state.

RIGHT AGAINST EXPLOITATION

- Prohibition of traffic in human beings and forced labour.
- No child below the age of fourteen years shall be employed to work in any factory or mine or engage in any other hazardous employment.

RIGHT TO FREEDOM OF RELIGION

- Every person in India shall have the freedom of conscience and shall have the right to Profess, practise and propagate religion, subject to public order.
- No person shall the compelled to pay any taxes, of any particular religion.
- No religious instruction shall be provided in any educational institution wholly maintained out of state fund.

CULTURAL AND EDUCATIONAL RIGHT

- Any section of the citizen residing in the territory of India or any part theof having a distinct language, script or culture of its own shall have the right to conserve the same.
- No citizen shall be denied admission into any educational institution matained by the state or receiving aid out of state fund on the ground only of religion, race, caste, langue or any of them.
- All minorities, whether based on religion or language, shall have the ht to establish and administer educational institution of their choice.

RIGHT TO CONSTITUTIONAL REMEDIES

- The Constitution guarantees the right to move the Supreme Court for thenforcement of the fundamental rights through certain constitutional writs. such as : Habeas Corpus, Mdamus, Certiorari, Prohibition and Quo-Warranto.

Fundamental Duties of Indian Citizen

ARTICLE 51 A

Fundamental Duties – It shall be the duty of every citizen of India–

(a) to abide by the Constitution and respect its ideals and institutions, t National Flag and the National Anthem ;

(b) to cherish and follow the noble ideals which inspired our national stggle for freedom :

(c) to uphold and protect the sovereignty, unity and integrity of India ;

(d) to defend the country and render national service when called upoto do so ;

(e) to promote harmony and the spirit of common brotherhood aongst all the people of India transcending religious, linguistic and regional or sectional diversities renounce practices derogatory to the dignity of women ;

(f) to value and preserve the rich heritage of our composite culture ;

(g) to protect and improve the natural environment including forests, kes, rivers, wild life and to have compassion for living creatures ;

(h) to develop the scientific temper, humanism and the spirit of inquirand reform;

(i) to safeguard public property and to abjure violence;

(j) to strive towards excellence in all spheres of individual and colltive activity so that the nation constantly rises to higher levels of endeavour and achievement.

(k) Who is a parent or guardian to provide opportunities for education his child on, and the case may be, ward between the age of six and fourteen years.

Written in accordance with the New Syllabus for H. S., Joint Entrance Examination, I. I. T., Tripura, A. I. E. E. E. and other Competitive Examination.

A TEXT BOOK OF PHYSICS

■ **For Class XII** ■

DR. MAHADEB DASKHAN

Former Reader, Physics Department
Ramakrishna Mission Residential College, Narendrapur

Author of

Physical Science (VII, VIII & IX-X); A Text Book of Physics (XI, XII);
Joint Entrance Physics (MCQ); Degree Practical Physics

FIRST EDITION

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✪ PREFACE TO THE FIRST EDITION ✪

The Text book of Physics (Class XII) is published. The book covers fully the syllabus prescribed by different Indian Higher Secondary Education Boards for Class XII of Higher Secondary Course. According to the new system of examination different H.S. Boards will conduct one final examination only on Paper-II i.e. on Class XII syllabus. Full marks 100 (Theoretical-80 and Practical-20).

The present volume is written following amended syllabus of Class XII which is divided into six parts. (i) Mechanics & Properties of matter-II (ii) Heat and Thermodynamics-II (iii) Waves and Physical optics (iv) Electrostatics (v) Current electricity-II (v) Modern Physics-II

The book has been written in SI units and the language of the book is kept as simple as possible. Following model questions of the new syllabus, objective questions. Short answer type questions and large number of solved numerical problems are included in each chapter.

This book will also help for the preparation of Joint Entrance Examination for admission to different Engineering and Medical Colleges in India.

I express my sincere gratitude to my publisher Sri Chanchal Kumar Bhowal of Calcutta Book House (P) Ltd., for his faith on me. I also express my thanks to the employees of this organisation for helping me for the publication of the book.

To conclude, I apologise in advance for the errors (if any) that may have inadvertently crept in the text.

Any suggestion for the improvement of the book will be warmly accepted and highly appreciated.

Author

Dated : 16th August, 2006
27/1, R.K. Palli
Sonarpur

SYLLABUS OF PHYSICS

CLASS XII

Full Marks – 100

THEORY – 80

Mechanics & Properties of Matter-II [10M / 10P]

(i) Dynamics of rotational motion **[5M / 5P]**

Rotational motion of a particle; angular velocity, angular acceleration, relation between angular velocity and linear velocity.

Angular momentum, relation between angular acceleration and torque (statement only), Moment of inertia and its physical significance (Formulae of moment of of inertia to be assumed for uniform bodies with simple geometrical shapes) radius of gyration (definition only). Rotational kinetic energy, conservation of angular momentum with some examples.

Centripetal force, centrifugal force (as a pseudo force).

(ii) Gravitation **[5P]**

Newton's Law of Universal Gravitation (statement and mathematical relation). Constant of gravitation (definition and value with units, no experimental determination). Examples of gravitational attraction between heavenly bodies (e.g. the Sun and the Earth, the Earth and the Moon) Gravitational attraction of the earth, relation between G and g. Variation of g. Gravitational potential energy near the surface of the earth, gravitational potential, escape velocity, Motion of planets and satellites, statement of Kepler's laws of planetary motion. Geostationary and polar satellites. Weightlessness in orbiting satellites.

Heat and Thermodynamics-II [10M / 10P]

(i) Thermodynamics **[5M / 5P]**

Heat as a form of energy; relation between calorie and erg, First Law of Thermodynamics, Intensive and extensive thermodynamic variables. Isothermal and adiabatic expansion of gases (brief discussion), reversible and irreversible processes. Specific heats of gases, relation between c_p and c_v.

(ii) Kinetic Theory of gases **[5M / 5P]**

Evidence of molecular structure of matter and of random molecular motion. Brownian movement (qualitative discussion). Basic assumptions of kinetic theory of ideal gases. Mean, rms and most probable speed. Pressure of an ideal gas (simple derivation). Charle's law, Boyle's law, Avogadro's law and pressure law from kinetic theory of gases. Kinetic energy of molecules and absolute temperature; definition of mean free path.

Waves and Physical Optics [16M / 16P]

(i) Waves **[11M / 11P]**

Elastic waves—longitudinal and transverse; characteristics of propagating waves, nature of the medium, wavelength, amplitude of wave, time period, frequency, velocity of wave and their relation. Properties of waves : Laws of reflection and refraction of waves with reference to sound wave. *Periods—3*

Sound wave : longitudinal elastic wave, velocity of sound wave, statement and explanation of Newton's formula and Laplace's correction. Dependence of velocity on temperature, pressure and humidity of air.

Superposition of waves—graphical and analytical representation : (i) two S. H. waves of nearly equal frequency and amplitude and moving in the same direction (beat) : determination of unknown frequency from beating frequency. (ii) Two S. H. waves of equal amplitude and frequency travelling in opposite directions (standing or stationary waves); characteristics of standing waves and comparison with progressive wave.

Transverse standing waves on stretched wire; laws of transverse vibration of strings, fundamental, harmonics.

Longitudinal standing waves in air column. (i) Vibration of air column in a tube closed at one end. Determination of velocity of sound or the frequency of a tuning fork by resonance with vibration of air column in a tube closed at one end. (ii) Vibration of an air column in a tube open at both ends.

Doppler Effect in sound propagation (derivation not required).

(ii) Physical Optics [5M / 5P]

Wave front and Huygen's Principle; Reflection and refraction of plane wave at a plane surface using wave fronts (qualitative idea); Interference—Young's double slit experiment and expression for fringe width. Coherent sources.

Electrostatics [12M / 12P]

(i) Introduction [3M / 3P]

Electricity from friction, two types of electricity, repulsion is a surer test of electrification, explanation of charging by rubbing in the light of electron theory. Conductor and insulator.

Dielectric polarisation — physical explanation.

Charge resides on the outer surface of a conductor—Electric screening. Effect of curvature on surface density of charge. Action of points—discharging action of points, spraying action of points, collecting action of points, lightning conductor.

(ii) Electrostatic field & Electric potential [6M / 6P]

Coulomb's law of force between two point charges; permittivity, e.s.u. and S.I. unit of charge, electric intensity; Electric field due to a point charge. Electric lines of force, properties of lines of force.Electric dipole, electric field due to a dipole and behaviour of a dipole in a uniform electric field.

Electric flux, statement of Gauss' Theorem and its application to find field due to uniformly charged infinitely long straight wire, thin spherical shell. Potential at a point in the electric field; Potential difference between two points; e.s.u. of potential, practical unit of potential, potential of a charged conductor, potential of the earth, relation between intensity and potential (qualitative).

(iii) Capacitance and capacitors [3M / 3P]

Capacitance of a conductor, factors affecting capacitance of conductor; capacitance of a spherical conductor, capacitors, parallel plate capacitors and capacitance, Unit of capacitance. Combination of capacitors in series and parallel. Types of capacitors. Van de Graff Generator.

Current Electricity-II [20M / 20P]

(i) Heating effect of current [4M / 4P]

Joule's law, mechanical equivalent of heat and its determination by electrical method. Electrical energy, power, unit of power and energy (to be taught in S.I. units). Board of Trade unit of electrical energy.

Thermoelectricity, Seebeck and Peltier effect, thermo-emf, thermo current, thermocouple, thermo-emf depends on difference of temperature of the two junctions of a thermocouple and nature of the metals of the thermocouple.

(ii) Electromagnetism-I [8M / 8P]

Concept of magnetic field, Oersted's experiment, Biot Savart law, Magnetic field due to an infinitely long current carrying straight wire and a circular loop; Ampere's circuital law and its application to straight and toroidal solenoids; Force on a moving charge in uniform magnetic and electric fields. Cyclotron frequency, force on current carrying conductor in a uniform magnetic field. Forces between two parallel current-carrying conductors-definition of ampere. Torque experienced by a current loop in a uniform magnetic field, moving coil galvanometer — its current sensitivity and conversion to ammeter and voltmeter, current loop as a magnetic dipole and its magnetic dipole moment; Magnetic field intensity due to a magnetic dipole (bar magnet). Tangent galvanometer : Bar magnet as an equivalent solenoid, magnetic field lines. Molecular theory of magnetism; Magnetic premeablity and susceptibility, Dia-, para- and ferromagnetic substances, examples, Terrestrial magnetism : Magnetic elements, Definitions, Declination, Dip and Horizontal component of the earth's magnetic field; their explanation (measurements not required).

(iii) Electromagnetism-II **[4M / 4P]**

Electromagnetic induction. Magnetic induction, magnetic flux, flux density, Faraday's laws of induction, induced emf; induced current; Lenz's law of electro-magnetic induction and its justification from the principle of conservation of energy. Determination of the direction of induced current by Lenz's law, emf induced in a conductor moving in a magnetic field, Fleming's left-hand rule.

(iv) Alternating current **[4M / 4P]**

Idea of alternating current (qualitative discussion only) Peak and rms values of alternating current / voltage. Elementary principles of dynamo and D. C. motor.

Modern Physics-II [12M / 12P]

(i) Electromagnetic waves **[2M / 2P]**

Electromagnetic waves and their characteristics (qualitative ideas only); transverse nature of electromagnetic waves. Electromagnetic spectrum (Radio, mocrowaves, infra-red, optical, ultraviolet, X-rays, γ-rays).

(ii) Semiconductors & Electronics **[5M / 5P]**

Elementary idea of energy bands in solids; intrinsic, *p* and *n*-type semiconductors, p-*n* junction diode and its characteristics, its use as half-wave and full-wave rectifiers, *pnp* and *npn* transistors, their CE characteristics. Binary numbers—basic concepts, AND, OR, NOT gates.

(iii) Quantum theory **[5M / 5P]**

Photoelectric phenomena, particle nature of radiation, Einstein's equation, threshold frequency, work function; energy and momentum of photon, photoelectric cell. Wave particle duality, de Broglie's hypothesis (qualitative idea only). Deduction of Bohr's formula and explanation of emission and absorption of radiation through electronic transition. X-ray emission, Moseley's law and atomic number.

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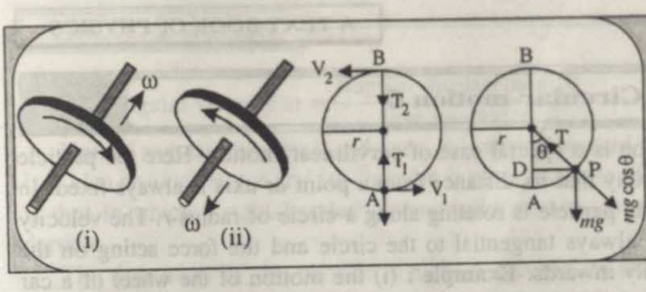


CHAPTER 1

CIRCULAR MOTION

● **TOPICS :** ○ Introduction, ○ Circular motion, ○ Angular displacement, ○ Angular velocity, ○ Average and instantaneous angular velocities, ○ Relation between angular and Linear velocity, ○ Angular acceleration, ○ Relation between angular and Linear acceleration, ○ Equations of rotation with constant angular acceleration, ○ Unit vectors along the radius and tangent of a circle, ○ Centripetal acceleration in a uniform circular motion, ○ Centripetal force and centrifugal reaction, ○ Non-inertial reference frame and pseudo force, ○ Example related to centrifugal force, ○ Practical Illustrations, ○ Effect of centrifugal force, ○ Rotation of a body in a vertical plane, ○ Motion of a particle inside a hollow tube or sphere, ○ Some important points, ○ Conical Pendulum, ○ Short answer type questions (with answers), ○ Exercise.

❂ 1.1. Introduction ❂

So far in this book we have considered the equations of linear motion and other dynamical formulae associated with a particle moving with uniform acceleration.

From Newton's second law we know that if no unbalanced force acts on a moving body, it will continue to move along a straight line with a constant velocity. Now, if force acts in the direction of motion or opposite to the direction of motion, an acceleration or retardation is produced in the body and the velocity of the body will be increasing or decreasing. But the direction of motion of the body remain unchanged, so the motion is rectilinear.

● **Example :** When a body is projected upwards, due to exactly oppositely directed gravitational force of the earth, the velocity of the body gradually decreases and at a particular height its velocity becomes zero. Then it begins to come down vertically with increasing velocity due to earth's gravity.

If on the other hand, the force acts at an angle to the direction of motion, the direction of motion of the body changes continuously and the motion is no longer linear.

● **Example :** Projectile motion. In this case the projectile is projected at an angle θ to the vertical ($\theta < 90^\circ$) The motion of the projectile is no longer linear ; it moves along a curve (parabolic). It is a case of curvilinear motion and the motion is confined in a plane.

Curvilinear motion may be of different types. In this chapter we shall consider a uniform circular motion. Here the particle moves along a circle with constant speed. Circular motion is a special type of rotational motion. Here the particle rotates about a fixed axis and its distance from the axis is always constant. The rotational motion of a rigid body about a fixed axis can be described by considering the circular motion of one of its constituent particles and then summing over all such particles.

There are numerous cases of objects moving in a curve about some fixed point or axis. The earth and the moon revolves continuously round the sun. Rim of the balance-wheel of a watch moves to and fro in a circular path about the fixed axis of the wheel. A car moves along a curve when it takes a bent of a road. Actually rotational motion is of much common occurrance in nature than rectilinear motion. In this chapter we shall confine our discussion on the motion of a particle round a circle of fixed centre with uniform speed.

❂ 1.2. Circular motion ❂

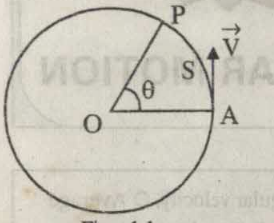


Fig. 1·1

Circular motion is a special case of curvilinear motion. Here the particle moves in such a way that its distance from a point or axis is always fixed. In fig 1.1, the moving particle is rotating along a circle of radius r. The velocity of the particle is always tangential to the circle and the force acting on the particle is radially inwards. **Example :** (i) the motion of the wheel of a car about a shaft, (ii) the motion of doors and windows about a hinge.

If the particle moving in a circle rotates through a definite angle in a given time, then the motion of the particle is called a uniform circular motion.

Axis of rotation :

It is the line passing through the centre of the circle and perpendicular to the plane of the circle is called the axis of rotation. For rotating a rigid body, the axis of rotation may pass through the body itself or it may lie outside the body.

- **Example :** axis of rotation of a wheel passes through its centre and normal to its plane. But when a stone is rotated by a string, its axis of rotation is at the other end of the string. So it is outside the stone.

A circular motion can be described in terms of **linear quantities** like linear displacement, linear velocity and linear acceleration. Also it can be described in terms of angular displacement, angular velocity and angular acceleration.

❂ 1.3. Angular displacement ❂

In the fig 1.1, the moving particle is at A and after a time t it comes to P. The radius OP makes angle θ with the initial line OA. The angle $\theta = \angle AOP$ is the angular position or displacement of the particle in time t.

❖ **Definition : In the case of circular motion, the angular displacement in a given time interval is defined as the angle through which the line joining the centre and the particle** (i.e. the radius) **rotates in that time interval.** It is expressed in **'radian'**.

We know that angle in radian is defined as the ratio of the arc length to the radius i.e.

$$\theta = \frac{\text{arc}}{\text{radius}} = \frac{s}{r} \quad \therefore \; s = r\theta \; \cdots\cdots\cdots\cdots\cdots\cdots \; (1.1)$$

The magnitude of the linear displacement i.e. $|\overrightarrow{AP}|$ is not equal to arc length s covered by the particle. However when P is very close to A then in the limit as the time interval Δt tends to become zero $|\overrightarrow{AP}| \to \Delta s$, when Δs represents a small distance covered by the particle.

If the particle moves in the anticlockwise direction, then the angular displacement is taken as positive and it is negative when the particle rotates in the clockwise direction.

Angular displacement is an axial vector and it is assumed to be acting along the axis of rotation. In the case clockwise rotation, angular displacement is directed along the axis away from the observer. On the other hand, in the case of rotation in the anti-clockwise direction it is directed towards the observer along the axis of rotation [Fig 1.2].

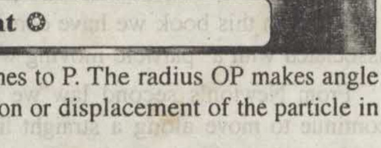


Fig. 1.2

❂ 1.4. Angular velocity ❂

Angular velocity of a rotating particle is defined as the angular displacement per unit time. It θ be the angular displacement in time t, then the angular velocity is $\omega = \frac{\theta}{t}$

$\therefore$ angular velocity $\omega = \dfrac{\text{Angular displacement}, \theta}{\text{time interval}, t}$ . . . . (1.2)

Unit of angular velocity is rad / sec. It is also expressed by revolution / min or rpm unit by engineers. It is also an axial vector and its direction is along the axis of rotation. The sense or direction of the vector is that in which a right handed screw would advance when turned in the direction of rotation of the particle.

The time taken by the particle to make one complete revolution is called its time period (T).

Now, in one complete rotation, angular displacement is 2π.

So, from the equation 1.2, $\omega = \dfrac{2\pi}{T}$ . . . . (1.3)

Again if the particle completes n revolution in 1 sec, then n is called frequency, So $n = \dfrac{1}{T}$

$\therefore$ $\omega = 2\pi n$ . . . . (1.4)

✪ 1.5. Average and instantaneous angular velocities ✪

The angular motion of particle may be uniform or accelerated, similar to the case of linear motion. In the case of non-uniform or accelerated angular motion, angular velocity changes with time. So we consider two types of angular velocities : (i) Average angular velocity and (ii) instantaneous angular velocity.

● **(i) Average angular velocity, $\overline{\omega}$:** In the case of non-uniform angular motion, **average angular velocity is defined as the ratio of the angular displacement to the elapsed time.**

If in a time t, the particle describes an angle θ then the average angular velocity during the time interval, is $\overline{\omega} = \dfrac{\theta}{t}$.

The particle moving along the circle passes from X to P in time t and its angular position now is θ. As the particle moves on the circle its angular position θ changes with time. During the small time interval from t to $t + \Delta t$, the particle passes from P to Q [Fig 1.3] and the angular displacement is $\Delta\theta$. Then the average angular velocity during this time interval is

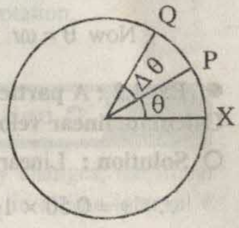


Fig. 1·3

$\overline{\omega} = \dfrac{\Delta\theta}{\Delta t}$ . . . . (1.5)

● **(ii) Instantaneous velocity, (ω) :** In the case of non-uniform angular motion, **angular velocity at any instant is called instantaneous angular velocity.**

To get the value of this angular velocity, the time interval is made progressively smaller, then in the limit when Δt approaches zero, the average velocity over the interval equal to the instantaneous velocity.

$\therefore$ instantaneous velocity, $\omega = \underset{\Delta t \to 0}{\text{Lt}} \dfrac{\Delta\theta}{\Delta t} = \dfrac{d\theta}{dt}$

So, the instantaneous rate of change of angular displacement is equal to the instantaneous angular velocity. Unless otherwise mentioned, the angular velocity will mean instantaneous angular velocity.

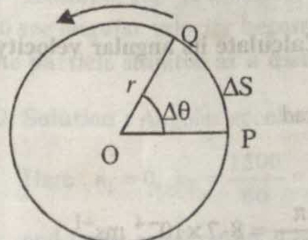


Fig. 1·4

✪ 1.6. Relation between angular and linear velocity ✪

A particle rotating about a point O, along a circle of radius r [Fig 1.4]. In a small time Δt, the particle passes from P to Q. So the angular displacement in time Δt is $\angle POQ = \Delta\theta$.

Linear displacement during this time Δt is $\Delta s = PQ$. If $\Delta\theta$ is measured in radian, $\Delta s = r.\,\Delta\theta$

$\therefore$ Angular velocity of the particle is $v = \dfrac{\Delta s}{\Delta t} = \dfrac{r.\Delta\theta}{\Delta t}$

$$\therefore \quad v = r\omega \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.7)$$

- In the case of non-uniform circular motion, instantaneous values of linear and angular velocities will be considered. Now, the instantaneous velocity is $v = \dfrac{ds}{dt}$ where $s = r\theta$

$$\therefore \quad v = \frac{d}{dt}(r\theta) = r\frac{d\theta}{dt} \quad [\text{ } r \text{ is a constant }]$$

$$\therefore \quad v = r\omega$$

- When a rigid body moves along a circle, angular velocities of each of the particles are same. But linear velocities of different particles are different. Greater the distance from the axis of rotation larger is the linear velocity. Evidently the linear velocity will be zero for a particle situated on the axis of rotation.

- **Example 1.1 : The angular velocity of a wheel is 300 rpm. Calculate (i) time period, (ii) frequency (iii) time for 60° angular displacement.**

○ **Solution :** (i) Angular velocity $\omega = 2\pi n = 2\pi \times 5 = 10\,\pi$ rad / s $\quad [n = 5 \text{ rps}]$

$\therefore$ time period, $\mathrm{T} = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{10\pi} = 0{\cdot}2$ sec.

(ii) frequency, $n = \dfrac{1}{\mathrm{T}} = 5$ Hz

(iii) Angular displacement $\theta = 60^\circ = \dfrac{\pi}{3}$ rad

Now $\theta = \omega t \quad \therefore \; t = \dfrac{\theta}{\omega} = \dfrac{\pi/3}{10\pi} = \dfrac{1}{30}$ sec.

- **Ex. 1.2 : A particle is moving along a circle of radius 50 cm with an angular velocity 1.5 rad / s. Calculate linear velocity and time period of the particle.**

○ **Solution :** Linear velocity is given by $v = r\omega$

$\therefore \quad v = 0{\cdot}50 \times 1{\cdot}5 = \mathbf{0{\cdot}75}$ ms^{-1}

and time period, $\mathrm{T} = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{1{\cdot}5} = \mathbf{4{\cdot}18\ sec}$

$r = 50$ cm $= 0{\cdot}5$ m
$\omega = 1{\cdot}5$ rad / s

- **Ex. 1.3 : A car is running along a circular track of radius 10 m with a speed of 54 Km / hr. What is the angular velocity of the car ?**

○ **Solution :** The required relation, $v = r\omega$

$\therefore \quad \omega = \dfrac{v}{r}$

$\therefore \quad \omega = \dfrac{15}{10} = 1{\cdot}5\ \text{rad s}^{-1}$

$r = 10$ m
$v = 54 \times \dfrac{5}{18} = 15\ \text{ms}^{-1}$

- **Ex. 1.4 : The length of the minute hand of a big clock is 50 cm. Calculate its angular velocity and linear velocity of the tip of minute hand.**

○ **Solution :** The angular displacement of the minute hand in 1 hr is 2π rad

$\therefore$ Its angular velocity, $\omega = \dfrac{2\pi}{3600} = \dfrac{\pi}{1800}$ rad / s

And the linear velocity of the tip of the minute hand $v = r\omega = 0{\cdot}5 \times \dfrac{\pi}{1800} = 8{\cdot}7 \times 10^{-4}\ \text{ms}^{-1}$

● **Ex. 1.5 : A stone is whirled along a horizontal circle by a string of length 2·0 m. If the stone makes 50 revolutions in 10 sec, calculate angular velocity and linear velocity of the stone.**

○ **Solution :** Angular velocity of the stone, $\omega = 2\pi n$ $\quad\therefore\quad \omega = 2\pi \times \frac{50}{10} = 10\pi = 31{\cdot}4 \text{ rad.s}^{-1}$

$\therefore$ linear velocity of the stone, $v = r\omega = 2 \times 31{\cdot}4 =$ **62·8 ms^{-1}**

1.7. Angular acceleration

When a particle executes non- uniform circular motion, its angular velocity is not constant but changes with time. The particle now possesses an angular acceleration. **The rate of change of angular velocity is called angular acceleration.**

Average angular acceleration ($\overline{\alpha}$) :

It is the ratio of the change in angular velocity to the time interval.

$$\therefore \quad \overline{\alpha} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\omega_2 - \omega_1}{t} \quad [\, t = t_2 - t_1 \,]$$

$\therefore$ If the time interval is small, $\overline{\alpha} = \frac{\Delta\omega}{\Delta t}$.

Instantaneous acceleration (α) :

In the limit ($\Delta t \to 0$) the instantaneous angular acceleration is given by $\alpha = \underset{\Delta t \to 0}{\text{Lt}} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$ ...(1.8)

$$\therefore \quad \alpha = \frac{d}{dt}\left(\frac{d\theta}{dt}\right) = \frac{d^2\theta}{dt^2}$$

Angular acceleration is also an axial vector and θ is directed along the axis of rotation.

It's unit is **rad s^{-2}**.

1.8. Relation between angular and linear acceleration

In the case of non-uniform circular motion of a particle as the angular velocity changes, the linear velocity also changes. Let the linear velocity of the particle changes from v_1 to v_2 in a time interval t.

Then its linear acceleration $a = \frac{v_2 - v_1}{t}$.

If the corresponding angular velocities be ω_1 and ω_2 then,

$v_1 = r\omega_1$ and $v_2 = r\omega_2$ [r = radius of the circular path]

$$\therefore \quad a = \frac{r\omega_2 - r\omega_1}{t} = \frac{r\,(\omega_2 - \omega_1)}{t} = r\alpha$$

$$\therefore \quad a = r\alpha \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 1.7(a)$$

$\therefore$ linear acceleration a = angular acceleration (α) × radius of the path (r) .

● **Example 1.6. A flywheel initially at rest starts rotating with an angular acceleration and after 30 sec angular velocity becomes 1500 rpm. Calculate angular acceleration and linear acceleration of the particle situated at a distance of 50 cm from the centre of the wheel.**

○ **Solution :** Angular acceleration, $\alpha = \frac{\omega_2 - \omega_1}{t} = \frac{2\pi\,(n_2 - n_1)}{t}$

Here $n_1 = 0$, $n_2 = \frac{1500}{60} = 25$ rps $\quad\therefore\quad \alpha = \frac{2\pi\,(25 - 0)}{30} = \frac{5}{3}\pi$ **rad / s**

and linear acceleration $a = r\alpha = 50 \times \frac{5\pi}{3}\text{ cm/s}^2 \quad \therefore\quad$ **$a = 2{\cdot}62 \text{ ms}^{-2}$**

❂ 1.9. Equations of rotation with constant angular acceleration ❂

The equations related to the rectilinear motion of a particle are : (i) $v = u + at$ (ii) $s = ut + \frac{1}{2}at^2$ (iii) $v^2 = u^2 + 2as$. For uniformly accelerated angular motion we can establish equations similar to the above equations connecting the angular displacement θ, angular velocity ω and angular acceleration α. The equations are : (i) $\omega = \omega_0 + \alpha t$, (ii) $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$, (iii) $\omega^2 = \omega_0{}^2 + 2\alpha\theta$ where ω_0 is the initial angular velocity.

● **Example 1.7 : A shaft rotating initially at 1725 rpm is brought to rest in 20 sec. Calculate the number of revolutions made by the shaft before stopping.**

○ **Solution :** $\omega_0 = 2\pi n = 2\pi \times \frac{1725}{60}$ rad / s, $\omega = 0$, $t = 20$ sec

Now, $\omega = \omega_0 + \alpha t = \alpha t$ $\quad\therefore\ \alpha = \frac{\omega_0}{t} = \frac{2\pi \times 1725}{60 \times 20} = 9{\cdot}03 \text{ rad / s}^2$

∴ Angular displacement during this time is $\theta = \frac{\omega_0{}^2}{2\alpha} = \left(\frac{1725\pi}{30}\right)^2 \times \frac{1}{2 \times 9{\cdot}03} = \mathbf{1805\ rad}$

∴ Number of revolutions, $n = \frac{\theta}{2\pi} = \frac{1805}{2 \times 3{\cdot}14} = \mathbf{287}$

● **Ex. 1.8 : After the switch is closed an electric fan gains the speed of 300 rpm after 5 full revolutions. Calculate : (i) angular acceleration of the fan and (ii) time after which the fan gains the speed of 300 rpm.**

○ **Solution :** (i) The required relation : $\omega^2 = \omega_0{}^2 + 2\alpha\theta$

Here $\omega_0 = 0$, $\omega = 2\pi n = 2\pi \frac{300}{60} = 10\pi$ rad / s, $\theta = 2\pi \times 5 = 10\pi$ rad

$\therefore\ \alpha = \frac{\omega^2}{2\theta} = \frac{(10\pi)^2}{2 \times 10\pi} = \mathbf{5\pi\ rad / sec^2}$

(ii) $\theta = \frac{1}{2}\alpha t^2$ $\quad\therefore\ t = \sqrt{\frac{2\theta}{\alpha}} = \sqrt{\frac{2 \times 10\pi}{5\pi}} = \mathbf{2\ sec}$

❂ 1.10. Unit vectors along the radius and tangent of a circle ❂

A particle is moving along a circle of radius r and at any time it is at A [Fig 1.5]. Let the centre O be the origin. OX and OY are the X-axis and Y-axis respectively. θ is the angular displacement. The radial unit vector or the unit vector along AB is $\hat{e}_r$ and the unit vector along the tangent to the circle at A is $\hat{e}_\theta$ in the direction of θ increasing. $\hat{e}_r$ is called radial unit vector and $\hat{e}_\theta$ the tangential unit vector. From the diagram $\overrightarrow{AB} = \hat{i}\,\text{AB}\cos\theta + \hat{j}\,\text{AB}.\sin\theta$

$\therefore\ \hat{e}_r = \hat{i}\cos\theta + \hat{j}\sin\theta$ .... (1.8)

Similarly, $\overrightarrow{AC} = -\hat{i}\,\text{AC}\sin\theta + \hat{j}\,\text{AC}.\cos\theta$

$\therefore\ \hat{e}_\theta = -\hat{i}\sin\theta + \hat{j}.\cos\theta$ .... (1.9)

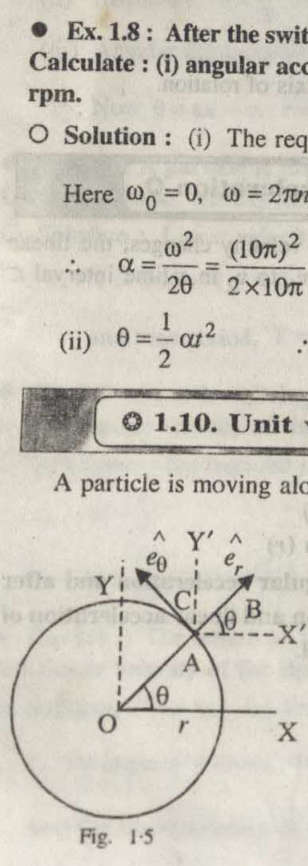


Fig. 1·5

✪ 1.11. Centripetal acceleration in a uniform circular motion ✪

Let a particle is moving along a circle of radius *r* with a uniform speed *v*. Although the magnitude of velocity or speed is constant, the velocity of the particle is not constant because the direction of velocity which is always tangential to the path, changes continuously. Actually the change of direction of velocity signifies the change of velocity of the particle. So, the particle moving with uniform circular motion possesses an acceleration. This acceleration gives rise to a change in the direction of motion but not the speed. As the speed does not change, the direction of acceleration should always be normal to the path of the particle i.e. towards the centre of the circular path. If the direction of acceleration were different, it would have a component along the direction of motion and the speed would change. Hence the acceleration and force acting on the particle is always directed towards the centre of the circular path. This acceleration is known as **centripetal acceleration** and the corresponding force is **centripetal force.**

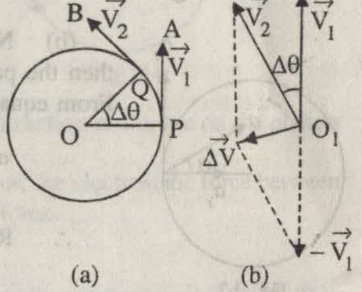
❖ **Definition : A particle moving along a circle with constant speed is acted upon by an acceleration directed towards the centre along the radius. This acceleration is termed as the centripetal acceleration.**

Expression for centripetal acceleration :

● **First method :** Consider the fig 1.6(a). At any instant, the particle at P where the velocity of the particle is $\vec{V}_1$ directed along the tangent PA. After a small time Δt the particle is at Q. Angular displacement in this time interval is $\Delta\theta$ the velocity of the particle at Q is $\vec{V}_2$ directed along the tangent QB [Fig 1.6 (a)]

Since the speed of the particle is uniform so, $|\vec{V}_1| = |\vec{V}_2| = V$

The direction of $\vec{V}_1$ and $\vec{V}_2$ are slightly different which means that the velocities of the particle at P and Q are different. The change in velocity as the particle passes from P to Q is $\overrightarrow{\Delta V} = \vec{V}_2 - \vec{V}_1$



(a) (b)

Fig. 1·6

$\therefore$ Centripetal acceleration, $a_c = \dfrac{|\overrightarrow{\Delta V}|}{\Delta t} = \dfrac{|\vec{V}_2 - \vec{V}_1|}{\Delta t}$

The change in velocity $\overrightarrow{\Delta V}$ is obtained by parallelogram law of addition of vectors [Fig 1.6 (b)] The vectors representing the velocities $\vec{V}_1$ and $\vec{V}_2$ are drawn from the common point O_1. $\vec{V}_1$ is reversed and the parallelogram is completed. Then diagonal gives the change in velocity $\overrightarrow{\Delta V}$.

$$\therefore \quad |\overrightarrow{\Delta V}|^2 = |\vec{V}_2|^2 + |\vec{V}_1|^2 + 2|\vec{V}_1|\ |\vec{V}_2| \cos(180^\circ - \Delta\theta)$$

$$= V^2 + V^2 - 2V^2 \cos\Delta\theta = 2V^2 . 2.\sin^2\left(\frac{\Delta\theta}{2}\right) = 4V^2 . \left(\frac{\Delta\theta}{2}\right)^2 \quad [\ \because \Delta\theta \text{ is small }]$$

$$\therefore \quad |\overrightarrow{\Delta V}| = V.\Delta\theta$$

$$\therefore \quad \text{Centripetal acceleration, } a_c = \frac{V.\Delta\theta}{\Delta t} = V\omega \quad [\ \omega = \text{angular velocity }]$$

$$\therefore \quad a_c = V.\frac{V}{r} = \frac{V^2}{r} = \omega^2 r \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ 1.9\ (a)$$

● **Second Method** (by calculus) **:** From the fig 1.6 (a) the position vector of the moving particle at Q at any time *t* is $\vec{r} = \overrightarrow{OQ} = (OQ)\hat{e}_r = r\left(\hat{i}\cos\theta + \hat{j}.\sin\theta\right)$.

So, the velocity of the particle at this instant is

$$\vec{V} = \frac{d\vec{r}}{dt} = \frac{d}{dt}\left[r\left(\hat{i}\cos\theta + \hat{j}\sin\theta\right)\right] = r\left[-\hat{i}\sin\theta.\frac{d\theta}{dt} + \hat{j}\cos\theta.\frac{d\theta}{dt}\right] = r\omega\,[-\hat{i}\sin\theta + \hat{j}\cos\theta]$$

Here $r\omega$ is the magnitude of velocity or speed of the particle and vector inside the bracket is a unit vector $\hat{e}_\theta$. Hence at any time, velocity of the particle is along the tangent to the circle. So $\vec{V} = V\,\hat{e}_\theta$

$\therefore$ The acceleration of the particle $\vec{a}_c = \frac{d\vec{V}}{dt}$

$$\therefore \quad \vec{a}_c = r\left[\omega\frac{d}{dt}(-\hat{i}\sin\theta + \hat{j}\cos\theta) + \frac{d\omega}{dt}(-\hat{i}\sin\theta + \hat{j}\cos\theta)\right]$$

$$= r\omega\left(-\hat{i}\cos\theta\,\frac{d\theta}{dt} - \hat{j}\sin\theta\,\frac{d\theta}{dt}\right) + r\frac{d\omega}{dt}\,\hat{e}_t$$

$$\therefore \quad \vec{a}_c = -\omega^2 r\,\hat{e}_r + \frac{dV}{dt}\,\hat{e}_\theta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.10)$$

Here $\hat{e}_r$ and $\hat{e}_\theta$ are respectively unit vectors along the radius and along the tangent.

(a) **Uniform circular motion :** When the particle moves along a circle with uniform speed, then $\frac{dV}{dt} = 0 \quad \therefore \quad \vec{a}_c = -\omega^2 r\,\hat{e}_r$. So, the acceleration a_c is along the radius towards the centre.

$\therefore$ The magnitude of the centripetal acceleration $a_c = -\omega^2 r = -\frac{V^2}{r}$.

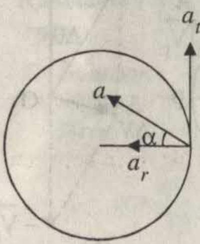


Fig. 1·7

(b) **Non-uniform circular motion :** If the speed of the particle is not uniform, then the particle will have both the radial and tangential acceleration [Fig 1.7]. From equation (1.10) radial acceleration,

$$a_r = -\omega^2 r = -\frac{V^2}{r} \text{ and tangential acceleration, } a_t = \frac{dV}{dt}$$

$$\therefore \quad \text{Resultant acceleration, } a = \sqrt{a_r^2 + a_t^2} = \sqrt{\left(\frac{V^2}{r}\right)^2 + \left(\frac{dV}{dt}\right)^2}$$

$$\text{and } \tan\alpha = \left(\frac{dV}{dt}\right)\Big/\left(\frac{V^2}{r}\right)$$

- **Example 1.9. A particle is moving along a circle of radius 20 cm with a uniform speed. If the particle takes 5 sec to make one complete revolution, calculate radial acceleration of the particle.**

○ **Solution :** Time period, T = 5 sec $\therefore$ angular velocity $\omega = \frac{2\pi}{T} = \frac{2\pi}{5}$ rad / sec

$\therefore$ radial acceleration, $a_r = \omega^2 r = \left(\frac{2\pi}{5}\right)^2 \times 20 \quad \therefore \quad a_r = \mathbf{31\cdot58\ cm\ s^{-2}}$

- **Ex. 1.10. A particle is moving along a circle of radius 500 m with non-uniform speed. The velocity of the particle increasing at the rate of 2 ms^{-2}. When the velocities of the particle is 30 ms^{-1} what is its total acceleration ?**

○ **Solution :** Centripetal acceleration of the particle moving in a circle is

$a_r = \frac{V^2}{r} = \frac{(30)^2}{500} = 1\cdot8\ \text{ms}^{-2}$ and tangential acceleration, $a_t = \frac{\Delta V}{\Delta t} = 2\ \text{ms}^2$

$\therefore$ Total acceleration, $a = \sqrt{a_r^2 + a_t^2} = \sqrt{(1\cdot8)^2 + (2)^2} = \mathbf{2\cdot7\ ms^{-2}}$

1.12. Centripetal force and centrifugal reaction

(A) Centripetal force :

When a particle moves along a circle, its direction of motion changes continuously. From Newton's first law we know that a body by itself can not change the direction of motion. So, a force must be applied on the body to change its direction of motion. As the speed of the body is uniform, the magnitude of velocity does not change. So, the force must be acting perpendicular to the direction of motion of the body. As the direction of motion is tangential to the circle, the force is radially inwards towards the centre. This force is called centripetal force.

❖ **Definition : When a particle moves along a circular path with a constant speed, a force directed radially inwards acts on it. This force is called centripetal force.**

The magnitude of the force is F_C = mass × centripetal acceleration.

$$\therefore\ F_C = \frac{mV^2}{r} = m\omega^2 r$$

It should be noted that centripetal force is always perpendicular to the direction of motion. So the force has no component in the direction of displacement of the particle. For this reason the centripetal force does not perform any work on the particle. It is called a **no-work force**. In fig 1.8, F_C is the centripetal force which acts on the body. As a results the body rotates along the circular path with constant speed.

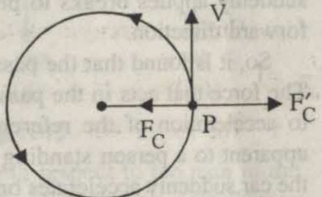


Fig. 1·8

- **Example :** (i) When a particle is rotated in a horizontal circle by a string, centripetal force is applied on the stone through the string. Length of the string is the radius of circle and the tension in the string is the centripetal force acting on the stone.

(ii) Planets moves round the sun in circular orbit. The gravitational attraction of the sun on the planets provides the necessary centripetal force.

(iii) Electrons in the atom revolves in a circular orbit around the nucleus, the electrostatic force between the electron and nucleus of the atom supplies the necessary centripetal force.

(B) Centrifugal reaction :

A particle rotates in a circular path with uniform speed due to a centripetal force acting on it. This force is applied on the rotating body by an external agency. Now, every action has equal and opposite reaction. So, the rotating object in turn, exerts an equal and opposite reaction force on the external agency which rotates the object. Evidently, the force is directed radially outwards from the centre of the circular path. This reaction is called centrifugal reaction. In the fig 1.9, F_C' is the **Centrifugal reaction**. From Newton's third law : $F_C = -F_C'$.

As action and reaction act on different objects, centripetal and centrifugal reaction also act on different objects. In the first example centripetal force F_C acts on the stone and the centrifugal reaction F_C' acts on the external agency. If the string suddenly breaks, both the forces vanish and the stone flies off tangentially with constant speed.

- Centrifugal reaction is a real force. Because it is produced due to interaction between different objects.

- **Example 1.11. A stone of mass 1 kg is tied to a string of length 50 cm. It is rotated in a horizontal circle by holding it at the other end at the rate 2 revolution per sec. What is the centripetal force acting with particle ?**

○ **Solution :** The centripetal force, $F_C = m\omega^2 r$

$\therefore\ F_C = 1 \times (4\pi)^2 \times 0{\cdot}5 = 8\pi^2 = \mathbf{78{\cdot}96\ N}$ | here, $m = 1$ kg ; $\omega = 2\pi n = 4\pi$ rad/s ; $r = 0{\cdot}5$ m

1.13. Non-inertial reference frame and pseudo force

We know that reference frame is a body with respect to which motion of other bodies are observed and the frame which is either rotating or moving with an acceleration is non-inertial frame. Newton's laws of motion are not valid in non-inertial frame.

- **Reason :** In the frame a new type of force other than those produced by the interaction between particles is present.

(A) Accelerated reference frame :

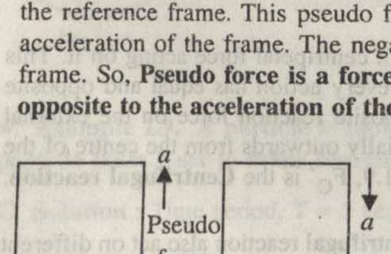
Suppose a body is moving along a straight line with uniform velocity. Now, the passengers in the car will not feel the motion of the car.

- **Reason :** Although the passengers have a velocity with respect to the road, but they have no velocity with respect to the car. If they look outside from the car, they will see that trees and houses etc on the road side are running in the opposite direction and the car will appear stationary to them.

Now, if the driver of the car suddenly accelerates the car, the passengers in the car will feel a push or impulsive force acting in the opposite direction of motion of the car. Similarly, if the driver of the car suddenly applies breaks to produce retardation, the passengers will feel the same force, but now in the forward direction.

So, it is found that the passengers feel the force in the direction opposite to the acceleration of the car. The force that acts in the passengers in the accelerated car is a **pseudo force**. This force is generated due to acceleration of the reference frame (in this case the car is the reference frame). This force is not apparent to a person standing on road side. The fact that the passengers in the car leans backward when the car suddenly accelerates or the passengers leans forward when the car suddenly decelerates, will appear to the observer as a case of inertia of rest or inertia of motion. But to the passengers in the car it is not an example of inertia, because they are not conscious about their state of motion with respect to the ground. The force experienced by the passengers is due to acceleration or retardation of the car and not due to action reaction between two objects. This force has no real existence and so it is called **Pseudo force.**

Evidently, a body in a non-inertial frame has an acceleration equal to the acceleration of frame but in the opposite direction. The body feels a force in the direction opposite to the direction of acceleration of the reference frame. This pseudo force is given by $\mathbf{F}' = -ma$ when m = mass of the body and 'a' is acceleration of the frame. The negative sign means that the force is opposite to the acceleration of the frame. So, **Pseudo force is a force experienced by a body in an accelerated frame in the direction opposite to the acceleration of the frame.**



(i) (ii)

Fig. 1·9

- **Pseudo force due to acceleration of the lift :** From experience we know that when a lift moves upwards with an acceleration, a person in the lift feels heavier. On the other hand, when the lift descends with an acceleration, the person feels lighter or loss in weight. The force acting on a person in an accelerated lift is an example of Pseudo force.

When the lift is moving upward with an acceleration 'a', the pseudo force acting on a body of mass m in the lift $F' = ma$ (downward). Now, the pseudo force acts in the direction of weight of the body [fig 1.9(i)]

$\therefore$ total downward force, $F = mg + ma = m(g + a)$

For this reason, a person in the lift feels heavier. On the other hand, when the lift descends with an acceleration, effective downward force is $F = mg - ma = m(g - a)$ [Fig 1.9 (ii)] so, the person in the lift now will feel lighter.

(B) Rotating frame and centrifugal force :

We have seen that a pseudo force $F' = -ma$ acts on a body of mass m situated in a reference frame which is moving with a linear acceleration 'a'. What will be the magnitude of the pseudo force if the reference frame rotates with a uniform angular velocity ?

Suppose a man is sitting inside a large closed box which is rotating with a constant angular velocity ω about a vertical Z-axis. [Fig 1.10 (i)]. A block of mass m is kept on the floor at a distance r from the Z-axis. Assume that the block is at rest with respect to the box. So it is rotating with an angular velocity ω with respect to the ground.

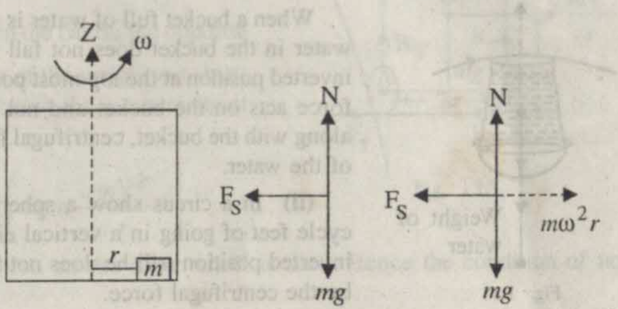
With respect to the ground the block is moving along a circle of radius r. It, therefore, has an acceleration $\frac{V^2}{r} = \omega^2 r$ towards the centre. The corresponding centripetal force on the box is $m\omega^2 r$. So, the different forces acting on the box with respect to the earth are

(a) its weight, mg

(b) normal reaction N of the floor

(c) frictional force on the floor, F_s

The different forces on the block is shown in fig 1.10 (ii). The necessary centripetal force is provided by F_S. Since the resultant force acts towards the centre, $F_S = m\omega^2 r$.



(i) (ii) (iii)

Fig. 1·10

Now, we consider the motion of the block with respect to the box or with respect to the man in the box. The block is at rest with respect to the man. So, the resultant force on the block is zero, according to Newton's law of motion. The weight and normal reaction balance each other, but the frictional force $F_S = m\omega^2 r$ acts on the box towards the centre. To make the resultant force zero, the existence of a pseudo force must be assumed. This force must be acting on the box away from the centre radically outward and its magnitude is $m\omega^2 r$. This pseudo force is called the centrifugal force. Hence with respect rotating reference frame or with respect to the man in the box, different forces acting on the block are :

(i) Wt. of block, mg (ii) Normal reaction, N (iii) Frictional for F_S and (iv) Centrifugal force, $m\omega^2 r$

These forces are shown in fig 1.11 (iii) As the body is at rest, from Newton's first law we get $F_S = m\omega^2 r$. So we get the same equation for friction as we got from the ground frame. When we consider the motion of the box with respect to the ground we apply second law of motion. But we apply first law when we consider the motion of the block with respect to the box.

When we consider a rotating reference frame with respect to an inertial frame, we require to imagine the existence of a force $m\omega^2 r$ acting radially outward on a body of mass m kept at a distance r from the axis of rotation in the rotating frame. Then we can apply Newton's laws of motion in the rotating frame. This radially outward pseudo force is called centrifugal force.

❖ **Definition of centrifugal force : Centrifugal force is a force which acts on a body rotating along a circular path and is directed radially outward.** This force is felt only by an observer rotating with the body. Evidently it is a pseudo force.

• Some important informations :

(i) Centrifugal force is produced due to acceleration of the frame.

(ii) Force produced due to interaction between different objects is a real force.

(iii) So, centrifugal force is a pseudo force.

(iv) This force has no existence in an inertial frame.

(v) The centripetal force has no existence to a person outside the rotating frame.

(vi) Centrifugal reaction and the centrifugal force are not same.

(vii) Centrifugal reaction is a reaction to centripetal force.

(viii) An observer in a rotating frame can feel the existence of both centripetal force and centrifugal force. But action and reaction always act on different bodies.

(ix) The centripetal force, centrifugal reaction and centrifugal force all have the same magnitude equal to mV^2 / r or $m\omega^2 r$.

1.14. Examples related to centrifugal force

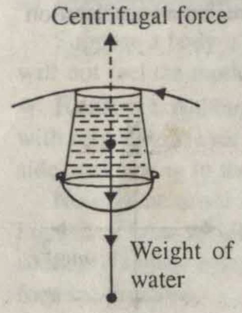


Fig. 1·11

(i) Rotation of a bucket full of water in a vertical plane.

When a bucket full of water is made to rotate quickly in a vertical plane, the water in the bucket does not fall from the bucket even when the bucket is in inverted position at the top most point of the circle [Fig 1.11]. Here the centripetal force acts on the bucket and not on the water in it. But as the liquid rotates along with the bucket, centrifugal force acts on it. This force balances the weight of the water.

(ii) In a circus show a spherical cage is erected for performing a motor cycle feet of going in a vertical circle. At the top most position, the rider is in inverted position still he does not fall. His weight acting downward is balanced by the centrifugal force.

● **Example 1.12. A bucket full of water is rotated in a vertical circle of radius 1.5 m by a rope. What should be the minimum speed of the bucket so that the water does not spill over at the highest point of circular path.**

○ **Solution :** At the highest point of the circular path, the water in the bucket will not spill over when the centrifugal force is just equal to the weight of water.

$$\frac{mv^2}{r} = mg \quad \therefore \quad v = \sqrt{rg} \quad \therefore \quad v = \sqrt{1\cdot5 \times 9\cdot8} = \mathbf{3\cdot83\ ms^{-1}}$$

1.15. Practical Illustrations

(A) Condition of no skidding at the bent of a road :

When a car moves on a straight horizontal road there is no problem. But when the car arrives at a turn and moves along a curved path the problem of skidding and overturning becomes important. First we consider the condition for no skidding.

Let a car of mass m is moving along a circular horizontal road of radius r [Fig 1.12]. At this moment different forces on the car are :

(i) Weight of the car mg acting downward.

(ii) Normal Reaction of the road, N.

(iii) Centrifugal force, $F_c = mV^2/r$ acting radially outward.

(iv) Friction force, F_S.

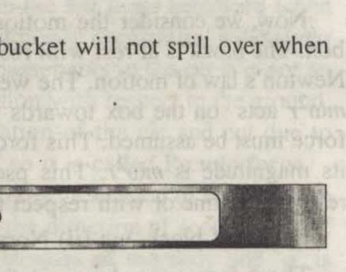


Fig. 1·12

Since the road is horizontal, normal reaction (N) of the road balances the weight of the car. Due to the centrifugal force the wheel of the car tends to skid outwards. This is prevented by the inward frictional force. Hence the condition for no skidding of the car on the horizontal road at the bent is

$$\frac{mV^2}{r} \le F_S \quad \text{or} \quad \frac{mV^2}{r} \le \mu mg \quad [\ \mu = \text{Coefficient of friction}\ ]$$

$$\therefore \quad V_{max} = \sqrt{\mu rg} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.11)$$

If the speed of the car exceeds V_{max}, the car will skid.

(B) Condition for no overturning of the car at the bent of a road :

Suppose a car is moving along a horizontal circular road of radius r at a speed v [Fig 1.13]. R_1 and R_2 are normal reactions on the wheels. F_1 and F_2 are the frictional forces on the wheels. The centre of gravity (G) of the car is at a height h from the ground and the separation between the wheels is $2a$. The frictional forces provide the necessary centripetal force and normal reactions balance the weight of the car.

$$\therefore \quad F_1 + F_2 = \frac{mv^2}{r} \text{ and } R_1 + R_2 = mg$$

Here the car is taking bent towards left. The moments of R_1 and $F_c = \frac{mv^2}{r}$ about the point B tends to overturn the car on the opposite side of the centre of rotation. But the moment of the weight mg about the same point B opposes the overturning. Taking the moment about the point G, we get $R_2 a = R_1 a + (F_1 + F_2)h$.

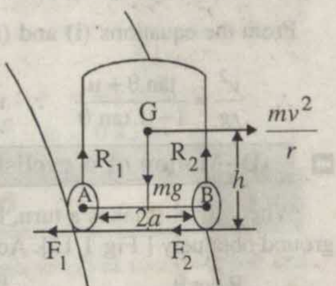


Fig. 1·13

$$\therefore \quad R_2 a = R_1 a + \frac{mV^2}{r}h \quad \text{or,} \quad R_1 = \frac{1}{2a}\left(mg\,a - \frac{mV^2}{r}h\right)$$

The car tends to overturn when the reaction on the inner wheel is zero. Hence the condition of no overturning as the bent of the road is

$$\frac{mv^2}{r}h \le mga \quad \therefore \quad v \le \sqrt{\frac{gra}{h}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1.12)$$

(C) Banking of roads :

We know that at the bent of a road frictional force provides the necessary centripetal force. But this force is not large. For this reason the speed of the car at the bent has to the lowered to avoid skidding; specially when the radius of curvature of the road is small. To overcome this difficulty, instead of depending on friction, the road is banked to provide additional centripetal force. Here the plane of the road is kept inclined to the horizontal i.e. the outer side of the road is slightly elevated with respect to the inner side. [Fig 1.14]. A part of the necessary centripetal force is supplied by the horizontal component of normal reaction of the surface. Rest of the centripetal force comes from friction of the road with the car. For a proper angle of banking, the horizontal component of reaction provides the necessary centripetal force and frictional force is not essential.

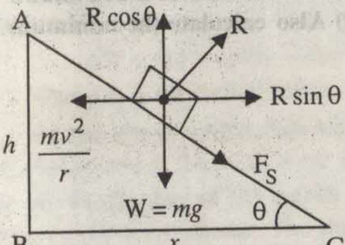


Fig. 1·14

In fig 1.14 different forces acting on a moving car at the bend of a road are shown. The angle of banking is θ. The vertical component of normal reaction R balances the weight of the car. The horizontal component $R\sin\theta$ acts as a centripetal force.

$$R\sin\theta = \frac{mv^2}{r} \text{ and } R\cos\theta = mg$$

When m = mass of the car and v = its speed. $\therefore \tan\theta = \frac{v^2}{rg}$

If h be the elevation and x be the breadth of the road then $\tan\theta = \frac{h}{x}$.

$$\therefore \quad \frac{h}{x} = \frac{v^2}{rg} \text{ or, } h = \frac{v^2 x}{rg} \quad \ldots\ldots (i) \quad \text{and} \quad v = \sqrt{\frac{hrg}{x}}$$

Hence the proper value of angle of banking depends on speed of the car and radius of curvature of the road.

At the higher speed than v given above, the force of friction must be considered to prevent skidding. The frictional force F_S (acting downward along the incline) is shown in fig 1.14. In this case we have

$$R\sin\theta + F_S\cos\theta = \frac{mv^2}{r} \quad \therefore \quad R(\sin\theta + \mu\cos\theta) = \frac{mv^2}{r}\ [\because\ F_S = \mu R] \ldots\ldots\ldots\ldots \text{(i)}$$

$$\text{and} \quad R\cos\theta = mg + F_S\sin\theta \quad \therefore \quad R(\cos\theta - \mu.\sin\theta) = mg \ldots\ldots\ldots\ldots \text{(ii)}$$

From the equations (i) and (ii) $\dfrac{v^2}{rg} = \dfrac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}$

$$\therefore \quad \frac{v^2}{rg} = \frac{\tan\theta + \mu}{1 - \mu.\tan\theta} \quad \therefore \quad v = \left[rg.\frac{\tan\theta + \mu}{1 - \mu\tan\theta}\right]^{1/2}$$

(D) Motion of a cyclist at the bent :

When a cyclist takes a turn, he always leans inwards automatically. At the bent, the cyclist presses the ground obliquely [Fig 1.15]. Additional centripetal force is generated because the reaction of the ground is also inclined to the horizontal. The horizontal component of the reaction provides the necessary centripetal force. From the fig.

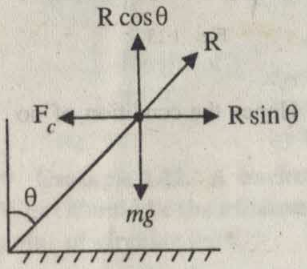


Fig. 1·15

$$R.\sin\theta = F_c = \frac{mv^2}{r} \quad \text{and} \quad R.\cos\theta = mg$$

Here m = mass of the cyclist, v = speed of the cyclist and r = radius of curvature of the road.

$$\therefore \quad \tan\theta = \frac{v^2}{rg}$$

This equation provides the angle of inclination of a cyclist with the vertical while moving with a speed v at the bent.

- **Example 1.13. The speed of a car while taking a circular turn of radius 150 metre is 100 km/hr. (i) Calculate the angle of banking so that the car does not skid. (ii) Also calculate the minimum value of coefficient of friction between the road and the tyre.**

○ **Solution :** (i) Velocity of the car, $v = 100 \times \frac{5}{18} = \frac{250}{9}\ \text{ms}^{-1}$

If the required angle of banking be θ, then

$$\tan\theta = \frac{v^2}{rg} = \frac{250 \times 250}{9 \times 9 \times 150 \times 9{\cdot}8} = 0{\cdot}5249$$

$\therefore\ \theta = 27{\cdot}7°$

$r = 150\ m$

$g = 9{\cdot}8\ \text{m/s}$

(ii) The minimum value of the coefficient of friction between the car and the road, $\mu = \frac{v^2}{rg} = 0{\cdot}5249.$

- **Ex. 1.14. A cyclist is moving along a circular road of radius 50 m with a speed of 36 km / hr. What is his angle of inclination with the vertical ? g = 10 ms^{-2}.**

○ **Solution :** The required relation, $\tan\theta = \dfrac{v^2}{rg}$

$$\therefore \quad \tan\theta = \frac{10 \times 10}{50 \times 10} = 0{\cdot}2 \quad \therefore \quad \theta = 11{\cdot}31°$$

∴ The angle of inclination = **11·31°**

Here,

r = 50 m,

v = 36 km/hr = 10 ms^{-1}

g = 10 ms^{-2}

- **Ex. 1.15. The radius of curvature at the turn of a horizontal road is 100 m. If the coefficient of friction between the road and the tyre be 0·3 what can be the maximum speed of the car so that it does not skid ?**

○ **Solution :** Here, $V_{max} = \sqrt{\mu rg}$

$\therefore \quad V_{max} = \sqrt{0\cdot 3\times 100\times 9\cdot 8} = 17\cdot 15\ \mathrm{ms}^{-1}$

∴ The maximum speed of the car for no skidding = **17.15 ms^{-1}**

- **Ex. 1.16. A vehicle is negotiating the bent of a horizontal road of radius 60 m. The separation between each pair of wheels is 1·4 m and the height of its c.g from the ground is 45 cm. What can be the maximum speed of the vehicle so that it does not overturn ?**

○ **Solution :** From equation 1.12; $v = \sqrt{\dfrac{gra}{h}}$;

$$v = \sqrt{\frac{9\cdot 8\times 60\times 0\cdot 7}{0\cdot 45}} = 30\cdot 25\ \mathrm{ms}^{-1}$$

$a = 0\cdot 7$ m, ; $v = 60$ m

g = 9·8 ms^{-2} ; $h = 0\cdot 45$ m

○ 1.16. Effect of centrifugal force ○

(i) Loss in wt of a body due to earth's diurnal rotation : The earth revolves round the sun and at the same time rotates about its axis with a time period of 24 hr. It is called diurnal motion. Suppose a body of mass m is situated on the surface of the earth at a place of latitude λ.

The actual weight of the body is earth's attraction W = mg and its acts towards the centre of the earth [Fig 1.16]. Now the body also rotates with the earth with the same time period.

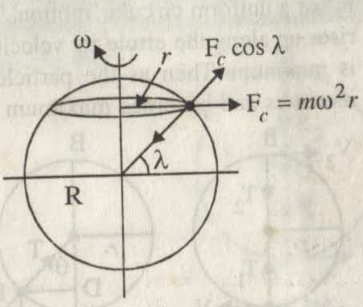


Fig. 1·16

So, the centrifugal force on the body is $F_c = m\omega^2 r$ acting radically outward. Hence the effective weight of the body is

$$W_\lambda = mg - F_c \cos\lambda = mg - m\omega^2 r\cos\lambda$$

Evidently, the loss in wt of the body due to earth's rotational motion is

$$\Delta W = m\omega^2 r\cos\lambda = m\omega^2 R\cos^2\lambda \quad [\, r = R\cos\lambda \,]$$

Where R is the earth's radius.

So, the loss in weight depends on the latitude of the place. At the equator ($\lambda = 0^\circ$) loss in weight is maximum and at the poles there is no loss in weight because at the poles $\lambda = 90^\circ$.

(ii) Flattening of the earth : The earth is not a perfect sphere, but is slightly flattened at the poles, bulging out at the equator. The shape of the earth is an oblate spheroid. It is due to earth's diurnal motion. We can explain the special shape of the earth as due to the action of the centrifugal force caused by the spinning motion of the earth around the axis passing through the poles. Every particle on the earth experiences a centrifugal force acting outward. This force is maximum at the equator and decreases gradually towards the poles. It completely vanishes at the poles. At the time of formation, the earth was in fluid state, the force of cohesion was small and could not balance the centrifugal force. Hence the earth bulges out at the equator. As the earth cools to its solid form, the shape does not change.

(iii) Centrifuge : It is a device used to separate light particles in suspension from the more dense liquid in which they are contained. It is essentially a vessel which can be rotated with a high speed. The mixture is taken in the container (or tube) of the centrifuge which then whirled at high speed in a horizontal circle. The light particles move towards the axis as explained below.

A mass m of the dense liquid at a distance r from the axis of rotation of the centrifuge requires a centripetal force $mr\omega^2$, where ω is the angular speed of the centrifuge. The required centripetal force is provided by the pressure gradient existing in the liquid. If in place of this mass of liquid, light particles of mass m_1 ($m_1 < m$) are present, then the existing centripetal force ($mr\omega^2$) on the particles will be greater than the required centripetal force ($m_1 r\omega^2$). As a result the light particles move towards the axis of rotation due to the net force $F = (m - m_1)\, r\omega^2$. When the centrifuge is stopped, the suspension of the container is found at the axis and the clear liquid at the top.

The heavy particles in the suspension will collect at the bottom of the container of the centrifuge, leaving clear liquid at the top.

- **Uses of centrifuge :**

(i) Cream is separated from the milk.

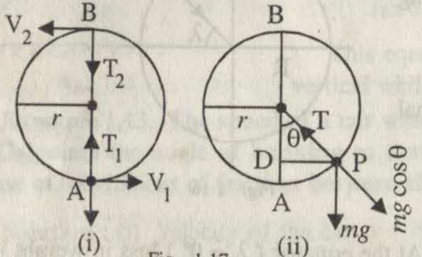
(ii) The wet clothes are dried by dry-cleaners in the drying machine.

(iii) In sugar industries sugar crystals are separated from the machines.

(iv) Ultra centrifuge (high speed centrifuge) are used to separate finest particles of the sediments from water or from highly viscous liquid.

❂ 1.17. Rotation of a body in a vertical plane ❂

Let a particle of mass m is tied to a long string and is rotated in a vertical plane by holding it at one end of the string. The weight of the particle is always vertically downward. But the centrifugal reaction always acts along the string radially outward. For this reason the motion of the string in a vertical circle is not a uniform circular motion. The tension in the string is not same at all positions. When the particle rises up along the circle its velocity gradually decreases. At the highest point, the velocity of the particle is minimum. Then as the particle comes down along the circle, the velocity of the particle gradually increases and becomes maximum at the lowest point.



(i) (ii)

Fig. 1·17

At the lowest point the tension in the string is sum total of the weight of the string and centrifugal force. At the highest point of the circle, the tension in the string is the difference between the centrifugal force and the weight of the particle [Fig 1.17 (i)]. If the velocity of the particle at the lowest point A is V_1 then the tension in the string

$$T_1 = \frac{mV_1^2}{r} + mg \qquad \cdots\cdots\cdots\cdots\cdots \text{(i)}$$

Also if the velocity at the highest point B be V_2, then the tension in the string

$$T_2 = \frac{mV_2^2}{r} - mg \qquad \cdots\cdots\cdots\cdots\cdots \text{(ii)}$$

In the limiting case i.e., when the particle is just able to continue its circular motion, we put $T_2 = 0$

Then from equation (ii), $V_2 = \sqrt{rg}$ ……………………(iii)

So, it is evident that if the velocity of the particle at the highest point be less than V_2 the string will slacken and the particle will no longer be able to remain in circular path and will fall down.

To maintain the minimum velocity V_2 at the highest point of the circle, the particle should be given a minimum velocity at the point A such that as the particle goes up along the circle, its velocity gradually decreases and becomes V_2 at the highest point. Let the required velocity at the lowest point A be V_1. Then applying the principle of conservation of energy at the point A and B of the circle we write

$$\frac{1}{2}mV_1^2 = \frac{1}{2}mV_2^2 + mg.2r$$

$$\therefore \quad V_1^2 = V_2^2 + 4rg = 5rg \quad \left[\because V_2 = \sqrt{rg}\right]$$

$\therefore\ V_1 = \sqrt{5rg}$ . . . . (iv) $\quad\therefore$ The required minimum velocity $V_{min} = \sqrt{5rg}$

From the equation (ii) if T_2 is positive the string will remain under tension and the particle will be able continue the circular path. On the other hand, when T_2 is negative, the string will slacken and particle will fall down.

Hence the particle will remain in the circle, if the velocity at the lowest point is $V_1 \geq \sqrt{5rg}$

At the lowest point if the minimum velocity of the particle be $V_{min} = \sqrt{5rg}$ then the tension in the string at the lowest point is

$$T_1 = \frac{m}{r}(5gr) + mg = 6mg \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(v)}$$

In general, the tension in the string at any point P [Fig 1.17 (ii)] is

$$T = mg\cos\theta + \frac{mV^2}{r} \qquad \begin{array}{l}[\ \theta = \text{angle made by the string with the vertical,} \\ \ \ V = \text{velocity of the particle at P}\ ]\end{array}$$

where $\cos\theta = \frac{r-h}{r}$ $\quad [h = AD]$

1.18. Motion of a particle inside a hollow tube or sphere

In the fig 1.18 cross section of the tube or the sphere is shown. At the lowest point the normal reaction is

$$R_1 = mg + \frac{mV_1^{\,2}}{r} = m\left(\frac{V_1^{\,2}}{r} + g\right)$$

and the highest point B, the normal reaction

$$R_2 = \frac{mV_2^{\,2}}{r} - mg = m\left(\frac{V_2^{\,2}}{r} - g\right)$$

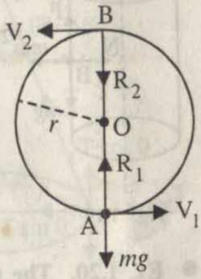


Fig. 1·18

1.19. Some important points

(i) When a particle moves in a vertical circle, it will be able to remain in the circle if the velocity at the lowest point be $V_1 = \sqrt{5gr}$.

(ii) Under this condition the velocity at the highest point will be $\sqrt{gr}$.

(iii) When the string is horizontal, the velocity of the particle is $\sqrt{3rg}$.

(iv) Tension in the string when it is at angle θ with the vertical is $T = mg\cos\theta + \frac{mV^2}{r}$.

- (a) at the lowest point $\theta = 0^\circ$, $T = 6$ mg
- (b) at the highest point, $\theta = 180^\circ$, $T = 0$
- (c) at the horizontal position, $\theta = 90^\circ$, $T = 3$ mg

● **Example 1.17. A stone is tied by a rope and is whirled in a vertical circle with uniform speed. The difference between the maximum and minimum tension in the string is 0·5 kg-wt. What is the mass of the stone ?**

○ **Solution :** Centrifugal force at the highest point, $F_c' = mg + T_1$ and centrifugal force at the lowest point, $F_c'' = T_2 - mg$. Since angular velocity is uniform, $F_c' = F_c''$

$\therefore\ T_2 - T_1 = 2mg$ or, $0\cdot5 \times g = 2mg \ \therefore\ m = 0\cdot25$ kg $\quad\therefore$ mass of the stone = 250 gm

- **Ex. 1.18. A horizontal disc is rotating about a vertical axis passing through its centre at the speed of 120 rpm. A small body is kept on the disc at a distance of 5 cm from its centre. Calculate the minimum value of the coefficient of friction between the body and the disc so that it does not fly off.**

○ **Solution :** A body of mass m kept at a distance of r from the centre of rotation will not fly off if the following condition is satisfied :

$$\mu mg \geq m\omega^2 r$$

$$\therefore \quad \mu \geq \frac{\omega^2 r}{g}$$

$$\therefore \quad \mu \geq \frac{(4\pi)^2 \times 0\cdot05}{9\cdot8}$$

$$\therefore \quad \mu \geq 0\cdot806 \quad \therefore \quad \mu_{\min} = 0\cdot806$$

Here,

$$\omega = 2\pi n = 2\pi \times 2 = 4\pi \text{ rad/sec}$$

$$r = 0\cdot05 \text{ m}$$

$$g = 9\cdot8 \text{ ms}^{-2}$$

So, the minimum value of coefficient of friction = **0.806**

- **Ex. 1.19. A cylindrical steel drum of inner radius 10 cm is rotating about its vertical axis. When the speed of the drum is 180 rpm a small steel ball remain attached at the inner surface of the drum and rotates with it. If the speed is below 180 rpm, the ball falls down. Calculate the coefficient of friction between the ball and the cylinder. $g = 980$ cm s^{-2}.**

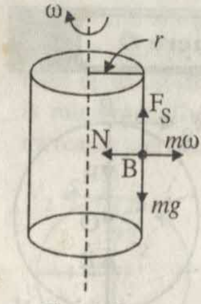


Fig. 1·19

○ **Solution :** The steel ball 'B' is in limiting condition and the difference forces acting on it are shown in the fig [1.19]. Considering equilibrium of the ball we write

$$F_S = mg \quad \text{or,} \quad \mu N = mg \text{ and } N = m\omega^2 r$$

$$\therefore \quad \mu m\omega^2 r = mg \quad \therefore \quad \mu = \frac{g}{\omega^2 r}$$

$$\therefore \quad \mu = \frac{980}{(6\pi)^2 \times 10} = 0\cdot276$$

$$g = 980 \text{ cm s}^{-2}$$

$$\omega = 2\pi n = 6\pi \text{ rad / s}$$

$$r = 10 \text{ cm}$$

$\therefore$ The coefficient of friction = **0·276**

- **Ex. 1.20. The radius of curvature of a convex bridge is 15 m. What can be the maximum velocity of the car at the highest point of the bridge so that it does not lose contact with the bridge ?**

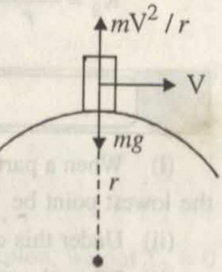


Fig. 1·20

○ **Solution :** Let V be the maximum velocity of the car for which it does not lose contact with the bridge at its highest point. Then

$$\frac{mV^2}{r} = mg$$

$$\therefore \quad V = \sqrt{rg} = \sqrt{15 \times 9\cdot8} \text{ ms}^{-1} = 12\cdot12 \text{ ms}^{-1} = 43\cdot64 \text{ km/hr}$$

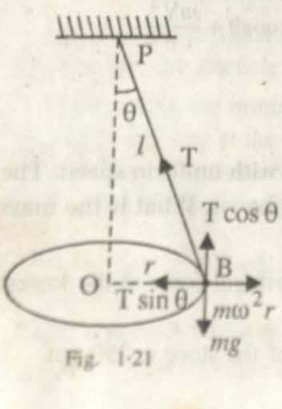


Fig. 1·21

1.20. Conical Pendulum

Conical pendulum is a small body suspended by a weightless and flexible string which can rotate in a horizontal plane about a vertical axis with uniform speed. So, the string of the conical pendulum describes the curved surface of a cone.

In fig 1.21, the bob (B) of the pendulum rotates along a horizontal circle of radius r with respect to the vertical axis passing through the centre O of the circle and the point of suspension P. The length of the string is l and θ is the semi-vertical angle.

Different forces acting on the bob are shown in fig. 1.21. Considering equilibrium of the bob

$$T\cos\theta = mg \quad \text{and} \quad T\sin\theta = m\omega^2 r$$

Now, $r = l\sin\theta \qquad \therefore \quad T\sin\theta = m\omega^2 l.\sin\theta$

$$\therefore \quad T = m\omega^2 l$$

$$\therefore \quad m\omega^2 l\cos\theta = mg \Rightarrow \omega = \sqrt{\frac{g}{l\cos\theta}}$$

$$\therefore \quad \text{time period,} \quad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l\cos\theta}{g}}$$

● **Example 1.21. A small bob of mass 5 gm is suspended by a string of length l=30 cm and describes a horizontal circle of radius r. Calculate the speed of the bob if the string makes an angle of 45° with the vertical axis. Also find (i) the time period of revolution and (ii) the tension in the string.**

○ **Solution :** The situation is shown in fig 1.22. The different forces on the bob are shown in fig. For the equilibrium of the bob,

$$T\cos 45^\circ = mg, \quad T\sin 45^\circ = \frac{mv^2}{r}$$

Now, $r = l\sin 45^\circ = \dfrac{30}{\sqrt{2}} = 21{\cdot}3 \text{ cm}$

also, $\tan 45^\circ = \dfrac{v^2}{rg} \quad \therefore \quad v = \sqrt{rg}$

$$\therefore \quad v = \sqrt{21{\cdot}3 \times 980} = 144{\cdot}48 \text{ cm s}^{-1}$$

(i) Now, time period of revolution $t = \dfrac{2\pi r}{v} = \dfrac{2\pi \times 21{\cdot}3}{144{\cdot}48} = 0{\cdot}926 \text{sec}$

(ii) Tension in the string, $T = \dfrac{mg}{\cos 45^\circ} = \dfrac{5\times 980}{1/\sqrt{2}} = 6909 \text{ dyne}$

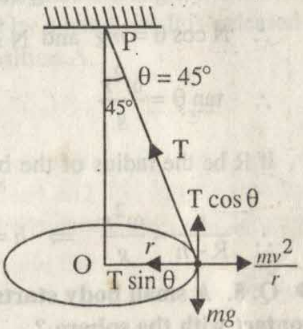


Fig. 1·22

● ● Short Answer Type Questions (with answers) ● ●

● **Q. 1. A stone tied to the end of a string is whirled in a circle. If the string breaks the stone flied away tangentially. Why ?**

○ **Ans.** The instantaneous velocity of the stone as it goes round the circular path is tangential to the circle. When the string breaks the centripetal force ceases to act. So, no force acts on the stone. Hence by Newton's first law, it will continue to move along a straight line. So it will fly off along the tangent to the circular path.

● **Q. 2. For a uniform circular motion does the direction of the centripetal force depend upon the sense of rotation ?**

○ **Ans.** The direction of the centripetal force does not depend, whether the body is moving in clockwise or anticlockwise direction. The force is always directed along the radius towards the centre of the circular path.

● **Q. 3. A bucket full of water is rotated in a vertical circle. Explain why the water does not fall ?**

○ **Ans.** The fig 1.23 shows a bucket containing water is whirled in a vertical circle. Let v be the velocity of the bucket at the top of the circular path.

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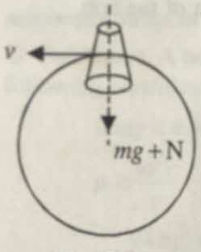

Fig. 1·23

The centripetal force required to rotate the bucket in vertical circle is $F_c' = \frac{mv^2}{r}$. Where m = mass of water and r is the radius of the path. Now, the weight of water plus the normal reaction N of the bottom of the bucket provide the necessary centripetal force. So,

$$N + mg = \frac{mv^2}{r} \qquad \therefore \quad N = \frac{mv^2}{r} - mg$$

The above expression shows that so long as v is larger than $\sqrt{gr}$ the water will not fall.

• Q. 4. A hemispherical bowl of radius 0·1 m is rotated about a vertical axis passing through the centre of the bowl with an angular velocity ω. A particle of mass $m = 10^{-2}$ kg placed inside the bowl also rotates with the bowl. If the height of the position of the particle from the bottom of the bowl be h, find the relation between h and ω. **[I. I. T 1993]**

○ **Ans.** Consider the fig 1.24, when the bowl with the particle rotates the position of the particle is A. The different forces acting on the particle are shown in fig.

$\therefore \quad N\cos\theta = mg$ and $N\sin\theta = m\omega^2 r$

$$\therefore \quad \tan\theta = \frac{\omega^2 r}{g}$$

If R be the radius of the bowl, $\tan\theta = \frac{r}{R-h}$

$$\therefore \quad \frac{r}{R-h} = \frac{\omega^2 r}{g} \Rightarrow h = R - \frac{g}{\omega^2} = 0{\cdot}1 - \frac{g}{\omega^2}$$

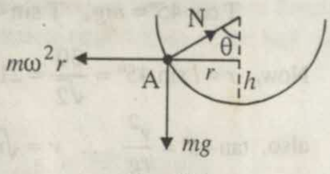

Fig. 1·24

• Q. 5. A small body starts falling from the top of a smooth sphere of radius R. When will it lose contact with the sphere ?

○ **Ans.** Consider the fig 1.25. From the highest point A, the small body starts falling. When the particle is at P, different forces acting on the body are shown in fig. If the speed of the body at the point P be v then

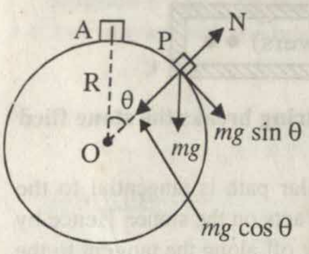

Fig. 1·25

$$mg\cos\theta - N = \frac{mv^2}{R} \quad [\text{ N = normal reaction }]$$

When the body loses contact with the sphere, N = 0

$$\therefore \quad mg\cos\theta = \frac{mv^2}{R} \qquad \therefore \; \theta = \cos^{-1}\left(\frac{v^2}{Rg}\right)$$

So, when the line joining the position of the particle and the centre of the sphere makes angle θ (given above) with the vertical, it will lose contact with the ground.

• Q. 6. A tube of length L is full of liquid of mass M and closed at both ends. The tube is rotated about its one end in the horizontal plane with an angular velocity ω. How much force will be exerted on the other end ? **[I. I. T. 1992]**

○ **Ans.** It may be assumed that the mass (M) of the liquid is concentrated at its centre.

The centrifugal force at the centre of mass of the liquid is $F_C = M\omega^2 . \frac{L}{2}$

$\therefore$ The force exerted by the liquid at the other end of the tube $= \frac{1}{2} M\omega^2 L$

● **Q. 7. Give the difference between the centrifugal reaction and centripetal force.**

○ **Ans.** Centrifugal reaction is the reaction to centripetal force. So it is a real force. On the other hand centrifugal force is the force acting on a body in a rotating frame. It is not a action-reaction force. So it is not a real force. It is called a pseudo force.

● **Q. 8. A vehicle of mass M is moving along a concave bridge with a velocity *v*. If the radius of curvature of the bridge be R, how much force is exerted at the lowest point of the bridge.**

○ **Ans.** When a vehicle moves over a concave bridge a centrifugal force acts on the vehicle. At the lowest point of the bridge this force acts in the vertically downward direction as the wt. of the vehicle [Fig 1.26]. So, the total force exerted on the bridge is $N = mg + \frac{mv^2}{R}$

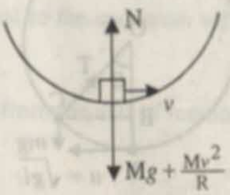

Fig. 1·26

● **Q. 9. A simple pendulum of mass *m* is deflected by an angle θ from its mean position and released. What is the tension in the string when it crosses the mean position ?**

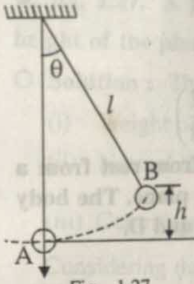

Fig. 1·27

○ **Ans.** Consider the fig 1.27. The pendulum is deflected by angle θ from its mean position A to the position B when it rises vertical by a height *h*. It is released at B. Let its velocity is *v* when it crosses its mean position A.

$$\therefore \quad \frac{1}{2}mv^2 = mgh \quad \therefore \quad v = \sqrt{2gh}$$

Now, from the figure $h = l - l\cos\theta = l(1 - \cos\theta)$

So, the tension in the string at the mean position A is

$$T = mg + \frac{mv^2}{l} = mg + \frac{m}{l}2gl(1 - \cos\theta)$$

$$T = mg(3 - 2\cos\theta)$$

• • Miscellaneous Examples • •

● **Example 1.22. A particle travels a semi circular path of radius 160 cm in 10 sec. Calculate (i) average speed, (ii) average velocity and (iii) average acceleration.**

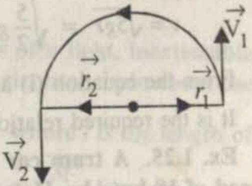

Fig. 1·28

○ **Solution :** (i) average speed, $v = \frac{\pi R}{t} = \frac{\pi \times 160}{10} = 50 \cdot 24 \text{ cm s}^{-1}$

(ii) average velocity, $\overline{V} = \frac{2R}{t} = \frac{2 \times 160}{10} = 32 \text{ cm s}^{-1}$

(iii) average acceleration $\bar{a} = \frac{|\vec{V}_2 - \vec{V}_1|}{t}$

$$\bar{a} = = \frac{1}{t}\left[\frac{\pi R}{t} - \left(-\frac{\pi R}{t}\right)\right] = \frac{2\pi R}{t^2} = \frac{2\pi \times 160}{100} = 10 \text{ cm s}^{-2}$$

● **Ex. 1.23. A heavy particle is suspended by a string from a fixed point. It is given a horizontal velocity of $\sqrt{gl}$ at the lowest point. When the tension in the string is equal to the weight of the particle, what is the velocity of the particle and the angle of inclination of the string ?**

○ **Solution :** As in fig. 1.29, OA is the string of length *l*. The particle attached to the string is given a velocity $u = \sqrt{gl}$ at the point A.

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Let at the point C, tension in the string is equal to the weight of the particle.

So T = mg. If the speed of the particle at C be v then,

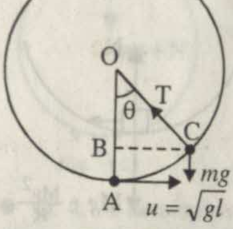

Fig. 1·29

$$T = mg\cos\theta + \frac{mv^2}{l} \quad \therefore\ mg - mg\cos\theta = \frac{mv^2}{l}$$

$$\therefore\quad v^2 = gl\,(1-\cos\theta)\ \ldots\ldots\ldots\ldots\ldots\ldots\ (i)$$

Applying the principle of conservation of energy at A and C

$$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 - mgh \quad [h = \text{AB}]$$

$$= \frac{1}{2}\,mu^2 - mgl\,(1-\cos\theta) \quad [\because h = l - l\cos\theta]$$

$$\therefore\quad v^2 = gl - 2gl\,(1-\cos\theta)\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (ii)$$

From equation (i) and (ii) $gl\,(1-\cos\theta) = gl - 2gl\,(1-\cos\theta)$

$$\therefore\quad \cos\theta = \frac{2}{3} \quad \therefore\quad \theta = \cos^{-1}\frac{2}{3}$$

$$\therefore\ v^2 = gl\left(1-\frac{2}{3}\right) = \frac{gl}{3} \quad \therefore\quad v = \sqrt{\frac{gl}{3}}$$

$\therefore$ Velocity of the bob $= \sqrt{\dfrac{gl}{3}}$ and angle of inclination of the string $= \cos^{-1}\left(\dfrac{2}{3}\right)$.

- **Ex. 1.24. A small body slides down a frictionless track after being released from rest from a point at a height *h*. The track ends in a circular loop of diameter D in the vertical plane. The body remains in contact with the loop as it moves along it. Find the relation between *h* and D.**

○ **Solution :** The small body is released from A at a height 'h' from the lowest point of the track B. Its velocity at the point B is obtained from

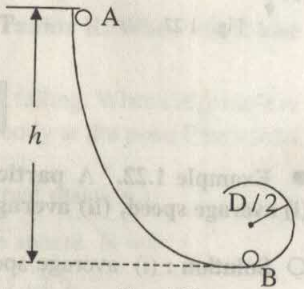

Fig. 1·30

$$\frac{1}{2}\,mv^2 = mgh$$

$$\therefore\quad v^2 = 2gh\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (i)$$

After B it goes round a circular loop of diameter D. So, its velocity at the lowest point must be

$$v = \sqrt{5gr} = \sqrt{\frac{5}{2}\,g\text{D}}\ \ldots\ldots\ldots\ldots\ldots\ (ii)$$

From the equation (i) and (ii) $2gh = \dfrac{5}{2}g\text{D} \quad \therefore\ \boldsymbol{h = \dfrac{5}{4}\,\text{D}}$

It is the required relation between h and D.

- **Ex. 1.25. A tram car of mass 6 × 10^5 kg is moving in a circular track of radius 120 m with a speed of 18 km / hr. How much lateral force it exerts on the rails ?**

○ **Solution :** The required centripetal force for the motion of the tram car in a circle is $F_C = \dfrac{mv^2}{r}$

The frictional force between the road and the wheel provides the centripetal force. So the lateral force exerted by the wheels of the car on the rails is

$$F'_C = \frac{mv^2}{r} = \frac{6\times10^5\times5^2}{120} = 1\cdot25\times10^5$$

$\therefore$ Lateral force = $\mathbf{1\cdot25\times10^5\ N}$

$m = 6\times10^5$ Kg
$v = 5\ \text{ms}^{-1}$
$r = 120$ m

- **Ex. 1.26. A coin placed on a horizontal table rotating at $33\frac{1}{3}$ rpm, revolves with the table without slipping, provided the coin is not more than 10 cm away from the axis. How far from the axis can the coin be placed so that it revolves with the table without slipping if the turn table rotates at 45 rpm ?** ($g = 980$ cm s^{-2}) **[J. E. E. 1995]**

○ **Solution :** When the coin is about to slide, the limiting friction will be equal to the maximum value of centrifugal force.

$\therefore$ limiting friction, $F_S = m\omega^2 r$

Here m is the mass, ω the angular velocity and r is the distance of the coin from the axis of rotation.

In the first case, $F_S = m{\omega_1}^2{r_1}^2$ and in the second case, $F_S = m{\omega_2}^2{r_2}^2$

$$\therefore \quad m{\omega_1}^2{r_1}^2 = m{\omega_2}^2{r_2}^2$$

$$\therefore \quad r_2 = r_1\left(\frac{\omega_1}{\omega_2}\right)^2 = \left(\frac{n_1}{n_2}\right)^2 . r_1 = \left(\frac{100}{3\times 45}\right)^2 \times 10 \quad \therefore \ r_2 = \mathbf{5.48\ cm}$$

- **Ex. 1.27. A particle is describing a horizontal circle inside a conical funnel [Fig 1.31]. If the height of the plane of the circle be 9.8 cm, calculate the velocity of the particle.**

○ **Solution :** The different forces acting on the particle is shown in fig 1.31. The forces are :

(i) Weight of the body, mg (downwards)

(ii) Normal reaction of the cone, R

(iii) Centrifugal force, $F_C = \frac{mv^2}{r}$ [v = speed of the particle]

Considering the equilibrium of the particle

$$R\cos\theta = \frac{mv^2}{r} \text{ and } R\sin\theta = mg$$

$$\therefore \quad \tan\theta = \frac{rg}{v^2} \quad \text{Also } \tan\theta = \frac{r}{h}$$

$$\therefore \quad \frac{r}{h} = \frac{rg}{v^2} \quad \therefore \quad v = \sqrt{gh} = \sqrt{9\cdot 8\times 9\cdot 8\times 10^{-2}} = \mathbf{0\cdot 98\ cm\ s^{-1}}$$

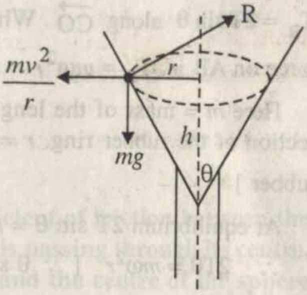

Fig. 1·31

- **Ex. 1.28. Two blocks of masses 6 kg and 10 kg are attached to the ends of a light inextensible string which passes over a frictionless ring at P. The light bob describes a horizontal circle about the heavier block (which is stationary at Q) as the centre. Show that $PQ = \frac{3l}{8}$ where l is the length of the string and the time period of revolution of the lighter block is $T' = \pi\sqrt{\frac{3l}{2g}}$ where g is the acceleration due to gravity.**

○ **Solution :** The arrangement is shown in fig 1.32.

T is the tension and θ is the angle between the segments of the string. It ω be the angular velocity of the lighter body, then

$$T = 10g\ , \quad T\cos\theta = 6g \ \text{ and } \ T\sin\theta = 6\,\omega^2 r$$

$$\therefore \quad \cos\theta = \frac{3}{5} \ \text{ and } \ \sin\theta = \frac{4}{5}$$

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Let PD = x, PQ = $x.\cos\theta$

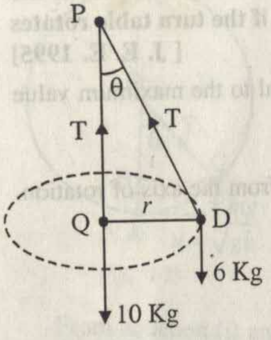

Fig. 1·32

$$\therefore \text{ length of the string, } l = PD + PQ = x(1+\cos\theta) \Rightarrow x = \frac{5l}{8}$$

$$\therefore PQ = x\cos\theta = \frac{5l}{8}\times\frac{3}{5} = \frac{3l}{8} \textbf{ (proved)}$$

$$\text{Now, } T.\sin\theta = 6\omega^2 r$$

$$\therefore 10g\times\frac{4}{5} = 6\omega^2 . x . \sin\theta \qquad [r = x\sin\theta]$$

$$= 6\omega^2\times\frac{5l}{8}\times\frac{4}{5} = 6\omega^2 . \frac{l}{2}$$

$$\therefore \omega = \sqrt{\frac{8g}{3l}} \quad \therefore \text{ time period } T' = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{3l}{8g}} = \pi\sqrt{\frac{3l}{2g}}$$

● **Ex. 1.29. A rubber ring is tightly fixed on a wheel of radius 40 cm. Now, the wheel is set into rotation at gradually increasing speed. When the rotational speed is 1200 rpm, the rubber ring slackens. What was the strain of the rubber ring before the wheel rotates ? Given, Y = $5{\cdot}5\times10^9$ dyne / cm^2 and density = 0·95 g / c.c**

○ **Solution :** We consider a small part of the ring AB = Δl, Δl subtends an angle 2θ at the centre at the wheel. If the longitudinal tension along the ring be T [Fig 1.33] then, radial component of the tension $T_R = 2T\sin\theta$ along $\overrightarrow{CO}$. When the wheel rotates, centrifugal force is generated. Now, the centrifugal force on AB is, $F_C = m\omega^2 r$.

Here m = mass of the length AB = Δl. A. ρ. [A = area of cross section of the rubber ring, r = radius of the wheel and ρ density of rubber]

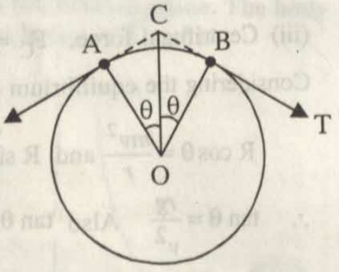

Fig. 1·33

At equilibrium $2T\sin\theta = m\omega^2 r$

$\therefore \quad 2T\theta = m\omega^2 r \quad [\because \theta \text{ small}]$ .... (i)

Now stress of the ring = $\dfrac{T}{A}$ and strain $= \dfrac{T}{AY} = \dfrac{1}{AY}.\dfrac{m\omega^2 r}{2\theta}$

Now, $\Delta l = r.2\theta \quad \therefore \theta = \dfrac{\Delta l}{2r}$

$$\therefore \quad \text{Strain} = \frac{1}{AY}(\Delta l.\,A.\rho)\,\omega^2\times\frac{r}{\Delta l}\times r = \frac{\rho}{Y}(\omega r)^2$$

$$\therefore \quad \text{Strain} = \frac{0{\cdot}95}{5{\cdot}5\times10^4}\times(40\pi\times40)^2 = 4{\cdot}36\times10^{-3}$$

ρ = 0·95 g/c.c.

Y = $5{\cdot}5\times10^9$ dyne/cm^2

ω = $2\pi n = 40\pi$ rad/s

r = 40 cm

● **Ex. 1.30. A small body is placed at the top of a smooth sphere of radius R. What horizontal velocity be given to the body so that it loses contact with the sphere when its height from the centre of the sphere is 4R/5.**

○ **Solution :** Let at the highest point A of the sphere, the body is given a horizontal velocity v_0. When it comes to the point B along the surface, it loses contact with the sphere. If v be the velocity of the sphere at B then,

$$mg\cos\theta = \frac{mv^2}{R} \Rightarrow \cos\theta = \frac{v^2}{Rg} \text{ .... (i)}$$

Applying the principle of conservation of energy at A and B, we get

$$\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = mg\,(\text{AC}) = mg\,(\text{R} - x)$$

$$\therefore \quad v^2 = v_0^2 + 2g\,(\text{R} - x)$$

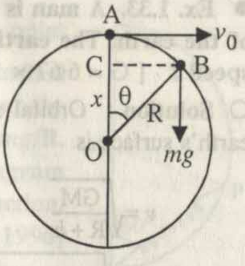

Fig. 1·34

Again $\cos\theta = \frac{x}{\text{R}}$, Now, from equation (i) $v^2 = \text{R}g\cos\theta$

$$\therefore \quad v_0^2 + 2g\,(\text{R} - x) = \text{R}g\,\frac{x}{\text{R}} = gx$$

$$v_0^2 = g\,(3x - 2\text{R}) = g\left(3.\frac{4\text{R}}{5} - 2\text{R}\right) \qquad \left[\text{As } x = \frac{4\text{R}}{5}\right]$$

$$v_0 = \sqrt{\frac{2\text{R}g}{5}}$$ It is the required velocity.

● **Ex. 1.31. The road way bridge over a canal is in the form of an arc of a circle of radius 50 ft. What is the maximum speed with which a car can cross the bridge without leaving the ground at the highest point ? [J. E. E 2001]**

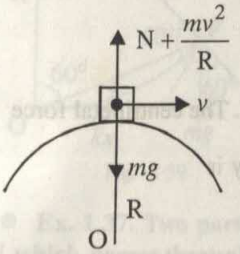

Fig. 1·35

○ **Solution :** Let the maximum speed of the car at the highest point of the bridge be v. Considering the equilibrium of the car. $\text{N} + \frac{mv^2}{\text{R}} = mg$

At the maximum speed of the car N = 0

$$\therefore \quad \frac{mv_m^2}{\text{R}} = mg \qquad v_m = \sqrt{\text{R}g}$$

$$\therefore \quad v_m = \sqrt{50 \times 32} = 40 \text{ ft/s}$$

● **Ex. 1.32. A small body is at the bottom of a hemisphere. The coefficient of friction between the body and the sphere is μ. The hemisphere is rotated about a vertical axis passing through its centre. When the body is at the highest position, the radius joining the body and the centre of the sphere makes angle θ with the vertical. Show that $\frac{\omega^2\text{R}}{g} = \frac{\sec\theta - \mu\,\text{cosec}\,\theta}{1 + \mu\tan\theta}$**

○ **Solution :** In the fig 1.36, the hemisphere is rotating about its vertical axis OA at a constant angular velocity ω. At the highest position of the body at P different forces acting on the particle are shown in fig. Considering equilibrium of the body,

$$mg\cos\theta + m\omega^2 r\sin\theta = \text{N}$$

and $$mg\sin\theta = m\omega^2 r\cos\theta + \mu\text{N}.$$

$$= m\omega^2 r\cos\theta + \mu\,(mg\cos\theta + m\omega^2 r\sin\theta)$$

$$\therefore \quad mg\,(\sin\theta - \mu\cos\theta) = m\omega^2 r\,(\cos\theta + \mu\sin\theta)$$

$$\therefore \frac{\omega^2 r}{g} = \frac{\sin\theta - \mu\cos\theta}{\cos\theta + \mu.\sin\theta}$$

or, $$\frac{\omega^2\text{R}\sin\theta}{g} = \frac{\sin\theta - \mu\cos\theta}{\cos\theta + \mu.\sin\theta}$$

$$\therefore \quad \frac{\omega^2\text{R}}{g} = \frac{1 - \mu.\cot\theta}{\cos\theta + \mu.\sin\theta} = \frac{\sec\theta - \mu.\text{cosec}\,\theta}{1 + \mu\tan\theta}$$

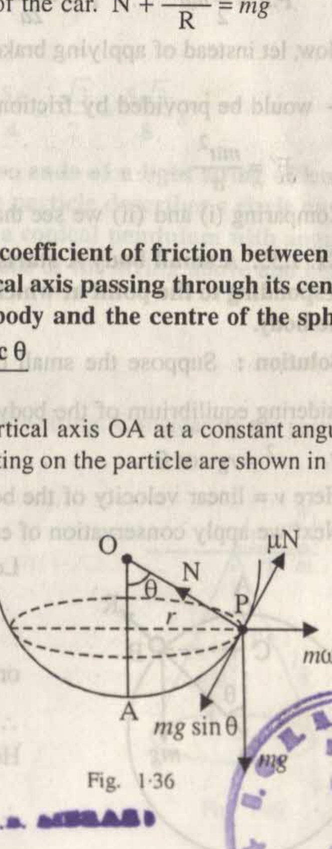

Fig. 1·36

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- **Ex. 1.33. A man is to be put into a circular orbit at a distance of $1{\cdot}6 \times 10^5$ m above the surface of the earth. The earth's radius is $6{\cdot}37 \times 10^6$ m and mass is $5{\cdot}98 \times 10^{24}$ kg. What is the orbital speed ?** [$G = 6{\cdot}67 \times 10^{-11}$ N m^2 / kg ^{2}] **[J. E. E. 1996]**

○ **Solution :** Orbital speed of an artificial satelite moving in a circular orbit at a height h above the earth's surface is

$$v = \sqrt{\frac{GM}{R+h}}$$

$$= \sqrt{\frac{6\cdot 67\times 10^{-11}\times 5\cdot 98\times 10^{24}}{6\cdot 37\times 10^{6}+0\cdot 16\times 10^{6}}}\ \text{m/s}$$

$$= 7\cdot 81\times 10^{3}\,\text{ms}^{-1} = 7\cdot 81\ \text{km s}^{-1}$$

$G = 6{\cdot}67 \times 10^{-11}$ N-m^2/Kg^2

$M = 5{\cdot}98 \times 10^{24}$ Kg

$R = 6{\cdot}37 \times 10^6$m

$h = 0{\cdot}16 \times 10^6$ m

∴ The orbital speed = **7·81 km s^{-1}**

- **Ex. 1.34. The driver of a car travelling at a velocity u suddenly sees a broad wall in front of him at a distance 'a'. Is it better to brake or to turn sharply ?**

○ **Solution :** Let the driver applies brake to stop the car. If F be the opposing force applied, then by conservation of energy principle

$$F.a = \frac{1}{2}mu^2 \quad \therefore\ F = \frac{mu^2}{2a} \ \ldots\ldots\ldots\ldots\ldots\ldots (i)$$

Now, let instead of applying brake the driven of the car takes a turn of radius 'a'. The centripetal force $\frac{mu^2}{a}$ would be provided by friction. Thus the frictional force that comes into play is

$$F' = \frac{mu^2}{a} \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (ii)$$

Comparing (i) and (ii) we see that $F' > F$. Hence it is safe to brake than to take a sharp turn.

- **Ex. 1.35. A small body A starts sliding off the top of a smooth sphere of radius r. Find the angle corresponding to the point at which the body breaks off the sphere ; Also calculate break off velocity of the body.**

○ **Solution :** Suppose the small body is initially at A and breaks off the sphere at an angle θ at B.

Considering equilibrium of the body at B, $mg\ \cos\theta - R = \frac{mv^2}{r}$. Now, when it breaks off, R = 0.

$$\therefore\quad v^2 = rg\cos\theta \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (i)$$

Here v = linear velocity of the body at B.

Next we apply conservation of energy at A and B.

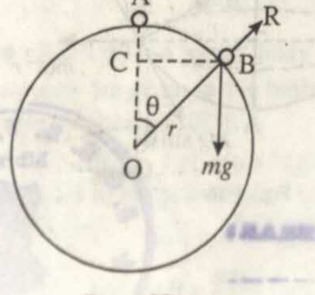


Fig. 1·37

Loss of potential energy = gain of kinetic energy.

$$\therefore\quad mg.(AC) = \frac{1}{2}mv^2$$

$$\text{or,}\quad mg\,(r - r\cos\theta) = \frac{1}{2}mv^2 \quad [AC = r - r\cos\theta]$$

$$\therefore\quad v^2 = 2gr\,(1-\cos\theta) \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (ii)$$

Hence from (i) and (ii), $\cos\theta = 2(1-\cos\theta)$

$$\therefore\quad \theta = \cos^{-1}\left(\frac{2}{3}\right)$$

The velocity at break-off point, $v^2 = 2gr\left(1-\frac{2}{3}\right) \quad \therefore\ v = \sqrt{\frac{2}{3}gr}$

● **Ex. 1.36. A smooth semi-circular wire track of radius R is fixed in a vertical plane [fig. 1.38]. One end of a massless spring of natural length $\frac{3R}{4}$ is attached to the lowest point O of the wire-track. A small ring of mass *m*, which can slide on the track, is attached to the other end of the spring. The ring is held stationary at point P such that the spring makes an angle of 60° with the vertical. The spring constant $k = mg/R$. Consider the instant when the ring is released and (i) draw the free body diagram of the ring, (ii) determine the tangential acceleration of the ring and normal reaction.**

[I. I. T. 1996]

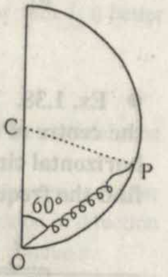


Fig. 1·38

○ **Solution :** It is clear from the free body diagram [fig. 1.39] OP = R.

So, spring has extended by $x = R - \frac{3R}{4} = \frac{R}{4}$.

From the free body diagram we write

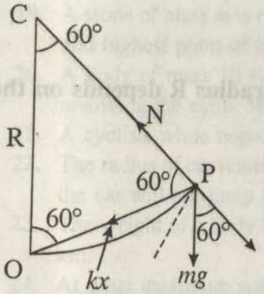


Fig. 1·39

$$kx\cos 60° + N - mg\cos 60° = \frac{mv^2}{R} \quad \text{.......... (i)}$$

and $\quad kx\sin 60° + mg\sin 60° = m\,a_t$ .... (ii)

Where a_t is the tangential acceleration

From (i), $N = mg\cos 60° - \frac{mg}{R} \times \frac{R}{4} \cdot \cos 60°$ $[\because\ v = 0$ initially$]$

$$= \frac{3mg}{4}\cos 60° = \frac{3}{8}mg$$

and from (ii), tangential acceleration is

$$a_t = \left(\frac{mg}{R} \cdot \frac{R}{4} + mg\right)\frac{\sin 60°}{m} = \frac{5g}{4} \times \frac{\sqrt{3}}{2} = \frac{5\sqrt{3}}{8}g$$

● **Ex. 1.37. Two particles each of mass *m*, are attached to the two ends of a light string of length *l* which passes through a hole at the centre of a smooth table. One particle describes a circle on the table with angular velocity w_1 and the other describes a circle as a conical pendulum with angular velocity w_2 below the table. If l_1 and l_2 are the lengths of the portions of the string above and below the table, then show that**

$$\frac{l_1}{l_2} = \frac{w_2^{\,2}}{w_1^{\,2}} \text{ and } \frac{1}{w_1^{\,2}} + \frac{1}{w_1^{\,2}} < \frac{l}{g}$$

○ **Solution :** The situation is shown in fig 1.40. T is the tension in the string. Considering the particle on the table

$$T = ml_1\,w_1^{\,2} \quad \text{.......... (i)}$$

and for the particle below the table

$$T\sin\theta = ml_2\sin\theta\ w_2^{\,2} \quad \text{.......... (ii)}$$

$$T\cos\theta = mg \quad \text{.......... (iii)}$$

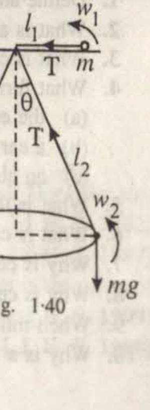


Fig. 1·40

From (i) and (ii), $\frac{l_1}{l_2} = \frac{w_2^{\,2}}{w_1^{\,2}}$ .... (iv)

Further, $l_1 = \frac{T}{mw_1^{\,2}}$ and $l_2 = \frac{T}{mw_2^{\,2}}$

$$\therefore \quad l = l_1 + l_2 = \frac{T}{m}\left(\frac{1}{w_1^{\,2}} + \frac{1}{w_2^{\,2}}\right)$$

$$\therefore \quad \frac{1}{w_1^{\,2}}+\frac{1}{w_2^{\,2}}=\frac{ml}{\mathrm{T}}=\frac{ml\cos\theta}{mg} \quad \text{[From (iii)]}$$

$$=\frac{l\cos\theta}{g}<\frac{l}{g}$$

● **Ex. 1.38. Two masses *m* and M are connected by a light string which passes through a hole O at the centre of the table. Mass *m* lies on the table and M hangs vertically. The mass *m* rotates along a horizontal circle with the hole as the centre. If *l* be the length of the horizontal portion of the string, find the frequency with which *m* must revolve so that M remains stationary.**

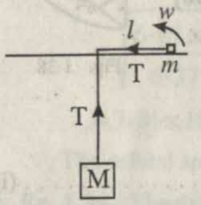


Fig. 1·41

○ **Solution :** Consider the fig 1.41. Let T be the tension in the string. Then,

$\mathrm{T} = \mathrm{M}g$ and $\mathrm{T} = ml\omega^2$

where ω is the required angular velocity of mass *m* so that M remains stationary.

$$\therefore \quad \omega=\sqrt{\frac{\mathrm{M}g}{ml}}$$

$$\therefore \quad \text{frequency,} \quad f=\frac{\omega}{2\pi}=\frac{1}{2\pi}\sqrt{\frac{\mathrm{M}g}{ml}}$$

● **Ex. 1.39. The kinetic energy K of a particle moving along a circle of radius R depends on the distance covered *s* as $\mathrm{K} = as^2$. Calculate the force acting on the particle.**

○ **Solution :** If *m* be the mass and *v* be the linear speed of the particle, then

$$\frac{1}{2}mv^2 = as^2 \qquad \therefore \quad v=\sqrt{\frac{2a}{m}}\cdot s$$

$$\therefore \quad \text{Tangential acceleration,} \quad a_t=\frac{dv}{dt}=\sqrt{\frac{2a}{m}}\,\frac{ds}{dt}=\sqrt{\frac{2a}{m}}\,v.$$

$$\text{and centripetal acceleration,} \quad a_c=\frac{v^2}{\mathrm{R}}=\frac{2as^2}{m\mathrm{R}}$$

$$\therefore \quad \text{Resultant acceleration,} \quad a=\sqrt{a_t^{\,2}+a_c^{\,2}}$$

$$\therefore \quad \text{Force on the particle,} \quad \mathrm{F}=ma=m\sqrt{\left(\frac{2as}{m}\right)^2+\left(\frac{2as^2}{m\mathrm{R}}\right)^2}=2as\sqrt{1+s^2/\mathrm{R}^2}$$

• E ◇ X ◇ E ◇ R ◇ C ◇ I ◇ S ◇ E •

◢ A. Short answer type questions :

1. Define angular velocity. How is it related to linear velocity ?
2. What is angular acceleration ? Give its relation with linear acceleration.
3. What is centripetal force ? What is the necessity of this force ?
4. What furnishes the required centripetal force in each of the following ?
   (a) the earth revolving round the sun.
   (b) a car turning a corner.
   (c) an electron revolving round the nucleus.
5. What is the name of the reaction of centripetal force ? Is this reaction real or fictitious ?
6. What is centrifugal force ? Mention its difference from centrifugal reaction.
7. Why is centrifugal force called pseudo force ? [H. S. 1997]
8. Why is centripetal force called 'no work' force ?
9. When milk is churned the cream separates from it and gets collected, where ?
10. Why is a force required to rotate a body along a circular path ? What is the name of this force ?

11. What is the analogous quantity in rotational motion to the force in the case of translation motion? [J. E. E. 1991]

12. A car is moving with a velocity v. The driver of the car suddenly sees a vertical wall in front of him. Is it better to apply brake or take a turn to avoid accident ? Explain.

13. What is moment of a force ? On which factor the turning effect of the force depend ?

14. What is couple ? What is meant by moment of the couple ?

15. In the case of linear motion, a body can move without any acceleration, but the circular motion the particle must have an acceleration. Explain.

16. A constant force F is applied on a body of mass m, such that the direction of the force is always perpendicular to the direction of motion of the body. Describe the motion of the body, specifying its acceleration in direction and magnitude. Also find the rate at which work is done by F, if the instantaneous speed has a value u. [J. E. E. '79]

17. When a car moves over a convex bridge with a high speed, the passengers in the lift feels lighter. Explain.

18. Two identical trains are moving in opposite directions over the rails along the equator with equal speed. Will both the trains exert same pressure on the rails ?

19. A stone of mass m is rotated in a vertical circle of radius r at constant speed ω. What is the tension at the lowest and highest point of the path.

20. A body of mass 10 Kg is suspended from a spring balance suspended from the ceiling of a artificial satellite revolving the earth. What will be the reading of the spring balance ?

21. A cyclists while negotiating a turn on a horizontal road leans inwards instinctively. Explain the reason.

22. The radius of curvature of a convex bridge in r. A car is crossing the river with the speed v. Show that if $v \leq \sqrt{rg}$ the car will not jump of the bridge at the highest point.

23. The weight of a body at the equator is zero. Calculate the number of days in a year. Radius of the earth = 6400 km.

24. At what maximum speed a vehicle can take a turn on a horizontal road without skidding ? The coefficient of friction between the vertical and the road is μ.

25. What is banking of a road ? Why is it done ?

26. At the bend why is the outer rail slightly elevated than the inner rail of rail lines ?

27. A simple pendulum is suspended from the ceiling of a vehicle which is moving along a circular track. Why the pendulum does not remain vertical ?

28. From the condition of no skidding of a vehicle at the bend of a horizontal road, obtain the expression for maximum velocity of the vehicle.

29. Why is a force needed to rotate a body along a circular path ? What is the direction of this force? On which factors its value depend ?

30. What is conical pendulum ?

31. Explain the following statements :
    (a) Centrifugal force is a pseudo force.
    (b) Centrifugal reaction is a real force.
    (c) Centripetal force is a 'no work' force.
    (d) The earth is a oblate spheroid.
    (e) A cyclist rounding a curve always leans inwards.
    (f) It is possible to rotate a bucket full water in a vertical circle without spilling water.
    (g) When a car turns sharply towards left, passengers lean towards right.
    (h) When milk is charmed, the cream separates from it.

32. Due to the force applied by the sun on the earth, earth moves round the sun in circular orbit. Does the sun performs any work for this motion of the earth ? Explain. [H. S. 2002]

33. What is a centripetal force ? Why is it called pseudo force ? [H. S. 1997]

34. What is centripetal force ? Radius of the earth is 6400 km. Due to earth's diurnal rotation, what is the value of centripetal acceleration at the equator ? [H. S. 1999]

35. What are centrifugal and centripetal force ? Give examples. [J. E. E. 1998]

B. Essay type questions :

1. Determine the expression for the centripetal force on a particle, moving round a circular path with uniform speed. Mention the relation between angular and linear velocity of this particle. **[H. S. 1996]**
2. What we mean by centripetal force and centrufugal force ? Why is centrifugal force called pseudo force ? **[H. S. 1993]**
3. Why is a force needed to rotate a body along a circular path ? What is the name of the force ? Obtain an expression for the force.
4. Duduce the condition of no skidding of a car at the bend of horizontal road.
5. What is banking of road ? Obtain the expression for the angle of banking in terms of speed and radius of curvature of the road.
6. A cyclist rounding a curve always leans inwards. Deduce the expression for this angle in terms of speed and radius of curvature of the road.
7. What we mean by centripetal and centrifugal force ? Explain why centrifugal force is called a no work force. **[J. E. E. '95, '98]**
8. A body is suspended by a string of length l. Show that if a horizontal velocity of $\sqrt{5gl}$ is given to the body at its lowest position, then it will be able to complete a vertical circle. Prove that the tension at the lowest point is six times the tension at the highest point of the circular path.
9. A stone of mass *m* is rotated in a vertical circle of radius R. Show that at the highest point the string will not slacken if the stone has a minimum velocity of $\sqrt{gr}$ at the highest point. **[H. S. 2000]**
10. Define centripetal force. Obtain an expression for it. **[H. S. 1998, 2002]**
11. A car is moving along a circular road of radius R. The banking angle is θ. Show that the car will not slip at bend if the speed of the car does not exceed $v = \left[\dfrac{rg\,(\mu + \tan\theta)}{1 - \mu\tan\theta}\right]^{1/2}$.

C. Simple numerical problems :

1. A particle revolves round a circular path at 42 rpm. Calculate its angular velocity. [**Ans.** 4.4 rad s^{-1}]
2. The speed of a wheel is 1200 rpm. Calculate : (i) frequency (ii) angular velocity (iii) time period. [**Ans.** (i) 20 Hz, (ii) 125.6 rad / s (iii) 0.05 s]
3. The breaking load of a string is 10 N. A stone of mass 50 g is whirled by the string in a horizontal plane. The length of the string is 50 cm. At what frequency of revolution the string will break? [**Ans.** $\frac{10}{\pi}$ rad s^{-1}]
4. The radius of curvature of a convex bridge over a canal is 20 m. At what maximum speed a car can cross the river without leaving contact with the bridge at its highest point ? $g = 9{\cdot}8\ ms^{-2}$ [**Ans.** 14 ms^{-1}]

   [**Hints :** $mg - N = \dfrac{mv^2}{r}$, $N \geq 0$]
5. A body of mass 1 kg is rotated by a string of length 50 cm in a vertical plane with a speed of 5 ms^{-1}. What are the tensions in the string at the highest and lowest point of its path ? [**Ans.** 46·2 N, 59·8 N] **[H. S. 1993]**
6. A spring of length 0·5 m and having force constant 10 Nm^{-1} is fixed at one end and a block of mass 0·5 kg is attached at the other end. The system is placed on a horizontal table. Now the block is rotated at an angular velocity of 2 rad / s. Show what will be the increase in length of the spring ? [**Ans.** 0·1 m]
7. A bucket full of water is rotated in a vertical circle of radius *r*. What should be the minimum velocity at the highest point so that the liquid does not spill. If the bucket has this minimum velocity at the highest point, what will be the tension in the rope at the lowest point of its path ? mass of water = *m* [**Ans.** $\sqrt{rg}$, 6 mg]
8. Two cars of masses m_1 and m_2 are moving along a circular track of radius r_1 and r_2 respectly. If their time period of revolution be same, what is the ratio of their angular velocities ? [**Ans.** $\omega_1 : \omega_2 = 1 : 1$]
9. The distance between the earth and the moon is $2{\cdot}85 \times 10^5$ km. If the time period of revolution of the moon round the earth be 27·3 days, calculate the acceleration of the moon with respect to the earth. [**Ans.** $2{\cdot}73 \times 10^{-3}\ ms^{-2}$]

10. An electric fan was rotating at 210 rpm. By the regulator its speed is increased to 630 rpm in 11 sec. What is the angular acceleration of the fan. How many revolutions the fan will complete in 11 sec ? [**Ans.** 4 rad s^{-1} ; 77]

11. A simple pendulum is hanging from the ceiling of a car. The car is running at a speed of 36 km/hr at the turn of a road. If the radius of curvature of the road be 10 m, what angle the string of the pendulum will make with the vertical ? $g = 10\ ms^{-2}$ [**Ans.** 45°]

12. A stone of mass 250 gm is rotated in a horizontal plane by a string of length 1 m at the speed of 30 rpm, (i) What is the tension in the string ? (ii) If the string can bear maximum tension of 100 N, with what maximum speed it can be rotated ? [**Ans.** (i) 2·47 N (ii) $v_{max} = 20\ ms^{-1}$]

13. A block of mass 1 kg is hanging from a string. The block is deflected by 30° from its mean position and then released. When the block cross the mean position what will be the tension of the string ? [**Ans :** 12·4 N]

14. The initial speed of a wheel is 400 rpm and after 4 sec the speed became 600 rpm. Calculate (i) angular acceleration and (ii) number of revolution made during this time. [**Ans.** (i) $\frac{5\pi}{3}$ rad s^{-2} (ii) $\frac{100}{3}$]

15. A body of mass 1 kg is rotated in a circle by a string of length 20 cm in a vertical plane. At what velocity at the highest point, the string will not slacken ? Corresponding to this velocity at the highest point, what will be its velocity at the lowest point of its path ? Also calculate tension in the rope at the lowest point. [**Ans.** 140 cm / s, 313 cm / s, 6 kg-wt]

16. The radius of a curved path is 80 m. Calculate the maximum speed of a car so that it does not skid while negotiating the curve. Coefficient of friction between the car and the road is 0·25. [**Ans.** 14 ms^{-1}]

17. A bulb is hanging from the roof of a train by a string the train moves in a curve the string makes an angle of 30° with the vertical. If the radius of the curved track be 100 m. calculate the velocity of the train. [**Ans.** 23·78 ms^{-1}]

18. A bucket full of water is rotated in a circle of radius 40 cm in a vertical plane. Calculate the minimum angular velocity of the bucket so that the water does not fall at the highest point of the circular path. [**Ans.** 4·95 rad s^{-1}]

19. A horizontal disc is rotating about a vertical axis passing through its centre at an angular velocity of 60 rpm. A small object is placed on the disc at distance of 10 cm from its centre. What should be coefficient of friction between the disc and the body so that it does not fly off ? [**Ans.** $\mu \geq 0{\cdot}41$]

20. A string can bear tension not more than 16 N. A stone of mass 50 gm is whirled by a string of length 50 cm in a horizontal plane. At what maximum rpm the stone can be rotated so that the string does not break ? [**Ans.** 240 rpm]

21. If the equatorial radius of the earth be 6000 km, what will be the acceleration at the equatorial region ? [**Ans.** 3·174 cm s^{-2}]

22. A stone is revolved in a circular path in a horizontal plaee by a string of length 0·5 m with uniform speed. If the string can bear a mzximum tension equal to 10 times the weight of the stone, calculate the maximum number of revolution per min so that the srring does not break.
If the string is rotated in a vertical circle by the same string what can be the maximum number of revolutions now ? [**Ans.** 2·23 rpm; 2.11 rpm]

23. A cylindrical drum of radius 9·8 cm is rotated with its axis vertical. When the speed of the drum in 200 rpm, a small body remains attached on the inner side of the drum, but the body falls if the speed of the drum be below this value. Calculate the coefficient of friction between the body and the drum. [**Ans.** $\mu = 0{\cdot}228$]

24. A small metal sphere of mass 200 m is hanging at the end of a string of length 1·5 m. Now the sphere is rotated in a horizontal circle of radius 50 cm. Calculate time period and tension in the string. [**Ans.** 2·39 sec ; 2·08 N]

25. The bridge over a small river is like an arc of a circle of radius 20 m. What will be the maximum speed of a car so that it does not lose contact with the bridge at the highest point ? [**Ans.** 14 ms^{-1}]

26. In a circus show a motor cyclist revolves along a vertical circle inside a big cage. Calculate the minimum speed of the cyclist so that he does not lose contact with the cage at the highest point. Given, radius of the cage is 8 m. [**Ans.** 8·85 m]

27. The centre of gravity of a cargo truck is at a height of 3 m above the ground. The separation between the wheels is 3m. If the truck moves along a circular path of radius 200 m, what can be the maximum speed of the truck so that it does not overturn ? [**Ans.** 31·3 ms^{-1}]

28. A simple pendulum of length 1 m is at rest at its mean equilibrium position. A horizontal velocity of 10 ms^{-1} is imparted to the bob. When the string of the pendulum becomes horizontal, what will be its speed ? Also calculate tension in the string at this position. Mass of the bob = 0·5 kg [**Ans.** 8·97 ms^{-1}; 40·2 N]

29. A string passes through a hole at the centre of horizontal table. Upper end of the string is tied to a mass *m* and at the lower end a body of mass M is hanging. If the mass *m* begins to rotate in a circle of radius *r* at a constant speed *v*, the mass M remains at rest, show that $v = \sqrt{Mgr/m}$.

30. A stone of mass 0·02 kg is whirled along a vertical circular path by a string of length 10 cm. At the highest point of the path the velocity of the particle is 1 ms^{-1}. Calculate (i) tension at the highest point (ii) tension at the lowest point and (iii) tension in the string when it is horizontal.
[**Ans.** (i) 4×10^{-3} N, (ii) 1·18 N; (iii) 0·592 N]

31. A smooth circular tube is fixed in a vertical plane. From the highest point of the tube a small particle of mass *m* dropped along the tube. When the angular displacement of the particle is θ, how much force the particle will exert on the tube ? [**Ans.** $(3 \cos \theta - 2)\ mg$]

32. A particle of mass 100 gm is suspended from the end of a weightless string of length 100 cm and is allowed to swing in a vertical circle. The speed of the particle is 200 cm / s when the string makes an angle of θ = 60° with the vertical, determine (i) the tension in the string and (ii) the speed of the particle when it is at the lowest position. Acceleration due to gravity = 980 cm / s^2
[**Ans.** (i) 0·89 N (ii) 3·715 ms^{-1}] [**J. E. E. 1993**]

33. A sphere of mass 'M' is hanging at the lower end of a string of length *l*. A bullet of mass '*m*' moving horizontally hits the sphere and stick to it. What should be the minimum velocity of the bullet so that the sphere will be able to go on a full circle ? [**Ans.** $\left(\frac{m+M}{m}\right)\sqrt{5gl}$]

34. If the distance of the earth from the sun becomes double of the present distance, calculate the number of days in a year [**Hints :** $T^2 \alpha r^3$] [**Ans.** 1032 days]

35. What should be the angular velocity of the earth so that the weight of a body at the equator be 60% of its present weight ? Radius of the earth = 6400 km [**Ans.** $7{\cdot}8 \times 10^{-4}$ rad / s]

36. From the top of a smooth sphere of radius *r*, a small body begins to slide down the surface of the sphere. At what angular displacement, the small body will loose contact with the sphere ? What will be its velocity at their moment ? [**Ans.** $\theta = \cos^{-1}\frac{2}{3}$; $v = \sqrt{\frac{2}{3}gr}$]

37. A car passes over a convex bridge. The centre of gravity of the car follows an arc if a circle of radius 88 ft. Assuming that the car has a weight of 2 tones, find the force the car will exert at the highest point of the bridge, if the velocity of the car be 30 miles per hr ; at what speed will the car loose contact with the road ?
[**Ans.** 44800 poundal ; 53·07 ft / sec] [**J. E. E. 1998**]

◢ D. Harder numerical problems :

1. A coin is placed on a horizontal turn table rotating at $33\frac{1}{3}$ rpm, revolves with the table without slipping provided the coin is not move that 10 cm away from the axis. How far from the axis can the coin in placed so that it revolves with the table without slipping if the turn table rotates at 45 rpm ? (g = 980 cm / s^2]
[**Ans.** 5·49 cm] [**J. E. E. 1995**]

2. A body is allowed to fall from a height *h* on an inclined plane and on reaching the bottom it moves along a circular track of radius *r*. What should be the minimum value of *h* so that the body will be able to describe a circle ? [**Ans.** $h = \frac{5}{2}r$]

3. A small body is resting on the top of a sphere of radius R. What horizontal velocity be given to it so that it will loose contact with the sphere when its height from the centre is 0·8 R ?

4. A small sphere of mass *m* is on the inner surface of a funnel whose semi-vertical angle is (π / 2 – θ). The coefficient of friction between the funnel and the sphere is μ. If the sphere be at a distance *r* from the axis and the funnel rotates about its vertical axis then, for what value of angular velocity of the funnel, the sphere will tend to move upwards along the funnel ?

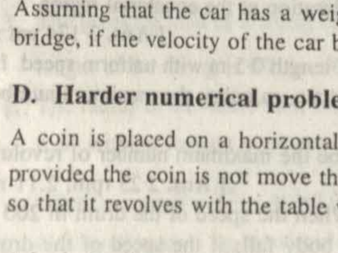


Fig. 1·42

[**Ans.** $\omega = \sqrt{\frac{g}{r}\left(\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}\right)}$]

[**Hints :** Considering the equilibrium of the body
$N\cos\theta = mg + \mu N\sin\theta$ and $N\sin\theta = m\omega^2 r - \mu N\cos\theta$]

5. The radius of the circular bent of a road is r and angle of banking θ. Show that the car will not skid if the maximum speed of the car is $v = \sqrt{\frac{rg(\mu + \tan\theta)}{1 - \mu\tan\theta}}$ when m = Coefficient of friction between the car and the road.

6. A car moves in a curve of radius of curvature R. The width between the wheels x and the height of the c.g. from the ground is h. With what velocity must the car move in order that the vertical force on the inside wheels shall be reduced to zero ? [**Ans.** $v = \sqrt{\frac{Rgx}{2h}}$]

7. A mass m on a frictionless table is attacked to a hanging mass M by a thread through a hole in a table. Find the condition between the given quantities so that when the mass m spins, the mass M will be at rest.

[**Ans.** $\frac{v^2}{r} = \frac{Mg}{m}$]

8. Two blocks of equal mass are connected by an inextensible string. The string passes through a hole on a smooth table. One of the blocks is on the table and the other is hanging below the table. The block on the table is revoluing along a circle of radius of 15 cm. At what angular speed it should rotate so that the block which is hanging will remain stationary ? [**Ans.** 77·2 rpm]

9. A body of mass 50 kg is tied at one end a string of length l. The body is oscillating in a vertical plane with angular amplitude θ_0. When the string makes an angle θ with the vertical what is tension in the string ? If the breaking load for the string be 80 kg what can be the maximum angular amplitude of the body ?

[**Ans.** 392 $(3\cos\theta - 2\cos\theta_0)$ N; $\theta_0 = 60°$]

10. A vertical rod is fixed on a rotating table at a distance r from the axis of rotation. From the upper end of the rod a body is suspended by a string of length l. If the string makes an angle θ with the vertical show that angular velocity of the table is $\omega = \sqrt{\frac{g\tan\theta}{r + l\tan\theta}}$.

11. A train is moving on rails which are bent in the form of a arc of a circle of radius 500 m. with a velocity of 60km/hr. Separation between the two rails is 1·70 m. Calculate the elevation of the outer rail with respect to the inner rail so that there is no lateral force on the outer rail, given $g = 9{\cdot}8$ m / sec ^{2}. [**Ans.** 9·86 cm]

12. Two blocks of mass $m_1 = 10$ kg and $m_2 = 5$ kg connected to each other by a massless inextensible string of length 0·3 m are placed along a diameter of a turn. table. The coefficient of friction between the table and m_1 is 0·5 while there is no friction between m_2 and table. The table is rotating with an angular velocity of 10 rad / sec about a vertical axis passing through its centre O. The mass m_1 is at a distance of 0·124 m from O. The masses are observed to be at rest with respect to the table. (i) Calculate the frictional force on m_1. (ii) What should be minimum angular speed of the turn table so that the masses will slip from the turn table ? **[I. I. T. '97]**

[**Ans.** (i) 36 Newton, (ii) 11·67 rad / sec]

13. A heavy spherical body is hanging from a string of length l. If the angular amplitude of rotation be θ_0. calculate the maximum tension in the string. Assume the weight of the body W and its amplitudes small.

[**Ans.** $T = W(1 + \theta^2_0)$]

14. The centre of gravity of a loaded vehicle is at a height 1·5 m from the ground and separation between two wheels is 2 m. At what maximum speed the vehicle can move on at the bent of a horizontal road so that it does not overturn ? [**Ans :** 25·62 m / sec,]

15. A particle of mass of m is rotating along a circle of radius R. Its velocity v changes with distances according to the relation $v = A\sqrt{s}$ where A is a constant. Calculate (i) accelaration of the particle. (ii) Maximum force on it.

[**Ans.** $a = \frac{1}{2}A^2\sqrt{\frac{4s^2}{R^2}}$; $F = ma$]

16. A rubber ring is rigidly fixed on a wheel of radius 80 cm. The wheel is then rotated about an axis with increasing rate. The ring became just slack on the wheel when it is rotated at 1200 rpm. What was the strain on the rubber before the wheel started rotation ? Given young modulus of rubber $Y = 5\cdot5 \times 10^{9}$ dyne / cm^{2} and density 0·95 gm / cc [**Ans.** $4\cdot36 \times 10^{-3}$]

17. A metallic chain of length 62·8 cm is fixed on a wooden wheel with the two ends of the chain connected together. When the wheel rotates at 60 rpm calculate the tension in the string. Assume mass of chain 40 gm. [**Ans.** 9·2 kg-wt]

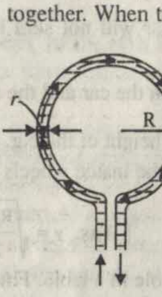


Fig. 1·43

18. A small body of mass 0·5 kg is placed at the top of a fixed sphere of radius 50 cm. The block is pushed a little to move along the sphere. Calculate (i) when the body makes 30° with the vertical what force it will exert on the sphere. (ii) when it loses contact with the sphere how much angle it will make with the vertical. (iii) At what distance from the point of contact of the sphere the body will fall on the ground ? [**Ans.** (i) 2·9 Newton, (ii) $\theta = \cos^{-1}\frac{2}{3}$, (iii) 0·73 m]

19. In the fig 1·43 water in flowing through a rubber ring Radius of the ring is R and radius of the tube r. ($r << R$) what force will be exerted on the rubber tube. [**Ans.** $\pi r^{2} v^{2}$]

20. A body of mass 100 gm is fixed at the lower end of a string of length l m and is rotated in a vertical circle about its other end. When the string makes 60° with the vertical its speed is 2 m / sec. Calculate (i) Tension in the string when $\theta = 60°$ (ii) Velocity of the body at lowest point. Given g = 9·8 m / sec ^{2} [**Ans.** (i) 0·89 Newton (ii) 3·13 m/sec]

21. A light rod of length l is placed vertically on the floor. A heavy sphere is placed on its upper end as shown in fig 1·44. From its vertical position the rod begins to fall. At what angle with the vertical the rod will not exert any force on the floor. [**Ans.** $\theta = \cos^{-1}\frac{2}{3} = 48°\cdot10'$]

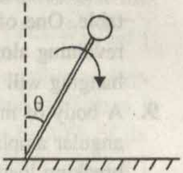


Fig. 1·44

22. A particle is moving along a circle of radius 1 cm. at a velocity $V = 2t$. Here V is measured in cm / sec and t in sec. Calculate (i) radical accelaration of particle after $t = 1$ sec. (ii) Tangential acceleration after $t = 1$ sec, (iii) Total acceleration after $t = 1$ sec. [**Ans :** (i) 4·0 cm / sec ^{2}, (ii) 2 cm / sec ^{2} (iii) $2\sqrt{5}$ cm / sec ^{2}]

23. Four identical balls each of mass 5 kg are placed on a horizontal turn table. Each ball is tied by a string of length 1 m forming a square with the ball at the corners of the square. The axis of rotation passes through the centre of the square. If the turn table rotates at $\frac{30}{\pi}$ rpm, what is the tension in the string ? [**Ans.** 2·5 N]

24. A horizontal rod of length l = 20 cm is rotated about a vertical aris at constant angular velocity. At the other end of the rod a string of length 20 cm is attached and at the lower end of the string a body of mass m is connected (fig. 1·45). If the string makes an angle $\theta = 30°$ with the vertical what is the angular velocity of rod. $g = 10$ m / sec ^{2} [**Ans.** $\omega = 4\cdot4$ rod / sec.]

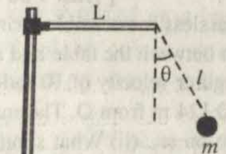


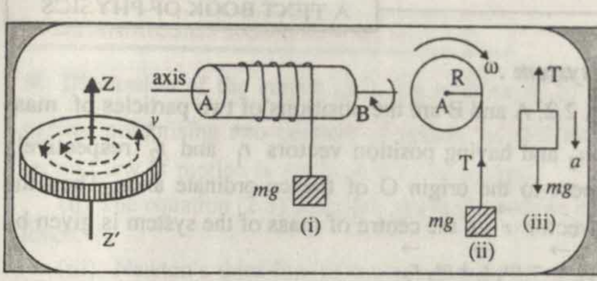
Fig. 1·45

25. A string passes through fixed vertical tube at one end a mass M and other end mass m are attached (fig. 1·45). The mass m is rotating in a horizontal circle. Length the string from the tube to the mass m is l and the string makes an angle θ with the vertical. If mass M is stationary calculate the frequency of revolution of m. [**Ans.** $n = \frac{1}{2\pi}\sqrt{\frac{g}{l\cos\theta}}$] [**I. I. T.**]

26. A metallic wire of length 2m and having mass 0·4 kg is rotated in a horizontal circle. If the breaking stress of the wire 8×10^{9} Nm^{-2} and area of cross section 8×10^{-6} m^{2} at what frequency of revolution the wire will break? At what point it will break ? [**Ans.** n = 63·6 Hz, at the upper end]

27. Angle of banking at the bend of a road is 10° and radius of curvature at the bent is 100 m. A car is moving at the bent. Coefficient of friction between car and road is 0·25. Calculate (i) At what maximum velocity of the car there will be no lateral pressure on the wheel ? (ii) Maximum velocity of the car so that it does not skid.

[**Ans.** (i) $V_0 = \sqrt{Rg\tan\theta} = 11\cdot1$ m / sec (ii) V max = 20·9 m / sec]



CHAPTER 2

DYNAMICS OF ROTATIONAL MOTION

- **TOPICS :** ○ Rotational motion, ○ Centre of mass, ○ Momentum conservation and centre of mass motion, ○ Examples of the motion of centre of mass, ○ Centre of mass of a rigid body, ○ Relation between linear motion of a particle of a rigid body and its rotation, ○ Torque and angular acceleration, ○ Angular momentum, ○ Relation between torque and angular momentum, ○ Law of conservation of angular momentum, ○ Power produced by a torque and work done, ○ Kinetic energy of rotation of a rigid body, ○ Two important theorems on moment of Inertia, ○ Examples of two dimensional rigid body motion, ○ Short answer type questions (with answers), ○ Exercise.

❂ 2.1. Rotational Motion ❂

Rotational motion is as common in our daily life as translational motion. The wheels, blades of a fan, spinning tops, minute and hour hands of a clock all perform rotational motion. In purely rotational motion, a body does not move from one place to another i.e. it does not have any translational motion. It simply turns through a certain angle.

❖ **Definition : A body is said to perform a purely rotational motion if every particle of the body moves in a circular path such that the centres of these circles lie on a straight line called the axis of rotation.**

However, we also find cases when the body describes both rotational and translational motion simultaneously. Example :

(i) The wheel of a car rotates about the axle and at the same time covers a linear distance.

(ii) The earth rotates about an axis passing through it and at the same time revolves round the sun.

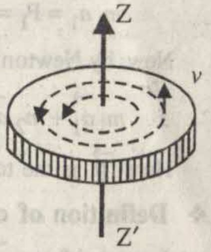


Fig. 2·1

If a body is constrained to rotate about a fixed axis, its constituent particles undergo circular motion about the axis of rotation. Obviously the radius of the circular paths are all differents. The body as a whole does not move but the constituent particles describe linear distances along different circular paths. Thus the motion of the constituent particles can be described in terms of linear quantities. But the rotational motion of the whole body can only be described in terms of angular quantities like angular displacement, angular velocity and angular acceleration. These quantities have been defined in the chapter of circular motion.

❂ 2.2. Centre of Mass ❂

We have the idea of centre of mass in the chapter of statics. Let us consider a system consisting of large number of particles all executing motion under an external force acting on it. It is found that **there is a point in the system where if whole mass of the system is supposed to be concentrated, the nature of motion of the system remains unchanged, when the forces acting on the system are applied directly at this point.** Such a point in the system is called **centre of mass** of the system.

➤ 2.2.1. *Centre of mass of a two particle system :*

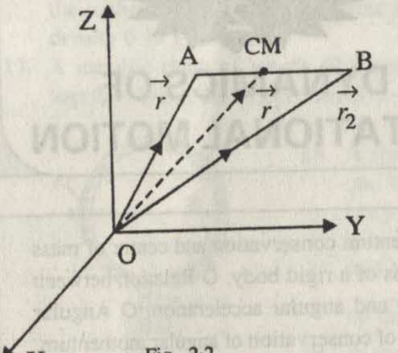


Fig. 2·2

In fig. 2.2, A and B are the positions of two particles of mass m_1 and m_2 and having position vectors $\vec{r_1}$ and $\vec{r_2}$ respectively with respect to the origin O of the co-ordinate axes. Then the position vector $\vec{r}$ of the centre of mass of the system is given by $(m_1 + m_2)\,\vec{r} = m_1\vec{r_1} + m_2\vec{r_2}$.

So, the product of total mass of the system and the position vector of the centre of mass is equal to the sum of the products of the individual masses and their respective position vectors,

$$\therefore\ \vec{r} = \frac{m_1\vec{r_1} + m_2\vec{r_2}}{m_1 + m_2} \quad \ldots\ldots\ldots\ldots (2.1)$$

● **Derivation :** Let $\vec{F_1} = \vec{F_{12}} + \vec{F_{1e}}$ be the force acting on the particle A causing an acceleration $\vec{a_1}$ of A. Here $\vec{F_{12}}$ is the force exerted on the particle A due to B and $\vec{F_{1e}}$ is the external force on the particle A. Evidently $\vec{F_{12}}$ is the internal force exerted by one part to the other part of the same system.

Similarly net force $\vec{F_2}$ acting on the particle B is $\vec{F_2} = \vec{F_{21}} + \vec{F_{2e}}$

When $\vec{F_{21}}$ is the force exerted by A on B. The net force $\vec{F_2}$ causes acceleration $\vec{a_2}$ of the particle B of mass m_2. So, the equation of motion of each particle is

$$m_1\vec{a_1} = \vec{F_1} = \vec{F_{12}} + \vec{F_{1e}} \quad \text{and} \quad m_2\vec{a_2} = \vec{F_2} = \vec{F_{21}} + \vec{F_{2e}}$$

Now, by Newton's third law $\vec{F_{12}} = -\vec{F_{21}}$

$$\therefore\ m_1\vec{a_1} + m_2\vec{a_2} = \vec{F_{1e}} + \vec{F_{2e}} = \vec{F} \quad \ldots\ldots\ldots\ldots (2.2)$$

Here $\vec{F}$ is the total external force acting on the system.

❖ **Definition of centre of mass :** The total mass of the system $M = m_1 + m_2$. Let us say that the total external force $\vec{F}$ acting on the system of mass M produces in it an acceleration $\vec{a}$. So, its equation of motion is $M\vec{a} = \vec{F}$ $\ldots\ldots\ldots\ldots$ (2.3)

From the equation (2.2) and (2.3) we get $M\vec{a} = m_1\vec{a_1} + m_2\vec{a_2}$

$$\therefore\ \vec{a} = \frac{m_1\vec{a_1} + m_2\vec{a_2}}{M} \quad \ldots\ldots\ldots\ldots (2.4)$$

Here $\vec{a}$ is called the acceleration vector of the centre of mass of the system. Now,

$$\vec{a} = \frac{d^2\vec{r}}{dt^2} \quad [\ \vec{r} = \text{Position of vector of CM}\ ]$$

$$\therefore\ \frac{d^2\vec{r}}{dt^2} = \frac{1}{M}\left(m_1\frac{d^2\vec{r_1}}{dt} + m_2\frac{d^2\vec{r_2}}{dt}\right)$$

$$\therefore\ \vec{r_1} = \frac{1}{M}\left(m_1\vec{r_1} + m_2\vec{r_2}\right) = \frac{m_1\vec{r_1} + m_2\vec{r_2}}{m_1 + m_2} \quad \ldots\ldots\ldots\ldots (2.5)$$

- **Discussion of the result :** (i) The equation (2.5) defines the position of the centre of mass of the system comprising two particle of masses m_1 and m_2 with position vectors $\vec{r_1}$ and $\vec{r_2}$. So Newton's second law of motion is applicable to the system.

(ii) The equation (2.3) discribes the motion of the total mass of the system under the effect of external force.

(iii) Newton's third law of motion enables us to get ride of the internal forces between the particles themselves.

(iv) The motion of a system can be studied either by applying Newton's second law of motion to the individual particles of the system along with the third law (for internal forces) or by studying the motion of the total mass located at the centre of mass of the system under the effect of external forces only.

(v) The centre of mass of a system of two particles always lies between the two particles and on the line joining them.

(vi) If the masses are equal i.e. $m_1 = m_2$. So, $r = \frac{1}{2}(r_1 + r_2)$

➤ 2.2.2. *Centre of mass-Generalisation to n-particles :*

Consider a system of n-particles of masses m_1, m_2, m_3 ....m_n with position vectors $\vec{r_1}, \vec{r_2}, \vec{r_3}$, . . . $\vec{r_n}$ with respect to the origin of the co-ordination system. Then the position Vector $\vec{r}$ of the centre of mass of the system of n - particles is given by

$$(m_1 + m_2 + m_3 + ...+ m_n)\,\vec{r} = m_1\vec{r_1} + m_2\vec{r_2} + ...+ m_n\vec{r_n}$$

$$\therefore \quad \vec{r} = \frac{m_1\vec{r_1} + m_2\vec{r_2} + ...+ m_n\vec{r_n}}{m_1 + m_2 + ...+ m_n}$$

$$\therefore \quad \vec{r} = \frac{\sum_{i=1}^{n} m_i\vec{r_i}}{\sum_{i=1}^{n} m_i} = \frac{1}{M}\sum_{i=1}^{n} m_i\vec{r_i} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots .2.5\text{ (a)}$$

Thus the position vector $\vec{r}$ of the centre of mass of a system of n particles is the weighted average of all the position vectors of all the particles of the system, the contribution of each particle being proportional to its mass.

The motion of the centre of mass is governed by the equation

$$M\vec{a} = \vec{F_1} + \vec{F_2} + ...+ \vec{F_n}$$

$$\therefore \quad M\vec{a} = \vec{F}_{(\text{tot})} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.6)$$

Where $M = \sum_{i=1}^{k} m_i$ is the total mass of the system, $\vec{a}$ is the acceleration of the centre of mass and $\vec{F}_{(\text{tot})}$ is the vector such of all the external forces acting on the system. The equation (2.6) states that **the centre of mass of a system of particles moves as if all the mass of the system were concentrated at the centre of mass and all the external forces were applied at that point.**

The internal forces between the particles cancel out in pairs by virtue of Newton's third law. So, if we are not concerned with the internal forces, internal structures and internal motion of the system, we can analysis the motion of the system by applying Newton's Second law of motion to the centre of mass of the system where whole mass is concentrated.

✪ 2.3. Momentum conservation and centre of mass motion ✪

We consider a system of n particles of masses $m_1, m_2, m_3 \ldots\ldots\ldots m_n$, moving with respective linear velocities $v_1, v_2, v_3 \ldots\ldots\ldots, v_n$. We also assume that total external force F (tot) acts on the system. The particles also exert internal forces on each other. However, the internal forces acting on the particles of the system just cancel out. So from equation (2.6)

$$M\vec{a} = 0 \qquad \therefore \ \vec{a} = 0$$

$$\therefore \ \frac{d}{dt}\left(\vec{V}_{cm}\right) = 0 \quad \therefore \ \vec{V}_{cm} = \text{Constant} \ldots\ldots\ldots\ldots\ldots\ldots (2.7)$$

V_{cm} is the velocity of the centre of mass of the system. Thus in the absence of any external force the velocity of the centre of mass remains constant.

Again from equation (2.6) in the case of isolated system, F (tot) = 0

$$\therefore \ \vec{F}_1 + \vec{F}_2 + \ldots\ldots\ldots \vec{F}_m = 0 \quad \therefore \ m_1\vec{a}_1 + m_2\vec{a}_2 + \ldots\ldots\ldots + m_n\vec{a}_n = 0$$

$$\text{or,} \ \frac{d}{dt}\left(m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots\ldots\ldots + m_n\vec{v}_n\right) = 0$$

Where $\vec{v}_1, \vec{v}_2, \ldots\ldots\ldots \vec{v}_n$ are the velocities of the particles of the system.

Thus $m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots\ldots\ldots + m_n\vec{v}_n =$ constant

$$\therefore \ \text{Total linear momentum} \ \vec{P} = \vec{P}_1 + \vec{P}_2 + \ldots\ldots\ldots + \vec{P}_n = \text{constant} \ldots\ldots\ldots (2.8)$$

Now, differentiating 2.5 (a) with respect to time,

$$\frac{d\vec{r}}{dt} = \frac{1}{M}\left(m_1\frac{d\vec{r}_1}{dt} + m_2\frac{d\vec{r}_2}{dt} + m_3\frac{d\vec{r}_s}{dt} + \ldots\ldots\ldots + m_n\frac{d\vec{r}_n}{dt}\right)$$

$$\therefore \ \vec{V}_{cm} = \frac{1}{M}\left(m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots\ldots\ldots m_n\vec{v}_n\right) \ldots\ldots\ldots\ldots\ldots\ldots (2.9)$$

$$M\vec{V}_{cm} = \vec{P}_1 + \vec{P}_2 + \ldots\ldots\ldots + \vec{P}_n = \vec{P} \ldots\ldots\ldots\ldots\ldots\ldots (2.10)$$

The equation (2.9) states that the total linear momentum of a system of particle is equal to the product of total mass of the system and the velocity of the centre of mass. Also the equation 2.8 describes the law of conservation of linear momentum for a system of n particles. It states that **if no external force acts on the system, the total linear momentum of the system remains constant.**

✪ 2.4. Examples of the motion of centre of mass ✪

(i) Let a shell or a special type of fire cracker is projected from earth and explodes in flight. Here earth's gravity is the external force and the system is the shell. Its projectary is shown in figure 2.3.

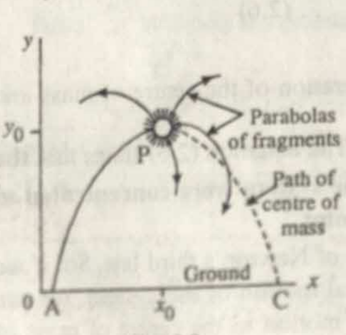


Fig. 2·3

Before explosion, the shell moves along a parabolic path AP and at P it explodes into several fragments. The fragments fly off in all directions. Now, what path will the centre of mass of the system follow ? Here the **force of explosion is the internal force** which can change the momenta of individual fragments but can not change the total momentum of the system. Only external force can change the total momentum. So, the centre of mass of the system will continue moving in the parabolic trajectory APC which is the path that the unexploded shell would have followed. But the fragments will follow different parabolic paths.

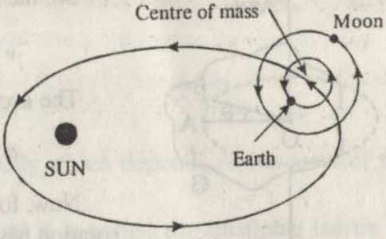


Fig. 2·4

(ii) Now, we consider the motion of the centre of mass of the earth-moon system. The moon revolves round the earth on a circular orbit and the earth-moon system goes round the sun in eliptical orbit [Fig 2.4]. In fact, the earth and the moon move in circular orbits about their centre of mass, while the centre of mass of the earth-moon system moves in an elliptical orbit round the sun.

Here it may be noted that for earth-moon system, the forces of attraction due to the sun are the external forces, while the forces of attraction between the earth and the moon are internal forces. The earth-moon system moves along elliptical orbit around the sun due to the torque produced by external forces.

(iii) Lastly we consider the spontaneous decay of radioactive nucleus. When a radioactive nucleus initially at rest, decays into two fragments, these fragments fly apart obeying the laws of conservation of linear momentum and energy. But the centre of mass of two fragments remains at rest.

2.5. Centre of mass of a rigid body

A body is said to the rigid if it does not undergo any change in shape by the action of a force i.e., the relative positions of the constituent particles do not change under the influence of the force. The body is not deformed by the applied force.

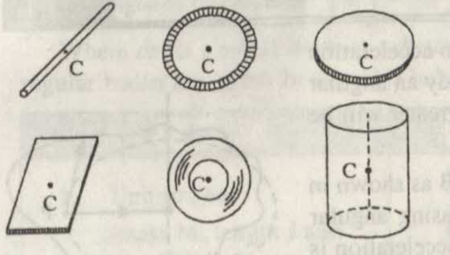


Fig. 2·5

The centre of mass of a rigid body having a regular geometric shape can be easily determined from the condition of symmetry. In the table below the positions of the centre of mass for bodies of different geometrical shapes are given. It may be noted that the centre of mass may not always be inside the body. For example in the case of a ring, hollow tube and hollow sphere the centre of mass lies in the hollow portion of the body.

The fig 2.5 shows the position of the centre of mass of some regular bodies.

Shape of a body	Position of centre of mass
1. Uniform rod	Middle point of a rod
2. Circular disc	Centre of the disc
3. Circular ring	Centre of the ring
4. Sphere	Centre
5. Cylinder	Middle point of the axis.
6. Triangular plane lamina	Point of intersection of the medians of the triangle

2.6. Relation between linear motion of a particle of a rigid body and its rotation

The point P of the rigid body G is rotating about a fixed axis [Fig. 2.6] . As the body rotates, this point moves in a circle. The radius '*r*' of the circle is the perpendicular distance of the particle from the axis of rotation. As the body rotates by angle θ, the radius vector also rotates by the same angle. The linear distance moved by the particle is $s = r\theta$ along the circle.

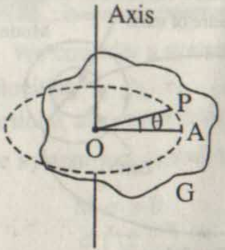


Fig. 2·6

So, the linear speed along the tangent is

$$v = \frac{ds}{dt} = r\frac{d\theta}{dt} = r\omega \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.11)$$

The acceleration along the tangent or the tangential acceleration is

$$a = \frac{dv}{dt} = r\frac{d\omega}{dt} = r\alpha \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.12)$$

Now, for different particles of the rigid body, the radius r of their circles of rotation has different values, but ω and α are same for all the particles. Hence the linear speed and tangential accelerations are different for different particles.

- **Example 2.1. A bucket is lowered into a well by a string passing over a fixed pulley of radius 5 cm and the rope does not slip on the pulley. Find the angular velocity and angular acceleration of the pulley at an instant when the bucket is going down at a speed of 10 cm s^{-1} and has an accleration of 4.0 ms^{-2}**

○ **Solution :** Since the rope does not slip on the pulley, the speed of the bucket is equal to the linear speed of the rim of the pulley.

∴ The angular velocity of the pulley is then $\omega = \frac{v}{r} = \frac{10}{5} = \mathbf{2\ rad/s}$

and the angular acceleration of the pulley is $\alpha = \frac{a}{r} = \frac{400}{5} = \mathbf{80\ rad\ s^{-2}}$

❂ 2.7. Torque and angular acceleration ❂

We know that when a force is applied to a moving body, an acceleration is generated. So also when a torque is applied to a rotating body an angular acceleration is produced. It is obvious that larger the torque, greater will be the angular acceleration.

Let a rigid body is capable of rotating about a fixed axis AB as shown in fig 2.7 When a torque is applied, it rotates with an increasing angular velocity. Let at any instant angular velocity is ω and angular acceleration is $\alpha = \frac{d\omega}{dt}$. The angular acceleration of all the particles is same, but their linear accelerations are different because the value of linear acceleration depends on the distance from the axis.

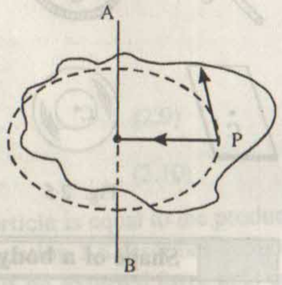


Fig. 2·7

Let m_1, m_2, m_3 .... be the masses of the particles constituting the body and their distances form the axis of rotation are $r_1, r_2, \ldots\ldots\ldots r_n$.

If F_1 be the force acting on m_1, then

$$F_1 = m_1\frac{dv_1}{dt} \quad [\ v_1 = \text{linear velocity of } m_1\ ]$$

$$= m_1\frac{d}{dt}(r_1\omega) = mr_1\frac{d\omega}{dt} = mr_1\alpha$$

∴ The moment of the force about the axis of rotation, $\tau_1 = F_1 r_1 = m_1 r_1^{\,2}\alpha$

Similarly, the moment of the force acting on the particle of mass m_2 is $\tau_2 = F_2 r_2 = m_2 r_2^{\,2}\alpha$

In the same way we can calculate the moment of the force acting on all other particles. Hence the total torque on the body is $\tau = \tau_1 + \tau_2 + \ldots\ldots\ldots + \tau_n$

$$= \left(m_1 r_1^{\,2} + m_2 r_2^{\,2} + \ldots\ldots\ldots m_n r_n^{\,2}\right)\alpha$$

$$= \alpha \sum_{i=1}^{n} m_i r_i^{\,2} \;=\; \mathrm{I}\alpha \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \; (2.13)$$

Here $\mathrm{I} = \sum_{i=1}^{n} m_i r_i^{\,2}$ is an important property of the rotating body which depends on the mass of the particles and their distribution with respect to the axis of rotation. It is called the rotational inertia or **moment of inertia** of the body.

➤ *2.7.1. Moment of Inertia :*

For rotational motion we have $\tau = \mathrm{I}\alpha = \mathrm{I}\dfrac{d\omega}{dt}$ and for linear motion $\mathrm{F} = ma = m\dfrac{dv}{dt}$. We know that mass is the measure of inertia for translational motion. The equivalent quantity I is the measure of inertia for rotational motion and is called moment of inertia. If we put $\dfrac{d\omega}{dt} = 1$, then $\tau = \mathrm{I}$. Hence moment of inertia of rotating body is defined as follows :

Moment of inertia of a body about an axis of rotation is numerically equal is the torque required to produce in it a unit angular acceleration.

Mathematically, $\mathrm{I} = \sum_{i=1}^{n} m_i r_i^{\,2}$. So its value depends on the distribution of mass about the axis of rotation.

In the language of calculus $\mathrm{I} = \int r^2 dm \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \; (2.14)$

Where *dm* is a small mass at a distance *r* from the axis. Formulae to calculate moment of inertia of regular bodies are given in the table below :

Body	Axis of rotation	Moment of Inertia
1. Uniform bar (mass M, length *l* and breadth *b*)	Through the centre ⊥ to its length	$\frac{\mathrm{M}}{12}\left(l^2 + b^2\right)$
2. Thin Circular ring of Mass M and radius *r*	Through the centre ⊥ to its plane.	$\mathrm{M}r^2$
3. Disc of Mass M and radius *r*	Do	$\frac{1}{2}\mathrm{M}r^2$
4. Cylindrical rod of mass M, length *l* and radius *r*	Through the centre and ⊥ to its length	$\mathrm{M}\left(\frac{l^2}{12} + \frac{r^2}{4}\right)$
5. Solid sphere of Mass M and radius *r*	Through the centre	$\frac{2}{5}\mathrm{M}r^2$
6. Spherical shell	Through the centre	$\frac{2}{3}\mathrm{M}r^2$

The following points be noted in connection with the moment of inertia :

(i) The moment inertia of the body depends on its mass.

(ii) Two bodies having same mass may have different moment of inertia. Because the moment of inertia not only depends on mass but also on the distribution of mass of the body.

(iii) A given body in general, has different moment of inertia about different axis of rotation.

Unit of moment of inertia on SI system : Kg. m^2 and dimension $[ML^2]$

➤ 2.7.2. *Radius of gyration (K) :*

Consider the figures shown in fig 2.7(a). In fig 2.7a(i) the rigid body B is made of particles of masses $m_1, m_2, \ldots\ldots m_n$ which are at the distance of $r_1, r_2, \ldots\ldots r_n$ from the axis rotation (AOR). Then the moment of inertia of the system of particle i.e., of the rigid body about the given axis is

$$I = \sum_{i=1}^{n} m_i r_i^2$$

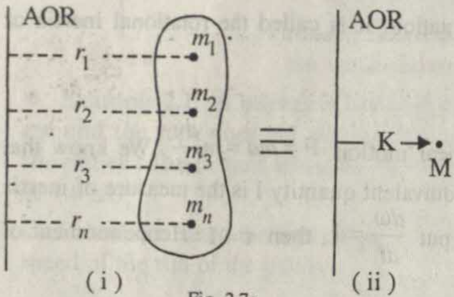


Fig. 2·7a

Now, we take a point at a distance K from the AOR when whole mass $M = \sum_{i=1}^{n} m_i$ is supposed to be concentrated [fig. 2.7a(ii)]. Then the moment of inertia of the point mass M about the given axis is

$$I = MK^2$$

Evidently, $MK^2 = \sum_{i=1}^{n} m_i r_i^2 \quad \therefore \quad K = \sqrt{\dfrac{m_1 r_1^2 + m_2 r_2^2 + \cdots\cdots + m_n r_n^2}{M}} \ \ldots \text{(i)} \quad \text{or,} \quad K = \sqrt{\dfrac{I}{M}}$

The quantity K is called radius of gyration of the system. Hence, **Radius of gyration of a rigid body about its axis of rotation is defined as the distance from the axis of rotation at which, if the entire mass of the body were concentrated, its moment of inertia about the given axis would be the same as with the actual distribution of mass.**

Now, if we put $m_1 = m_2 = \cdots\cdots m_n = m$ then

$$K = \sqrt{\frac{m\,(r_1^2 + r_2^2 + \cdots\cdots + r_n^2)}{mn}} = \sqrt{\frac{r_1^2 + r_1^2 + \cdots\cdots + r_n^2}{n}}$$

So, radius of gyration equals Root Means Squares (RMS) of the distances of the particles from the axis.

So K is some kind of an average effective distance of the particle from the axis of rotation.

- **Example : Moment of inertia of a solid cylinder about one of its diameter is**

$$I = \frac{2}{5}Mr^2 = M\left(\frac{2}{5}r^2\right) \quad \therefore \quad K^2 = \frac{2}{5}r^2 \quad \therefore \quad K = \sqrt{\frac{2}{5}} \cdot r$$

Using this relation we can calculate radius of gyration of different rigid bodies.

- **Ex. 2.2. A torque of 7·5 N-m is applied to a circular disc of mass 6 kg and diameter 50 cm. It can rotate about an axis passing through the centre and perpendicular to its plane. Calculate angular acceleration of the disc.**

○ **Solution :** The required reaction : $\tau = I\alpha$

$$\tau = 7{\cdot}5 \text{ N-m} \quad \text{and} \quad I = \frac{1}{2}Mr^2 = \frac{1}{2} \times 6 \times 0{\cdot}25 = 0{\cdot}75 \text{ Kg. m}^2$$

$\therefore$ angular acceleration, $\alpha = \dfrac{\tau}{I} = \dfrac{7{\cdot}5}{0{\cdot}75} = 10 \text{ rad / sec}^2$

- **Ex. 2.3. A flywheel of moment of inertia 0·3 kg m^2 is rotating at the rate of 200 rpm. What torque is needed to stop in 10 sec ?**

○ **Solution :** Angular retardation, $\alpha = \frac{\omega_0 - \omega}{t} = \frac{2\pi(n_0 - n)}{t}$

$$\therefore \quad \alpha = \frac{2\pi n_0}{t} \; [\because n = 0]$$

$$= \frac{2\pi \times 200}{60 \times 10} = 2 \cdot 093 \text{ rad / s}^2$$

$\therefore$ Required torque $\tau = I\alpha = 0 \cdot 3 \times 2 \cdot 093 = \mathbf{0 \cdot 63\ N\text{-}m^2}$

2.8. Angular momentum

❖ **Definition : The turning movement of a particle about the axis of rotation is called the angular momentum of the particle and is measured by the product of the linear momentum and the perpendicular distance of the line of action from the axis of rotation.**

So the angular momentum, L = linear momentum × perpendicular distance from the axis of rotation.

Its SI, unit is kg m^2 s^{-1} and dimensional formula is [M. L^{2}T^{-1}]

Let a particle of mass m is moving along a curve and its linear momentum $\vec{P} = m\vec{V}$ when it is at Q, its position vector is $\vec{r}$ so, the angular momentum of the particle is $L = mVr\sin\theta = rP\sin\theta$

Where θ is the angle between r and P.

$$\therefore \quad \vec{L} = \vec{r} \times \vec{P} \cdots\cdots\cdots\cdots (2.15)$$

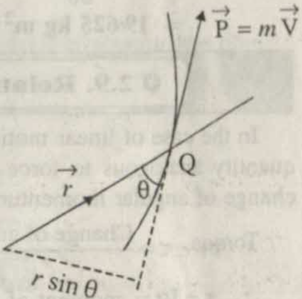


Fig. 2·8

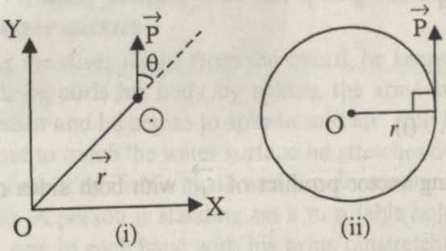


Fig. 2·9

Now, consider the fig 2.9(i) when the motion of the particle is in the plane XY and its angular momentum is along Z- axis and the direction is denoted by ⊙ symbol.

- **Special cases :** In fig 2.9 (ii) the particle is moving along a circle of radius r and its linear momentum P = mV,

So, the moment of momentum or angular momentum is

$$L = rP = rmV$$

$$\therefore \quad L = mr^2\omega \;\; [\text{When } V = r\omega, \;\; \omega = \text{angular velocity}]$$

$$\therefore \quad L = I\omega \;\; [\,I = \text{moment of inertia}\,]$$

- **Case of rigid body :** In the same way we can calculate the angular momentum of a rigid body which consists of n particles. Let $m_1, m_2, \ldots\ldots m_n$ be the masses and $r_1, r_2, \ldots\ldots r_n$ be the distance of n particle from the axis of rotation. Then the angular momenta of the particles are $L_1 = m_1 r_1{}^2\omega$, $L_2 = m_2 r_2{}^2\omega, \ldots\ldots\ldots\ldots L_n = m_n r_n{}^2\omega$.

So, the total angular momentum of the rotating rigid body is

$$L = L_1 + L_2 + \ldots\ldots + L_n$$

$$= m_1 r_1{}^2\omega^2 + m_2 r_2{}^2\omega + \cdots\cdots + m_n r_n{}^2\omega$$

$$= \left(\sum_{i=1}^{x} m_i r_i{}^2\right)\omega = I\omega \quad \left[I = \sum_{i=1}^{n} m_i r_i{}^2\right]$$

$$\therefore \quad L = I\omega \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (2.16)$$

I is the moment of inertia of the body. Comparing L = Iω with the well known result of linear momentum P = mV for translatory motion, we find that I is equivalent to m.

Now if ω = 1 then I = L

So, **the moment of inertia of a rigid body about at axis is numerically equal to the angular momentum of the rigid body when rotating with unit angular velocity about that axis.**

- **Example 2.4. A sphere of mass 20 kg and diameter 0·25 m is revolving about one diameter at the rate of 1500 rpm. Calculate (i) Moment of inertia and (ii) Angular momentum.**

○ **Solution :** (i) Moment of inertia, $\mathrm{I}=\frac{2}{5}\mathrm{M}r^2$

$$\therefore\quad \mathrm{I}=\frac{2}{5}\times 20\times\left(\frac{0\cdot 25}{2}\right)^2=0\cdot 125\ \text{kg-m}^2$$

(ii) Angular momentum, L = Iω = 2 πnI

$$\therefore\quad \mathrm{L}=2\pi\times\frac{1500}{60}\times 0\cdot 125\left[n=\frac{1500}{60}\,rps\right]$$

$$= \mathbf{19{\cdot}625\ kg\ m^2\ /\ s}$$

$$\mathrm{M}=20\ \text{kg}$$

$$r=\frac{0\cdot 25}{2}\,m$$

✪ 2.9. Relation between torque and angular momentum ✪

In the case of linear motion, rate of change of linear momentum is the force acting on the body. The quantity analogous to force is the torque for rotational motion. So, we can say, torque is the rate of change of angular momentum.

$$\text{Torque, } \tau=\frac{\text{Change of angular momentum}}{\text{time}}\qquad \therefore\quad \tau=\frac{d\mathrm{L}}{dt}=\frac{d}{dt}(\mathrm{I}\omega)=\mathrm{I}\frac{d\omega}{dt}$$

$$\therefore\quad \tau=\mathrm{I}\alpha=\text{ moment of inertia}\times\text{angular acceleration.}$$

- **Proof :** From Newton's second law of motion $\vec{\mathrm{F}}=m\,\vec{a}$

$$\therefore\quad \vec{\mathrm{F}}=m\frac{d\vec{\mathrm{V}}}{dt}=\frac{d}{dt}\left(m\,\vec{\mathrm{V}}\right)\quad[\because m=\text{constant}]$$

$$\therefore\quad \vec{\mathrm{F}}=\frac{d\,\vec{\mathrm{P}}}{dt}\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ \text{(i)}$$

Where $\vec{\mathrm{P}}$ is the linear momentum of the particle. Taking vector product of $\vec{r}$ with both sides of equation (i), we get $\vec{r}\times\vec{\mathrm{F}}=\vec{r}\times\frac{d\,\vec{\mathrm{P}}}{dt}$.

$$\text{or,}\quad \vec{\tau}=\vec{r}\times\frac{d\,\vec{\mathrm{P}}}{dt}\ \left[\because \vec{\tau}=\vec{r}\times\vec{\mathrm{F}}\right]\ \ldots\ldots\ldots\ldots\ldots\ldots\ \text{(ii)}$$

Again, angular momentum $\vec{\mathrm{L}}=\vec{r}\times\vec{\mathrm{P}}$

$$\therefore\quad \frac{d\,\vec{\mathrm{L}}}{dt}=\frac{d}{dt}\left(\vec{r}\times\vec{\mathrm{P}}\right)=\frac{d\,\vec{r}}{dt}\times\vec{\mathrm{P}}+\vec{r}\times\frac{d\,\vec{\mathrm{P}}}{dt}$$

$$=\vec{\mathrm{V}}\times\vec{\mathrm{P}}+\vec{r}\times\frac{d\,\vec{\mathrm{P}}}{dt}=m\left(\vec{\mathrm{V}}\times\vec{\mathrm{V}}\right)+\vec{r}\times\frac{d\,\vec{\mathrm{P}}}{dt}$$

$$=\vec{r}\times\frac{d\,\vec{\mathrm{P}}}{dt}\ \left[\because\ \vec{\mathrm{V}}\times\vec{\mathrm{V}}=0\right]\ \ldots\ldots\ldots\ldots\ldots\ \text{(iii)}$$

$$\text{Here from (ii) and (iii),}\quad \frac{d\,\vec{\mathrm{L}}}{dt}=\vec{\tau}\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (2.17)$$

So, torque is rate of change of angular momentum.

The equation of linear motion that corresponds to equation (2.17) is $\frac{d\vec{P}}{dt} = \vec{F}$.

So equation (2.17) is the rotational analogue to Newton's Second law of linear motion.

- **Physical meaning of angular momentum :** The angular momentum $\vec{L} = \vec{r} \times \vec{P}$

$\therefore$ $L = rP \sin\theta = P\,(r \sin\theta)$

$\therefore$ angular momentum, L = linear momentum × moment arm.

So, angular momentum is moment of linear momentum. The physical meaning of angular momentum is that it is a measure of turning motion of an object.

✪ 2.10. Law of conservation of Angular momentum ✪

This law states that if no external torque acts on a system, the total angular momentum of the system remains conserved. This law of conservation of angular momentum in rotational motion is as important as the law of conservation of linear momentum in translational motion.

- **Deduction :** We know that the external torque acting on a system about an axis is equal to the time rate of change of angular momentum of the system about the same axis.

So $\tau = \frac{dL}{dt}$ If no external torque acts $\tau = 0$, So, $\frac{dL}{dt} = 0$

$\therefore$ L = Constant or, **$I\omega$ = Constant.** or, $I_1\omega_1 = I_2\omega_2$

It is the mathematical form of the law of conservation of angular momentum.

➡ Application of the law of conservation of angular momentum :

This conservation law has many applications in our practical life. A few examples are given below.

(i) **A diver jumping from the spring board sometimes exhibits sommersaults in air before touching the water surface.**

As the diver jumps from the board, he keeps his legs and arms outstretched. After leaving the spring board, he curls his body by rolling the arms and the legs inwards. Due to this, his moment of inertia decreases and he begins to spin in mid air with larger angular speed. As he is about to touch the water surface he stretches out his limbs. This increases the moment of inertia again and the diver enters water at a gentle speed.

(ii) A person is standing on a turn table holding a pair of heavy dumb bells, one in each hand with his arms outstretched [Fig 2.10]. The table is rotating with a certain angular frequency. The person suddenly draws the weights towards his chest. The speed of rotation is found to increase considerably.

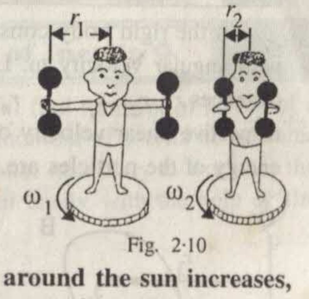


Fig. 2·10

(iii) Dancers on skates can spin faster by folding their arms.

(iv) **The angular velocity of a planet revolving in an elliptical orbit around the sun increases, when it comes nearer the sun and vice versa.** When the planet moving along an elliptical orbit is near the sun, its moment of inertia about the axis through the sun decreases and so its angular speed increases. On the other hand, when it is far away from the sun, the moment of inertia about the same axis increases and so its angular speed decreases.

- **Example 2.5. What would be the duration of the day, if the earth suddenly shrinks to $\frac{1}{64}$ of its initial volume, mass remaining constant ?**

○ **Solution :** According to the principle of conservation of linear momentum $I_1\omega_1 = I_2\omega_2$

$$\therefore \quad \frac{2}{5}\mathrm{M}r_1^{\,2}.\frac{2\pi}{\mathrm{T}_1}=\frac{2}{5}\mathrm{M}r_1^{\,2}\times\frac{2\pi}{\mathrm{T}_2} \quad \therefore \quad \frac{\mathrm{T}_1}{\mathrm{T}_2}=\left(\frac{r_1}{r_2}\right)^2 \quad \ldots\ldots\ldots\ldots \text{(i)}$$

$$\text{Now,} \quad \frac{4}{3}\pi r_1^{\,3}=64\times\frac{4}{3}\pi r_2^{\,3} \quad \therefore \quad \frac{r_1}{r_2}=4 \quad \therefore \quad \mathrm{T}_2=\mathrm{T}_1\cdot\left(\frac{r_2}{r_1}\right)^2=24\times\left(\frac{1}{4}\right)^2=\mathbf{1{\cdot}5\ hrs}$$

∴ The duration of the day will be **1·5 hrs.**

- **Ex. 2.6. The angular momentum of rotation of a body is 75·36 J.s. and its rate of rotation is 24 rev. per sec. Calculate its moment of inertia.**

○ **Solution :** $\mathrm{L}=\mathrm{I}\omega \qquad \mathrm{L}=75{\cdot}36\ \mathrm{J}.s. \qquad \omega=2\pi n=2\pi\times24=48\pi\ \mathrm{rad/s}$

$$\therefore \quad \mathrm{I}=\frac{75{\cdot}36}{48\times3{\cdot}14}=0{\cdot}5\ \mathrm{kg\text{-}m^2}$$

2.11. Power produced by a torque and work done

Let a rigid body is rotating about an axis. Due to a torque acting on it an angular acceleration is produced in the body and its kinetic energy gradually increases. Power produced by the torque is equal to the rate of doing work or equal to the rate of increase of kinetic energy.

$$\therefore \quad \text{Power,} \quad \mathrm{P}=\frac{d\mathrm{W}}{dt}=\frac{d\mathrm{E}}{dt}$$

Here W = work done and E = increase of kinetic energy.

$$\therefore \quad \mathrm{P}=\frac{d}{dt}\left(\frac{1}{2}\mathrm{I}\omega^2\right)=\mathrm{I}\omega\,\frac{d\omega}{dt}=\mathrm{I}\alpha\omega=\tau\omega$$

∴ Power = torque × angular velocity.

Now, the work done for small angular displacement $d\mathrm{W}=\tau\,d\theta$

$$\therefore \quad \text{Total work done,} \quad \mathrm{W}=\int_0^{\theta}\tau.d\theta$$

2.12. Kinetic energy of rotation of a rigid body

Let the rigid body consists of n number of particles . It is rotating about the axis PQ as in Fig. 2.11 with angular velocity ω. Let the masses of the particles be $m_1, m_2, \ldots\ldots\ldots m_n$ and are situated at distances $r_1, r_2, \ldots\ldots\ r_n$ from the axis of rotation. Now the angular velocity of each particle is ω. The respective linear velocity of the particles is $v_1=r_1\omega$, $v_2=r_2\omega$ and $\ldots\ldots v_n=r_n\omega$. Hence the Kinetic energy of the particles are,

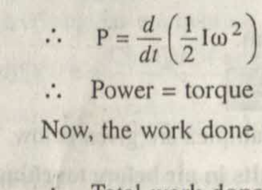


Fig. 2·11

$$\mathrm{E}_1=\frac{1}{2}m_1v_1^{\,2}=\frac{1}{2}m_1r_1^{\,2}\omega^2,\ \mathrm{E}_2=\frac{1}{2}m_2r_2^{\,2}\omega^2 \quad \text{and so on.}$$

So, the total kinetic energy of the rotating body is

$$\mathrm{E}_k=\frac{1}{2}m_1r_1\omega^2+\frac{1}{2}m_2r_2\omega^2+\ldots\ldots+\frac{1}{2}m_nr_n\omega^2$$

$$=\frac{1}{2}\omega^2\left(m_1r_1^{\,2}+m_2r_2^{\,2}+\ldots\ldots+m_nr_n^{\,2}\right)$$

$$=\frac{1}{2}\omega^2\sum_{i=1}^{n}m_ir_i^{\,2}=\frac{1}{2}\mathrm{I}\,\omega^2 \quad \text{when} \quad \mathrm{I}=\sum_{i=1}^{n}m_ir_i^{\,2}$$

$$\therefore \quad \mathrm{E}_k=\frac{1}{2}\,\mathrm{I}\omega^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(2.18)}$$

➤ 2.12.1. *Angular Impulse (Rotational Impulse) :*

We know, measurement of torque, $\tau = I\alpha = I\frac{d\omega}{dt}$ $\therefore$ $I\,d\omega = \tau\,dt$

Here I is the moment of inertia and torque acts for an infinitasional time *dt* producing *dω* change in angular velocity.

Integrating we get, $I\int_{\omega_1}^{\omega_2} d\omega = \int_0^t \tau\,dt$ or, $I(\omega_2 - \omega_1) = \int_0^t \tau\,dt$ or, $\Delta L = \int_0^t \tau\,dt$

$\therefore$ Change in angular momentum = Rotational impulse

● **Example 2.7. A solid cylinder of mass 20 kg rotates about its axis with an angular speed 100 rad /sec. The radius of the cylinder is 0.25 m. What is the rotational kinetic energy of the cylinder ? What is the magnitude of angular momentum of the cylinder ?**

○ **Solution :** (i) Kinetic energy of the rotating cylinder $= \frac{1}{2} I\omega^2$

$\therefore$ $E_k = \frac{1}{2} \times \frac{5}{8} \times (100)^2 = 3125\,\text{J}$

$I = \frac{1}{2} Mr^2 = \frac{1}{2} \times 20 \times (0{\cdot}25)^2 = \frac{5}{8}\,\text{kg m}^2$; $\omega = 100\,\text{rad/s}$

(ii) Angular momemtum of the cylinder is $L = I\omega = \frac{5}{8} \times 100 = \mathbf{62{\cdot}5\ kg\ m^2 s^{-1}}$

● **Ex. 2.8. Four small bodies P, Q, R and S which can be considered as particles are connected by rods of negligible masses as shown in fig 2.12. Find the moment of inertia of the system (i) about an axis coinciding with rod QR and (ii) about an axis passing through P and perpendicular to the plane of the diagram.**

○ **Solution :** Mass of each the small bodies placed at each corner is 30 gm.

(i) M. I. about the axis coinciding with QR

$I = 30 \times (PQ)^2 + 30 \times (RS)^2 = 30 \times 25 + 30 \times 25 = \mathbf{1500\ g\text{-}cm^2}$

(ii) M. I. about the axis passing through P and perpendicular to the plane of the diagram is

$$I = 30\,(PQ)^2 + 30\,(PR)^2 + 30 \times (PS)^2$$
$$= 30 \times 25 + 30 \times \left(5\sqrt{2}\right)^2 + 30 \times (25)$$
$$= 750 + 1500 + 750 = \mathbf{300\ g\text{-}cm^2}.$$

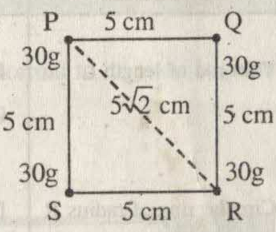


Fig. 2·12

✪ 2.13. Two important theorems on moment of Inertia ✪

In case of rigid bodies having regular shapes, it is always convenient to find moment of inertia of a body about the axis through the centre of mass. Sometimes we require moment of inertia about some other axis. Now, to find the moment of inertia about some other axis it is not necessary to work out the problem from the basic principles because the same could be determined easily with the help **of the theorems of parallel and perpendicular axis.**

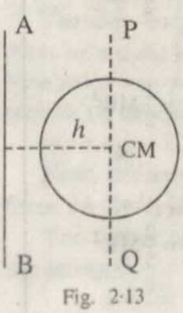


Fig. 2·13

(a) Theorem of parallel axis :

It states that the moment of inertia of a rigid body about any axis is equal to the moment of inertia about a parallel axis through its centre of mass plus the product of the mass of the body and the square of the perpendicular distance between the axes.

Let I be the moment of inertia of a body of mass M about any axis AB. Also I_C be the moment of inertia of the body about a parallel axis PQ which passes through the centre of mass (CM) of the body. If *h* be the distance between the two axes [Fig 2.13].

Then, according to the theorem of parallel axes, $I = I_C + Mh^2$ . . . . (2.19)

This relation shows that the moment of inertia about any parallel axis is always greater than the moment of inertia about a parallel axis passing through the centre of mass.

(b) Theorem of perpendicular axis :

It states that the moment of inertia of a laminar body about an axis perpendicular to its plane is equal to the sum of the moments of inertia of the lamina about any two mutually perpendicular axes in its plane and intersecting each other at the point where the perpendicular axis passes through it.

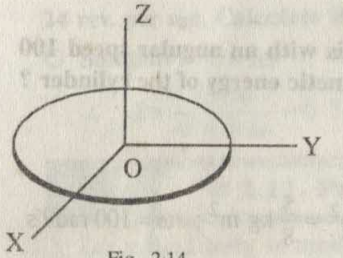


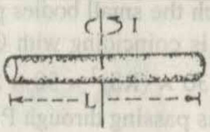
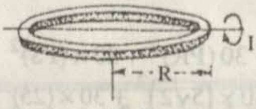
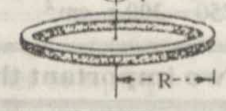
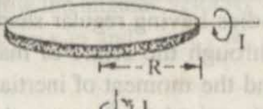
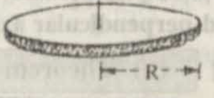
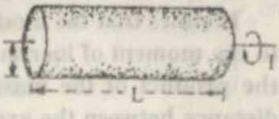
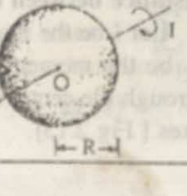
Fig. 2·14

Let the axis OZ is perpendicular to the plane lamina and passes through O [Fig 2.14]. OX and OY are two mutually perpendicular axis in the plane of the lamina and intersecting at O. If I_x, I_y and I_z be the moments of inertia of the plane lamina about the axes OX, OY and OZ respectively, then according to the theorem of perpendicular axis

$$I_z = I_x + I_y \quad \ldots\ldots (2.20)$$

Moments of Inertia of a few rigid bodies :

The following table gives moment of inertia of some rigid bodies of regular steps about a particular axis.

Body	Axis	Figure	Moment of Inertia
Thin rod of length L	Passing through centre and perpendicular to the rod.		$I = \frac{1}{12}ML^2$
Circular ring of radius R	Diameter		$I = \frac{1}{2}MR^2$
Circular ring of radius R	Passing through its centre and perpendicular to its plane		$I = MR^2$
Circular disc of radius R	Diameter		$I = \frac{1}{4}MR^2$
Circular disc of radius R	Passing through its centre and perpendicular to its plane.		$I = \frac{1}{2}MR^2$
Right circular solid cylinder of radius R and length L	Symmetry axis		$I = \frac{1}{2}MR^2$
Solid sphere of radius R	Diameter		$I = \frac{2}{5}MR^2$

● **Example 2.9. The moment of inertia of a solid sphere of mass M and radius R about an axis through its centre is $\frac{2}{5}MR^2$. What is the moment of inertia about an axis tangential to the surface of the sphere ?**

○ **Solution :** Here $I_C = \frac{2}{5}MR^2$. By the theorem of parallel axis, the moment of inertia about an axis tangential to the sphere is $I = I_C + MR^2 = \frac{2}{5}MR^2 + MR^2 = \frac{7}{5}MR^2$

● **Ex. 2.10. A disc of mass 2 kg has radius of 0·5 m. What is the moment of inertia (i) about an axis passing through the centre of mass and perpendicular to its plane, (ii) about an axis passing through its edge and perpendicular to its plane.**

○ **Solution :** (i) $I_C = \frac{1}{2}MR^2 = \frac{1}{2} \times 2 \times (0{\cdot}5)^2 = \mathbf{0{\cdot}25\ kg\text{-}m^2}$

(ii) $I = I_C + MR^2 = 0{\cdot}25 + 2 \times (0{\cdot}5)^2 = \mathbf{0{\cdot}75\ kg\text{-}m^2}$

● **Ex. 2.11. The moment of inertia of a uniform circular disc of mass M and radius R about any of its diameter is $\frac{1}{4}MR^2$. What is the moment of inertia of the disc about an axis passing through its centre and normal to the disc ?**

○ **Solution :** The moment of inertia of the disc about any diameter is $\frac{1}{4}MR^2$. We consider two perpendicular diameters, one along the X-axis and the other along the Y-axis.

$$\therefore \quad I_x = I_y = \frac{1}{4}MR^2$$

Then the moment of inertia about the axes passing through the centre and normal to its plane is given by $I_c = I_x + I_y$ [by perpendicular axis theorem]

$$= \frac{1}{4}MR^2 + \frac{1}{4}MR^2 = \frac{1}{2}MR^2$$

✪ 2.14. Examples of two dimensional rigid body motion ✪

We shall consider two examples of two dimensional motion of a rigid body.

I. Motion of a point mass tied to a string wound on a cylinder :

It is an example of the rigid body motion where the centre of mass of the body remains at rest. We take a cylinder of mass M and radius R which can rotate freely about its axis. A very light string is wound on the cylinder and a small mass *m* is suspended at the lower end of the string [Fig 2.15] As the small mass begins to fall the cylinder will start unwinding. As the mass *m* falls, the cylinder rotates with a certain angular acceleration α and the mass falls with a linear acceleration '*a*'.

The free body diagrams of the cylinder and the mass *m* are shown in fig (ii) and (iii). According to Newton's second law of motion, the equation of motion of the mass is

$ma = mg - T$ [T = tension of the string] . . . . . (i)

Now, the tension of the string exerts a tangential force on the cylinder and makes it to rotate.

The torque on the cylinder due to the tension of the string is

$\tau = TR$ . . . . (ii)

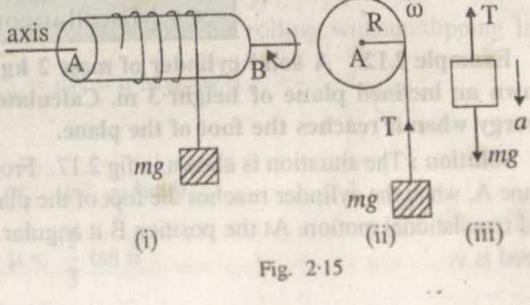


Fig. 2·15

Also $\tau = I\alpha$, when I is the moment of inertia of the cylinder about the axis of rotation.

$$\therefore \quad I\alpha = TR \text{ or, } T = \frac{I\alpha}{R} = \frac{Ia}{R^2} \quad \therefore \; ma = mg - \frac{Ia}{R^2} \qquad [\,\alpha = \text{angular acceleration}\,]$$

$$\therefore \quad a\left[m + \frac{I}{R^2}\right] = mg \text{ or, } a = \frac{mg}{m + \frac{I}{R^2}} = \frac{mgR^2}{mR^2 + 1}$$

So, the acceleration of the point mass is less than the acceleration due to gravity. The tension T on the string is

$$T = \frac{Ia}{R^2} = \frac{I}{R^2} \times \frac{mgR^2}{mR^2 + I} = \frac{mg}{1 + \frac{mR^2}{I}} \quad \therefore \; T = \frac{mg}{1 + \frac{2m}{M}} \quad \left[I = \frac{1}{2}MR^2\right]$$

It shows that tension in the string is less than *mg*.

Lastly the angular acceleration of the cylinder is $\alpha = \frac{TR}{I} = \frac{mg}{1 + \frac{2m}{M}} \times \frac{R}{\frac{1}{2}MR^2} = \frac{mg}{mR + \frac{1}{2}MR}$

It is to be noted that this analysis is also applicable to a mass tied to a string wound on a wheel or heavy pulley.

II. Cylinder rolling on an inclined plane without slipping :

It is an example where a rigid body is in motion and the centre of mass also moves with an acceleration.

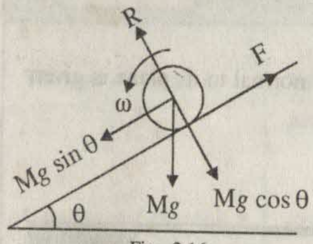


Fig. 2·16

Here a cylinder of mass M and radius R is rolling down an inclined plane of inclination θ [fig 2.16]. The different forces acting on the cylinder is shown in the fig. Mg is the weight, F is the frictional force and R is the normal reaction. If '*a*' be the linear acceleration of the cylinder then, by Newton's second law of motion,

$$Ma = Mg\sin\theta - F \ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

Also the cylinder rolling down the plane rotates about its axis of symmetry. The torque on the cylinder, $\tau = I\alpha \ldots\ldots$ (ii)

When I is the moment of inertia and α is the angular acceleration about the axis of rotation.

Also $\tau = FR \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$ (iii)

$$\therefore \quad FR = I\alpha \text{ or, } F = \frac{I\alpha}{R} = \frac{Ia}{R^2} \ldots\ldots\ldots \text{(iv)}$$

$$\therefore \quad \text{From (i), } Ma = Mg\sin\theta - \frac{Ia}{R^2} \quad \therefore \; a = \frac{\mathbf{Mg\sin\theta}}{\mathbf{M} + \frac{\mathbf{I}}{\mathbf{R^2}}} \quad \text{But } I = \frac{1}{2}MR^2 \; \therefore \quad a = \frac{\mathbf{2}}{\mathbf{3}}\mathbf{g\sin\theta}$$

$$\therefore \quad \text{From (iv), } F = \frac{1}{2}MR^2 \cdot \frac{1}{R^2} \times \frac{2}{3}g\sin\theta = \frac{\mathbf{1}}{\mathbf{3}}\mathbf{Mg\sin\theta}$$

• • Miscellaneous Examples • •

● **Example 2.12. A solid cylinder of mass 2 kg and radius 0.1 m rolls down an inclined plane of height 3 m. Calculate its rotational kinetic energy when it reaches the foot of the plane.**

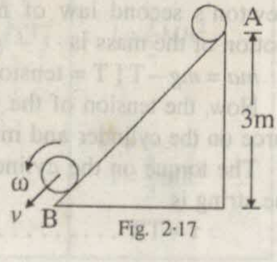


Fig. 2·17

○ **Solution :** The situation is shown in fig 2.17. From the top of the inclined plane A, when the cylinder reaches the foot of the plane, it has both rotational and translational motion. At the position B it angular velocity is ω and linear speed is *v*.

Hence from the conservation of energy,

$$Mgh = \frac{1}{2} I\omega^2 + \frac{1}{2} Mv^2 = \frac{1}{2} \times \frac{1}{2} MR^2\omega^2 + \frac{1}{2} MR^2\omega^2 = \frac{3}{4} MR^2\omega^2 \qquad \therefore \quad \omega^2 = \frac{4gh}{3R^2}$$

$\therefore$ The rotational kinetic energy at the bottom is

$$E_r = \frac{1}{2} I\omega^2 = \frac{1}{2} \times \frac{1}{2} MR^2 \times \frac{4gh}{3R^2} = \frac{1}{3} Mgh = \frac{1}{3} \times 2 \times 9{\cdot}8 \times 3 = \mathbf{19{\cdot}6\ J}$$

● **Ex. 2.13. A solid cylinder of mass M and radius R rolls down the inclined plane from height *h* without slipping. What is the speed of the centre of mass when it reaches the bottom ?**

○ **Solution :** By the law of conservation of energy : $Mgh = \frac{1}{2} MV_{cm}^2 \left(1 + \frac{K^2}{R^2}\right)$

Now, for a cylinder, $\frac{K^2}{R^2} = \frac{1}{2}$ $\therefore V_{cm} = \sqrt{\frac{4}{3} gh}$ It is the required velocity.

● **Ex. 2.14. A cylinder of mass 10 kg is rolling perfectly on inclined plane of inclination 30º. Find the force of friction between the cylinder and the inclined plane.**

○ **Solution :** The required relation, $F = \frac{1}{3} mg \sin\theta \quad \therefore \quad F = \frac{1}{3} \times 10 \times 9{\cdot}8 \times \sin 30^\circ = \mathbf{16{\cdot}3\ N}$

● **Ex. 2.15. A string is wound on a wheel of diameter 0·3 m. The axis of the wheel is horizontal. A block of mass 0·5 kg is attached at the lower end of the string and is allowed to fall from rest. If the block falls 1·5 m in 4 sec. What is the angular acceleration of the wheel ? Also calculate moment of inertia of the wheel.**

○ **Solution : 1st part :** We first calculate the linear acceleration of the falling mass. Its initial velocity $u = 0$ and it has fallen 1·5 m in 4 sec.

Now, $s = ut + \frac{1}{2} at^2 \quad \therefore \quad a = \frac{2s}{t^2} = \frac{2 \times 1{\cdot}5}{(4)^2} = \mathbf{0{\cdot}1875\ m/s^2}$

So, the angular acceleration of the wheel, $a = R\alpha$

$$\therefore \quad \alpha = \frac{a}{R} = \frac{0{\cdot}1875}{0{\cdot}15} \qquad \left[R = \frac{0{\cdot}3}{2} = 0{\cdot}15m\right]$$

$$= 1{\cdot}25 \text{ rad s}^{-2}$$

$\therefore$ The required angular acceleration of the wheel $\alpha = \mathbf{1{\cdot}25\ rad\ s^{-2}}$

2nd part : Now torque on the wheel, $\tau = MgR$; also $\tau = I\alpha \quad \therefore \quad I\alpha = MgR$

$$\therefore \quad I = \frac{MgR}{\alpha} = \frac{0{\cdot}5 \times 9{\cdot}8 \times 0{\cdot}15}{1{\cdot}25} = \mathbf{0{\cdot}588\ kg\text{-}m^2}$$

● **Ex. 2.16. Show that a cylinder will slip on an inclined plane of inclination θ if the coefficient of static friction between the plane and the cylinder is less than $\frac{1}{3}\tan\theta$.**

○ **Solution :** We know that the value of the frictional force needed for rolling without slipping is $F = \frac{1}{3} Mg\sin\theta$. Now, Maximum frictional force $F_{max} = \mu N = \mu\, Mg\cos\theta$

So rolling will take place when $F \le F_{max}$

$$\therefore \quad \frac{1}{3} Mg\sin\theta \le \mu\, Mg\cos\theta \quad \therefore \quad \frac{1}{3}\tan\theta \le \mu \quad \text{or,} \quad \mu \ge \frac{1}{3}\tan\theta$$

So, the cylinder will slip on the inclined plane if $\mu < \frac{1}{3}\tan\theta$

● **Ex. 2.17. You are given two cylinders of the same size, mass and appearance. One of the cylinders is hollow and the other is solid. How will you distinguish between the two ?**

○ **Solution :** Linear acceleration of a cylinder rolling down an inclined plane is given by

$$a=\frac{g\sin\theta}{1+\dfrac{I}{MR^2}}$$ where the terms have usual meaning.

Let for the solid cylinder a_s is the linear acceleration down the place then

$$a_s=\frac{g\sin\theta}{1+\dfrac{1}{2}\dfrac{MR^2}{MR^2}}=\frac{2}{3}\,g.\sin\theta\ \ldots\ldots\ldots\ldots\ldots\ (i)$$

Now for the hollow cylinder a_h is the linear acceleration and $I=MR^2$

$$\therefore\quad a_h=\frac{1}{2}\,g\sin\theta\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (ii)$$

Evidently, $a_s>a_h$. So the solid cylinder will reach the bottom first.

• E ◇ X ◇ E ◇ R ◇ C ◇ I ◇ S ◇ E •

◢ A. Short answer type questions :

1. What is the rotational analogue of mass in the case of linear motion ?
2. The translational motion is along a fixed direction. What remains constant in rotational motion ? **[J. E. E. 1991]**
3. Define centre of mass. Is it a fixed point ?
4. Is centre of mass a reality ?
5. Does the centre of mass of a body always lie inside the body ?
6. What is the centre of mass of a triangular lamina ?
7. Give the location of the centre of mass of a sphere, cylinder, ring and cube each of uniform mass density.
8. Write down the expression for the position vector of the centre of mass of a system consisting of two particles in terms of their masses and position vectors.
9. Torque and work are both defined as force times distance. Explain how do they differs.
10. There is a stick half wood and half steel. It is pivoted at the wooden end and a force is applied at right angles to its length at the steel end. Next, it is pivoted at the steel end and the same force is applied at the wooden end. Will the angular acceleration be same at the two cases ?

    [**Hint :** $\tau=I\alpha$; so $\alpha=\dfrac{\tau}{I}$, Here τ is same for both cases, but $I_1>I_2$, hence $\alpha_1<\alpha_2$]
11. A solid and a hollow sphere have the same radius and identical mass. Which has larger moment of inertia ?

    [**Ans.** Hollow sphere]
12. You are given a sphere and an inclined plane. Explain how you can determine whether or not the sphere is hollow ?
13. Explain why the moment of inertia is different for different axis ?
14. If the ice at the poles of the earth were to melt, how would this affect the length of the day ?

    [**Hints :** As the ice melts, water shifts from poles to equator and the moment of inertia decreases. But angular momentum remain constant. So time of rotation increases.]
15. For a description of translational motion, all the mass of the body may be assumed to be concentrated at the centre of mass. Give a reason why the same statement cannot hold with regard to rotational motion.

◢ B. Essay type questions :

1. (a) Define moment of inertia and explain in its physical significance.

   (b) Obtain relation between torque applied to a rigid body and angular acceleration produced .
2. Show that the work done by a torque τ in rotating a body through an angle θ is given by $\tau\ \theta$.

3. Define angular momentum. Show that the torque applied equals to the rate of change of angular momentum.
4. Prove that the Kinetic energy of a body rotating about an axis is $\frac{1}{2}MK^2\omega^2$ where the symbols have usual meaning.
5. Define centre of mass. Show that the total linear momentum of a system of *n* particles is equal to product of total mass of the system and the velocity of centre of mass.
6. (a) What is the relation between linear momentum and angular momentum ?
   (b) Give the physical meaning of angular momentum.
   (c) State and explain the principal of conservation of linear momentum.

◢ C. Simple numerical problems :

1. A body of mass 50 kg is rotated along a circle of radius 100 cm. If its speed of rotation be 5 rpm, calculate its angular momentum. [**Ans.** 26·17 kg m^2 / sec]
2. A force of 100 N is applied tangentially to the rim of a wheel of diameter 1·0 m. If the moment of inertia of the wheel is 100 kg-m^2, calculate : (i) angular acceleration (ii) the angular speed, 5*s* from rest, (iii) number of revolutions made in 5*s*, (iv) kinetic energy of the wheel. [**Ans.** 0·5 rad s^{-2} (ii) 2·5 rad s^{-1} (iii) 4 (iv) 312·5 J]
3. The radius of a disc is 20 cm and its moment of inertia about the axis passing through its centre is 0.1 kg-m^2. It is allowed to roll down the plane of inclination 30° from rest. What will be its angular velocity when it moves 3 m down the plane? Mass of the disc is 5 kg. [**Ans.** 22·13 rad /s]
4. The angular velocity of a fly wheel decreases from 600 rpm to 300 rpm in 5s. Calculate (i) angular acceleration, (ii) number of revolution completed in 5 sec, (iii) the time after which the wheel stops.
   If the moment of inertia of the wheel the 0·5 kg-m^2, calculate torque acting on it.
   [**Ans.** (i) – 6·28 rod / s^2 (ii) 37·5 rev, (iii) 10 sec; 3·14 N-m]
5. A disc of mass 5 kg has a radius of gyration of 20 cm. Find its kinetic energy of rotation when it is rotating at 3 rps. [**Hints :** $E=\frac{1}{2}MK^2\omega^2$] [**Ans.** 35·5 J]
6. A sphere of mass 25 kg and radius 0·2 m is rotating about an axis passing through its centre at a speed of 2100 rpm. Calculate (i) moment of inertia, (ii) Angular momentum[**Ans :** (i) $I=\frac{2}{5}MR^2 = 0{\cdot}4$ kg. m^2 (ii) 88 kg m^2 / sec]
7. Initial angular velocity of a disc is 78 rpm. It is stopped in 3 sec. What is the angular acceleration of the disc and number of revolutions it will complete before it stops. [**Ans.** – 0·27 rad / s ^{2}; 19·5 rev.]
8. What is the rotational kinetic energy of the earth ? Assume mass of the earth 6·0 × 10^{24} kg and its radius = 6·4 × 10^3 km. Earth is a uniform sphere. [**Ans.** 2.6 × 10^{29} J]
9. A sharpening wheel of stone starts revolving with an acceleration of 3.0 rad s^{-2}. Calculate angular displacement and angular speed of the wheel after 2 sec. [**Ans.** 6 rad ; 6·0 rad s^{-1}]
10. The radius of wheel is 1.5 m. Its constant angular acceleration is 10.0 rad / s^2 and its initial angular velocity is $\frac{66}{\pi}$ rpm. Calculate angular velocity, angular displacement after 2 sec. What is the linear velocity and linear acceleration of a point at the rim of the wheel ? [**Ans.** 22·2 rad / s, 24·4 rad; 33·3 ms^{-1} and 15 ms^{-2}]
11. A force of 100 N is applied at the edge of a wheel of radius 0.5 m and moment of inertia 100 kg-m^2. Calculate; (i) angular acceleration (ii) angular velocity after 5 sec (iii) Number of revolution after 5 sec. (iv) rotational Kinetic energy. [**Ans.** (i) 0·5 rad / s^2 (ii) 2·5 rad / s (iii) 4 (iv) 312·5 J]
12. When 100 J of work is done on a flywheel, its angular speed increases from 60 rpm to 180 rpm. Calculate its moment of inertia. [**Ans.** 0·633 kg-m^2]
13. The constant angular velocity of the motor of an engine is 200 rad / s. If a torque of 180 N-m is applied on the wheel, calculate power of the engine. [**Ans.** $P=\tau\omega=36\times10^3$ watts]
14. A circular disc of mass m and radius *r* is set rolling on a table. If ω be the angular velocity, show that its total energy is given by $\frac{3}{4}mr^2\omega^2$.
15. The position vectors of two particles of mass 100 g and 300g are $(2\hat{i}+5\hat{j}+10\hat{k})$ and $(-6\hat{i}+4\hat{j}-2\hat{k})$. Calculate its centre of mass. [**Ans.** $\left(-4\hat{i}+\frac{17}{4}\hat{j}+\frac{7}{4}\hat{k}\right)$]

16. 484 J of energy is spent to increase the speed of a fly wheel from 60 rpm to 360 rpm. Calculate the moment of inertia of the wheel. [**Hints :** $\frac{1}{2} I \left(w_2^{\,2} - w_1^{\,2} \right) = 4821 \text{ J}$] [**Ans.** 0·7 kg-m^2]

17. A circular ring of radius 20 cm and mass 1 kg is rotating about an axis passing through its centre and perpendicular to its plane at the rate of 10 rev per sec. Calculate angular momentum of the wheel. [**Ans.** L = 0·8 π kg m^2 s^{-1}]

18. Mass remaining constant if the volume of the earth decreases to $\frac{1}{64}$ part of its initial volume. Calculate number of hours in a day. [**Ans.** 1·5 hr]

19. The mass and radius of a cylinder are 3 kg and 40 cm respectively. It is pulled by a string wound over it by a force of 30N. What is the angular acceleration of the cylinder ? Also calculate linear acceleration of the string. [**Hints :** torque, $\tau = FR = I\alpha$] [**Ans.** α = 25 rad / s^2 ; a = 10 ms $^{-1}$]

20. The mass of a flywheel of radius 0·2 m is 25 kg. Its speed of revolution is 240 rpm. How much torque is required to stop it is 20 sec ? If the torque is produced by a tangential force applied at the rim of the wheel, calculate the force that has to be applied. [**Ans.** (i) $\tau = 0 \cdot 2\pi$ N-m (ii) 3·14 N]

21. A circular ring and a circular disc has same mass and radius. They are rotated about an axis passing through its centre. Calculate the ratio of their radii of gyration.

22. A solid sphere is moving over a horizontal surface with a speed of 20 ms^{-1}. If the sphere is now allowed to move up an inclined plane of inclination 30°, to what maximum height the sphere will rise ? [**Ans.** 28·6 m]

23. A flywheel of mass 500 kg and radius 1 m is rotating at the rate of 500 rpm. If the whole mass of the flywheel be at its centre, calculate (i) its angular velocity (ii) moment of inertia (iii) rotational kinetic energy [**Ans.** (i) 52·36 rad/s, (ii) 500 kg-m^2 (iii) 68·54 × 10 ^{4} J]

24. A circular disc of moment of inertia is 10^{-3} kg-m^2 is rotating about an axis passing through its centre at the rate of 40 rev. per min. Now a piece of wax of mass 20 g is attached with the disc at a distance of 10 cm from its axis. What will be its new angular velocity ? [**Hints :** $I\omega_0 = I\omega_1 + mr_1^{\,2}\omega_1$] [**Ans.** 34 rev / min]

25. A sphere of mass 0·1 kg and radius 2·5 cm is rolling on a plane surface, without sliding, with a linear speed of 10 ms^{-1}. Find its (i) rotational energy, (ii) total energy. [**Ans.** (i) 2 J (ii) 7 J]

26. A grind stone is in the form of a circular disc of diameter 0.4 m and mass 5 kg. If a constant torque $\frac{\pi}{4}$ N–m is applied for 4 sec, what is the angular velocity of the grind stone if it starts from rest ? At what rate is work done by the torque at the end of 4 sec ? [M. I = $\frac{1}{2}$ MR^2] [**Ans.** 10 π rad / s; 2.5 π^2 watt]

27. A flywheel of moment of inertia 0·1 kg-m^2. What constant torque is required to increase its speed from 3 rps to 9 rps after 18 revolutions ? [**Ans.** 1·256 N–m]

28. A cord is wound round the circumference of a wheel of diameter 0·3 m. The axis of the wheel is horizontal. A mass of 0·5 kg is attached at the end of the cord and it is allowed to fall from rest. If the weight falls 1·5 m in 4 sec, what is the angular acceleration of the wheel ? Also find the moment of inertia of the wheel. [**Ans.** 1·25 rad / s^2; I = 0·588 kg-m^2]

◢ D. Harder numerical problems :

1. A rope is wound on a circular disc of moment of inertia 0·1 kg-m^2. The axis of the wheel is horizontal. A tangential force of 20 N on the rope rotates the wheel about its axis. If the radius of the wheel be 0·1 m, calculate its acceleration. Now a mass of 2 kg is suspended at the lower end of the rope. What will be the angular acceleration ? [**Ans.** 20 rad / s^2; 16.7 rad / s^2]

2. The moment of inertia of a wheel of radius 0·2 m about an axis passing through its centre is 0·1 kg m^2, it is now allowed to roll down an inclined plane of inclination 30°. What will be its angular velocity after it moves down by 2 m along the plane ? Given mass of the disc = 5 kg. [**Ans.** 18·07 rad / s]

3. The moment of inertia of a disc is 5 × 10^{-4} kg-m^2. It is rotating about a vertical axis passing through its centre at a speed of 40 rev per min. A piece of wax of mass 20 gm is attached at a distance of 8 cm from its centre. Calculate its new speed. [**Ans.** 32 rpm]

4. The moment of inertia of a flywheel is $0{\cdot}1$ kg-m^2. A cord is wound along the circumference and a mass of 2 kg is attached at the lower end. If the radius of the wheel the 0.1 m, calculate the acceleration of the body. [**Ans.** $1{\cdot}67$ m / s^2]

5. A mass m is suspended from the lower end of the string wound over a horizontal cylinder of mass M and radius R [Fig 2.18]. Initially the mass is at a height *h* from the ground and is allowed to fall. What will be the angular velocity of the cylinder when the mass m touches the ground ?

$$\left[\textbf{Ans :}\ \omega = \frac{1}{R}\sqrt{\frac{2gh}{\left(1+\frac{M}{2m}\right)}}\right]$$

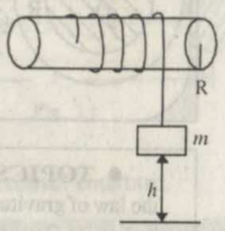


Fig. 2·18

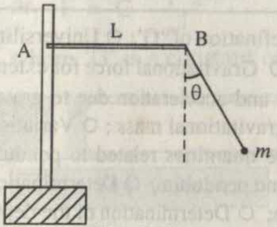


Fig. 2·19

6. A horizontal rod AB of length L = 20 cm is rotating about a vertical axis with a constant angular velocity. A string of length 20 cm is fixed at the end B of the rod and a mass m is attached at the free end of the string [Fig 2.19]. If the strong makes an angle of $\theta = 30^\circ$ with the vertical. Calculate angular velocity of the rod. $g = 10$ ms^{-2} [**Ans.** w = $4{\cdot}4$ rad / sec]

7. A weightless string is wound on a circular disc of radius 10 cm and mass $0{\cdot}5$ kg [Fig 2.20]. A body of mass 2 kg is attached at the lower end of the string. When the body starts falling what will be its acceleration ? Also calculate tension in the string.
[**Hints :** *ma* = *mg* – T and TR = Iα] [**Ans.** $\alpha = 8{\cdot}7$ m / s^2 ; T = $2{\cdot}2$ N]

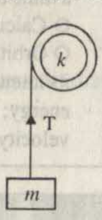


Fig. 2·20

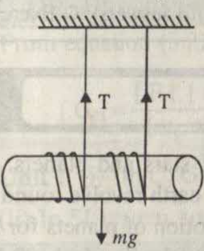


Fig. 2·21

8. Consider the fig. 2.21, when a cylinder of mass M is hanging horizontally by means of two cords. The cords are wound on the cylinder and the ends are fixed at the ceiling of a room. Calculate (i) downward acceleration of the cylinder and tension on the cords. [**Ans.** $a = \frac{2}{3}g$; $T = \frac{1}{6}Mg$]
[**Hints :** Mg – 2T = ma; 2TR = I α]

9. A wheel of radius 6 cm is rotating about a horizontal axis passing through its centre. A string is wound over it and from its lower end a mass of 200 g is suspended and is allowed to fall. The mass descends 1 m after 5 sec. Calculate angular velocity and moment of inertia of the wheel.[**Ans.** $1{\cdot}33$ rad / s^2 ; $87{\cdot}48 \times 10^4$ gm. cm^2]

10. A metre stick is hold vertically with one end on the floor and is then allowed to fall. Find the velocity of the other end, when it hits the floor, assuming that the end on the floor does not slip. [**Ans.** $5{\cdot}4$ ms^{-1}]

11. A man stands at the centre of a rotating platform holding his arm extended horizontally with a weight in each hand, the platform is making one rev. per sec. The moment of inertia of the whole system is 6 kg-m^2. When the man pulls his hands to his sides, the moment of inertia becomes one-third of its initial value. What is the new angular velocity of the platform ? Has the kinetic energy been conserved ? If not, explain it. Assume that there is no friction in the platform. [**Ans.** 6 π rad / s; K.E is increased by 237 J]

12. A thin uniform bar lies on a frictionless horizontal surface and free to move in any way on the surface. Its mass is $0{\cdot}16$ kg and length is $\sqrt{3}$ m . Two particles, each of mass $0{\cdot}06$ kg are moving on the same surface and towards the bar in a direction perpendicular to the bar, one with a velocity of 10 ms^{-1} and the other with 6 ms^{-1} [Fig 2.22]. The first particle strikes the bar at point A and the other at point B. Points A and B are at a distance of $0{\cdot}5$ m from the centre of the bar. The particles strike the bar at the same instant of time and stick to the bar on collision. Calculate the loss of energy of the system in the above collision process. [**Ans.** 2.72 J] [**I. I. T. 1989**]

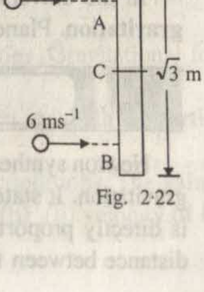
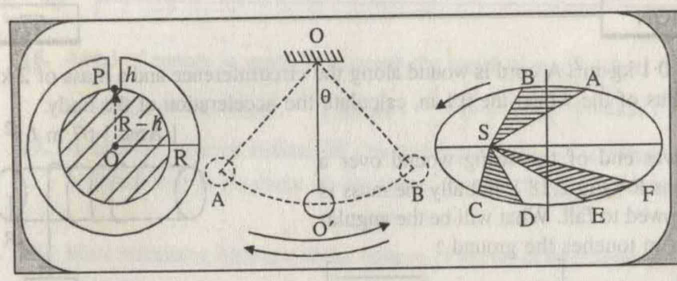


Fig. 2·22



CHAPTER 3

GRAVITATION AND GRAVITY

● **TOPICS :** ○ Introduction; ○ Newton's Law of Gravitation ; ○ Defination of 'G'; ○ Universility of the law of gravitation; ○ Some important properties of Gravitational force; ○ Gravitational force for extended bodies; ○ Gravitational attraction between sun and its planets ; ○ Gravity and acceleration due to gravity ; ○ Determination of mass of the sun ; ○ Weight of a body ; ○ Inertial and gravitational mass ; ○ Variation of acceleration due to gravity ; ○ Simple pendulum; ○ Definitions of some quantities related to pendulum; ○ Laws of simple pendulum; ○ Time period of a simple Pendulum; ○ Second pendulum; ○ Determination of 'g' at a place; ○ Determination of height of a mountain by simple pendulum; ○ Determination of the depth of a mine by simple pendulum; ○ Loss or gain of time by simple pendulum; ○ Planetary motion and Kepler's law; ○ Calculation of mass of the sun; ○ Satellites : Natural and Artificial ; ○ Principle of launching a satellite; ○ Orbital velocity and period of revolution of a satellite; ○ Geostationary Satellite; ○ Gravitational field and its intensity; ○ Intensity of gravitational field due to earth; ○ Gravitational potential; ○ Gravitational potential energy; ○ Gravitational potential energy of a system of masses; ○ Energy of an orbiting satellite; ○ Escape velocity; ○ Weighlessness in orbiting satellite; ○ Short answer type questions (with answers); ○ Exercise.

✪ 3.1. Introduction ✪

Since ancient times, scientists were very eager to know the mistries of stars and planets. In 1543 Polish scientist Copernicus first pointed out that all the planets including the earth revolve round the sun. After Copernicus. Danish Astronometer Tycho Brahe keenly observed the motion of planets for 20 years and collected large amount of data regarding the motion of planets and heavenly bodies. In 1600 Johannes Kepler carefully analysed the Tycho's data and showed that all planets move around the sun in elliptical orbits with the sun at one of its foci of the ellipse. He gave three laws governing the motion of planets around the sun which are now known as **Kepler's laws of planetary motion**. But the nature of the force influencing the motion of planets round the sun was unknown to him. In an attempt to explain Kepler's laws, Sir Isaac Newton put forward the famous **law of Universal Gravitation**.

In the previous chapters we studied the kinematics of a body falling freely under gravity of the earth. We learnt of a very important property of the gravitations force, namely, that it is a **Conservative force**.

The four fundamental forces of nature are : the frictional, gravitational, electromagnetic and nuclear forces. The gravitation force happens to be the weakest among them. But at the astronomical level, when masses are enormous, the force plays an important role from initiating the birth of a star, controlling the orbits of planets to determining the structure and evolution of the entire universe.

In the universe any two particles attracts each other. This universal force of attraction is called **gravitation.** Planets move round the sun in different orbits due to gravitational force.

✪ 3.2. Newton's law of Gravitation ✪

Newton synthesised all the available concepts on gravitaion and put forward the famous law of universal gravitation. It states : Any two particles anywhere in the universe attract each other with the force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them, the direction of force being along the line joining the particles.

Two particles m_1 and m_2 are at a separation 'r' and if the gravitional force of attraction between them be F, then

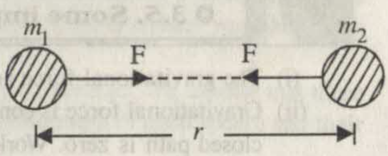


Fig. 3.1

$F \propto m_1m_2$ and $F \propto \frac{1}{r^2}$

Combining we get, $F \propto \frac{m_1m_2}{r^2}$

or, $F = G\frac{m_1m_2}{r^2}$ .... (3.1)

where 'G' is a constant of proportionality and is known as **Universal gravitational constant.**

❂ 3.3. Definition of 'G' ❂

(A) Definition :

From equation (3.1), $G = \frac{Fr^2}{m_1m_2}$ .... (3.2)

So, if $m_1 = m_2 = 1$ and $r = 1$ then G = F

Thus, gravitational constant is the force between two particles of unit masses separated by unit distance.

(B) Dimension and unit of 'G' :

From equation (3.2) we can deduce the dimensional formula for 'G' as

$$[G] = \frac{[F][r^2]}{[m_1 m_2]} = \frac{[MLT^{-2}][L^2]}{[M^2]} = [M^{-1}L^3T^{-2}]$$

- **Unit :** (i) In C. G. S. system, unit of 'G' is $gm^{-1}\ cm^3\ s^{-2}$ or dyne . $cm^2\ gm^{-2}$

(ii) In SI system, unit of 'G' is $kg^{-1}\ m^3\ s^{-2}$ or N . $m^2\ kg^{-2}$

(C) Value of 'G' :

(i) In C. G. S. system value of G is $6{\cdot}67 \times 10^{-8}$ cgs. It means that if two particles each of mass 1 gm are at a separation of 1 cm, they attract each other with a force of $6{\cdot}67 \times 10^{-8}$ dyne.

(ii) In SI system the value of G is $6{\cdot}67 \times 10^{-11}$ SI. If meant that if two particles each of mass 1 kg are at a separation of 1 m, they attract each other with a force of $6{\cdot}67 \times 10^{-11}$ N.

- **N. B.** Gravitational force is much weaker than coulomb electrostatic force. For example, electric force between two electrons kept at a separation of 1 cm in air is $2{\cdot}3 \times 10^{-19}$ dyne. But gravitational force between them is $5{\cdot}5 \times 10^{-62}$ dyne. Evidently electric force is nearly 10^{48} times stronger.

❂ 3.4. Universility of the law of gravitation ❂

1. It is applicable to all bodies— big or small. The law is equally applicable to subatomic particles as well as celestrial and terrestrial bodies.

2. It depends only on the masses of the bodies and not on any other properties. Gravitational force is not influenced by the state of the body—whether it is a solid, liquid or gas.

3. Gravitational force does not depend on any of the properties of the medium in which the particles are situated.

4. It does not depend on temperature, magnetic field or any other physical conditions. The value of 'G' always remains constant, G is an universal constant like (i) planck's constant (h) (ii) velocity of light in vacuum (C) (iii) electronic charge (e) (iv) Gas constant (R).

✪ 3.5. Some important properties of gravitational force ✪

(i) The gravitational force acts along the line joining two particles. So it is a central force.

(ii) Gravitational force is conservative. For this reason, work done by the gravitational force over a closed path is zero. Work done does not depend on path followed.

(iii) Like magnetic and electric force, the gravitational force obey inverse square law.

(iv) Gravitational force is always attractive. It cannot be repulsive.

(v) This force is independent of the medium.

(vi) Gravitational force depends only on the masses of the bodies.

(vii) It is a mutual force. It depends on the product of the masses and not on individual mass.

● **Example 3.1. The distance between two spheres of masses 40 kg and 15 kg is 20 cm. If gravitational force between them is 0·1 mg, calculate the value of 'gravitational constant'.**

○ **Solution :** From the law of gravitation, $F = G\dfrac{m_1 m_2}{r^2}$

$$\therefore \quad G = \frac{Fr^2}{m_1 m_2}$$

$$= \frac{9\cdot8\times10^{-7}\times(0\cdot2)^2}{40\times15}$$

$$\therefore \quad G = 6\cdot54\times10^{-11}\ \text{SI}$$

Here

$F = 0\cdot1\ \text{mg} = 9\cdot8\times10^{-7}$ N.

$r = 0\cdot2$ m

$m_1 = 40\ kg,\quad m_2 = 15\ kg$

● **Ex. 3.2. Two particles of equal mass are moving along a circle of radius R due to their mutual gravitational force. Calculate the velocity of each particle.**

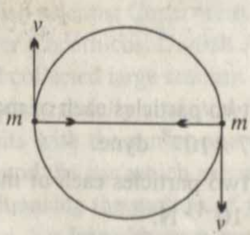


Fig. 3.2

○ **Solution :** The particles moves in such a way that they always remain at the two extreme points of a diameter. So the force on each particle acts along the diameter. So the force on each particle is $F = \dfrac{Gm^2}{4R^2}$ [m = mass of each particle].

This force provides the necessary centripetal force.

$$\therefore \quad \frac{mv^2}{R} = \frac{Gm^2}{4R^2} \qquad \therefore \quad v = \sqrt{\frac{Gm}{4R}}.$$

✪ 3.6. Gravitational force for extended bodies ✪

The law of gravitation represented by the equation (3.1) applies only when the masses are very small in dimension. But in reality every mass has some extension. For extended objects if the separation is very large, we can neglect their dimension in comparison with the distance separating them and consider them as point masses. In such a case the masses of the bodies are then assumed to be concentrated at their respective centres of mass.

But if the separation between two extented bodies is not very large then we have to divide the two bodies into point masses and then find out the forces of attraction between every pair of such point masses. The resultant attraction is obtained by summing all these forces vectorially. Evidently the process is rather complicated. In the case of regular shaped bodies the method becomes very simple.

For a uniform solid sphere or for a uniform spherical shell, the force of attraction at points outside is found to be the same as that due to a particle situated at the centre of the sphere or shell. The particle has the same mass as the sphere or shell. Thus for the calculation of force of attraction at a point outside the sphere or shell, whole mass is assumed to be at the centre and the distance is calculated from the centre upto the point.

Similarly, to calculate the gravitational force of attraction between two spheres, we assume that there are two particles at the centres. The masses of the particles are equal to the mass of each sphere and their separation is equal to the distance between the centres of the spheres.

3.7. Gravitational attraction between sun and its planets

All the nine planets of the sun revolve round the sun in different elliptical orbits which may be considered as circular as an approximation. The gravitational attraction between the sun and its planets provides the centripetal force necessary for the planets to describe circular motion round the sun.

As the sun and the planets are spheres we shall assume that their masses are at the centres and distance between them is equal to centre to centre distance. So in order to determine the gravitational attraction between sun and the earth (a planet) we regard each of them as a point of equal mass situated at its centre. Hence the gravitational attraction between the sun and the earth is

$$F = \frac{G M_s \; M_e}{r^2}$$

$$\therefore \quad F = \frac{6 \cdot 67 \times 10^{-11} \times 1 \cdot 99 \times 10^{30} \times 5 \cdot 96 \times 10^{24}}{\left(1 \cdot 497 \times 10^{11}\right)^2} = \mathbf{3 \cdot 53 \times 10^{22} N}$$

M_e = mass of the earth = $5 \cdot 96 \times 10^{24}$ kg

M_s = Mass of the sun = $1 \cdot 99 \times 10^{30}$ kg

r = average distance between sun and earth = $1 \cdot 497 \times 10^{11}$ m

3.8. Gravity and acceleration due to gravity

The force of attraction between any two material objects is gravitation. In particular, the force of attraction of the earth on a body situated near or on the surface of the earth is called gravity. So, gravity is the special case of gravitation. Hence Earth's gravitation is gravity.

Like gravitation, gravity is also mutual. It means that the force with which the earth attracts a body, the body exerts same force on the earth. But we observe that if a body is allowed to fall freely, it falls vertically downward towards the earth. The earth does not move towards the body. The reason is : the mass of the earth extremely large compared to the mass of the terrestrial body. Any motion of the earth under the action of the force is not perceptible. We can show this in the following way :

Let M be the mass of the earth and m is the mass of the terrestrial body. If the force of attraction between the earth and the body be F, then acceleration of the earth towards the body is $a_e = \frac{F}{M}$ and acceleration of the body towards the earth is $a_b = \frac{F}{m}$

as $M >> m, \quad a_e << a_b$

So, acceleration of earth is negligible and body will fall towards the earth with an acceleration which is called **acceleretion due to gravity**. It is denoted by g.

Acceleration due to gravity (g) :

When a body falls freely downward from a certain height due to earth's gravity, it falls with increasing velocity. So due to gravity a falling body possesses an acceleration. This acceleration is called acceleration due to gravity.

Let a body of mass m is near the earth's surface at a distance r from the centre of the earth [Fig 3.3]

So, $r = h + \mathrm{R}$ $(h << \mathrm{R})$

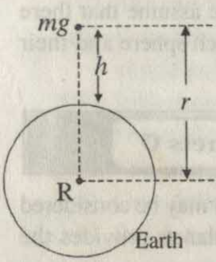


Fig. 3.3

According to Newton's law of gravitation, the force of attraction between the earth and the body is

$$\mathrm{F} = \frac{\mathrm{G}\mathrm{M}m}{r^2} \qquad \ldots\ldots (3.3)$$

When M is the mass of the earth assumed to be concentrated at its centre.

Now, earth's attraction is the weight of the body

$$\therefore \quad \mathrm{W} = \mathrm{F} = mg \qquad \ldots\ldots (3.4)$$

$$mg = \frac{\mathrm{G}\mathrm{M}m}{r^2} \quad \therefore \quad g = \frac{\mathrm{G}\mathrm{M}}{r^2}$$

As G and M are constant, $g \propto \dfrac{1}{r^2}$

So, acceleration due to gravity is inversely proportional to the square of the distance from its centre,

Now, if the body is on the earth's surface $r = \mathrm{R}$

$$\therefore \quad g = \frac{\mathrm{G}\mathrm{M}}{\mathrm{R}^2} \qquad \ldots\ldots (3.5)$$

Determination of mean density of the earth :

If ρ be the mean density of the earth, mass of the earth is $\mathrm{M} = \dfrac{4}{3}\pi \mathrm{R}^3\rho$

From equation (3.5), $g = \dfrac{\mathrm{G}}{\mathrm{R}^2}\,\dfrac{4}{3}\pi \mathrm{R}^3\rho = \dfrac{4}{3}\pi\rho \mathrm{R}\mathrm{G}$

$$\therefore \quad \rho = \frac{3\,g}{4\,\pi\,\mathrm{G}\,\mathrm{R}} \qquad \ldots\ldots (3.6)$$

Value of mean density of the earth can be calculated by the equation (3.6).

Here $g = 9{\cdot}8\ ms^{-2}$, $\mathrm{G} = 6{\cdot}67 \times 10^{-11}$ SI, $\mathrm{R} = 6400 \times 10^3 m$

$$\therefore \quad \rho = \frac{3 \times 9{\cdot}8}{4\pi \times 6{\cdot}67 \times 10^{-11} \times 6400 \times 10^3} = 5{\cdot}52 \times 10^3\ kg\text{-}m^{-3}$$

● **Example 3.3. If the earth is a solid iron sphere of radius 6·37 × 10^6 *m* and having density 7·86 g/c.c., calculate the value of acceleration due to gravity on the earth's surface. Gravitational constant G = 6·58 × 10^{-8} C. G. S. unit** **[W. B. H. S. 2000]**

○ **Solution :** The required relation $\rho = \dfrac{3g}{4\pi \mathrm{R}\mathrm{G}} \Rightarrow g = \dfrac{4}{3}\pi\rho \mathrm{R}\mathrm{G}$

$$\therefore \quad g = \frac{4}{3} \times 3{\cdot}14 \times 7{\cdot}86 \times 6{\cdot}37 \times 10^8 \times 6{\cdot}58 \times 10^{-8} \quad \therefore \quad g = \mathbf{1380{\cdot}55\ cm\ s^{-2}}$$

● **Ex. 3.4. Mass remaining constant if the radius of the earth decreases by 1%, what is the percentage change in acceleration due to gravity on the surface of the earth ?** **[J. E. E. 1998]**

○ **Solution :** If the mass of the earth remains constant, then

$$g \propto \frac{1}{\mathrm{R}^2} \quad \therefore \quad \frac{g'}{g} = \frac{\mathrm{R}^2}{(0{\cdot}99\mathrm{R})^2} \quad [\ \mathrm{R}' = 0{\cdot}99\mathrm{R}\ ]$$

$$\therefore \quad \frac{g'}{g} = \frac{1}{0{\cdot}98} \Rightarrow g' = 1{\cdot}02\,g$$

$\therefore$ Percentage change in value of g is, $\dfrac{g' - g}{g} = 0{\cdot}02 = 2\%$

3.9. Determination of mass of the sun

Assume that the earth is moving round the sun in a circular orbit with a constant angular velocity ω. To maintain the rotational motion of the earth, gravitational force of the sun on earth provides the necessary centripetal force.

$$\therefore \quad \frac{G M_s M_e}{r^2} = M_e \omega^2 r$$

Here M_s = mass of the sun, M_e = mass of the earth and r = radius of earth's orbit.

$$\therefore \quad M_s = \frac{r^3 \omega^2}{G} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.7)$$

Now if T be the time period of revolution of the earth then, $\omega = \frac{2\pi}{T}$.

$$\therefore \quad M_s = \frac{4\pi^2 r^3}{GT^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.8)$$

Here, $r = 1{\cdot}5 \times 10^{11} m$, $\quad G = 6{\cdot}67 \times 10^{-11}$ SI, $\quad T = 365 \times 86400$ sec

$$\therefore \quad M_s = \frac{4\pi^2 \times \left(1{\cdot}5 \times 10^{11}\right)^3}{6{\cdot}67 \times 10^{-11} \times (365 \times 86400)^2} \text{ kg} = \mathbf{2 \times 10^{30}\ kg} \quad \textbf{(Approx)}$$

3.10. Weight of a body

Weight of a body is the force with which it is attracted towards the centre of the earth. Let a body of mass 'm' be near or on the surface of the earth. Then the weight of the body is given by

$$W = G\, \frac{Mm}{R^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.9)$$

Now, acceleration due to gravity on the earth's surface, $g = \frac{GM}{R^2}$

$$\therefore \quad W = mg \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.10)$$

∴ wt. of the body = Its mass × acceleration due to gravity.

- **Example 3.5. Mass and diameter of the earth are respectively 81 times and 4 times the mass and diameter of the moon. Compare the values acceleration due gravity on the surface of the earth and on the surface of the moon.** **[W. B. H. S. 1987]**

○ **Solution :** It M be the mass and R be the radius of a terrestrial body. Then the value of acceleration due to gravity on the body is $g = \frac{GM}{R^2}$

So, in care of the earth, $g_e = \frac{GM_e}{R_e^2}$ and in case of the moon, $g_m = \frac{GM_m}{R_m^2}$

$$\therefore \quad \frac{g_e}{g_m} = \frac{M_e}{M_m} \times \left(\frac{Rm}{Re}\right)^2$$

$$= 81 \times \left(\frac{1}{4}\right)^2 = 5{\cdot}06$$

$$\therefore \quad g_e = g_m = 5{\cdot}06 : 1$$

Now, $\frac{Me}{M_m} = 81$

$$\frac{Rm}{Re} = \frac{1}{4}$$

- **Ex. 3.6. Mass remaining unchanged if the radius of the earth be half of its present radius, how does the weight of a body change ?**

○ **Solution :** We know weight of a body, $W = mg = \frac{GMm}{R^2}$

where M is the mass and R is the present radius of the earth.

$$\therefore \quad W \propto \frac{1}{R^2} \quad [\text{ when M is constant }]$$

$$\therefore \quad \frac{W_1}{W_2} = \frac{{R_2}^2}{{R_1}^2}$$

$$\text{Here } R_2 = \frac{R_1}{2} \quad \therefore \quad \frac{R_2}{R_1} = \frac{1}{2} \quad \therefore \quad \frac{W_1}{W_2} = \left(\frac{1}{2}\right)^2 \Rightarrow W_2 = 4W_1$$

So, the weight of the body will increase four times.

◎ 3.11. Inertial and gravitational mass ◎

● **(i) Inertial mass :** From Newton's second law of motion, $F = ma$. Here F is the applied force, 'm' is the mass and 'a' is the acceleration produced. From this equation we can determine the mass by measuring the acceleration produced in it by a known constant force.

$$\text{Now,} \quad m = \frac{F}{a}.$$

So, dividing the force by the acceleration we get the mass of the body. The mass so determined is called **inertial mass** of the body. Hence mass may be regarded as a quantitative measure of inertia.

$$\text{Again,} \quad a \propto \frac{1}{m} \quad [\because F = \text{constant}]$$

So, for a given force if m is large, 'a' will be small. It means that this mass has the property to oppose acceleration. For this reason, m in the Newton's law of motion is called inertial mass.

$$\text{Also } F = ma = m_1 a_1 \quad \therefore \quad \frac{m}{m_1} = \frac{a_1}{a}.$$

Thus we can compare the inertial masses of the two bodies by measuring acceleration produced in them by a constant force.

● **(ii) Gravitational mass :** From Newton's law of gravitation, the gravitational force between two particles of mass m_1 and m_2 kept at a separation r is $F = \frac{Gm_1 m_2}{r^2}$

This relation tells that the gravitational force on a body is proportional to its mass. The consequence of this proportionality is that we can measure the mass of a body by measuring the gravitational force exerted on it by a massive body, such as earth. This can be done with the help of a spring balance. The mass m so determined is called gravitational mass of the body.

From different experiments, it is proved that inertial mass and gravitational mass are equivalent.

◎ 3.12. Variation of acceleration due to gravity ◎

The value of acceleration due to gravity is maximum on the surface of the earth. The value is $g = 9 \cdot 8\ ms^{-2}$. The acceleration due to gravity changes as we go up (effect of altitude) or below (effect of depth) the surface of earth. It also changes with latitude of the place due to rotation of the earth round its. axis. Further, the earth is not exactly spherical in shape. The radius of the earth is more at equator than at the poles. The acceleration due to gravity varies also due to the shape of the earth.

(A) Variation of acceleration due to gravity with altitude :

A small body of mass 'm' is situated at a height 'h' from the surface of the earth. M is the mass and R is the radius of the earth. Whole mass of the earth is concentrated at its centre.

So, the attraction of the earth on the body is $F = \dfrac{GMm}{(R+h)^2}$

If acceleration due to gravity at the height 'h' be g'

Then the weight of the body at this height is $W = mg'$

Now, earth's attraction is the weight of the body. So, $W = F$

$$\therefore \quad mg' = \frac{GMm}{(R+h)^2}$$

$$\therefore \quad g' = \frac{GM}{(R+h)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.11)$$

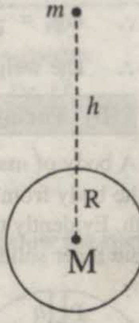


Fig. 3.4

On the earth's surface acceleration due to gravity, $g = \dfrac{GM}{R^2}$

$$\therefore \quad \frac{g'}{g} = \frac{R^2}{(R+h)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.11a)$$

$$\therefore \quad g' = g \cdot \frac{R^2}{R^2\left(1+\frac{h}{R}\right)^2} = g\left(1+\frac{h}{R}\right)^{-2}$$

Expanding the above by Binomial theorem, we have

$$g' = g\left[1-\frac{2h}{R} + \text{ terms containing higher power of } \frac{h}{R}\right]$$

Usually h is quite small compared to R. So higher powers of $\frac{h}{R}$ can be neglected. Hence

$$g' = g\left(1-\frac{2h}{R}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.12)$$

From equations (3.11a) and (3.12) it follows that as we go up from the earth's surface, the value of acceleration due to gravity decreases. The equation (3.12) is used to find the value of g at a height $h << R$.

When h is comparable to the radius of the earth, the equation (3.11a) has to be used to get the new value of g.

$\therefore$ when $h << R$, decrease in value of g, $\Delta g = g - g' = \dfrac{2hg}{R} \quad \ldots\ldots\ (3.13)$

● **Example 3.7. At what height from the earth's surface the value of acceleration due to gravity decreases by 25% of its value on the surface of the earth ? [R = 6400 km]**

○ **Solution :** Let at a height h from the surface of the earth the value of acceleration due to gravity decrease by 25% of its value on the surface of the earth,

$$\therefore \quad g' = g \cdot \frac{R^2}{(R+h)^2} \qquad \text{Here } \frac{g'}{g} = 0{\cdot}75 = \frac{3}{4} \qquad \therefore \quad \frac{3}{4} = \frac{R^2}{(R+h)^2}$$

$$\therefore \quad \frac{R}{R+h} = \frac{\sqrt{3}}{2} = 0{\cdot}866 \qquad \therefore \quad h = 990{\cdot}1 \text{ km.}$$ It is the required height.

- **Ex. 3.8. A body weighs 63N on the surface of the earth. How much it will weigh at a height equal to the radius of the earth ?**

○ **Solution :** Here $mg = 63$ N, $h = R/2$

Now, $g' = g \cdot \dfrac{R^2}{(R+h)^2} = g \cdot \dfrac{R^2}{\left(R+\dfrac{R}{2}\right)^2} = \dfrac{4}{9} g.$

$\therefore \quad mg' = \dfrac{4}{9} mg = \dfrac{4}{9} \times 63 = 28\text{N}$

$\therefore$ The weight of the body at the given height = 28 N

(B) Variation of acceleration due to gravity with depth :

A body of mass m is situated at P at a depth h below the surface of the earth [Fig. 3.5]. Now distance of the body from the centre of the earth is $(R - h)$. Draw a sphere of radius $(R - h)$ concentric with the earth. Evidently the body lies on the inner surface of the spherical shell of thickness h and on the surface of the inner solid sphere of radius $(R - h)$.

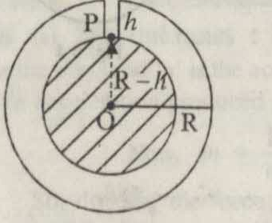


Fig. 3.5

Now, it can be proved that the spherical shell will exert no force on the body. So, gravitational force on the body is only due to the inner sphere of radius $(R - h)$. So, by the law of gravity, the force of attraction on the body of mass m is

$F' = \dfrac{GM'm}{(R-h)^2}$ where M' is the mass of the inner sphere. If we consider the earth to be a homogeneous sphere then, $M' = \dfrac{4}{3}\pi (R-h)^3 \rho$

If g' be the acceleration due to gravity at the depth h then, $F' = mg'$

$$\therefore \quad mg' = \frac{GM'm}{(R-h)^2} \Rightarrow g' = \frac{GM'}{(R-h)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

Now, on the surface of the earth, $g = \dfrac{GM}{R^2}$

$$\therefore \quad \frac{g'}{g} = \frac{M'}{M} \cdot \frac{R^2}{(R-h)^2} = \frac{\frac{4}{3}\pi (R-h)^3 \rho}{\frac{4}{3}\pi R^3 \rho} \times \frac{R^2}{(R-h)^2}$$

$$\therefore \quad \frac{g'}{g} = \frac{R-h}{R} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3.14)}$$

So, the acceleration due to gravity inside the earth varies directly as the distance from the centre of the earth. At the centre of the earth ($h = R$) acceleration due to gravity is zero. The weight of the body is thus zero at the centre of the earth.

Decrease in acceleration due to gravity at a depth h is

$$g - g' = g\frac{h}{R} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3.15)}$$

Variation of acceleration due to gravity with distance from the centre of the earth.

From our previous discussions we observe that, (i) the acceleration due to gravity is maximum at the surface of the earth.

(ii) For a point outside the earth, the value of g varies inversely as the square of its distance from the centre of the earth $\left[g' \propto \frac{1}{(R+h)^2}\right]$.

(iii) For point inside the earth, g varies directly as the distance from the centre of the earth $[g' \propto (R-h)]$. The variation of 'g' with distance from the centre of the earth is shown in Fig 3.6.

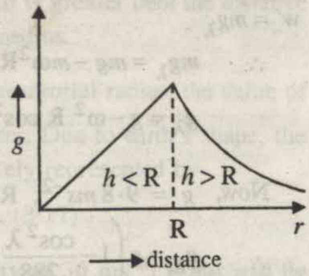


Fig. 3.6

● **Example 3.9. At what depth below the earth's surface, the value of acceleration due to gravity will be 25% of its value on the earth's surface ? Assume the earth to be a homogeneous sphere of radius R.**

○ **Solution :** The required relation is $\frac{g'}{g}=\frac{R-h}{R}$ Here $\frac{g'}{g}=\frac{1}{4}$.

$$\therefore \quad \frac{R-h}{R}=\frac{1}{4} \Rightarrow h=\frac{3}{4}R=\mathbf{0\cdot75\,R}$$

● **Ex. 3.10. Find the percentage decrease in weight of a body, when taken 16 km below the surface of the earth. Radius of the earth 6400 km.**

○ **Solution :** Here R = 6400 km, h = 16 m.

From the equation (3·14), $g-g'=g\frac{h}{R}$

$$\therefore \quad \frac{g-g'}{g}=\frac{h}{R}=\frac{16}{6400}=\frac{1}{400}$$

$$\therefore \quad \text{percentage decrease in weight} = \frac{mg-mg'}{mg}\times100=\frac{g-g'}{g}\times100=\frac{1}{400}\times100=\mathbf{0\cdot25\%}$$

(C) Variation of acceleration due to gravity due to earth's diurnal rotational motion :

The earth is continuously rotating from west to east about its own axis. All the bodies on the surface of the earth, except those lying on the poles move along a circular path, the centres of the circle being situated on the axis of the earth. Consequently all the bodies on the earth's surface (except at the poles) experience a centrifugal force due to which they tend to fly off. This centripetal force is maximum at the equator and zero at the poles. The magnitude of the centrifugal force varies with the latitude of the place. Since this force acts opposite to the force of gravity, a part of the force of gravity is spent to balance the centrifugal force and the apparent weight of the body is less than the true weight. So, the apparent value of acceleretion due to gravity will be less than its actual value.

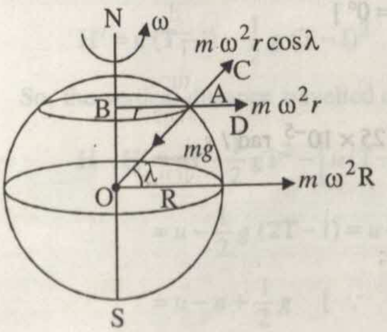


Fig. 3.7

Let a body of mass m be situated on the surface of the earth at A of latitude λ [Fig 3·7]. Angular velocity of earth is ω. All the bodies on the earth's surface will rotate with anguler velocity ω and so, the body at the point A moves along a circle AB = r with constant angular velocity ω. If R be the radius of the earth. $r = R\cos\lambda$

Now, centrifugal force on mass m is, $F_c = m\omega^2 r$ (along AD) and the weight of the body W = mg acts along AO.

The component of centrifugal force along AC is $F_c \cos\lambda$

So, the effective weight of the body at A, $w' = mg - F_c\cos\lambda = mg - m\omega^2 r\cos\lambda$

$$\therefore \quad w' = mg - m\omega^2 R\cos^2\lambda \quad [\because r = R\cos\lambda]$$

If g_λ be the effective value of acceleration due to gravity at A, then effective weight of the body is $w' = mg_\lambda$

$$\therefore \quad mg_\lambda = mg - m\omega^2 R\cos^2\lambda$$

$$g_\lambda = g - \omega^2 R\cos^2\lambda = g\left(1 - \frac{\omega^2 R}{g}\cos^2\lambda\right) \cdots\cdots\cdots \text{(3·16)}$$

Now, $g = 9\cdot8\, ms^{-2}$, $R = 6400\times10^3 m$ and $\omega = \frac{2\pi}{86400} = 7\cdot25\times10^{-5}$ rad / s

$$\therefore \quad g_\lambda = g\left(1 - \frac{\cos^2\lambda}{288}\right)$$

Since $g_\lambda < g$, it follows that acceleration due to gravity decrease due to rotational motion of the earth.

- **Special cases :** (a) at the equator $\lambda = 0°$, $g_\lambda = g - \omega^2 R$

  (b) at the poles, $\lambda = 90°$, $g_\lambda = g$

So, on account of earth's spinning motion, decrease in the value of g is maximum at the equator and zero at the poles.

- **Example 3.11. Show that if the earth stops rotating, the value of acceleration due to gravity at the equater will increase by 3·38 cm / s². Earth's radius R = 6·4 × 10⁸ cm.**

○ **Solution :** Due to earth's rotational motion, the effective value of acceleration due to gravity is $g_\lambda = 8 - \omega^2 R$. So if the earth stops rotating, the acceleration due to gravity increases by $\Delta g = \omega^2 R$.

Here $\omega = 7\cdot25\times10^{-5}$ rad / s, $R = 6\cdot4\times10^8$ cm

$$\therefore \quad \Delta g = \left(7\cdot25\times10^{-5}\right)^2 \times 6\cdot4\times10^8 = 3\cdot38 \text{ cm / s}^2$$

- **Ex. 3.12. Show that if the angular velocity of the earth be nearly 17 times its present angular velocity, then a body at the equator will appear weightless.**

○ **Solution :** The effective value of acceleration due to gravity due to earth's rotational motion at a place on the surface of the earth of latitude λ is

$$g_\lambda = g - \omega^2 R\cos^2\lambda = g - \omega^2 R \quad [\because \text{ at equator } \lambda = 0°]$$

$$= g\left(1 - \frac{\omega^2 R}{g}\right)$$

Now, $R = 6400\times10^3 m$, $g = 9\cdot8\, ms^{-2}$ and at present $\omega = 7\cdot25\times10^{-5}$ rad / s

$$\therefore \quad \frac{\omega^2 R}{g} = \frac{1}{288} \cdots\cdots\cdots\cdots \text{(i)}$$

Let $\omega = \omega_0$ when a body at the equator will appear weightless ;

then, $g_\lambda = 0$ and $\frac{\omega_0{}^2 R}{g} = 1 \cdots\cdots\cdots \text{(ii)}$

$\therefore$ From equations (i) and (ii), $\frac{\omega_0{}^2}{\omega^2} = 288$ $\therefore$ $\omega_0 = \sqrt{288}\,\omega = 17\,\omega$

Effect of shape of the earth on the value of *g* :

The value of acceleration due to gravity is maximum on the surface of the earth. But this value is not same at all places on the surface of the earth. The reason is : the earth is not a perfect sphere but is ellipsoidal in shape. The distance of the equator from the centre of the earth is greater than the distance of the poles from the centre. Earth's equatorial radius is greater than polar radius.

Now, $g \propto \frac{1}{R^2}$ (when mass is constant). As polar radius is less than equitorial radius, the value of acceleration due to gravity at the poles is higher than its value at the equator. Due to earth's shape, the value of acceleration due to gravity with latitude of the place is approximately represented by

$$g' = 978\cdot0\,(1+5\cdot31\times10^{-3}\sin^2\lambda) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3\cdot17)$$

- **Example 3.13. A body is projected vertically upwards with a velocity of 30 ms^{-1}. What will be its velocity when it rises half of the maximum height. g = 10 ms^{-2}**

○ **Solution :** The maximum height attained by the body is

$$H = \frac{u^2}{2g} = \frac{(30)^2}{2\times10} = \mathbf{45}\ \boldsymbol{m}$$

Let at a height $h = \frac{H}{2} = \frac{45}{2} = 22\cdot5\,m$, the velocity of the body is v m/s. Then

$$v^2 = u^2 - 2gh = (30)^2 - 2\times10\times22\cdot5 = 450$$

$$\therefore \quad v = \sqrt{450} = 21\cdot21 \text{ ms}^{-1}$$

- **Ex. 3.14 : Show that the distance travelled by a body projected vertically upwards in the last second of its vertical motion is equal to the distance travelled by a freely falling body during the first second.**

○ **Solution :** If a body is projected vertically upwards with a velocity, u its time of ascent, $T = \frac{u}{g}$. The maximum height attained by the body is $H = uT - \frac{1}{2}gT^2$.

Height attained by the body during (T–1) sec is

$$H' = u(T-1) - \frac{1}{2}g(T-1)^2$$

So, the vertical distance travelled during the last second

$$H - H' = uT - \frac{1}{2}gT^2 - \left[u(T-1) - \frac{1}{2}g(T-1)^2\right]$$

$$= u - \frac{1}{2}g(2T-1) = u - gT + \frac{1}{2}g$$

$$= u - u + \frac{1}{2}g \quad [\because \quad u = gT]$$

$$\therefore \quad H - H' = \frac{1}{2}g \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (i)$$

Now, the distance travelled in the first second of the motion by a freely falling body is

$$h = \frac{1}{2}gt^2 = \frac{1}{2}g(1)^2 = \frac{1}{2}g \quad \ldots\ldots\ldots\ldots\ldots\ldots (ii)$$

Hence from (i) and (ii) $H - H' = h$

3.13. Simple Pendulum

A simple pendulum is a heavy mass suspended by a light, inextensible and perfectly flexible string from a rigid support. It can execute to and fro oscillation.

Evidently, such a pendulum cannot be realised in practice. A simple pendulum is nearly achieved by suspending a small and heavy metal sphere by a very long flexible and light thread. The weight of the thread is negligible compared to the weight of the metal sphere.

3.14. Definitions of some quantities related to pendulum

- **(i) Point of suspension :** The point of the rigid support from where the pendulum is suspended is called point of suspension.
- **(ii) Effective length :** The distance between the point of suspension and the centre of gravity of the pendulum bob is called the effective length of the pendulum, If l' be the length of the string and r be the radius of the spherical bob, then effective length of the pendulum is $l = l' + r$

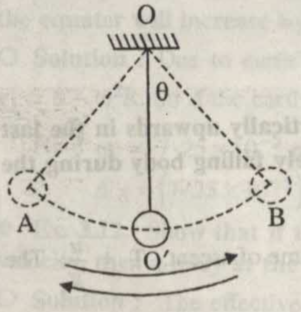


Fig. 3.8

- **(iii) Amplitude :** The maximum displacement of the bob (O′A or O′B) on either side of the mean position O′ is called its amplitude of oscillation. It can also be measured by the angle between the mean position and the extreme position on either side is called the angular amplitude.

  So in the fig, $\theta = \angle O'OA$ or $\theta = \angle O'OB$ is the angular amplitude.
- **(iv) Complete oscillation and Time period (T) :** When the pendulum starts from one extreme point of its path and again comes back to the same extreme point, the pendulum is said make one complete oscillation.

  The time required to complete one full oscillation is called the **time period or** the period of oscillation.
- **(v) Frequency :** The number of full oscillations executed in one second is called frequency.

  In T sec the pendulum completes one oscillation. So the number of oscillations is 1 sec, $n = \frac{1}{T}$.

  $\therefore$ frequency $n = \frac{1}{T}$.

3.15. Laws of simple pendulum

The motion of simple pendulum obeys some simple laws which are known as laws of simple pendulum. These laws tells how the motion of the pendulum depends on angular amplitude, the effective length of the pendulum, acceleration due to gravity at the place and the mass of the bob. The four laws of motion of the pendulum are :

- **(i) Law of isochronism :** When the angular amplitude of oscillation does not exceed 4° the time period of oscillation of a simple pendulum of fixed length and at a given place is always constant, being independent of amplitde.
- **(ii) Law of length :** When the angular amplitude of oscillation does not exceed 4°, the period of oscillation of a simple pendulum at a given place is directly proportional to the square root of the effective length of the pendulum. i.e. $T \alpha \sqrt{l}$.

● **(iii) Law of acceleration :** When the angular amplitude does not exceed 4°, the period of oscillation of a simple pendulum of a fixed length is inversely proportional to the square root of the acceleration due to gravity of the place, i. e. $T \propto \frac{1}{\sqrt{g}}$

● **(iv) Law of mass :** When the angular amplitude does not exceed 4°, the period of oscillation of a simple pendulum of given length at a particular place does not depend on the mass of the bob.

⭘ 3.16. Time period of a simple pendulum ⭘

If l be the effective length of the pendulum and g be the acceleration due to gravity at a place, then by the laws of simple pendulum

$$T \propto \sqrt{l} \quad \text{and} \quad T \propto \sqrt{\frac{1}{g}}$$

combining these laws, $T \propto \sqrt{\frac{l}{g}} \quad \therefore \quad T = k\sqrt{\frac{l}{g}}$

k is a constant of proportionality whose value is taken to be 2π.

$$\therefore \quad T = 2\pi\sqrt{\frac{l}{g}}\,.$$

3.17. Second pendulum ⭘

It is a simple pendulum whose time period is 2 sec. It takes exactly 1 sec to complete one swing i.e. to go from one extreme end to the other extreme end during oscillation.

Since time period $T = 2\pi\sqrt{\frac{l}{g}}$, for a seconds pendulum $2 = 2\pi\sqrt{\frac{l}{g}}\,.$

$\therefore \quad l = \frac{g}{\pi^2}$ Now, taking $g = 980 \text{ cm/s}^2$

the length of the second pendulum is $l = \frac{980}{9 \cdot 87} = 99 \cdot 3$ cm

▲ Some uses of simple pendulum ▲

3.18. Determination of '*g*' at a place ⭘

Time period of simple pendulum of length l at a place where acceleration due to gravity is g is

$$T = 2\pi\sqrt{\frac{l}{g}} \quad \text{or,} \quad T^2 = 4\pi^2 \frac{l}{g}$$

$$\therefore \quad g = 4\pi^2 \cdot \frac{l}{T^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3{\cdot}18)$$

So, by knowing $\frac{l}{T^2}$, g can be determind.

(i) Different values of l are adjustted and for each value of l, time period of oscillation (T) is determind. Then for each value of l, $\frac{l}{T^2}$ is calculated and mean value of all $\frac{l}{T^2}$ values is determined.

This average value of $\frac{l}{T^2}$ is substituted in equation (3.18) and value of 'g' is obtained.

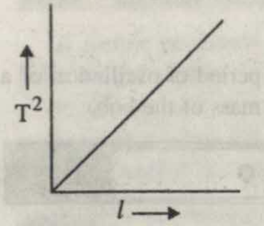


Fig. 3.9

(ii) Also we can calculate 'g' from $l - T^2$ graph. This $l - T^2$ graph will be a straight line passing through origin [Fig 3·9].

From the graph for a new value of l the value of T^2 s obtained and $\frac{l}{T^2}$ is calculated.

Putting this value of $\frac{l}{T^2}$ in equation (3·17) the value of 'g' is calculated.

✪ 3.19. Determination of height of a mountain by simple pendulum ✪

We can determine the height of a mountain or a tall building by a simple pendulum. For this we measure the periods oscillation of a simple at the bottom and the top of a mountain. Let these two time periods be T and T′. If g and g′ be the values of acceleration due to gravity at the bottom and at the top of the mountain then, $\frac{T'}{T} = \sqrt{\frac{g}{g'}}$ .... (i)

Now, $g = \frac{GM}{R^2}$ and $g' = \frac{GM}{(R+h)^2}$

Here M = mass of the earth and R = radius and h = height of the mountain

$$\therefore \quad \frac{g}{g'} = \frac{(R+h)^2}{R^2} \Rightarrow \sqrt{\frac{g}{g'}} = \frac{R+h}{R} \text{ .... (ii)}$$

From equation (i) and (ii) $\frac{T'}{T} = \frac{R+h}{R} = 1 + \frac{h}{R}$

$$\therefore \quad h = R\left(\frac{T'}{T} - 1\right) \text{ .... } (3{\cdot}19)$$

Since in equation, all other quantities are known h can be calculated.

✪ 3.20. Determination of the depth of a mine by simple pendulum ✪

The depth of a mine can be determind in the same way as the height of a mountain. In this case, time period of a simple pendulum is determined at the surface of the earth and then at the bottom of the mine. If these time periods be T′ and T respectively, then $\frac{T'}{T} = \sqrt{\frac{g}{g'}}$ where g = value of acceleration due to gravity on the earth and g' = value of acceleration due to gravity at the bottom of the mine.

Now $\frac{g}{g'} = \frac{R}{R-h}$ [R = radius of the earth and h = depth of the mine]

$$\therefore \quad \frac{T'}{T} = \sqrt{\frac{R}{R-h}} \Rightarrow \left(\frac{T}{T'}\right)^2 = \frac{R-h}{R} = 1 - \frac{h}{R}$$

$$\therefore \quad h = \left[1 - \left(\frac{T}{T'}\right)^2\right] R \text{ .... } (3{\cdot}20)$$

3.21. Loss or gain of time by simple pendulum

Time period of a simple pendulum depends on its effective length (l) and on the value of acceleration due to gravity (g) of to place. So if l and g change, its time period will also change. So the pendulum will go slow or fast.

(A) Due to change in length of the pendulum :

Let the pendulum gives correct time when the length is l and the time period is T. Now, the length has increased to l' and the time period becomes T′. Then

$$\frac{T'}{T}=\sqrt{\frac{l'}{l}}\,. \quad \text{If length increases by } \Delta l, \quad l'=l+\Delta l$$

$$\therefore \quad \frac{T'}{T}=\sqrt{\frac{l+\Delta l}{l}}=\left(1+\frac{\Delta l}{l}\right)^{\frac{1}{2}}=1+\frac{1}{2}\frac{\Delta l}{l}\left[\because \ \frac{\Delta l}{l}<<1\right]$$

$$\therefore \quad \frac{T'-T}{T}=\frac{1}{2}\frac{\Delta l}{l}$$

It is the loss of time per second. So the loss of time per day,

$$\Delta T=86400\times\frac{1}{2}\frac{\Delta l}{l}=43200\ \frac{\Delta l}{l} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3{\cdot}21)$$

The pendulum will gain the same time per day if its length decreases by Δl.

(B) Due to change in acceleration due to gravity :

Let the simple pendulum gives correct time when the value of acceleration due to gravity be g and the correct time period is T. Now, let acceleration due to gravity changes to g' and $g'=g-\Delta g$ and if T′ be the new time period then,

$$\frac{T'}{T}=\sqrt{\frac{g}{g'}}=\sqrt{\frac{g}{g-\Delta g}}=\sqrt{\frac{g}{g\left(1-\frac{\Delta g}{g}\right)}}=\left(1-\frac{\Delta g}{g}\right)^{\frac{1}{2}}$$

$$\therefore \quad \frac{T'}{T}=1+\frac{1}{2}\ \frac{\Delta g}{g} \qquad \left[\because \ \frac{\Delta g}{g}<<1\right]$$

$$\therefore \quad \frac{T'-T}{T}=\frac{1}{2}\ \frac{\Delta g}{g}$$

It is the loss of time per second. So the loss of time per day is

$$\Delta T=86400\times\frac{1}{2}\frac{\Delta g}{g}=43200\ \frac{\Delta g}{g} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3{\cdot}22)$$

● **Example 3.15. The length of a pendulum is 1 m. It completes 20 oscillates is 40 sec. Calculate acceleration due to gravity at the place.**

○ **Solution :** Time period of the pendulum, $T=\frac{40}{20}=2$ sec.

$$\text{Now,} \quad T=2\pi\sqrt{\frac{l}{g}} \qquad \therefore \quad g=\frac{4\pi^2}{T^2}\,l$$

$$\therefore \quad g=\frac{4\times9{\cdot}87\times1}{(2)^2} \qquad [\ l=1\,m,\ T=2\text{ sec.}\ ]$$

$$\therefore \quad g=9{\cdot}87\ ms^{-2}$$

- **Ex. 3.16. If the length of seconds pendulum increases by 2%, how much time will it lose of gain per day ?**

○ **Solution :** The required relation $\Delta T = 43200\,\dfrac{\Delta l}{l}$ [vide equation 3·21]

Here $\dfrac{\Delta l}{l} = 0\cdot02$

$\therefore \quad \Delta T = 43200 \times 0\cdot02 = 864$ sec.

So, the pendulum will lose 864 sec per day.

- **Ex. 3.17. A pendulum gives correct time when acceleration due to gravity is 980 cm /s^2. It is taken to a place where acceleration due to gravity is 978 cm / s^2. Calculate its loss of time per day.**

○ **Solution :** The required relation : $\Delta T = 43200\,\dfrac{\Delta g}{g}$

Here $\Delta g = 980 - 978 = 2$ cm/s^2 and $g = 980$ cm/s^2

$\therefore \quad \Delta T = 43200 \times \dfrac{2}{980} = 88\cdot16$. So loss of time/day = 88·16 sec.

- **Ex. 3.18. A faulty second's pendulum loses 10 sec per day. How its length be changed so that it may give correct time again ?** $g = 9\cdot8\, ms^{-2}$

○ **Solution :** Evidently, time period of the pendulum is greater than 2 sec. Since time period increase with increases of length $\left(T \;\alpha\; \sqrt{l}\right)$, so to get correct time the length of the given pendulum has to be shortened.

Let the present length is l_1 and actual length be l

$\therefore$ the length should be decreased by $x = (l_1 - l)$ to get correct time.

Now, $\dfrac{T_1}{T} = \sqrt{\dfrac{l_1}{l}} = \sqrt{\dfrac{l+x}{l}} = \left(1+\dfrac{x}{l}\right)^{\frac{1}{2}} = 1 + \dfrac{1}{2}\,\dfrac{x}{l} \quad [x << l]$

$\therefore \quad \dfrac{T_1 - T}{T} = \dfrac{1}{2}\,\dfrac{x}{l}$. It is the loss of time per sec.

So, the loss of time per day is $\Delta T = 86400 \times \dfrac{1}{2}\,\dfrac{x}{l} = 43200\dfrac{x}{l}$

$\therefore \quad x = \dfrac{\Delta T \cdot l}{43200} = \dfrac{\Delta T \cdot g}{43200\,\pi^2} \quad \left[\because \; l = \dfrac{g}{\pi^2}\right]$

$\therefore \quad x = \dfrac{10 \times 980}{43200 \times 9\cdot87}$ cm $=$ **0·023 cm**

$\therefore$ the length of the pendulum has to be decreased by 0·023 cm.

- **Ex. 3.19. A pendulum gives correct time on the surface of the earth, when it is taken to the top of a tower it loses 20 sec per day. Calculate height of the tower. Radius of the earth 6400 km.**

○ **Solution :** The required height is $h = R\left(\dfrac{T'}{T} - 1\right)$ [vide equation 3.19]

Here, R = 6400 km, $T' = \dfrac{86400}{86380} \times 2 = \dfrac{4320}{4319} \times 2$ and T = 2 sec.

$\therefore \quad h = 6400 \times \left(\dfrac{4320}{4319} - 1\right) = 6400 \times \dfrac{1}{4319} = 1\cdot48$ km

● **Ex 3·20. Two simple pendulums having time periods 1·8 sec and 2 sec. are set into oscillation simultaneously. After what time faster pendulum will execute 1 oscillation more than the slower one. Calculate the number of complete oscillations made by the faster pendulum during this time.** **[H. S. 1996]**

○ **Solution :** Let after t sec. the faster pendulum makes one oscillation more than the slower one.

In time t the number of oscillation executed by the first pendulum $n_1 = \frac{t}{1 \cdot 8}$ and the number of oscillations completed by the other pendulum, $n_2 = \frac{t}{2}$

∴ By question, $\frac{t}{1 \cdot 8} - \frac{t}{2} = 1 \Rightarrow t = \mathbf{18\ sec.}$

∴ The number of oscillations completed by the faster pendulum in time $t = 18$ sec is

$$n = \frac{t}{1 \cdot 8} = \frac{18}{1 \cdot 8} = \mathbf{10}$$

● **Ex. 3.21. A pendulum of length 60 cm is attached with an aeroplane which is moving upwards making an angle of $\alpha = 30°$ with the horizontal with an acceleration of $a = 4\ \mathrm{ms^{-2}}$. What is the time period of the pendulum ?** **[J. E. E. 1990]**

○ **Solution :** Horizontal acceleration of the plane, $a_x = a \cos 30°$ and effective vertical acceleration, $a_y = g + a \sin 30°$.

∴ Resultant accelaration, $g' = \sqrt{a_x^{\,2} + a_y^{\,2}}$

$$\therefore \quad g' = \sqrt{(a \cos 30°)^2 + (g + a \sin \theta)^2} \qquad [a = 4\ ms^{-2}]$$

$$= \sqrt{(2\sqrt{3})^2 + (9 \cdot 8 + 2)^2} = \sqrt{151 \cdot 24} = 12 \cdot 3\ ms^{-2}$$

∴ time period, $\mathrm{T} = 2\pi \sqrt{\frac{l}{g}} = 2 \times 3 \cdot 14 \times \sqrt{\frac{0 \cdot 6}{12 \cdot 3}} = \mathbf{1 \cdot 38\ sec.}$

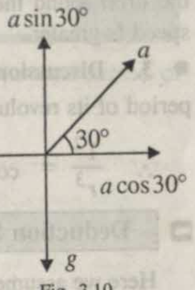


Fig. 3.10

✪ 3.22. Planetary motion and Kepler's law ✪

In 498 AD, the Indian mathematician and the astronomer Aryabhatta made a large number of astronomical observations and proposed that earth revolves around the sun and at the same time it rotates about its own axis. In 1543, the Polish astronomer Nicolus Copernicus advanced the idea that not only the earth, but also the planets move round the sun in circular orbit around the sun.

German astronomer Johannes Kepler after a thorough and careful study of the vast accumutation of data collected by the Danish astronomer Tycho Brahe concerning the motions of the planets round the sun, was able to unify the observations into three emperical laws related to planetary motion. These laws are known as **Kepler's law** of planetary motion which form the basis of the famous. Newton's law of universal gravitation. The laws are :

1. Each planet revolves round the sun in an elliptical orbit with the sun at one focus of the ellipse.

2. The line joining the sun and the planet sweeps out equal areas in equal times i.e. the areal velocity of planet around the sun always remains constant.

3. The square of the period of revolution of any planet around the sun is proportional to the cube of its mean distance from the sun.

- **1. Discussion of the first law :** The first law known as law of orbits, gives the idea regarding nature of the orbits of the planets round the sun. In fig 3.11. Orbit of one planet is shown. The orbit is elliptical with the sun at one focus S_1 where S_1 and S_2 are the two focii of the orbit. AB and CD are the major and minor axes of the orbit. Evidently, the distance of the planet from the sun is not constant. The point A, where the planet is nearest to the sun is called **Perihelion** and the point B, where it is farthest from the sun is known as **apihelion**.

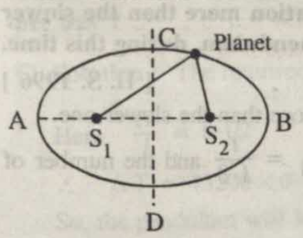


Fig. 3.11

While moving along the elliptic orbit, the speed of a planet increases when it comes nearer to the sun. Thus the speed of a planet is greatest at the A (Perihelion) and least at B (apihelion). In the case of the earth, the maximum and minimum speeds are 18·8 miles / sec and 18·2 miles / sec. The near equality of these two speeds show that the earth's orbit is nearly circular.

- **2. Discussion of the second law :** The second law also known as law of area, specifies that the speed of the planet varies with its position on the orbit. In fig 3.12 the shaded regions represents the areas described by the line joining the planet to the sun in equal time interval. From the fig, all the areas SAB, SCD and SEF are equal. Evidently, the speed of a planet in the orbit round the sun is not constant. Nearer the planet to the sun, its speed is greater.

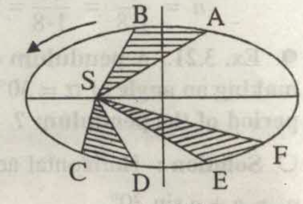


Fig. 3.12

- **3. Discussion of the third law :** If r be the mean distance of the planet from the sun and T the time period of its revolution, then by Kepler's third law $T^2 \alpha r^3$

$\therefore \frac{T^2}{r^3} =$ constant for all planets.

Deduction from Newton's law of gravitation :

Here we assume that the planet's orbit is circular. Let mass of the planet be m and radius of the orbit r. Mass of the sun is M. The force of attraction of the sun on earth is $F = \frac{GMm}{r}$. This force provides the necessary centripetal force. $F_c = \frac{mv^2}{r}$

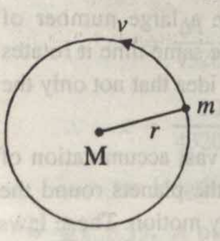


Fig. 3.13

$$\therefore \frac{mv^2}{r} = \frac{GMm}{r^2} \Rightarrow v^2 = \frac{GM}{r} \ldots\ldots\ldots\ldots\ldots (i)$$

If T be the time period of revolution of the planet around the sun.

$$v = \frac{2\pi r}{T} \quad \therefore \quad v^2 = \frac{4\pi^2 r^2}{T^2} \ldots\ldots\ldots (ii)$$

Hence from equation (i) and (ii), $\frac{4\pi^2 r^2}{T} = \frac{GM}{r}$

$$\therefore \quad T^2 = \left(\frac{4\pi^2}{GM}\right) r^3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.23)$$

$$\therefore \quad T^2 \propto r^3 \quad \left[\because \frac{4\pi^2}{GM} \text{ is a constant} \right]$$

It is Kepler's third law .

● **Example 3.22 : The radius of the circular orbit of the earth round the sun is $1{\cdot}5 \times 10^8$ km and period of revolution 365 days. The radius of the moon's orbit round the earth is $3{\cdot}8 \times 10^5$ and period of revolution is 27 days. Calculate the ratio of the masses of the sun and the earth.**

○ **Solution :** It T_1 be the period of revolution of the earth round the sun, then

$$T_1^{\,2} = \left(\frac{4\pi^2}{GMs}\right) r_1^{\,3} \quad [\ r_1 = \text{radius of the earth's orbit},\ \ M_s = \text{mass of the sun}\ ]$$

Similarly in the case of the moon, its time period of revolution is

$$T_2^{\,2} = \left(\frac{4\,\pi^2}{GMe}\right) r_2^{\,3} \quad [\ r_2 = \text{radius of the moon's orbit},\ M_e = \text{mass of the earth}\ ]$$

$$\therefore \quad \frac{T_1^{\,2}}{T_2^{\,2}} = \frac{M_e}{M_s} \times \frac{r_1^{\,2}}{r_2^{\,2}}$$

$$\therefore \quad \frac{M_s}{Me} = \frac{T_2^{\,2}}{T_1^{\,2}} \cdot \frac{r_1^{\,2}}{r_2^{\,2}} = \left(\frac{27}{365}\right)^2 \times \left(\frac{1{\cdot}5 \times 10^8}{3{\cdot}8 \times 10^5}\right)^3 = 3{\cdot}37 \times 10^5$$

$$M_s : Me = 3{\cdot}37 \times 10^5 : 1$$

3.23. Calculation of mass of the sun

Let a planet revolve round the sun of mass M in a circular orbit of radius *r*. If its time period of revolution be T

$$T^2 = \frac{4\,\pi^2}{GM}\, r^3 \qquad \therefore \quad M = \frac{4\,\pi^2\, r^3}{GT^2}$$

If we consider our earth as one of the planets of sun then,

$r = 1{\cdot}5 \times 10^{11}\ m$, T = 365 days = 365 × 86400 sec.

$$\therefore \quad M = \frac{4 \times 9{\cdot}87 \times (1{\cdot}5 \times 10^{11})^3}{6{\cdot}67 \times 10^{-11} \times (365 \times 86400)^2} = 2 \times 10^{30}\ \text{kg}$$

(a) Calculation of mass of the earth :

We can calculate the mass of the earth by considering the motion of the moon round the earth.

The radius of the orbit of the moon, $r = 3{\cdot}84 \times 10^5$ km. and the period of revolution of the moon, T = 27·3 days = 27·3 × 86400 sec.

Then mass of the earth can be calculated from

$$M = \frac{4\,\pi^2\, r^2}{GT^2} = \frac{4 \times 9{\cdot}87 \times (3{\cdot}84 \times 10^5)^3}{6{\cdot}67 \times 10^{-11} \times (27{\cdot}3 \times 86400)^2} = 6{\cdot}0 \times 10^{24}\ \text{kg}$$

(b) Gravitational force between the sun and the earth :

The gravitational attraction between the sun and the earth is $F = \dfrac{GMs\,Me}{r^2}$

mass of the sun, $Ms = 2 \times 10^{30}$ kg, mass of the earth $Me = 6 \times 10^{24}$ kg and a Average distance between sun and the earth, $r = 1{\cdot}5 \times 10^{11}$ m.

$$\therefore \quad F = \frac{6{\cdot}67 \times 10^{-11} \times 2 \times 10^{30} \times 6 \times 10^{24}}{\left(1{\cdot}5 \times 10^{11}\right)^2} = 3{\cdot}56 \times 10^{22}\ \text{N}$$

(c) Gravitational force between the earth and the moon :

The gravitational attraction between the sun and the earth is $F = \dfrac{GMe\,M_m}{r_1^3}$

mass of the earth $Me = 6 \times 10^{24}$ kg, mass of the moon, $M_m = 7 \cdot 33 \times 10^{23}$ kg

The average distance between the earth and the moon, $r_1 = 3 \cdot 84 \times 10^8$ m

$$\therefore \quad \text{Gravitational force,} \quad F = \frac{6 \cdot 67 \times 10^{-11} \times 6 \times 10^{24} \times 7 \cdot 33 \times 10^{22}}{\left(3 \cdot 84 \times 10^8\right)^3} = 1 \cdot 99 \times 10^{20}\,\text{N}$$

$$\therefore \quad \frac{\text{Gravitational force between sun and earth}}{\text{gravitational force between earth and moon}} = \frac{3 \cdot 56 \times 10^{22}}{1 \cdot 99 \times 10^{20}} = 178 \cdot 8$$

3.24. Satellites : Natural and Artificial

A heavenly body revolving round the planet in a stable orbit is called a natural satellite.

In solar system there are nine planets of which earth is one. These planets have satellites revolving around them. The planets mercury, venus and pluto do not have any satellite. The earth has one, mars and neptune—two each, uranous—five, saturn—ten and Jupiter has twelve satellites. These heavenly bodies going around a planet are called natural satellites. Moon is natural satellite of the earth. It goes round the earth in about 27·3 days in an almost circular orbit of radius 3·84 × 10^5 km.

A satellite launched by man and put in stable orbit round a planet is called an **artificial satellite**.

Since 1957 many artificial satellites of the earth have been launched in their orbits. Many satellites were launched by different countries.

India also joined the space club on April 19, 1975 by putting in orbit **Aryabhatta**. India has launched many satellites.

3.25. Principle of launching a satellite

Consider a tower of height several kilometers on the earth's surface. Now a stone is thrown from the top of the tower with a horizontal velocity. After moving along a parabolic path it may hit the earth at P_1 [Fig 3·14]. If the stone is thrown with greater horizontal velocity it will hit the earth at P_2 away from P_1. If the horizontal velocity of projection is increased gratually, the stone will hit the ground at increasing greater distance. Ultimately at a certain horizontal velocity of projection, it may so happen that stone be in state of free fall i.e. earth's attraction on it is just equal to centripetal force necessary to move along a circle. The result will be that the stone will follow a circular path around the earth. It is now an artificial satellite and the horizontal velocity of projection is its orbital velocity.

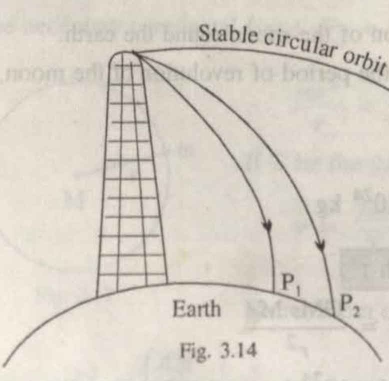


Fig. 3.14

Artificial satellites are actually launched from the surface of the earth by rockets. Behind the satellite there are a number of rockets. These rockets controls the direction, magnitude and velocity of the satellite. On reaching a predetermined height the satellite acquired necessary horizontal velocity. Now, the satellite is put into stable orbit and will rotate around the earth in a circular orbit. Its velocity is called **orbital velocity**.

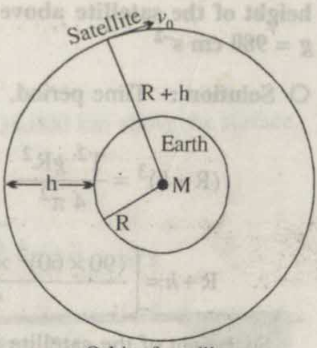
✪ 3.26. Orbital velocity and period of revolution of a satellite ✪

(A) Orbital velocity (V_0) :

We now ask if there is any prescribed value of the velocity of the satellite, for it to be able to continue to move in an orbit ? The answer is, yes. The reason is that for a body to move in a circular orbit, a centripetal force (which depends on velocity) must be provided.

Orbital velocity of a satellite is the velocity required to put the satellite in its orbit round the earth. We shall now calculate the value of orbital velocity, V_0.

Suppose a satellite of mass *m* goes round the earth in a circular orbit of radius *r* with a uniform speed V_0. If the height of the satellite above the earth's surface be *h*, then $r = R + h$, where R is the radius of the earth.



Orbit of satellite

Fig. 3.15

So, the centripetal force necessary for the satellite is $F_c = \dfrac{mv_0^2}{r}$

Now, the gravitational attraction between the earth and the satellite provides this centripetal force to keep the satellite in circular orbit.

$$\therefore \quad \frac{GMm}{r^2} = \frac{mv_0^2}{r} \qquad \therefore \quad v_0 = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{R+h}} \quad \ldots\ldots\ldots\ldots\ldots (3.24)$$

Thus the orbital velocity of a satellite is independent of the mass of the satellite, but depends on the radius of the orbit and mass of the planet around which it revolves.

Now, the acceleration due to gravity on the surface of the planet is

$$g = \frac{GM}{R^2} \qquad \therefore \qquad GM = gR^2$$

$$\therefore \quad v_0 \sqrt{\frac{gR^2}{R+h}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.25)$$

If the satellite is a few hundred kilometer above the earth's surface (say, 100 to 300 km), we can replace (R + h) by R.

$$\therefore \quad v_0 = \sqrt{\frac{gR^2}{R}} = \sqrt{gR} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.26)$$

To calculate v_0 for a satellite near the earth's surface we put $g = 9{\cdot}8 \text{ m/s}^2$, $R = 6400 \times 10^3$ m

$$\therefore \quad v_0 = \sqrt{9{\cdot}8 \times 6400 \times 10^3} \text{ ms}^{-1} = 7{\cdot}92 \text{ km s}^{-1}$$

(B) Period of revolution of the satellite :

It is the time taken by a satellite to go once round the earth. If T be the time period, then

$$T = \frac{\text{circumference of the orbit}}{\text{orbital velocity}} = \frac{2\pi(R+h)}{v_0}$$

$$\therefore \quad T = \frac{2\pi(R+h)^{\frac{3}{2}}}{\sqrt{gR^2}} = \frac{2\pi}{R}\sqrt{\frac{(R+h)^3}{g}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.27)$$

If the satellite is near the earth's surface then R + *h* = R

$$\therefore \quad T = \frac{2\pi}{R}\sqrt{\frac{R^3}{g}} = 2\pi\sqrt{\frac{R}{g}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.28)$$

Putting $R = 6400 \times 10^3 \text{m}$ and $g = 9{\cdot}8 \text{ m/s}^2$, we get

$$T = 2\pi \sqrt{\frac{6400 \times 10^3}{9 \cdot 8}} \text{ sec} = 84 \text{ min} = 1 \text{ hr } 24 \text{ min}$$

● **Example 3.23. An earth's satellite makes a circle around the earth in 90 minutes. Calculate the height of the satellite above the surface of the earth. Given, radius of the earth is 6400 km. and $g = 980 \text{ cm s}^{-2}$**

○ **Solution :** Time period, $T = 2\pi \sqrt{\frac{(R+h)^3}{gR^2}}$

$$(R+h)^3 = \frac{T^2 \, gR^2}{4\pi^2} \qquad \therefore \quad R+h = \sqrt[3]{\frac{T^2 \, gR^2}{4\pi^2}}$$

$$\therefore \quad R+h = \left[\frac{(90 \times 60)^2 \times 4 \cdot 8 \times (6400 \times 1000)^2}{4 \times (9 \cdot 87)}\right]^{\frac{1}{3}} \text{ metre} = 6668 \text{ km}$$

So, height of the satellite is $= 6668 - 6400 = 268$ km

❂ 3.27. Geostationary Satellite ❂

The earth revolve from west to east round its own axis in a day or in 24 hour. So, its period of revolution is 24 hrs. Now if a satellite is made to revolve from west to east in a circular orbit concentric and coplanar with the equitorial plane of the earth, the satellite may appear to be moving with respect to the earth. However, if the period of revolution of the satellite is made just 24 hours i.e. equal to the period of revolution of the earth about its axis, then the satellite will stay over the same place on the earth. Hence it will appear stationary with respect to the earth. Such a satellite is known as **geostationary** satellite. Hence a satellite will always appear to be over the same place relative to an observer on the earth, is a geo stationary satellite.

The orbit of the geostationary satellite is known as parking orbit. Such a satellite is very useful for global communication.

A satellite will appear stationary if the following conditions are satisfied :

(i) The satellite should revolve in an orbit concentric and coplanar with the equitorial plane.

(ii) Its sense of rotation should be same as that of the earth i.e. from west to east.

(iii) Its period of revolution should be same as that of the earth i.e. 24 hrs.

● **Calculation of the height of the parking orbit :**

Let mass of the earth M ; mass of the satellite m ; radius of the parking orbit $r = R + h$; orbital speed, v_0 and time period, T.

Now, earth's gravitational attraction on the satellite provide the centripetal force.

$$\therefore \quad \frac{mv_0^2}{r} = \frac{GMm}{r^2} \quad \therefore \quad v_0 = \sqrt{\frac{GM}{r}} = \sqrt{\frac{gR^2}{r}} \qquad [\because \ GM = gR^2]$$

Now, the period of revolution, $T = \frac{2\pi r}{v_0}$

$$\therefore \quad T = \frac{2\pi r \sqrt{r}}{\sqrt{gR^2}} = \frac{2\pi r^{\frac{3}{2}}}{\sqrt{gR^2}}$$

$$\therefore \quad r = \sqrt[3]{\frac{gR^2\,T^2}{4\pi^2}}\ ;$$

Here $g = 9\cdot8\ ms^{-2}$, $R = 64\times10^5\ m$, $T = 86400$ sec then,

$$r = \left[\frac{9\cdot8\times\left(6\cdot4\times10^6\right)^2\times(86400)^2}{4\times9\cdot87}\right]^{\frac{3}{2}} \text{ metres } = 42400 \text{ km}$$

$\therefore$ The radius of the parking orbit is **42400** km.

So, a geostationary satellite should revolve at a height of 42400 – 6400 = 36,000 km above the surface of the earth.

- **Velocity of geostationary satellite :**

The orbital velocity of geostationary satellite is $v = \frac{2\pi r}{T} = \frac{2\pi\ (42400)}{86400} = 3\cdot1\ \text{km/s}$

3.28. Gravitational field and its intensity

The gravitational attraction between two masses exists even when the masses are at a distence from each other. For this reason, gravitational interaction are sometimes called as **'action at a distance'** To explain this interaction between masses, we introduce the concept of field.

According to this field concept, a material body sets up its gravitational field around it. If another material body is placed inside the field, it will experience a force. So we define gravitational field as :

The space around a material body in which its gravitational attraction can be experienced is called gravitational field.

It is the gravitational field due to which a body exerts force on any other material body placed inside the field.

Intensity of the field :

The intensity of gravitational field of a body at any point in the field is defined as the force experienced by a unit mass placed at that point.

If a small mass m placed in the field experiences a force F, then intensity of the field is $E = \frac{F}{m}$.

$\therefore\ F = Em$

Gravitational intensity at a point due to a point mass :

We consider a point mass m. To calculate intensity at a point at a distence r, we put a unit mass at the point. In fig 3.16, P is the point where we want to calculate the intensity. We place a unit mass at P. Now, the gravitational force on the unit mass kept at P is

$$F = \frac{Gm\times1}{r^2} = \frac{Gm}{r^2}$$

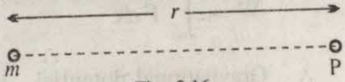


Fig. 3.16

By definition, F is the field intensity E.

$$\therefore \quad E = \frac{Gm}{r^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3\cdot29)$$

In vector notation, $\vec{E} = \frac{Gm}{r^2}\,\hat{r}$ [$\hat{r}$ is a unit vector along the line joining the masses]

3.29. Intensity of gravitational field due to earth

Let M be the mass of the earth. P is a point at a distance r from the centre 'O' of the earth where we want to calculate the gravitational intensity. We place a small mass m at the point P. We assume that whole mass of the earth is at its centre.

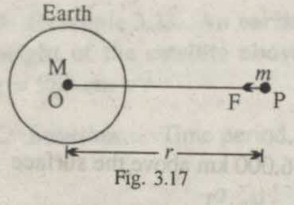


Fig. 3.17

According to Newton's law of gravitation, the force on the mass m at P is $F = \dfrac{GMm}{r^2}$

Then, the gravitational field intensity due to the earth at P is

$$E = \frac{F}{m} = \frac{GM}{r^2} \text{ directed along PO} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3{\cdot}30)$$

It follows that intensity decreases as the distance r increases and becomes zero at infinity.

Again acceleration due to gravity at P is given equation (3.30). So it follows that intensity of the gravitational field at a point is equal to acceleration due to gravity at that point. So, gravitational field at a point is $\dfrac{GM}{r^2}$ means that a mass placed at the point will accelerate towards the earth with an acceleration $a = \dfrac{GM}{r^2}$

The gravitational field on the surface of the earth can be obtained by putting $r = R$ in equation (3·29). So, the gravitational field at a point on the surface of the earth is

$$E_0 = \frac{GM}{R^2} = g \quad [\ g = \text{acceleration due to gravity on the earth's surface}\ ]$$

3.30. Gravitational potential

Gravitational potential at a point in the gravitational field is defined as the work done to bring a unit mass from infinity upto the point.

If W be to the work done to bring a mass m, then gravitational potential is $V = \dfrac{W}{m}$.

It is a scalar quantity and its unit is J kg^{-1} in SI system and in C. G. S. system the unit is erg gm^{-1}

- **Potential due to a point mass :** Let m be the mass of the particle. We like to calculate the gravitational potential at distance r from the point mass.

Now a material object of unit mass is brought from infinity into the gravitational field of the particle. When the unit mass is at a distance x from the particle, the gravitational force on the unit mass is

$$F = \frac{Gm}{x^2}.$$

The work done to displace the unit mass by a small distance in the direction of the force is, $dW = F\,dx$

Hence the total work done to bring the unit mass from infinity upto the distance r is

$$W = \int_{\infty}^{r} F\,dx$$

$$\therefore \quad \text{Gravitational potential, } V = \int_{\infty}^{r} \frac{Gm}{x^2}\,dx = -\frac{Gm}{r}$$

$$\therefore \quad V = -\frac{Gm}{r} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3{\cdot}31)$$

- **Potential due to earth :** Since earth is a spherical body, it can be treated as a particle, whole mass of the earth being at its centre.

Then the gravitational potential at a point at a distance r ($r > R$, radius of the earth) from the centre of the earth is $V = -\frac{GM}{r}$ .... (3·32)

where M is the mass of the earth and $r = R + h$.

So, gravitational potential at a point (i) is always negative (ii) increases as one moves towards infinity. It is zero at infinity. (iii) On the surface of the earth it is given by $V = -\frac{GM}{R}$.

- **Potential difference :** The potential difference between two points A and B in a gravitational field is the work done by the external agency in moving unit mass from A to B. If W_{AB} be the work done to move a mass *m* from A to B then the potential difference between two points is : $V_B - V_A = \frac{W_{AB}}{m}$. It has same unit as potential.

- **Example 3.24. Three point masses each of mass *m* are placed at the vertices of an equilateral triangle of side '*a*'. Calculate gravitational field and potential due to three masses at the centroid of the triangle.**

○ **Solution :** Situation is shown in fig 3.18. Three particles each of mass *m* are placed at the vertices A, B and C of the equilateral triangle ABC. O is the centroid of the triangle. Evidently,

$$AP = AB \, . \sin 60° = \frac{\sqrt{3}}{2} a. \text{ and } AO = BO = CO = \frac{2}{3}\,(AP) = \frac{a}{\sqrt{3}}$$

(i) To find, gravitational field at O, we measure force on unit mass placed at the point O due to three equal masses placed at equal distance $AO = BO = CO = \frac{a}{\sqrt{3}}$. The three force are E_A, E_B and E_C. They are all equal and by symmetry the resultant force is zero.

So, the resultant field at O is E = 0.

(ii) Now, the gravitational potential at O is

$$V_g = -\frac{Gm}{AO} - \frac{GM}{BO} - \frac{GM}{CO}$$

$$\therefore \quad V_g = -\frac{3\,GM}{\frac{a}{\sqrt{3}}} = -3\sqrt{3}\ \frac{GM}{a}.$$

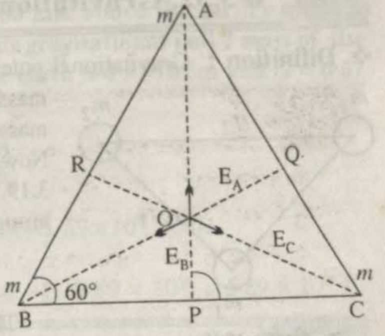


Fig. 3.18

3.31. Gravitational potential energy

The gravitational potential energy of a body at a point is defined as the amount of work done in bringing the body from infinity upto the point.

So, the potential energy of a body of mass *m* when it is brought from infinity upto a point at a distance r from another point mass m' is

$$E_p = (\text{potential at the point due to } m') \times \text{ mass } (m)$$

$$= -\frac{Gm'}{r} m = -\frac{Gm'm}{r}.$$

Similarly, potential energy of a mass m at a point situated at a height *h* from the surface of the earth is $E_p = \frac{-GMm}{R+h}$. (M = mass of the earth and R = radius of the earth.)

If the body is situated on the surface of the earth, then its potential energy is

$$E_p = -\frac{GMm}{R} = -\frac{gR^2m}{R} \qquad \left[\because \ g = \frac{GM}{R^2}\right]$$

$$\therefore \quad E_p = -mgR \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 3{\cdot}33$$

- **Special discussion :** We take the gravitational potential energy of a body of mass m at a height h from the earth's surface as $E_p = mgh \ (h << R)$. In this case, the potential energy at a point on the surface of the earth is taken to be zero. We can deduce the relation as follows :

Potential energy of a mass m on the surface of the earth $= \frac{-GMm}{R}$

∴ Potential energy at a height h is $E_p = \frac{-GMm}{R+h}$.

$$\therefore \quad E_p = \frac{-GMm}{R\left(1+\frac{h}{R}\right)} = \frac{-GMm}{R}\left(1-\frac{h}{R}\right) \qquad [\because \ h << R]$$

$$= \frac{-GMm}{R} + \frac{GM}{R^2}\cdot mh = \frac{-GMm}{R} + mgh. = \text{Potential energy at the surface } + \text{ mgh.}$$

If potential energy at the surface is taken to be zero then $E_p = mgh$

✪ 3.32. Gravitational potential energy of a system of masses ✪

❖ **Difinition :** Gravitational potential energy of a system of masses is the work done to bring these masses from infinity to form the system. Let the system consists of three masses m_1, m_2 and m_3. Initially they were at infinity from each other. Now, we have to calculate the work done to form the system shown in fig 3.19. m_1 was at the position as shown in the fig. Now, the mass m_2 is brought from infinity at a distance r_1 from m_1. The work done is

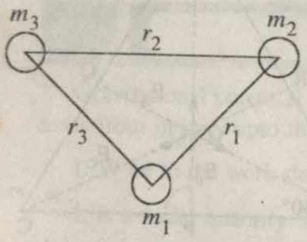


Fig. 3.19

$$W_1 = \frac{-Gm_1m_2}{r_1}$$

Then the mass m_3 is brought from infinity. So it is at a distance r_2 from m_2 and at r_3 from m_1.

Work done to bring m_3 at a distance r_2 from m_2, $W_2 = \frac{-Gm_2r_3}{r_2}$ and the work done to bring m_3 at a distance r_3 from m_1, $w_3 = -\frac{Gm_1m_3}{r_3}$

∴ total work done or the potential energy of the system is $E_p = -\left[\frac{Gm_1m_2}{r_1} + \frac{Gm_2m_3}{r_2} + \frac{Gm_1m_3}{r_3}\right]$

- **Example 3.25. The radius of the earth is 6·37 × 10⁸ cm, mean density 5·5 gm / c.c. and G = 6·67 × 10⁻⁸ C. G. S. Calculate potential at a point on the earth's surface.**

○ **Solution :** We consider the earth as a solid sphere. The potential at any point on the surface is

$$V = -\frac{GM}{R} = -\frac{G}{R} \times \frac{4}{3}\pi R^3 \rho = -\frac{4}{3}\pi\rho G R^2$$

$$\therefore \quad V = -\frac{4}{3} \times 3{\cdot}14 \times 5{\cdot}5 \times 6{\cdot}67 \times 10^{-8} \times \left(6{\cdot}37 \times 10^{8}\right)^2$$

$$= -62{\cdot}27 \times 10^{10} \quad \text{erg / gm.}$$

3.33. Energy of an orbiting satellite

When a satellite revolves around a planet in its orbit, it has both potential energy and kinetic energy.

Now, the gravitational potential energy of the satellite of mass m when it is in the orbit at a height h from the surface of the earth is

$$E_p = -\frac{GMm}{R+h}. \quad [\text{ vide art. } 3{\cdot}32\ ]$$

To calculate kinetic energy we assume that the orbital velocity of the satellite be v_0. Since earth's attraction on the planet provide the necessary centripetal force, we write

$$\frac{mv_0^{\ 2}}{R+h} = \frac{GMm}{(R+h)^2} \quad \therefore \quad mv_0^{\ 2} = \frac{GMm}{R+h}.$$

$\therefore$ Kinetic energy of the satellite, $E_k = \frac{1}{2}\,\frac{GMm}{R+h}$.

$\therefore$ Total energy of the satellite, $E = E_p + E_k$

$$\therefore \quad E = -\frac{1}{2}\,\frac{GMm}{R+h}.$$

So, the total energy of the satellite is negative. For this reason we say that the planet and the orbiting satellite form a **bound system**.

● **Example 3.26. A satellite is revolving the earth at a height 400 km. above the earth's surface. How much energy it should be supplied so that it will escape earth's gravitational pull ? mass of the satellite = 200 kg, mass of the earth = 6·0 × 10^{24} kg, radius of the earth 6·4 × 10^6 m and G = 6·67 × 10^{-11} SI.**

○ **Solution :** Total energy of the satellite in the orbit, $E = \frac{-GMm}{2\,(R+h)}$

$$\therefore \quad E = -\frac{6{\cdot}67\times10^{-11}\times6{\cdot}0\times10^{24}\times200}{2\left(6{\cdot}4\times10^{6}+0{\cdot}4\times10^{6}\right)} = -\frac{6{\cdot}67\times6\times2}{2\times6{\cdot}8}\times10^{9} = -5{\cdot}89\times10^{9}\ \text{Joule}$$

$\therefore$ Energy required for the satellite to escape from gravity is = 0 – (– 5·89 × 10^9) = 5·89 × 10^9 J

3.34. Escape velocity

In general, if we throw a stone vertically upwards, it rises to a certain maximum height and finally returns to the point of projection. If the velocity of projection is increased, maximum height attained also increases. If we gradually go on increasing the velocity of projection of the stone, at a particular velocity it may so happen that the stone never returns to the point of projection. Now, the stone has overcome earth's gravity and lost in space. This particular velocity is called escape velocity.

❖ **Definition : The minimum velocity with which a body must be projected so as to enable it just to over come the gravitational pull and lost in space, is called escape velocity.**

Expression for escape velocity :

We apply principal of conservation of energy to obtain the expression for escape velocity, v_e. The gravitational potential energy of a body of mass m on the surface of the earth is

$E_{p_1} = -\frac{GMm}{R}$ [M = mass of the earth, R = radius of the earth] when the body escapes earth's attraction, its potential energy becomes $E_{p_2} = 0$

$\therefore$ Increase of potential energy, $\Delta E_p = E_{p_2} - E_{p_1} = 0 - \left(-\dfrac{GMm}{R}\right) = \dfrac{GMm}{R}$

Now, if the escape velocity is v_e, its kinetic energy is $E_k = \dfrac{1}{2}mv_e^2$

So, by conservation of energy, $E_k = \Delta E_p$ or, $\dfrac{1}{2}mv_e^2 = \dfrac{GMm}{R}$.

$$\therefore \quad v_e = \sqrt{\frac{2\,GM}{R}} = \sqrt{\frac{2\,gR^2}{R}} = \sqrt{2gR} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3{\cdot}34)$$

Thus we see that like orbital velocity, escape velocity does not depend on mass of the body to be projected.

- **Discussion :**

- (i) **Value of v_e for earth :** In this case $R = 64 \times 10^5 m,\ g = 9{\cdot}8\ m/s^2$

$$\therefore \quad v_e = \sqrt{2 \times 9{\cdot}8 \times 64 \times 10^5}\ \text{m/s} = \mathbf{11{\cdot}2\ km / sec}$$

- (ii) **Relation between v_e and v_0 :** The orbital velocity of a satellite which is near the earth's surface is $v_0 = \sqrt{gR}$

$$\therefore \quad v_e : v_0 = \sqrt{2gR} : \sqrt{gR} = \sqrt{2} : 1$$

- (iii) In the case of a body projected from a point at a height *h* from the earth's surface, we write

$$\frac{1}{2}\,mv_e^2 = \frac{GMm}{R+h} \qquad \therefore \qquad v_e = \sqrt{\frac{2GM}{R+h}}$$

- (iv) squaring both sides of equation (3·34), we have $v_e^2 = 2gR = (2R)\,g$

$\therefore \quad v_e^2 =$ diameter of the earth × acceleration due to gravity.

So, the square of the escape velocity is equal to the product of diameter of the earth and acceleration due to gravity on the surface of the earth.

- (v) If the earth is supposed to be a sphere of uniform density, ρ

then $M = \dfrac{4}{3}\pi R^3 \rho$

$$\therefore \text{ escape velocity,} \quad V_e = \sqrt{\frac{2\,GM}{R}} = \sqrt{\frac{2G}{R} \cdot \frac{4}{3}\pi R^3 \rho}$$

$$\therefore \quad V_e = 2R\sqrt{\frac{2\pi\rho G}{3}}$$

$$= \frac{2\sqrt{2}}{\sqrt{3}}\,R\sqrt{\pi\rho G} = 1{\cdot}63\,R\sqrt{\pi\rho G} \quad \ldots\ldots\ldots\ldots\ldots (3{\cdot}35)$$

It is the expression for escape velocity in terms of density and radius of the earth.

- (vi) The expression for escape velocity has been derived by neglecting the resistance of earth's atmosphere. At such high speed $v_e = 11{\cdot}2$ km / s, air resistance is also very high. In practice the actual escape velocity on the surface of the earth is much higher than given by equation 3·34.

- (vii) **Light gases are rarer in earth's atmosphere :** Light gases like hydrogen, helium etc. are rare in earth's atmosphere. It can be explained by the concept of escape velocity. If a particle has velocity greater than escape velocity for the earth, then they will escape from earth's atmosphere.

At the early stage of formation of the earth, the temperature was quite high. So, the r. m. s. velocities of molecules of lighter gases were very large and a larger part of these gases had velocities greater than the escape velocity. Hence they were lost in space. This explains when the lighter gases are rare in earth's atmosphere.

- (viii) **Moon has no atmosphere :** We can also explain it by the concept of escape velocity. As the mass of the moon is less than earth's mass, so the escape velocity from the moon is quite small. It is about 2·4 km / sec. Initially when the temperature of the moon was high, the r. m. s. velocities of the gas molecules was of the same order as the escape velocities. So they escaped from the moon.

❂ 3.35. Weighlessness in orbiting satellite ❂

Weight of a body is the earth's gravitational attraction. So if the earth's attraction can be balanced, then the body becomes weightless. If a body rests on a surface, its weight acts downwards i.e. the body exerts a force on the surface. Then by Newton's third law, the surface exerts an equal and apposite reaction on the body. We feel this reaction as our weight.

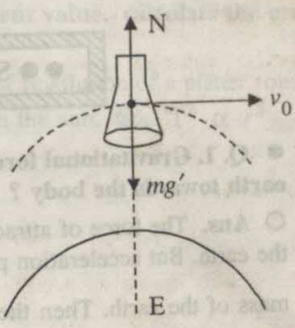


Fig. 3.20

- **Example :** If a lift descends freely then everything in the lift has downward acceleration $a = g$. Hence the reaction $R \doteq m(g - a) = m(g - g) = 0$. As a result a person if present in the lift will feel weightless.

Let an artificial satellite is revolving the earth in a stable orbit of radius r ($r > R$, radius of the earth).

Centripetal acceleration, of the satellite, $a_c \doteq \frac{v_0^2}{r}$ where v_0 = orbital speed of the satellite. So, the centripetal force, $F_c = ma_c$.

Let an astronaut be inside the satellite. The forces acting on him are (i) his weight mg' (g' = acceleration due to gravity at the position of the satellite) and (ii) oppositely directed reaction force N applied by the surface on which he stands. Obviously, the resultant of these low forces produce the necessary centripetal force.

$$\therefore \quad mg' - N = ma_c$$

Now, orbital velocity, $v_0 = \sqrt{g'r}$

$$\therefore \quad N = m(g' - a_c) = m\left(g' - \frac{v_0^2}{r}\right) = m(g' - g') = 0.$$

$\therefore$ the normal reaction is zero. So, the gravitational force on the astronaut is zero. As a result he will feel himself weightless. This is the condition of **weightlessness** inside a satellite.

- **Ex. 3.27. Two material objects each of mass 6·4 kg are at a separation of 0·16 m. From the point P equidistant from both the objects, a particle is released. The distance of P from the line joining the two objects at A and B. [Fig 3.21] is 0·06 m. When the point from P cross the point Q, what will be its velocity. Here Q is the mid point of the line AB.**

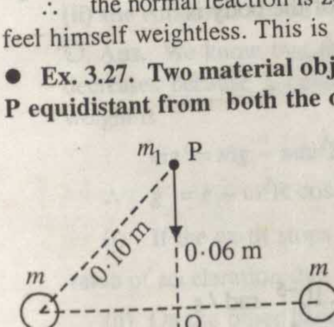


Fig. 3.21

○ **Solution :** The situation is shown in fig. m_1 is the mass of the particle released from P. Now, by the principle of conservation of energy, sum total of kinetic energy and potential energy of the particle at P will be equal to the kinetic and potential energy at Q.

$$O + E_{p_1} = \frac{1}{2}m_1v^2 + E_{p_2} \quad [\text{ where } v = \text{velocity of the particle at Q.}]$$

$$\therefore \quad E_{p_1} = \frac{-2\,G\,mm_1}{AP} \quad \text{and} \quad E_{p_2} = \frac{-2\,G\,mm_1}{AQ}$$

$$\therefore \quad \frac{-2\,Gm\,m_1}{AP} = \frac{1}{2}\,m_1\,v^2 + \frac{-2\,G\,m\,m_1}{AQ}$$

$$\therefore \quad \frac{1}{2}v^2 = 2\,Gm\left(\frac{1}{AQ} - \frac{1}{AP}\right)$$

$$\therefore \quad v^2 = 4 \times 6\cdot 67 \times 10^{-11} \times 6\cdot 4 \times \left(\frac{1}{0\cdot 08} - \frac{1}{0\cdot 10}\right)$$

$$= 42\cdot 7 \times 10^{-10}$$

$$\therefore \quad v = 6\cdot 53 \times 10^{-5}\ ms^{-1}$$

$AP = \sqrt{(0\cdot 08)^2 - (0\cdot 06)^2} = 0\cdot 10m$

$AQ = 0\cdot 08\ m$

$m = 6\cdot 4\ \text{kg}$

●● Short Answer Type Questions (with answers) ●●

● **Q. 1. Gravitational force is a mutual force. Then why a body falls towards the earth and not the earth towards the body ?**

○ **Ans.** The force of attraction of the earth on a body is equal to the force of attraction of the body on the earth. But acceleration produced in them are not equal. Let m be the mass of the body and M be the mass of the earth. Then the mutual gravitational force between the earth and the body is $F = \frac{GMm}{r^2}$ [r = distance of the body from the centre of the earth]

Now, the acceleration of the body towards the earth is $a_1 = \frac{F}{m} = \frac{GM}{r^2}$ and the acceleration of the earth towards the body is $a_2 = \frac{F}{M} = \frac{Gm}{r^2}$

since $M >> m, \ \frac{a_1}{a_2} = \frac{M}{m} >> 1.$

The acceleration of the body towards the earth is much greater. For this reason, a body falls towards the earth and not the earth towards the body.

● **Q. 2. If the earth stops rotating, what is percentage increase in weight of a body at the equator?**

○ **Ans.** Due to the earth's diurnal rotational moion, a body on the earth's surface experiences a centrifugal force due to which the body loses a part of its weight. Effective weight of the body is

$$w' = w - m\,\omega^2 R\ \cos^2\lambda \quad [\ \lambda = \text{ latitude of the places }]$$

If the earth stops rotating; $w = 0$ the $w' = w$

Increase in weight of the body, $\Delta w = m\omega^2 R\ \cos^2\lambda$.

So, the percentage increase the weight of the body at the equator is

$$\frac{\Delta w}{w} = \frac{m\,\omega^2 R}{mg} \times 100 \quad [\lambda = 0°]$$

$$= \frac{\omega^2 R}{g} \times 100$$

$$= \left(7\cdot 27 \times 10^{-5}\right)^2 \times \frac{6400 \times 10^3}{9\cdot 8} \times 100$$

$$= 0\cdot 34\ \%$$

$\omega = 7\cdot 27 \times 10^{-5}$ rad / s

$R = 6400 \times 10^3\ m$

$g = 9\cdot 8\ m / s^2$

● **Q. 3. If the mass of the earth be eighty times the mass of the moon and the radius of the earth is four times the radius of the moon, compare the values of acceleration due to gravity on the surface of the earth and on the surface of the moon.**

○ **Ans.** Acceleration due to gravity on the surface of the earth, $g_e = \frac{GM_e}{R_e^2}$ and on the surface of the moon, $g_m = \frac{GM_m}{R_m^2}$

$$\therefore \frac{g_e}{g_m} = \frac{M_a}{M_m} \times \frac{R_m^2}{R_e^2} = 80 \times \left(\frac{1}{4}\right)^2 = 5 \quad \therefore g_e = g_m = 5 : 1.$$

● **Q. 4. If the distance of the sun and the earth be half of its present value, calculate the number of days in a year.**

○ **Ans. From Kepler's third law :** The square of the time period of revolution of a planet round the sun is proportional to the cube of the mean distance of the planet from the sun. So, $T^2 \alpha r^3$

$$\frac{T_1^2}{T^2} = \frac{r_1^2}{r^3} \Rightarrow T_1^2 = T^2 \cdot \frac{r_1^3}{r^3}$$

$$\therefore T_1^2 = 365 \times \left(\frac{1}{2}\right)^3 = \frac{365}{2\sqrt{2}} = \mathbf{129} \text{ days.}$$

$\therefore$ No. of days in a year = 129 days.

● **Q. 5. Mass remaining same if the radius of the earth decreases by 1% what is the percentage change of acceleration due to gravity ?** **[H. S. 1998]**

○ **Ans.** We know, at the surface of the earth, $g \; \alpha \; \frac{1}{R^2}$ [R = earth's radius]

$$\therefore g = \frac{k}{R^2} \quad [\ k = \text{ a constant }]$$

$\therefore$ differentiating, $\frac{\Delta g}{g} = -2\frac{\Delta R}{R}$ Here $\frac{\Delta R}{R} = 1\,\%$

$$\therefore \frac{\Delta g}{g} = -2 \times 1\,\% = -2\%.$$

$\therefore$ Acceleration due to gravity will **decrease by 2%**.

● **Q. 6. What will happen to the value of acceleration due to gravity, if (i) the earth stops rotating, (ii) the rotaional speed of the earth increases.** **[J. E. E. 1986, 1996]**

○ **Ans.** We know that the effective weight of a body on the surface of the earth (except at the poles) decreases because a centrifugal force act on the body due to earth's roational motion. The effective weight is

$$mg' = mg - m\omega^2 R \cos^2 \lambda \quad [\ \lambda = \text{latitude of the place }]$$

$$\therefore g' = g - \omega^2 R \cos^2 \lambda \quad [\ \omega = \text{angular velocity of the earth }]$$

(i) If the earth stops rotating, $\omega = 0$. No centrifugal force acts on the body. Now, $g' = g$. So, the value of acceleration due to gravity increases by $\Delta g = \omega^2 R \cos^2 \lambda$.

(ii) On the other hand, if the rotational speed of the earth increases, angular velocity also increases. Then centrifugal force increases. So, effective value of acceleration due to gravity g' decreases.

If must be mentioned that the value of acceleration due to gravity will not be affected due to variation of ω.

● **Q. 7. From the top of a tower, a ball is thrown upwards and another ball is thrown downwards with the same velocity. Which ball will reach the ground with greater velocity ? [J. E. E. 1996]**

○ **Ans :** Both the balls will reach the ground with the same velocity.

● **Reason :** Let both the balls are thrown with the same initial velocity u. The first ball which is thrown upwards, will have the same velocity u (downwards) when it come back to the point of projection. Evidently both the balls will have the same initial downward velocity u. They travel the same vertical distance (equal to the height of the tower) with the same acceleration equal to g. So they will reach the ground with the same velocity.

● **Q. 8. From a point two stones are allowed to fall, one starts falling with zero initial velocity and the other starts with an initial horizontal velocity. Which stone will touch the ground earlier ? [J. E. E. 1988]**

○ **Ans.** Both the stones will touch the ground at the same time.

● **Explanation :** For the first stone, initial vertically downward velocity $u = 0$. If the height of the place be h and if it takes t_1 sec to reach the ground then, $h = \frac{1}{2} g t_1^2 \quad \therefore \quad t_1 = \sqrt{\frac{2h}{g}}$.

Again, the second stone has an initial horizontal velocity, but no vertical component of velocity. So its initial vertically downward velocity $u = 0$.

So it takes a time t_2 to descend the height h than, $h = \frac{1}{2} g t_2^2 \quad \therefore \ t_2 = \sqrt{\frac{2h}{g}}$.

Since $t_1 = t_2$, both the stones will take same time to reach the ground.

● **Q. 9. What will be the value of acceleration due to gravity, if the length of a seconds pendulum is taken as unity ? [J. E. E. 1988]**

○ **Ans.** Time period of a simple pendulum is $T = 2\pi \sqrt{\frac{l}{g}}$

for the seconds pendulum T = 2 sec. So $l = g/\pi^2$

Now, if $l = 1$ then $g = \pi^2 = \mathbf{9 \cdot 87}$ **unit of length / sec²**

● **Q. 10. The mass and diameter of a planet are twice those of earth. What will be the period of oscillation of a pendulum on this planet, if it is seconds pendulum on earth ? [J. E. E. 1995]**

○ **Ans.** Time period of a simple pendulum $T \propto \frac{1}{\sqrt{g}}$

$\therefore \frac{T_p}{T_e} = \sqrt{\frac{g_e}{g_p}}$ [T_e = time period on earth, T_p = time period on the planet]

Now, accleration due to gravity on the earth's surface, $g_e = \frac{GM_e}{R_e^2}$ and deceleration due to gravity on the surface of the planet, $g_p = \frac{GM_p}{R_p^2}$

$$\therefore \quad \frac{T_p}{T_e} = \sqrt{\frac{GM_e}{R_e^2} \times \frac{R_p^2}{GM_p}} = \sqrt{\frac{1}{2} \times 4} = \sqrt{2} \qquad \therefore \quad T_p = T_e \sqrt{2} = 2\sqrt{2} \text{ sec.}$$

● **Q. 11. A lift is moving upwards with an acceleration f. What will be the period of a simple pendulum suspended from the ceiling of the lift ? If the lift now falls freely what will its time period ? [J. E. E. 1992]**

○ **Ans. 1st part :** When the lift is stationary the initial time period of the pendulum is $T = 2\pi \sqrt{l/g}$ when the lift moves upward with an acceleration f, the effective upward acceleration will be $g' = g + f$

So, its new time period, $T' = 2\pi\sqrt{\frac{l}{g'}} = 2\pi\sqrt{\frac{l}{g+f}}$

Evidently $T' < T$. So, the time period will decrease.

2nd part : If the lift falls freely, its downward acceleration $f = g$. So the effective acceleration is $g_1 = g - f = g - g = 0.$ Hence the time period will be infinity. It means that the pendulum will stop oscillating.

● **Q. 12. If the mass and radius of the earth be half of its present mass and radius, what will be the length of a day ? Calculate also the number of days in a year.**

○ **Ans : First part :** According to the principle of conservation of angular momentum, $I_1\omega_1 = I_2\omega_2$

or, $I_1 \cdot \frac{2\pi}{T_1} = I_2 \times \frac{2\pi}{T_2} \Rightarrow \frac{I_1}{T_1} = \frac{I_2}{T_2} \quad \therefore \quad T_2 = \frac{I_2}{I_1} \cdot T_1$

Now, initial moment of inertia $I_1 = \frac{2}{5}MR^2$ and $T_1 = 24$ hrs. and final moment of inertia

$$I_2 = \frac{2}{5}\,\frac{M}{2}\left(\frac{R}{2}\right)^2 = \frac{I_1}{8},\ T_2 = ?$$

$\therefore \quad T_2 = \frac{1}{8} \times 24 =$ **3 hrs.**

$\therefore$ Now, the number of hours in a day is **3 hrs.**

2nd part : Period of revolution of the earth round the sun does not change. So, the length of the year does not change. Hence the number of days in a year is $365 \times 8 =$ **2920.**

● **Q. 13. A person in an artificial satellite orbiting the earth in a stable orbit feels weightlessness. Explain.** **[H. S. '95, '92, '90]**

○ **Ans.** Vide Art 3.36.

● **Q. 14. If the value of gravitational constant G decreased gradually, what will its effect on the motion of the moon ? Explain clearly.** **[J. E. E. 1990]**

○ **Ans.** Force of attraction of earth on moon, $F = \frac{GMm}{r^2}$ If the value of G remain unchanged, the moon will revolve round the earth in an stable orbit.

On the other hand, if the value of G gradually decreases, earth's attraction will gradually decrease. As a result, the radius of the moon's orbit will gradually increase.

So the moon will now rotate in a orbit whose radius increases continuously [Fig 3.22].

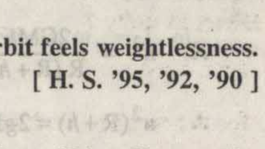


Fig. 3.22

● **Q. 15. A satellite carrying an astronaut is placed in orbit by a rocket. What changes in weight will the astronaut feel (i) during launch, (ii) in orbit ? Explain your answer.** **[J. E. E. 1997]**

○ **Ans.** (i) At the time of launching the astronaut will feel heavier.

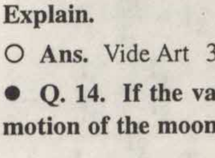


Fig. 3.23

● **Reason :** When the satellite moves upwards with an acceleration, the astronaut will feel the reaction as his weight. now, the reaction acting on him is $N = m(g + a)$ where 'a' is the upward acceleration of the satellite.

Evidently $R > mg$ Hence he will feel heavier.

(ii) When the satellite moves in a stable orbit, the astronaut will feel weightless.

Reason : The different forces acting on the astronaut are (i) his own weight mg towards the centre of the earth and (ii) outward normal reaction. So the necessary centripetal force is

$$F_c = mg - N \quad \therefore \quad \frac{mv_0^2}{R} = mg - N$$

But $v_0 = \sqrt{gR}$ (orbital velocity)

$$\therefore \quad \frac{m}{R}(gR) = mg - N \qquad \therefore \quad N = 0$$

So, the apparent weight of the astronaut is zero.

- **Q. 16. An object is projected vertically upward with a velocity u. Show that the maximum height reached by the object is $h = \frac{u^2R}{2gR - u^2}$. Calculate escape velocity from it.**

○ **Ans.** Let the mass of the object be m. Then its initial kinetic energy, $E_k = \frac{1}{2}mu^2$. It potential energy on the surface of the earth, $E_p = -\frac{GMm}{R}$.

At the maximum height h its kinetic energy, $E_k' = 0$ and potential energy, $E_p' = \frac{-GMm}{R+h}$

Then by the principle of conservation of energy $\frac{1}{2}mu^2 + \frac{-GMm}{R} = -\frac{GMm}{R+h}$

$$\therefore \quad \frac{1}{2}mu^2 = GMm\left[\frac{1}{R} - \frac{1}{R+h}\right] = \frac{GMm\,h}{R\,(R+h)}$$

$$\therefore \quad u^2 = \frac{2GMh}{R\,(R+h)} = \frac{2gR^2h}{R\,(R+h)} \qquad \left[\because\ GM = gR^2\right]$$

$$\therefore \quad u^2(R+h) = 2gRh \quad \therefore \quad h = \frac{u^2R}{2gR - u^2}$$

2nd part : Now, $u = v_e$ (escape velocity) and $h = \infty$

$$\therefore \quad 2gR = v_e^2 \qquad \therefore \quad v_e = \sqrt{2gR}$$

- **Q. 17. Will the escape velocity be same for a body at rest on the surface of the earth and for a body revolving the earth ?**

○ **Ans :** The value of escape velocity will not he same in both cases.

Reason : For a body at rest on the surface of the earth, the escape velocity is

$$v_e = \sqrt{2gR} = \sqrt{\frac{2GM}{R}}.$$

But for a body revolving the earth in a stable orbit near the earth's surface, its total energy is

$$E = -\frac{GMm}{R} + \frac{1}{2}m\,v_0^2 = \frac{-GMm}{R} + \frac{1}{2}m \cdot \frac{GM}{R} = -\frac{1}{2}\frac{GMm}{R}$$

If the escape velocity be v_e', then

$$\frac{1}{2}m\,(v_e')^2 = \frac{1}{2}\cdot\frac{GMm}{R} \qquad \therefore \quad v_e' = \sqrt{\frac{GM}{R}} \qquad \therefore \quad v_e > v_e'$$

- **Q. 18. What is the change in gravitational potential energy of a body of mass m when it is raised to a height equal to earth's radius. Value of acceleration due to gravity on the earth's surface = g.**

○ **Ans :** Gravitational potential energy of a body of mass m on the surface of the earth is

$$E_p = -\frac{GMm}{R}.$$

Its potential energy at a point above the earth's surface at distance r ($r > R$, earth's radius) from the centre of the earth is

$$E'_p = -\frac{GMm}{r}.$$

$\therefore$ Gravitational potential energy at a height equal to earth's radius from its surface is

$$E'_p = -\frac{GMm}{2R} \qquad [r = h + R = 2R]$$

$\therefore$ Increase of potential energy, $\Delta E_p = E_p{}' - E_p$

$$\Delta E_p = GMm\left[\frac{1}{R} - \frac{1}{2R}\right] = \frac{1}{2}\frac{GMm}{R}$$

$$\therefore \quad \Delta E_p = \frac{gR^2 m}{2R} = \mathbf{\frac{1}{2}\, mgR}$$

● **Q. 19. If the radius of the earth shrinks by $\frac{1}{2}$ %, what will be the change in length of a day ? Assume the earth to the a uniform sphere and its moment of inertia, $I = \frac{2}{5}MR^2$, M = mass and R = redius of the earth.**

[J. E. E.]

○ **Ans :** Here the principle of conservation of linear momentum is applicable. So, $I_1\,\omega_1 = I_2\,\omega_2$

$$\therefore \quad I_1 \frac{2\pi}{T_1} = I_2 \frac{2\pi}{T_2}$$

$$\therefore \quad \frac{I_1}{T_1} = \frac{I_2}{T_2}$$

$$\therefore \quad I_2 = \frac{I_2}{I_1} \cdot T_1 = \frac{99}{100} \times 24 \text{ hrs.}$$

$$I_1 = \frac{2}{5}MR^2$$

$$T_1 = 24 \text{ hrs}$$

$$I_2 = \frac{2}{5}M\left(R - \frac{R}{200}\right)^2 = \frac{2}{5}MR^2 \times \frac{99}{100}$$

$$T_2 = ? \qquad \frac{I_2}{I_1} = \frac{99}{100}$$

$\therefore$ Decrease in length of the day is $T_1 - T_2 = 24 - \frac{99 \times 24}{100} = 0{\cdot}24$ hrs. $=$ 14 min 24 sec.

●● Miscellaneous Examples ●●

● **Example 3·28. Calculate escape velocity from the surface of the moon. Given, radius of the moon is $1{\cdot}7 \times 10^6$ m, mass of the moon $= 7{\cdot}35 \times 10^{22}$ kg ; $G = 6{\cdot}67 \times 10^{-11}$ N m^2 kg^{-2}**

○ **Solution :** We known escape velocity $v_e = \sqrt{\frac{2\,GM}{R}}$

$$\therefore \quad v_e = \sqrt{\frac{2 \times 6{\cdot}67 \times 10^{-11} \times 7{\cdot}35 \times 10^{22}}{1{\cdot}6 \times 10^6}}$$

$$= 2{\cdot}4 \times 10^3\ ms^{-1} = \mathbf{2{\cdot}4\ km\,s^{-1}}$$

$$G = 6{\cdot}67 \times 10^{-11}\,\text{SI}$$

$$M = 7{\cdot}35 \times 10^{22}\ \text{kg}$$

$$R = 1{\cdot}6 \times 10^6\ \text{m}$$

● **Ex 3·29. A rocket is projected vertically upward with a velocity of 10 km/s. To what maximum height it will rise ? Radius of the earth = 6400 km. Neglect all types of frictions.**

○ **Solution :** Total energy of the rocket at the time of projection is

$$E = E_k + E_p = \frac{1}{2} mv^2 + \frac{-GMm}{R}.$$

At the highest point, kinetic energy, $E_k = 0$ [as $v = 0$]

∴ Total energy of the rocket at the highest point is

$$E' = \text{Potential energy at the highest point} = -\frac{GMm}{R+h}.$$

By conservation of energy principle, $E = E'$

$$\therefore \quad \frac{1}{2} mv^2 + \frac{-GMm}{R} = \frac{-GMm}{R+h}.$$

$$\therefore \quad \frac{1}{2}v^2 = \frac{GM}{R}\left(1 - \frac{R}{R+h}\right) = \frac{gR^2}{R} \cdot \frac{h}{R+h}.$$

$$\therefore \quad h = \frac{v^2 R}{2gR - v^2}$$

$$v = 10^4 \, ms^{-1}$$
$$R = 64 \times 10^5 m$$
$$g = 9 \cdot 8 \, ms^{-1}$$

$$\therefore \quad h = \frac{10^8 \times 6 \cdot 4 \times 10^6}{2 \times 9 \cdot 8 \times 6 \cdot 4 \times 10^6 - 10^8} = 2 \cdot 5 \times 10^7 \text{ m} = \mathbf{2 \cdot 5 \times 10^4} \textbf{ km}$$

● **Ex. 3·30. How much work is required to be done for a rocket of mass 2000 kg to shift from its initial orbit of radius 2R to another orbit of radius 3R. R = earth's radius = 6400 km.**

○ **Solution :** We know that total energy of a satellite which is revolving in a orbit of radius r is,

$$E = -\frac{1}{2} \frac{GMm}{r}.$$

∴ Total energy of the satellite when $r = 2R$, $E_1 = -\frac{1}{2} \frac{GMm}{2R}$

and total energy of the satellite when $r = 3R$, $E_2 = -\frac{1}{2} \frac{GMm}{3R}$

$$\therefore \quad \text{Energy difference, } \Delta E = E_2 - E_1 = \frac{1}{2} \frac{GMm}{R}\left(\frac{1}{2} - \frac{1}{3}\right) = \frac{GMm}{12R}$$

$$\therefore \quad \Delta E = \frac{mg\,R}{12} \quad \left[\because \ g = \frac{GM}{R^2} \right]$$

$$\therefore \quad \text{Required work} = \frac{mgR}{12} = \frac{1}{12} \times 2000 \times 9 \cdot 8 \times 6 \cdot 4 \times 10^6 = \mathbf{1 \cdot 045 \times 10^{10}} \textbf{ J}$$

● **Ex. 3·31 : A satellite of mass 300 kg is revolving around the earth in a orbit at a height of 700 km from the surface of the earth.**

Calculate : (i) kinetic energy (ii) potential energy and (iii) total energy. Mass of the earth 6×10^{24} kg, earth's radius = $6 \cdot 4 \times 10^6$ *m* and G = $6 \cdot 67 \times 10^{-11}$ Nm^2 kg^{-2}.

○ **Solution :** Distance of the satellite from the centre of the earth $r = R + h = 7 \cdot 1 \times 10^6$ m

(i) kinetic energy of the satellite, $E_k = \frac{1}{2} \frac{GMm}{r}$

$$\therefore \quad E_k = \frac{6 \cdot 67 \times 10^{-11} \times 6 \times 10^{24} \times 300}{2 \times 7 \cdot 1 \times 10^6} = 8 \cdot 7 \times 10^9 \text{ J}$$

(ii) Potential energy, $E_p = \frac{-GMm}{r} = -2(E_k) = -17\cdot4\times10^9$ J

(iii) Total energy, $E = E_k + E_p = (8\cdot7 - 17\cdot4)\times10^9 = \mathbf{-8\cdot7\times10^9}$ **J**

● **Ex. 3·32. Two bodies of masses M and *m* are initially at infinite distance apart. Due to mutual attraction they approach each other. When they are at a separation *r*, show that their velocity of approch is** $\sqrt{\frac{2G}{r}(M+m)}$.

○ **Solution :** Initially the bodies are at infinite distance apart. So their initial potential energy is zero. Also, initial kinetic energy is zero as the bodies was initially at rest.

∴ Total initial energy $= 0$.

Due to mutual gravitational force when they are at a separation *r*, let the velocity of the mass M is V and that of mass *m* is *v*. So their relative velocity of approach, $v_R = V + v$

Now, by the principle of conservation of momentum,

$$0 = mv - MV \quad [\because \ v \text{ and } V \text{ are in the opposite direction}]$$

$$\therefore \quad V = \frac{mv}{M} \qquad \therefore \quad v_R = V + v = \frac{mv}{M} + v = v\left(\frac{m}{M}+1\right)$$

Kinetic energy when the bodies are at a separation *r* is

$$E_k = \frac{1}{2}mv^2 + \frac{1}{2}MV^2 = \frac{1}{2}mv^2\left(1+\frac{m}{M}\right) = \frac{1}{2}\,\frac{m{v_R}^2}{1+\frac{m}{M}} = \frac{1}{2}\,\frac{Mm\,{v_R}^2}{M+m}.$$

Now, potential energy at this position, $E_p = -\frac{GMm}{r}$

By the principle of conservation of energy

$$E_k + E_p = 0 \qquad \therefore \quad \frac{1}{2}\,\frac{Mm{v_R}^2}{M+m} - \frac{GMm}{r} = 0.$$

$$\therefore \quad \frac{Mm\ {v_R}^2}{2\,(M+m)} = \frac{GMm}{r} \qquad \therefore \quad v_R = \sqrt{\frac{2G}{r}\ (M+m)}\cdot$$

● **Ex. 3.33. A man on earth can jump to a maximum height of 2m. To what maximum height a man will be able to jump in another planet whose density is $\frac{1}{3}$ of the density of earth and radius is $\frac{1}{4}$ of the radius of the earth.**

○ **Solution :** Let the man can jump to a height h_e on earth and to a height h_p on the planet. If the initial kinetic energy the same in both case then his potential energy at the highest point be also same on both the planets.

$$\therefore \quad m\,g_p\,h_p = m\,g_e\,h_e \qquad \therefore \quad h_p = \frac{g_p}{g_e}h_e$$

Now, acceleration due to gravity on earth is $g_e = \frac{GM_e}{{R_e}^2}$

$$\therefore \quad g_e = \frac{G}{{R_e}^2}\cdot\frac{4}{3}\pi {R_e}^3\rho = \frac{4\pi}{3}\,GR_e\rho_e$$

Similarly, in the case of the planet, $g_p = \frac{4\pi}{3}\ GR_p\ \rho_p$

$$\therefore \quad \frac{g_e}{g_p} = \frac{\mathrm{R}_e}{\mathrm{R}_p} \times \frac{\rho_e}{\rho_p}$$

$$\therefore \quad h_p = \frac{\mathrm{R}_e}{\mathrm{R}_p} \times \frac{\rho_e}{\rho_p} \times h_e = 4 \times 3 \times 2 = \mathbf{24}\, m.$$

● **Ex. 3.34. When a seconds pendulum is taken from the surface of the earth to the bottom of a mine, the pendulum loss 50 sec per day. Calculate the depth of a mine. Radius of the earth = 6400 km.**

○ **Solution :** $\Delta \mathrm{T} = 43200 \dfrac{\Delta g}{g}$

If the depth of the mine be h, then the acceleration due to gravity at the bottom of a mine is

$$g' = g\left(1 - \frac{h}{R}\right) \quad \therefore \quad \frac{\Delta g}{g} = \frac{h}{\mathrm{R}}.$$

∴ loss of time per day, $\Delta \mathrm{T} = 43200 \dfrac{h}{\mathrm{R}}$.

$$\therefore \quad h = \frac{\Delta \mathrm{T} \,.\, \mathrm{R}}{43200} = \frac{50 \times 6400}{43200} = 7 \cdot 4 \ km.$$

∴ depth of the mine = **7·4 km.**

● **Ex. 3.35. In the balloon expedition, a seconds pendulum is taken inside the balloon. When the balloon reaches the highest point, the pendulum is found to lose 400 sec per day. Calculate the height of the balloon, if radius of the earth be 6450 km.**

○ **Solution :** With increase in height, acceleration due to gravity decreases. Period of oscillation of the pendulum increases, because $\mathrm{T} \propto \dfrac{1}{\sqrt{g}}$.

If the pendulum loses ΔT sec per day, then $\Delta \mathrm{T} = 43200 \dfrac{\Delta g}{g}$.

If height of the balloon be h where acceleration due to gravity be g' then

$$g' = g \cdot \frac{\mathrm{R}^2}{(\mathrm{R}+h)^2} = g\left(1 - \frac{2h}{\mathrm{R}}\right) \quad [h << \mathrm{R}]$$

$$\therefore \quad g - g' = \frac{2hg}{\mathrm{R}} \quad \therefore \quad \frac{\Delta g}{g} = \frac{2h}{\mathrm{R}}$$

∴ loss of time per day, $\Delta \mathrm{T} = 43200 \dfrac{2h}{\mathrm{R}}$

$$\therefore \quad h = \frac{\Delta \mathrm{T} \cdot \mathrm{R}}{2 \times 43200} = \frac{400 \times 6450}{2 \times 43200} = \mathbf{29 \cdot 87}\ km$$

● **Ex. 3.36. A body is projected vertically upwards with a kinetic energy which is half of the minimum kinetic energy required to escape earth's gravitational attraction. Calculate the maximum height attained by the body. Take R = radius of the earth.** **[I. I. T. 1997]**

○ **Solution :** We know escape velocity, $v_e = \sqrt{2g\mathrm{R}}$

∴ corresponding kinetic energy $\mathrm{E} = \dfrac{1}{2} m v_e^{\,2} = \dfrac{1}{2} m v_e^{\,2} = mg\mathrm{R}$

∴ kinetic energy imparted to the body, $\mathrm{E}_1 = \dfrac{1}{2} mg\mathrm{R}$ (by question)

When the body reaches to the maximum height h,

$$\frac{1}{2}mg\text{R}+\frac{-\text{GM}m}{\text{R}}=-\frac{\text{GM}m}{\text{R}+h} \quad \therefore \quad \frac{1}{2}mg\text{R}=\frac{\text{GM}m}{\text{R}}\left(1-\frac{\text{R}}{\text{R}+h}\right)$$

$$\therefore \quad \frac{1}{2}\,g\text{R}=\frac{g\text{R}^2}{\text{R}}\cdot\frac{h}{\text{R}+h}\left[\because \quad \text{GM}=g\text{R}^2\right]$$

$$\therefore \quad \frac{1}{2}=\frac{h}{\text{R}+h}. \quad \therefore \quad h=\text{R}.$$

● **Ex. 3.37. A satellite is revolving the earth on a circular orbit near the earth's surface. What additional velocity be given to the satellite so that it may escape earth's gravity ?**

Radius of the earth = 6400 km and $g = 9{\cdot}8\ ms^{-2}$.

○ **Solution :** As the satellite is near the earth's surface, its orbital velocity, $v=\sqrt{g\text{R}}$. Also escape velocity from earth is $v_e=\sqrt{2g\text{R}}$

$\therefore$ Additional velocity required for escape of the satellite

$$\Delta v=v_e-v=\left(\sqrt{2}-1\right)\sqrt{g\text{R}}=0{\cdot}4\times\sqrt{9{\cdot}8\times6400\times10^3}\ m/s$$

$$\therefore \quad \Delta v=3{\cdot}36\ km/s.$$

● **Ex. 3.38. A geostationary satellite is revolving the earth whose height from the earth's surface is 6R where R is the radius of the earth, what will be its time period if its height from the earth's surface be 2·5 R.** **[J. E. E. 1987]**

○ **Solution :** The required relation $\text{T}^2 \ \alpha \ r^3$

$$\therefore \quad \left(\frac{\text{T}_1}{\text{T}_2}\right)^2=\left(\frac{r_1}{r_2}\right)^3.$$

$$\therefore \quad \text{T}_2=\left(\frac{3{\cdot}5\text{R}}{7\text{R}}\right)^{3/2}=\left(\frac{1}{2}\right)^{3/2}\ \text{day}$$

$$\therefore \quad \text{T}_2=\mathbf{8{\cdot}48}\ \text{hrs.}$$

$\text{T}_1 = 1$ day
$\text{T}_2 = ?$
$r_1 = \text{R} + 6\text{R} = 7\text{R}$
$r_2 = 3{\cdot}5\ \text{R}$

● **Ex. 3.39. A juggler is showing juggling with four balls. He is throwing and catching the balls in such a way that all the four balls are in motion. If all the balls rises to a height of 90 cm, calculate the velocity of projection of each ball.**

When he is throwing the fourth ball, determine the position of the other balls.

○ **Solution : 1st Part :** Each of the four balls rises to a height of 90 cm. So, the velocity of projection of each ball is

$$u=\sqrt{2gh}=\sqrt{2\times980\times90}=420\ \text{cm/s}$$

2nd part : Time of flight of each ball, $\text{T}=\dfrac{2u}{g}=\dfrac{2\times420}{980}=\dfrac{6}{7}$ sec.

Time interval between the throwing of any two balls $=\dfrac{3}{14}$ sec.

So, we can say when the fourth ball is thrown, third ball has been thrown $\dfrac{3}{14}$ sec earlier.

The position of the third ball, $s_3=ut-\dfrac{1}{2}gt^2$

$$s_3=420\times\frac{3}{14}-\frac{1}{2}\times980\times\left(\frac{3}{14}\right)^2=67{\cdot}5\ \text{cm}$$

similarly second ball has been thrown $\frac{3}{14}+\frac{3}{14}=\frac{3}{7}$ sec earlier.

∴ the height of the second ball, $s_2 = 420\times\frac{3}{7}-\frac{1}{2}\times 980\left(\frac{3}{7}\right)^2 = 90$ cm.

lastly, first ball has been thrown $\frac{9}{14}$ sec earlier

∴ the height of the first ball, $s_1 = 420\times\frac{9}{14}-\frac{1}{2}\times 980\times\left(\frac{9}{14}\right)^2 = \mathbf{67\cdot 5}$ cm.

Hence the position of the three balls are 67·5 cm, 90 cm, 67·5 cm.

- **Ex. 3.40. A planet is revolving the sun in a circular orbit. If the planet is suddenly stopped show that it falls to the sun after a time $\frac{\sqrt{2}}{8}$T. Where T is the period of revolution of the planet.**

○ **Solution :** We have the relation, $m\,\omega^2 R = \frac{GMm}{R^2} \Rightarrow \omega^2 = \frac{GM}{R^2}$.

$$\therefore \quad \left(\frac{2\pi}{T}\right)^2 = \frac{GM}{R^3} \quad \therefore \quad T^2 = \frac{4\pi^2 R^3}{GM} \quad \cdots\cdots\cdots\cdots \text{(i)}$$

Let the planet fall by a distance x t sec after it is dropped.

$$\therefore \quad m\,\frac{dv}{dt} = \frac{GMm}{(R-x)^2} \quad \text{or,} \quad v\,\frac{dv}{dx} = \frac{GM}{(R-x)^2}$$

Integration $\frac{1}{2}v^2 = \frac{GMm}{(R-x)} + k$ [k = constant]

Again, when $x = 0$, $v = 0$. ∴ $k = -\frac{GM}{R}$

At $x = 0$, $v = 0$ ∴ $k = -\frac{GM}{R}$ ∴ $\frac{1}{2}v^2 = \frac{GM}{(R-x)} - \frac{GM}{R} = \frac{GM\,x}{R\,(R-x)}$

$$\therefore \quad v = \sqrt{\frac{2GM}{R}} \times \sqrt{\frac{x}{R-x}} \Rightarrow \frac{dx}{dt} = \sqrt{\frac{2GM}{R}} \times \sqrt{\frac{x}{R-x}}$$

$$\therefore \quad \sqrt{\frac{R-x}{x}} \cdot dx = \sqrt{\frac{2GM}{R}}\; dt$$

$$\therefore \quad \int_0^R \sqrt{\frac{R-x}{x}}\, dx = \sqrt{\frac{2GM}{R}} \int_0^t dt \quad \cdots\cdots\cdots\cdots \text{(ii)}$$

Let $x = R\cdot\sin^2\theta$ ∴ $dx = 2R\ \sin\theta\ d\theta\ \cos\theta$

∴ L. H. S. of equation (ii),

$$\int_0^R \sqrt{\left(\frac{R}{x}-1\right)}\, dx = \int_0^R 2R\ \cos^2\theta\ d\theta = R \int_0^R (1+\cos 2\theta)\, d\theta = R.\ \frac{\pi}{2}$$

Now, from equation (ii), $R\cdot\frac{\pi}{2} = \sqrt{\frac{2GM}{R}}\cdot t$

$$\therefore \quad t = \frac{\pi R}{2}\sqrt{\frac{R}{2GM}} = \sqrt{\frac{\pi^2 R^3}{8\,GM}} = \frac{\sqrt{2}}{8}\,T.$$

EXERCISE

A. Short answer type questions :

1. What is the difference between gravity and gravitation ?

2. State Newton's law of gravitation. What gravitational constant ? What is its unit in C. G. S. system ? **[H. S. 2002]**

3. Why G is called universal constant ?

4. What is meant by "the value of G is $6{\cdot}67 \times 10^{-11}$ SI" ?

5. Define acceleration due to gravity. Give its relation with G.

6. What do you mean by acceleration due to gravity ? Indicate where on the surface of the earth, acceleration will be maximum. **[H. S. 2003]**

7. What is escape velocity ? Is its value the same on the surface of the earth and the moon ? **[H. S. 2003]**

8. The force on earth due to a 1 kg lead ball and the force on 1 kg lead ball by the earth—which force is greater ? Explain your answer.

9. Gravitational force is a mutual force. Then why does a small body falls towards the earth ; but not the earth towards the body. Explain.

10. What is meant by free fall ? Why does a body appear weightless during free fall ?

11. Two bodies are allowed to fall at the same time from the roof of a house. One is dropped from rest and the other is projected with a horizontal velocity. Which stone will reach the ground earlier ? **[J. E. E. 1988]**

12. Mass remaining unchanged, if the readius of the earth shrinks to half of its present value, what will be the acceleration due to gravity on the earth's surface now ? [**Hint :** $g \propto \frac{1}{R^2}$; $\frac{g'}{g} = 4$]

13. What will be the value of acceleration due to gravity g if (i) earth stops rotating, (ii) rotational speed of the earth increases. **[J. E. E. '86, '96]**

14. From the top of a tower two balls are thrown—are vertically upwards and another vertically downward with the same velocity. Which are will touch the ground with greater velocity ? **[J. E. E. 1986]**
[**Hint :** same velocity]

15. If the length of a seconds pendulum is taken to be unity what will be the value of acceleration due to gravity? [**Ans.** 9·87 unit of length/sec^2] **[J. E. E. 1988]**

16. The mass and diameter of a planet are twice those of the earth what will be the time period of a second pendulum on earth ? [**Ans.** 2·828 sec] **[J. E. E. '90, '95]**

17. A lift is moving upwards with an acceleration f. What will be the changed time period of a simple pendulum suspended from the ceiling of the lift ? If the lift falls freely with an acceleration equal to the acceleration due to gravity what will be the time period now ? **[J. E. E. 1992]**

18. Why is tide not observed in a big lake ? **[J. E. E. 2000]**

19. Show graphically the variation of acceleration due to gravity with distance from the centre of the earth. **[H. S. 2001]**

20. What is escape velocity ? What is relation between the escape velocity with the orbital velocity of a satellite near the earth's surface ? **[H. S. 2001]**

21. What is geostationary satellite ? Why does a body appear weightless in an artificial satellite ? **[H. S. 2002]**

22. The acceleration due to gravity in a planet is 196 cm/s^2. If it is safe to jump from a height of 2m on earth what will be the safe height of jump on the planet. **[Ans.** 10 m]

23. Show graphically how does the acceleration due to gravity change from the centre of the earth inside and outside of earth. **[H. S. '99]**

24. What is the difference between mass and weight ? **[H. S. 2000]**

25. We can measure weight by spring balance and mass by common balance. Explain. **[H. S. 2000]**

26. How does the time period of a pendulum change if the diameter of the pendulum increase or length of the pendulum decrease ?

27. Why moon has no atmosphere ?

28. The weight of a body at the centre of the earth is zero. Explain.

29. A pendulum is taken from equator to the pole. Will it go slow or fast ? Explain.

30. Mass remaining unchanged if the radius of the earth decreases by 1%, what is the percentage change in acceleration due to gravity ?
31. From the top of an smooth inclined plane a block is allowed to fall down the plane. At the same time another block is dropped vertically downward from the same point. Which one will reach the ground earlier and whose velocity will be greater on reaching the ground ?
32. When a body is projected vertically upwards, its momentum first decreases, becomes zero at the highest point and then increases again. Is the principle of conservation of linear momentum violated here ? Explain.
33. How does the weight of a body on the surface of the earth changes due to earth's rotational motion ?
34. At what angular velocity of the earth a body at the equator will appear weightless ?
35. A second's pendulum is kept in a lift which is moving upwards with an acceleration '*a*'. What will be its change in time period ?
36. The point of suspension of a simple pendulum is moving horizontally with an acceleration. How will its time period change now ?
37. If the earth stops rotating how does the weight of a body change ?
38. If the angular velocity of the earth increases how does the weight of a body change ?
39. Between the moon and the earth, in which case escape velocity is greater ? **[J. E. E. 1998]**
40. Show that a satellite follows the following law :
    Cube of the radius of the orbit is proportional to the square of the time period of revolution.
41. A body on earth is attracted both by the earth and the sun. At mid day these two forces of attraction are opposite, but at midnight these two forces are in the same direction. Will the weight of the body at mid night be greater ?
42. If the value of the gravitational constant gradually decreases, what will be its effect on the motion of the moon ? Explain clearly. **[J. E. E. 1990]**
43. Two satellites are revolving the earth in orbits at the same height from the surface. The mass of one satellite is double the other. Which of the satellites will have greater speed ?
44. Mass remaining unchanged, if the radius of the earth be twice its present radius, show that the weight of a body on the earth's surface radius to 25% of its initial weight.
45. Show that in the case of vertical motion, time of ascent is equal to time of descent.
46. Calculate the difference in the value of acceleration due to gravity at the pole and at the equator due to earth's rotational motion. Radius of the earth R = 6400 km.
47. If the angular velocity of the earth be such that a body at the equator appears weightless then what will be the length of the day ? Radius of the earh 6400 km, acceleration due to gravity 980 cm/s^2.
48. Why are light gasses like hydrogen and helium rare in earth's atmosphere ?
49. A body is dropped from a orbiting setellite. What will happen to the body ?
50. A car is moving along a circular road with constant speed. What will be the time period of pendulum suspended from the ceiling of the car ?
51. A planet is revolving the sun in a circular orbit. Show graphically the variation of its potential and kinetic energy.
52. The earth is revolving the sun in a circular orbit. Show that area described the line joining the sun and the earth per unit time is constant. **[J. E. E. 1987]**
53. The earth's radius is R and acceleration due to gravity on the earth's surface is g. How much work is done to raise a body of mass m by height *h*. ? $\left[\textbf{Ans.}\quad mgh\,/\left(1+\frac{h}{\mathrm{R}}\right)\right]$

◢ B. Essay type questions :

1. (a) What we mean by 'Gravitation' and 'gravity'. State and explain Newton's law of gravitation. Why is gravitational constant called universal ?
   (b) Define acceleration due to gravity. Show by graph how does the acceleration due to gravity change with distance from the centre of the earth both inside and outside the earth. **[H. S. 1998]**
   (c) How does the acceleration due to gravity changes due to earth's diurnal motion. **[H. S. 1998]**
2. (a) What is the condition of putting a satellite at a height h from the earth's surface. Mass and radius of the earth an M and R respectively. Why is a body inside a satellite weightless ? **[H. S. 2000]**
   (b) Obtain the relation between the acceleration due to gravity and gravitational constant.

3. (a) Prove that the value of acceleration due to gravity decreases with height from the surface of the earth.
(b) Deduce an expression for the acceleration due to gravity at a depth '*h*' inside the earth.

4. (a) Between mass and weight of a body which one is more intrinsic.
(b) How can we measure mass and weight of a body ?
(c) What are inertial and gravitational mass ?

5. (a) What is a seconds pendulum ? Calculate its length if $g = 10\ \text{m/s}^2$
(b) How can you measure height of a mountain and depth of a mine by a simple pendulum ?

6. Show that the weight of a body at the centre of the earth is zero. **[H. S. 2004]**

7. What do you mean by acceleration due to gravity ? Determine how the acceleration due to gravity changes with the latitude of the place and from it, indicate where on the earth's surface acceleration will be maximum. **[H. S. 2003]**

8. What is escape velocity ? Is its value the same on the surface of the earth and the moon ? **[H. S. 2003]**

9. If the earth ceases to rotate for some reason, calculate the percentage change in the weight of a body situated on the equator. [Radius of earth = 6400 km and $g = 9{\cdot}8\ \text{mc}^{-2}$] **[H. S. 2004]**

10. (a) Why does a body appear weightless in a satellite ?
(b) Deduce an expression for the orbital speed of an artificial satellite. Calculate the orbital velocity when the satellite is near the earth's surface.

11. (a) State Kepler's laws of planetary motion. How can you establish Keplers law from the Newton's law of gravitation ?
(b) What is the gravitational potential energy of a body of mass *m* in the surface of the earth ?
(c) How much work is done to raise the body from earth's surface to a height equal to earth's radius ?

12. What is geostationary satellite ? Calculate its height from the earth's surface. What is parking orbit ?

13. Define escape velocity. Calculate the value of the escape velocity on the surface of the earth. How is this velocity related to the orbital velocity of a satellite near the earth's surface ?

14. Show by calculation that the orbital velocity of a satellite revolving round a planet in a circular orbit depends on the average density of the planet.

15. (a) A body in a satellite is weightless. Explain. **[H. S. 2000]**
(b) A satellite along with a astronaut is put into an orbit. What changes in weight will the astronaut feel (i) during launching of the satellite (ii) when in stable orbit ? **[J. E. E. 1997]**

◢ C. Simple numerical problems :

1. Two solid metal spheres having radii 10 cm and 1 cm are placed so that the centres are at a separation of 1 m. If the gravitational force between the spheres be $15{\cdot}5 \times 10^{-6}$ dyne, calculate the value of gravitational constant G. Density of the metal = 11·5 gm / c.c. [**Ans.** $6{\cdot}68 \times 10^{-8}$ C.G.S]

2. Calculate the value of acceleration due to gravity on the surface of the earth from the following data : $G = 6{\cdot}6 \times 10^{-11}\ \text{Nm}^2\ \text{kg}^{-2}$, average density of the earth $5\times10^3\ \text{kg}\,m^{-3}$; radius of the earth $= 6\cdot4\times10^6$ m. [**Ans.** $8\cdot85\,ms^{-2}$]

3. Calculate the mass of the earth from the following data : acceleration due to gravity $= 9{\cdot}8\ \text{ms}^{-2}$, $G = 6{\cdot}67 \times 10^{-11}$ SI and radius of the earth 6400 km. [Ans $5{\cdot}96 \times 10^{24}$ kg] **[H. S. Tripura, 1992]**

4. A stone is dropped form the top of a tower of height 400 m. Simultaneously another stone is projected vertically upward from the ground with the velocity of $100\ \text{ms}^{-1}$. When and where will the stones meet ?
[**Ans.** 4 sec after projection ; 78·4m from the top of the tower] **[W. B. H. S. 2001]**

5. A small satellite is revolving round a planet having density 10 g/c.c. Radius of the orbit of the satellite is slightly greater than the radius of the planet. Calculate the time period of the satellite. ($G = 6{\cdot}68 \times 10^{-8}$ C.G.S units) [**Ans.** 1 hr 26 min] **[W. B. H. S. 2002]**

6. The separation between the centres of two spheres of masses 50 kg and 10 kg is 30 cm. The gravitational force between the spheres is $\frac{1}{24}$ mg-wt. If radius of the earth be 6×10^6 m, calculate mean density of the material of the earth. $g = 9{\cdot}8\ \text{ms}^{-2}$ [**Ans.** 5·31 g/ c.c.]

7. At what height from the surface of the earth, the value of acceleration due to gravity be 1% of its value on the earth's surface ? Radius of the earth $= 6\cdot4\times10^8$ cm. [**Ans.** $5\cdot7\times10^3$ m]

8. At what height from the surface of the earth, the value of the acceleration due to gravity be $\frac{1}{4}$ th of its value on the surface of the earth ? Radius of the earth $= 6 \cdot 4 \times 10^6$ m. [**Ans.** $4 \cdot 8 \times 10^6$ km]

9. At what height and at what depth the value of the acceleration due to gravity be half of its value on the surface ? Radius of the earth = 6400 km. [**Ans.** 2608 km ; 3150 km]

10. Density of earth 5·5 g/c.c. ; $G = 6 \cdot 7 \times 10^{-11}$ SI unit and radius of the earth = 6400 km. Calculate the value of acceleration due to gravity. [**Ans.** 9·9 m/s^2]

11. A balloon is rising vertically upward. When its height is 390 m from the ground a stone is dropped. The stone reaches the ground after 10 sec. Calculate the velocity of the balloon when the stone was dropped ? $g = 9{\cdot}8\ m/s^2$. [**Ans.** 10 m/s] **[W. B. H. S. 1984]**

12. A particle is allowed to fall freely from a height 19·6 m. Calculate the time taken by the particle to cover (i) first 1m, (ii) last 1m of its motion. [**Ans.** 0·45 sec. (ii) 0·05 sec]

13. A particle is thrown vertically upward with a velocity *u*. After *n* seconds, another particle is projected from the same point and in the same vertical line with a velocity *v*. If the two particles meet each other at the greatest height for the first particle, show that $2(v-u)(u-ng) = n^2 g^2$. **[W. B. H. S. 2004]**

14. A stone is dropped freely from a height of 90 m. Simultaneously another stone is projected vertically upwards along the same line with a velocity of 30 m/s. When and where will the stones meet ? [**Ans.** 3 sec ; 45·9 m]

15. The distance of separation between two spheres of masses 800 kg and 600 kg is 25 cm. Calculate the gravitational potential at a point which is at a distence of 20 cm from the first sphere and 15 cm from the second sphere. [**Ans.** $-5 \cdot 3 \times 10^{-7}$ J kg^{-1}]

16. A man can jump to a maximum height of 1·5 m on earth. What should be the radius of the other planet so that the man can escape gravitational pull of the planet by jumping in the same way as on the earth ? Assume that the earth and the planet have the same density. Radius of the earth $6 \cdot 4 \times 10^6$ m. [**Ans.** $3 \cdot 1 \times 10^3$ m]

17. Calculate the minimum velocity to be given to a body so that it will escape earth's gravitational attraction. Given, $G = 6 \cdot 63 \times 10^{11} N\text{-}m^2\ kg^{-2}$, mass of the earth $5 \cdot 97 \times 10^{24}$ kg, radius of the earth $= 6 \cdot 37 \times 10^6$ m. [**Ans.** 11·2 km/s]

18. A boy is throwing balls vertically upward. When one ball reaches the maximum height, he throws the other ball. If he throws two balls per second, calculate the maximum height attained by each ball. $g = 10\ m/s^2$ [**Ans.** 1·25 m]

19. A block is sliding down a smooth inclined plane form its top. At the same time another block is let fall vertically downdward from the same point. (i) which block will reach the ground earlier and (ii) which block will have greater velocity on reaching the ground ? [**Ans.** (i) second block (ii) same velocity]

20. The distance between the sun and the earth is $1 \cdot 5 \times 10^{11}$ m. $G = 6 \cdot 66 \times 10^{-11}\ Nm^2\ kg^{-2}$ and time period of revolution of earth round the sun is 365 days. Calculate the mass of the sun. [**Ans.** $1 \cdot 97 \times 10^{30}$ kg]

21. What should be the angular velocity of the earth so that a body at the equator will appear weightless ? Average density of the earth $= 5 \cdot 6$ gm/c.c. and $G = 6 \cdot 67 \times 10^{-8}$ C.G.S. [**Ans.** $1 \cdot 95 \times 10^{-3}$ red / sec]

22. A ball of mass 10 kg is projected vertically upward with a velocity of 2 ms^{-1} from the top of a tower of height 10 m. What will be its kinetic energy when it is about the touch the ground ? $g = 10\ m/s^2$ [**Ans.** 10^3 J] **[J. E. E. 1994]**

23. A shell is fired from a gun vertically upward, with a velocity of 300 ms^{-1}. Calculate (i) maximum height attained by the shell (ii) time of asent (iii) velocity of the shell 15 sec after firing. [**Ans.** (i) 4592 m (ii) 30·6 sec. (iii) 153 m/s] **[W. B. H. S. 1999]**

24. What should be the length of a simple pendulum on the moon so that its time period is equal to the time period of the simple pendulum on earth. (mass of the earth is 80 time that of the moon and radius of the earth is four time that of the moon). $\left[\textbf{Ans. } l_m = \frac{1}{5} l_e\right]$ **[J. E. E. 1992]**

25. The wt. of a body on earth 63 N. What would be its weight at a height equal to half of earth's radius ? [**Ans.** 28N]

26. The diameter of a planet is four time that of the earth. If a seconds pendulum on earth is taken to that planet, what will be its time period ? [**Ans.** 1 sec]

27. At what depth below the earth's surface, the value of acceleration due to gravity be 1% less than its value on the earth's surface ? Earth's radius = 6400 km. [**Ans.** 64 km]

28. The time period of second pendulum is same at the top of a mountain and at the bottom of a mine. If the height of the mountain be H and the depth of the mine be *h*, calculate the value of the ratio H : *h*. [**Ans.** H : *h* = 1 : 2]

29. A second pendulum gives correct time at the bottom of a building of height 1482m. If the pendulum is taken to the top of the building how much time will it lose per day ? Earth's radius = 6400 km. [**Ans.** 20 sec]

30. The time period of a simple pendulum is 2·85 sec. If the length of the pendulum is increased by 20 cm, its time period becomes 2·99 sec. From these observations, calculate the value of acceleration due to gravity. [**Ans.** $9{\cdot}89\ \mathrm{ms}^{-2}$]

31. A second pendulum loses 20 sec per day. How its length has to be changed so that it may give correct time again ? [**Ans.** length to decrease by 0·046 cm] [**J. E. E 2003**]

32. The moon revolve the earth with time period 27·3 days. Show that the centripetal acceleration of the moon is equal to the acceleration due to gravity at the surface of the moon. [Radius of the earth $= 6\cdot 4\times10^6$ m , radius of the moon's orbit $= 3\cdot 8\times10^8$ m.] [**J. E. E. 2003**]

33. A seconds pendulum gives correct time at a place where acceleration due to gravity is 981 cm/sec^2. It is taken to a place where the acceleration due to gravity is 980·32 cm/sec^2. Calculate the loss or gain of time by the pendulum. [**Ans.** 30 sec.]

34. A pendulum which gives correct time on earth is taken into a lift which is moving upward with an acceleration 2·5 m/s^2. How much time will it gain per minute ? g = 10 ms^{-2}. [**Ans.** 7 sec fast]

35. An artificial satellite is revolving the earth at a height of 700 km from the earth's surface. Calculate the orbital velocity of the satellite. Radius of the earth = 6400 km, g = 9·8 m/s^2. [**Ans.** $7\cdot 5\times10^5$ km / sec]

36. A man jumps from a plane with parachute and descends 50 m freely. When parachute opens the man now descends with a retardation of 2·0 m/s^2. At the time when he touches the ground, the velocity is 3 ms^{-1}. Calculate the time of fall. Also calculate the height from which the man jumps. [**Ans.** 17·3 sec ; 293 m]

37. When the length of a simple pendulum is increased by 1 cm the time period increases by 0·01 sec. What is the initial length of the pendulum ? [**Ans.** 100·6 cm]

38. The radius of a planet is 600 km and density 5×10^3 kg / m^3. Calculate the maximum temperature of the plane at which the oxygen will not escape from planet. [**Ans.** 1018°C]

[**Hints :** $V_0 = \sqrt{\frac{8}{3}G\,\pi r^2\,\rho} = \sqrt{\frac{3RT}{M}}$.]

39. An artificial satellite is moving in a circular orbit around the earth at a height equal to earth's radius R = 6400 km. It is stopped suddenly and allowed to fall freely. With what speed, the satellite will reach the earth ? [**Ans.** 7·92 km / s]

40. If the time period of a satellite be 2 hrs, what is its height from the surface of the earth ? Earth's radius, R = 6400 km. [**Ans.** 1680 km]

41. A satellite is revolving the earth at a height of 3200 km from the surface of the earth. If it is suddenly stopped in the orbit, with what speed will the satellite reach the surface of the earth ? Earth's radius = 6400 km. [**Ans.** 6·47 km/s]

42. When the length of a simple pendulum is increased by *l*, its time period changes from T_1 to T_2. Show that

$$g=\frac{4\pi^2 l}{T_2^{\,2}-T_1^{\,2}}$$

43. The mass and diameter of the planet jupiter an 309 times and 11 times respectively compared to those of the earth. Calculate the value of acceleration due to gravity on the planet. [**Ans.** 25 m/s^2]

44. A man is put in a circular orbit at a height of $1\cdot 6\times10^5$ m above the earth's surface. Radius of the earth is $6\cdot 37\times10^6$ m and mass $5\cdot 96\times10^{24}$ kg. What is the orbital velocity of the man ? $\left(G=6\cdot 67\times10^{-11}\ \mathrm{N}\,m^2/\mathrm{kg}^2\right)$ [**J. E. E. 1992**]

45. A satellite is revolving the earth at a height of 700 km from the surface. Calculate orbital velocity of the satellite (Earth's radius 6300 km, $g = 9{\cdot}8$ m/s^2) [**Ans.** 7·454 km/s] **[J. E. E. 1992]**

46. The mass and diameter of a planet are twice those of the earth. If a seconds pendulum is taken from the earth to that planet, what will be its time period ? [**Ans.** $2\sqrt{2}$ sec] **[I. I. T.]**

47. The value of acceleration due to gravity on earth is 10 ms^{-2}. What will be the value of acceleration due to gravity on the planet mars if its mass be $\frac{1}{5}$ times the earth's mass and radius $\frac{1}{2}$ times the earth's radius. [**Ans.** 8 ms^{-2}]

48. The time period of a simple pendulum is 1·96 sec. If its length is increased by 10 cm, its time period increases by 0·1 sec. From this observation, calculate the value of acceleration due to gravity. [**Ans.** 9·81 ms^{-2}]

49. A simple pendulum makes 40 oscillations in 2 minutes. If the diameter of the bob of the pendulum be 3·28 cm, calculate its length. [**Ans.** 2·22 m]

50. Weight of a body on the surface of the earth is 20 kg. What will its weight in the following cases : (i) at a height equal to earth's radius (ii) at a height equal to twice the earth's radius. [**Ans.** 5 kg ; 2·2 kg]

51. Show that if the angular velocity of the earth be 17 times the present angular velocity of the earth then a body at the equator will be weightless. Also calculete the length of the day under this condition. [**Ans.** 1·4 hrs]

52. The mass of a planet is $\frac{1}{10}$ of the mass of the earth and its radius $\frac{1}{2}$ of earth's radius. Compare the acceleration due to gravity on the two planets. [**Ans.** 2 : 5]

53. A plane is descending at an angle of 45° with the horizontal with a retardation of 2 m /s^2. What is the time period of a simple pendulum of length 100 cm kept in the plane ? [**Ans.** 1·87 sec]

◢ D. Harder numerical problems :

1. Three identical particles each of mass m are located at the vertices of an equilateral triangle with side l. At what speed must they move if they all revolve under the influence of one another's gravity in a circular orbit circumscribing the triangle while still preserving the equilateral triangle ? What is the time period of the circular motion ? $\left[\textbf{Ans. } v = \sqrt{\frac{Gm}{l}}\ ;\ \ T = 2\pi\sqrt{\frac{l^3}{3Gm}}\right]$ **[I. I. T. 86]**

2. An artificial satellite is revolving in a circular orbit around the earth with a speed equal to half of escape velocity from earth. (i) Determine the height of the satellite above the earth's surface. (ii) if the satellite is stopped suddenly in the orbit and allowed to fall freely on to the earth, find the speed with which it hits the surface of the earth. R = 6400 km[**Ans.** (i) h = R = 6400 km (ii) 7·92 km/s] **[I. I. T. '90, J. E. E '96]**

3. The masses and radii of the earth and the moon are M_1, R_1 and M_2, R_2 respectively. Their centres are at distance d apart. Find the minimum speed with which a particle of mass m should be projected from a point midway between the two centres so as to escape to infinity. $\left[\textbf{Ans. } v = \sqrt{\frac{4G(M_1 + M_2)}{d}}\right]$ **[J. E. E. '88]**

4. Two spheres having masses 800 kg and 600 kg are at a separation of 0·25 m. What is the gravitational intensity at a point which is at a distence of 20 cm from the first sphere and at a distance of 15 cm from the second sphere. [**Ans.** $2 \cdot 22 \times 10^{-6}$ N kg^{-1}]

5. A lift is moving upward with an acceleration of 2 ms^{-2}. When the velocity of the lift is 8 m/s a nail from the ceiling of the lift starts falling. If the height of the lift be 3m, after what time the nail will reach the floor of the lift. [**Ans.** 0·71 sec]

6. A stone is projected vertically upwards with a velocity of $\sqrt{gR}$ where R is the radius of the earth. Calculate the maximum height attained by the stone. [**Ans.** h = R]

7. Two points P and Q are located on the same vertical line, P is above the point Q. A body is projected vertically upwards from the point Q such that it can rise upto P. Simultaneously another body is dropped from P. Show that when the bodies meet in space they have equal and oppositely directed velocities. Also show that the distances they travel during this time are in the ra0tio 3 : 1.

8. A rocket is fired vertically upwards from the ground with the acceleration 10 m/s^2. If the fuel is exhausted after 1 min, calculate the maximum height attained by the rocket. [**Ans.** 36367·5 m] [**I. I. T. '76**]

9. A body falling freely from rest travels half of the total distance during the last second of its motion. Calculate. (i) height from which the body is dropped, (ii) time of fall. [**Ans.** (i) 56·65 m (ii) 3·4 sec]

10. A satellite is revolving the earth in a circular orbit. If the height of the orbit from surface of the earth of be h calculate the time period of the satellite. Consider variation of g with height. $\left[\textbf{Ans.}\ \ T = 2\pi\sqrt{\frac{R}{g}}\left(1+\frac{3h}{2R}\right)\right]$

11. Two satellites A and B of equal mass are revolving the earth along circular orbits. The height of the orbits from the surface of the earth are R and 3R respectively. (R = radius of the earth). Compare the kinetic and potential energies of the two satellites. [**Ans.** 2 : 1 ; 1 : 2]

12. A small ball of mass 100 g is suspended by a string of length 200 cm from a rigid support. Now the ball is rotated in a horizontal circle of diameter 100 cm. Calculate frequency of revolution and tension in the string. [**Ans.** n = 0·38 Hz ; T = 115·47 gm-wt.]

13. A satellite is revolving the each in a circular orbit near the earth's surface. What additional velocity be given to the satellite so that in can escape earth's gravity ? Radius of the earth = 6400 km ; g = 9·8 m/s^2. [**Ans.** 3·36 × 10^3 m/s] [**Roorkee 1988**]

14. A simple pendulum is constructed by suspending a sphere of mass 1 kg by a copper wire of length 5 m and diameter 0·08 cm. Calculate the time period of the pendulum. Now a sphere of mass 10 kg is taken instead of 1 kg sphere. Calculate the change in time period, if any. Young's modulus of Copper $= 12\cdot4\times10^{10}\ \mathrm{N}/m^2$. [**Ans.** 4·488 sec ; 0·0035 sec] [**Roorkee 1998**]

15. The mass and radius of a planet are half of those of the earth. If the temperature on the surface of the planet be 800 K, show by calculation whether oxygen gas can exist in the atmosphere of the planet. Escape velocity from earth 11·2 km/sec. Boltzman constant. K = 1·38 × 10^{-23} J k^{-1}, molecular mass of oxygen $= 5\cdot3\times10^{-26}$ kg.] [**Ans.** Yes] [**Roorkee 1984**]

16. The radius of a planet is twice the radius of the earth. But average density of the planet is equal to that of the earth. If the escape velocity from the earth be v_e show that the escape velocity from the planet is $v_p = 2v_e$.

17. The gravitational potential at a point situated at a certain height from the surface of the earth is $-5\cdot2\times10^7\ \mathrm{J\ kg^{-1}}$ and acceleration due to gravity 6·4 m/s^2. Radius of the earth 6400 km. Calculate the height of the point from the earth's surface. [**Ans.** 1600 km]

18. An artificial satellite of mass 200 kg is revolving the earth in a circular orbit of radius 6670 km. Calculate kinetic energy, potential energy and total energy of the satellite (mass of the earth = 6·0 × 10^{24} kg ; G = 6·67 ×10^{-11} Nm^2 / kg^2 $\left[\textbf{Ans.}\ 6\cdot0\times10^9\ \mathrm{J}\ ;\ -12\cdot0\times10^9\ \mathrm{J}\ ;\ -6\cdot0\times10^9\ \mathrm{J}\right]$

19. The distance of a planet from the sun is four times the distance of the earth from the sun. Calculate the time period of revolution of planet round the sun. [**Ans.** 8 yrs]

20. The period of revolution of moon round the earth is 30 days. Calculate the distance of the moon from the earth. (G = 6·67 × 10^{-11} Nm^2 /kg^2 mass of the earth 6·0 × 10^{24} kg. [**Ans.** 4·08 × 10^5 km] [Roorkee '84]

21. What is the escape energy of body of mass 100 kg on the surface of the earth ? Earth's radius $= 6\cdot4\times10^6\ m$ and $g = 10\,m/s^2$ [**Ans.** $6\cdot4\times10^9$ J]

22. A particle is moving along a circle under the influence of a centripetal force $F_c = -\frac{k}{r^2}$. Calculate total energy of the particle $\left[\textbf{Ans.}\ -\frac{k}{2r}\right]$ [**I. I. T**]

23. A body is revolving the earth in a circular orbit. height of the orbit from the surface of the earth is h. Work done to put the body in the orbit is equal to its kinetic energy. Show that $h = R/2$; R = radius of the earth.

24. A body was falling from a height H. At a height h from the ground it hits an inclined plane and then moves horizontally. For what value of the ratio $\frac{h}{H}$. the body will take maximum time to reach the ground ?

[**Ans.** 1 : 2] [**I. I. T. 1986**]

25. Show that Kepler's second law can be derived from the principle of conservation of angular momentum.

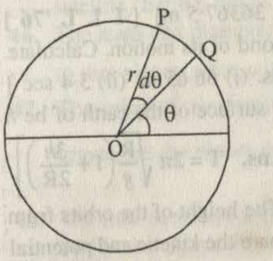


Fig. 3.24

[**Hint :** In small time dt the planet moves from P to Q.

Area discussed , $dA = \frac{1}{2} r \cdot r\,\theta = \frac{1}{2} r^2 \theta$

$\therefore$ areal velocity, $\frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt} = \frac{1}{2} r^2 \omega = \frac{1}{2} m r^2 \omega = \frac{L}{2m}$.

Where L = angular momentum. If L = constant, $\frac{dA}{dt}$ = constant,

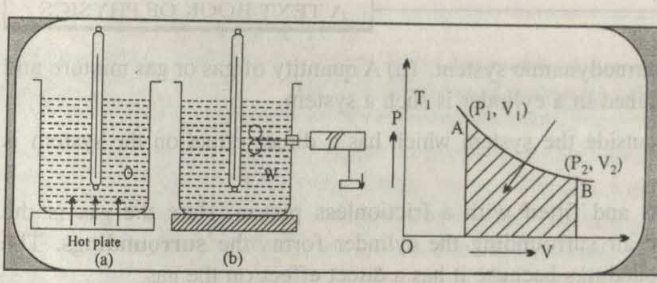
So, areal velocity is constant]

26. Distance between the centres of two stars is 10 cm. The masses of these stars are M and 16M and their radii a and $2a$ respectively. A body of mass m is fired straight from the surface of the larger star towards the smaller star. What should be the minimum initial speed to reach the surface of the smaller star ? Obtain the expression in terms of G, M and a. $\left[\textbf{Ans. } \frac{3}{2}\sqrt{\frac{5GM}{a}}\right]$ **[I. I. T. 1996]**

HEAT AND THERMODYNAMICS – II

• TOPICS DISCUSSED •

1. THERMODYNAMICS 109
2. KINETIC THEORY OF GASES 139



CHAPTER 1

THERMODYNAMICS

● **TOPICS :** ○ Introduction, ○ Definition of some terms used in Thermodynamics; ○ Internal energy of a system; ○ Heat and Work; ○ Work done by a gas during expansion against a constant pressure; ○ Work done by a gas under varying pressure; ○ Work done in a cyclic process; ○ First law of Thermodynamics; ○ Significance of the first law of Thermodynamics; ○ Specific heats of gases; ○ C_p is greater than C_v.; ○ Difference between the two specific heats; ○ Importance of the ratio of two specific heats of a gas; ○ Thermodynamic process; ○ Slopes of isothermals and adiabatics; ○ Reversible and Irreversible process; ○ Inadequacy of the first law; ○ Second law of Thermodynamics; ○ Significance of the law; ○ Difference between the first law and second law of thermodynamics; ○ Short answer type questions (with answers); ○ Exercise.

✪ 1.1. Introduction ✪

Thermodynamics is a branch of Physics which deals with the relation between heat and mechanical energy. It is primarily concerned with the transformation of heat into mechanical work and vice versa. Thermodynamics deal with only experimentally observables such as temperature, pressure, volume etc. and take no account of the structure of system. For this reason thermodynamical methods are specially useful when finer details of the structure of the system are lacking. The principles of thermodynamics are very general and exact and have been successfully applied not only to problems in Physics and engineering but also to those in chemistry, such as chemical reaction, chemical equilibria, elctrochemistry etc.

There are two ways of studying the bahaviour of a system. One is **microscopic behaviour** which deal with the study of atoms and molecules of the substance, constituting the system. The second way is to study the **macroscopic behaviour** where we study the average behaviour of extremely large number of atoms and molecules constituting the system.

Temperature, pressure, volume, internal energy etc. with which thermodynamics deals, actually describe the macroscopic behaviour of large number of atoms or molecules of the system. We speak of temperature or pressure of a gas rather than the temperature and pressure of a single atom or molecule. So, thermodynamics is concerned with macroscopic behaviour rather than microscopic behaviour of the system.

Evidently, thermodynamics deals with gross (or macro) characteristics of the system, taking no account of the atomic constitution of matter. It is indeed remarkable that highly accurate results can be obtained with the help of a few laws of thermodynamics.

Thermodynamics is based on two general laws of nature which governs the conversion of heat into work and vice versa. The first law represents the relation between heat and mechanical work while the second law defines the manner in which the energy changes take place.

✪ 1.2. Definition of some terms used in Thermodynamics ✪

● **(i) Thermodynamics system :** A definite quantity of matter bounded by some closed surface is known as system. The boundaries of a system may be real and imaginary and through which matter does not pass. But the existence of the boundaries of the system is essential in order to visualise the system from the rest of the universe. A system may contain one or more kinds of matter and may be homogeneous or hetrogeneous.

- **Example :** (i) water in a beaker is a thermodynamic system. (ii) A quantity of gas or gas mixture and a mixture of water and water vapour contained in a cylinder is such a system.

- **(ii) Surroundings :** Everything outside the system which has a direct effect on the system is called surroundings.

Consider a cylinder containing a gas and fitted with a frictionless piston. Here the gas is the thermodynamic system. The atmospheric air surrounding the cylinder forms the **surroundings.** The moveable piston is also a part of the surroundings because it has a direct effect on the gas.

A system is said to be **closed** when it can exchange only energy and not matter with the surroundings. Systems which can exchange both matter and energy with the surroundings are known as **open systems.**

A system is said to be **isolated** when it can exchange neither energy nor matter with its surroundings.

- **(iii) Thermodynamic variables or parameters :** The state of a system is determined by the quantities—volume (V), pressure (P), temperature (T), internal energy (U) etc. These quantities are known as thermodynamic variables or parameters or co-ordinates.

Any change in one of the variables causes a change in thermodynamic state of a system.

- **(iv) Thermodynamic state of a system :** The state of a system is its condition or position determined by the thermodynamics co-ordinates. For example, the thermodynamic state of a gaseous system can be determined by its pressure, volume and temperature. Of these three quantities any two are variables, the third is automatically fixed. If we consider a mass of a certain gas in a vessel and fix its pressure and temperature at some predetermined values, the value of volume at equilibrium is fixed. So, volume is the function of temperature and pressure so that only two out of three are independent.

- **(v) Intensive property :** The property that does not depend on the extent of the system is called intensive property of the system. For example, pressure and temperature.

- **(vi) Extensive property :** The property that depends on the extent of the system is called extensive property of the system. For example, volume, internal energy and mass.

- **(vii) Thermodynamic process :** When a system undergoes a change of state, a thermodynamic process said to take place. When the water in a beaker is heated, pressure and volume practically remains unchanged, but temperature of water increases gradually. It means that thermodynamic process has occurred in the system.

A process is named after he fundamental property that remains constant in the process. For example, in isothermal process temperature remains constant. In isochoric process volume is constant, isobaric process is the process that occurs at constant pressure. Adiabatic process is the process of constant heat.

- **(viii) Thermodynamic equilibrium :** If an isolated system is left to itself, the pressure and temperature at a point approach to a steady value. When these quantities attains a steady value, the system is said to be in a state of **thermodynamic equilibrium.** In such a system, the values of thermodynamic co-ordinates (temperature, pressure, volume) are same at all points.

- **(ix) Equation of state :** For a homogeneous system, it is usually enough to specify only three parameters, namely, pressure, volume and temperature. The mathematical relationship between these parameters is known as the equation of state of the system. In the case of an ideal gas, the equation of state is PV = RT for one mole and PV = *n*RT for *n* mole.

Where P, V and T are respectively pressure, volume and temperature of the gas and R is the universal gas constant. In such a case, only two parameters are specified, the third is automatically fixed. Evidently, the thermodynamic state of a simple homogeneous system is described by specifying any two of the three parameters.

✪ 1.3. Internal energy of a system ✪

A matter consists of large number of minute particles called molecules which are in a state of constant rapid motion. Hence they possess kinetic energy. The total kinetic energy of the molecules of the system is called the **internal kinetic energy** while the potential energy arising due to intermolecular attraction is called the **internal potential energy.** The sum of the kinetic and potential energies of all the molecules of a system is called its **internal energy** and usually denoted by U.

❖ **Definition : The sum of the kinetic and potential energies of the constituent particles (molecules) of a system in thermodynamic equilibrium is called its internal energy.**

It is stored within the matter. Internal energy includes strain energy stored in a deforming elastic body and released when unloaded, pressure energy stored in a gas and released when pressure is reduced, chemical energy which can be released in a chemical reaction etc. Temperature rise increases the internal energy.

A thermodynamic system in a given state has one and only one value of internal energy. Hence internal energy is a function of thermodynamic coordinates defining a state. A gas is a simple, homogeneous system whose equilibrium state can be defined in terms of any two of the thermodynamic co-ordinates pressure (P), volume (V) and temperature (T). So internal energy of a gas is a function of P and V, P and T or V and T. **In the case of perfect gas, however, internal energy is a function of temperature alone.**

● **A discussion :** Heat is a form of energy. So, when heat is given to a body its internal energy increases. When work is done on a body, internal energy of the body also increases. Evidently, we can increase the internal energy of a body in two way :

● (i) **By supplying heat :** When heat is given to a body (say, gas), kinetic energy of its molucules increases. As a result, the temperature of the body increases and change of state takes place. Internal energy of the body increases.

● (ii) **By performing work :** When work is done on a body, its internal energy increases. For example assume some gas is enclosed in a cylinder fitted with piston. When a force is applied on the piston, gas is compressed and its volume decreases. Work done on the gas is stored in it in the form of internal energy and temperature of the gas rises.

Hence heat supplied to a gas or work done on the gas have the same effect.

✪ 1.4. Heat and Work ✪

We may define heat as **energy in transit.** It should be noted the energy possessed internally by a body is not called heat—it is simply called internal energy. The term 'heat' is reserved for that energy which flows from a hot body to a cold body due to difference of their temperatures.

When two bodies at different temperatures are kept in contact with each other, the two temperatures approach each other and after sometimes becomes equal, Hence

The energy that flows from one body to another due to the difference in their temperatures is known as heat energy.

If a system loses energy by working against external mechanical force, it is said to do work. Thus work is the transfer of energy without a temperature difference while heat is a transfer of energy due to temperature difference only.

In thermodynamics, 'work' is defined as energy in transit not by virtue of temperature difference but by virtue of change of state or configuration.

- **Equivalence of heat and work :** When two physical quantities produce the same effect in a system, they are supposed to be equivalent to each other. Let 1g of water at 14·5°C is placed in contact with a hot plate [Fig 1·1a]. Energy flows from hot plate to water due to temperature difference. This energy in transit is called heat. The amount of energy that flows when temperature of water rises by 1°C is called 1 calorie.

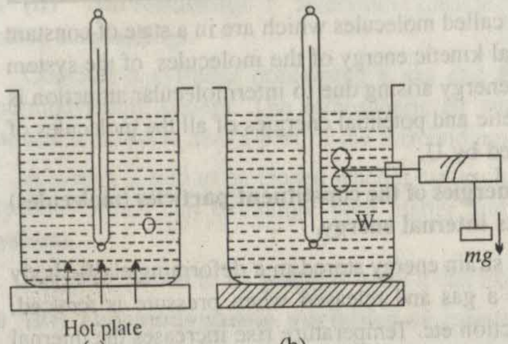


Fig. 1·1

Next the hot plate is removed and a paddle wheel is dipped in water in the beaker [Fig. 1.1(b)]. The wheel is rotated with the help of a falling weight. Here ' work-flow' (not heat flow) will take place from the weight to the water. Rise of temperature of water will be observed. Hence **'heat flow'** and **work flow** produce same effect. Hence we can say that heat and work are equivalent to each other. It is found that water in the beaker is heated through 1°C when the falling weight performs 4·2 joule work. So I calorie of heat is equivalent to 4·2 Joule of work.

❂ 1.5. Work done by a gas during expansion against a constant pressure ❂

When a gas expands it does work on the surroundings. Let a certain amount of gas is enclosed in a cylinder fitted with a moveable piston [Fig. 1.2]. The piston is at the initial position B. At this position pressures on both sides of the piston is same. If P be the pressure of the gas and area of cross section of the piston be A. Then the force applied by the gas on the piston, F = PA.

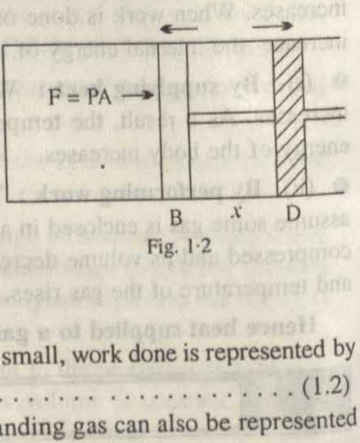


Fig. 1·2

Now, if heat is supplied to the gas at constant pressure the gas pushes the piston outwards. Let the piston comes to the position D. If BD = x, then the work done by the gas is

$$W = FA = PAx = P(V_2 - V_1) \ldots\ldots\ldots\ldots\ldots (1.1)$$

Here $A.x = V_2 - V_1$ = increase in volume. If the change in volume be small, work done is represented by

$$\Delta w = P.\Delta V \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.2)$$

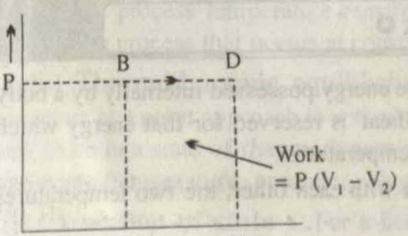


Fig. 1·3

The work done by the expanding gas can also be represented graphically. For this we draw P – V graph which is a straight line [Fig 1·3]. P – V graph of a substance is also called **indicator diagram.**

The work done by the expanding gas from volume V_1 to V_2 is $W = P(V_2 - V_1)$. It is also equal to the area of the rectangle. BD V_2V_1. Hence the work done by a gas is equal to the area under the graph and volume axis.

❂ 1.6. Work done by a gas under varying pressure ❂

When a gas expands against a pressure, work is done by the gas. When the pressure is variable (P–V) diagram for the gas is shown in fig 1.4. Initial state of the gas is represented by R and after expansion final state is denoted by Q. Work done by the gas, W = area RQ V_2V_1. To calculate the area we consider a small area *abcf* (shaded). Its width $df = dV$ denotes small change in volume and its height $ab = P$, pressure on the gas assumed to be constant. So, the work done by the gas for a small increase in volume dV under a constant pressure is $dW = PdV$.

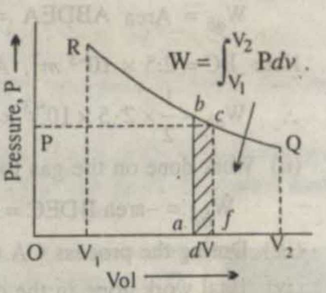


Fig. 1·4

Hence the total work done for change of volume from V_1 to V_2 is

$$W = \int_{V_1}^{V_2} PdV \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.3)$$

When the volume increases, work is done by the gas. It is also called **positive work**. On the other hand when the gas contracts ; work is done on the gas, it is called **negative work**.

1.7. Work done in a cyclic process

When a system (a gas or any working substance) after passing through different states return to the original state it is said to have gone through a cyclic process. In this type of process the indicator diagram or (P – V) diagram is shown in fig 1·5.

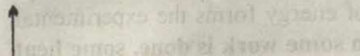


Fig. 1·5

In the cyclic process the work done by the system is numerically equal to the area enclosed by the curve. So, in this case work done is W = area ACBDA.

- **Proof :** The work done by the gas along the forward path ACB is, W_{ab} = area ACBEFA (+ ve work)

The work done along the return path BDA is W_{ab} = – area BEFADB (– ve work) Hence the network done by the gas is

$$W = W_{ab} + W_{ba} = \text{area ACBEFA} - \text{area BEFADA.}$$
$$= \textbf{area ACBDA.}$$

It must be mentioned that in the cyclic process, no change of internal energy of the gas takes place. So, $\Delta U = 0$ as the gas returns to its initial state.

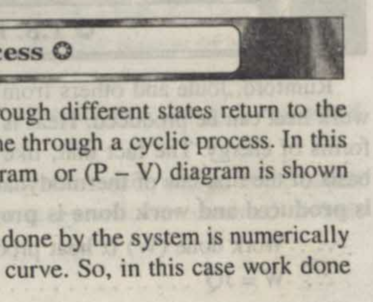


Fig. 1·6

- **Example 1.1. In fig 1·6, (P–V) diagram for a gas is shown in fig. Here pressure is measured in P*a* and volume in m^3. What is the work done by the gas.**

○ **Solution :** The work is done at constant pressure . The measurement of work done is W = area ABCD

$\therefore \quad W = AB \times AD = 3 \times 2 = \textbf{6 J.}$

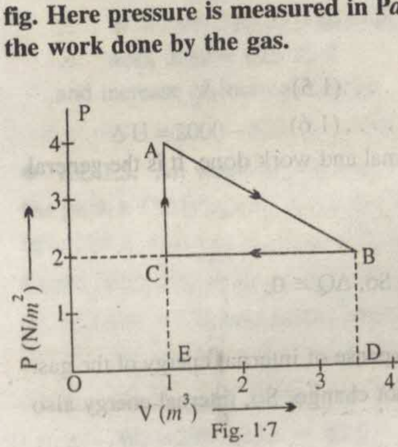


Fig. 1·7

- **Ex. 1.2. In the cyclic process shown in fig 1·7 (P–V) graph is the triangle ABC. The co-ordinates of the point A, B and C are respectively (4, 1), (2, 4) and (2, 1). Pressure in N/m^2 and volume in litre. Calculate the work done in the following cases : (i) For the expansion along AB. (ii) For the contraction along BC and (iii) For the path CA. (iv) Total work done in the cyclic process ABC.**

○ **Solution :** (i) Work done by the gas for the expansion from A to B is

$$W_{ab} = \text{Area ABDEA} = \frac{1}{2}BC \times AC + DE \times EC$$

Here $BC = 2{\cdot}5 \times 10^{-3}\ m^3$, $AC = 2\ Nm^{-2}$, $DE = 2{\cdot}5 \times 10^{-3}\ m^3$, $EC = 2\ Nm^{-2}$

$$\therefore \quad W_{ab} = \frac{1}{2} \times 2{\cdot}5 \times 10^{-3} \times 2 + 2{\cdot}5 \times 10^{-3} \times 2 = \mathbf{7{\cdot}5 \times 10^{-3}\ J}$$

(ii) Work done on the gas for the process BC is

$$W_{bc} = -\text{area BDEC} = -DE \times EC = -3 \times 10^{-3} \times 2 = -6 \times 10^{-3}\ J$$

(iii) During the process CA no work is done as it does not enclose any area.

(iv) Total work done in the cyclic process = $(7{\cdot}5 - 6) \times 10^{-3} = \mathbf{1{\cdot}5 \times 10^{-3}\ J}$

❂ 1.8. First law of Thermodynamics ❂

Rumford, Joule and others from their experimental investigation confirmed that at the expenditure of work heat can be produced. Heat is a form energy and it can perform work. It can be converted to other forms of energy. The fact that, like mechanical work, heat is a form of energy forms the experimental basis of the first law of thermodynamics. The first law states than, **when some work is done, some heat is produced and work done is proportional to the heat produced.**

∴ Work done (W) α heat produced (Q)

∴ $W = JQ$ . . . . (1.4)

J is called the mechanical equivalent of heat. It is defined as the work done to produce unit quantity of heat. Its value is found to be $J = 4{\cdot}2\ J\ cal^{-1}$. Obviously, the first law gives the equivalence of heat and work.

- **General statement of the law :** Clausius put the law in a wider sense and showed that the first law of thermodynamics is actually the alternative form of principle of conservation of energy.

Whenever a heat is supplied to a system, a part of it is used to increase the temperature of the system which is equivalent to at increase of internal energy of the system and the rest is used to do some external work.

Let a working substance or system is in state A having internal energy U_A. It is now given a heat Q. Let the system performs a work W and goes to the state B having internal energy U_B.

Before change of state total energy of the system = $U_A + Q$

After change of state total energy of the system = $U_B + W$

Now, by conservation of energy principle, $U_A + Q = U_B + W$

∴ $Q = U_B - U_A + W$ . . . . (1.5)

In differential form, $\Delta Q = \Delta U + \Delta W$ . . . . (1.6)

So, heat absorbed by a body is equal to the sum of change of internal and work done. It is the general form of the law.

- **Discussion on the equation 1.6 :**

(i) If the process be adiabatic no heat is supplied to the system, So, $\Delta Q = 0$.

∴ $\Delta U + \Delta W = 0$ ∴ $\Delta W = -\Delta U$

It means that in an adiabatic process the system does work at the expense of internal energy of the gas.

(ii) During isothermal process, temperature of the system does not change. So, internal energy also remain unchanged. So $\Delta U = 0$ and $\Delta W = \Delta Q$

(iii) In the case of cyclic process $\oint \Delta U = 0$ [as U is a state function]

$$\therefore \quad \oint dQ = \oint dW + \oint dU = \oint dW.$$

$\therefore$ In the case of cyclic process, heat supplied is equal to work done. Hence the first law may be stated as follows :

If heat is supplied to a system which is capable of doing work, then the quantity of heat energy absorbed by the system will be equal to the sum of the increase of internal energy of the system and external work done by it.

⚪ 1.9. Significance of the first law of Thermodynamics ⚪

The significance of the first law lies in the fact that it establishes a relation between heat and work. The law points out that definite amount of work has to be done to get a definite amount of heat and vice versa. So to get work, energy has to be spent. Work or heat cannot be obtained from nothing.

- **Example 1.3. An object of mass 5 kg is dropped from a height of 0·5 km. If all energy is converted to heat, how much heat in calorie will be obtained ?**

○ **Solution :** Here all the initial potential energy will be fully converted into kinetic energy of the object just before touching the floor. After impact with the floor, kinetic energy will be converted to heat energy.

$\therefore$ Heat produced, $Q = \dfrac{W}{J} = \dfrac{mgh}{J}$.

$$\therefore \quad Q = \frac{5 \times 9 \cdot 8 \times 500}{4 \cdot 2} = 5833 \cdot 3 \text{ cal}$$

- **Ex. 1.4. The temperature of one mol of an ideal gas rises by 75 K when a heat of 2×10^3 cal is supplied to it at constant pressure. Calculate work done and change of internal energy of the gas. $R = 8{\cdot}31 \text{ J mol}^{-1}\text{K}^{-1}$.**

○ **Solution :** Change of volume at constant pressure,

$$V_2 - V_1 = \frac{nR}{P}\left(T_2 - T_1\right) \text{ or, } \Delta V = \frac{nR}{P}\Delta T$$

$\therefore$ work done at constant pressure, $W = P.\,\Delta V = n\,R\,\Delta T$

In this case, $n = 1,\ R = 8 \cdot 31 \text{ J mol}^{-1}\text{ K}^{-1},\ \Delta T = 75\text{K}.$

$\therefore \quad W = 1 \times 8 \cdot 31 \times 75 = \mathbf{623 \cdot 25\ J}$

$\therefore$ work done = **623·25** J

and increase of internal energy, $\Delta U = Q - W$

$\therefore \quad \Delta U = 2000 - 623 \cdot 25 = 1376 \cdot 75 \text{ J}$

- **Ex. 1.5. An working substance is brought from its initial state A to the final state B following the path A CB [Fig 1·8]. Heat absorbed is $\Delta Q = 420$ J and work done $W = 168$ J. How much work will be done if the path ADB is followed ? Given, heat absorbed in this case 294 J.**

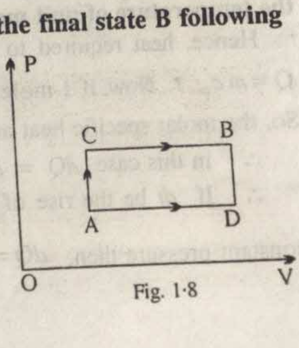


Fig. 1·8

○ **Solution : For the path ACB :** $Q_1 = W_2 + (U_B - U_A)$

$\therefore \quad U_B - U_A = Q_1 - W = 420 - 168 = 252$ cal.

$\therefore$ For the path ADB, work done, $W_2 = Q_2 - (U_B - U_A)$

$\therefore \quad W_2 = 294 - 252 = \mathbf{42\ J}$

1.10. Specific heats of gases

In general, specific heat of a substance is defined as the heat required to raise the temperature of unit mass of the substance by one degree. So, Sp. heat, $s = Q/t$. Where Q = heat required to raise the temperature of unit mass by $t°$. This definition of specific heat is sufficient for solids and liquids, but not sufficient for gases. It is evident from the following discussion.

Under isothermal expansion, the gas absorbs heat, but its temperature remains constant. So, under isothermal condition specific heat is, $s = \dfrac{\text{heat absorbed}}{\text{rise of temperature}} = \dfrac{Q}{0} = \infty$

Again, under adiabatic expansion, no heat is supplied to the gas. But there is a change of temperature. So, spacific heat under adiabatic expansion is

$$s = \frac{\text{heat absorbed}}{\text{rise of temperature}} \cdot \frac{0}{\Delta T} = 0.$$

So, it is evident that if the pressure and volume is not specified, specific heat of a gas varies from zero to infinity. The reason is : when a gas is given a heat its temperature rises and at the same time pressure and volume also changes. Rise of temperature of a fixed mass of a gas when a heat given to it depends on external condition. If the pressure is kept constant, rise of temperature will be less than the case when volume is kept constant. Because at constant pressure a part of the heat is spent to perform external work, but at constant volume no external work is done.

Hence it is customary to define specific heat of a gas by keeping either pressure constant or volume constant. Accordingly a gas possesses two specific heats—(i) specific heat at constant volume (c_v) and (ii) specific heat at constant volume (c_P).

❖ Definitions :

- **(i) The specific heat at constant volume (c_v) is defined as the amount of heat required to raise the temperature of unit mass of a gas by 1°, the volume being kept constant.**

Hence heat required to rise the temperature of m gm of the gas at consant volume by t°C at constant volume is $Q = m\, c_V\, t$.

Now, if 1 mole of the gas is considered then the specific heat is called molar specific heat.

$\therefore$ Molar specific heat at constant volume, $C_v = M\, c_v$ [M = molecular mass]

From the first law of thermodynamics $dQ = dU + Pdv$

When volume is kept constant $dv = 0$. $\therefore$ $dQ = dU$

If dT be the rise of temperature of unit mass of the gas when a heat dQ is given to it, then

$$dQ = c_v dT$$

$$\therefore\ c_v \cdot dT = dU \quad \therefore\ c_v = \left(\frac{dU}{dT}\right)_v = \left(\frac{dQ}{dt}\right)_v$$

- **(ii) The specific heat at constant pressure C_P is defined as the amount of heat required to raise the temperature of unit mass of a gas by 1°, the pressure being kept constant.**

Hence, heat required to raise the temperature of m gm of the gas by t°C at constant pressure is $Q = m\, c_P\, t$. Now, if 1 mole of the gas is considered, then the specific heat is called molar specific heat. So, the molar specific heat at constant pressure is $C_P = M\, c_P$.

$\therefore$ in this case $dQ = dU + P\,dV$ [$\because dV \neq 0$]

$\therefore$ If dt be the rise of temperature when dQ amount of heat is given to unit mass of the gas at constant pressure then, $dQ = c_p\, dT$ $\therefore$ $c_p = \left(\dfrac{dQ}{dT}\right)_p$

- **Units :** (i) In C.G.S. system unit of c_p or c_v is cal g^{-1} $°C^{-1}$ and unit of C_p or C_v is cal mol^{-1} $°C^{-1}$

  (ii) In SI, unit of c_p or, c_v is $J \cdot Kg^{-1}\ K^{-1}$ and of C_P or, C_V is $J \cdot mol^{-1}\ K^{-1}$
- **Relation between the two units : 1 cal g^{-1} $°C^{-1}$ = 4200 J Kg^{-1} K^{-1}**

❂ 1.11. C_p is greater than C_v ❂

From the first law of thermodynamics we know : $Q = U_2 - U_1 + W$

$\therefore$ Heat supplied to the gas = change of internal energy + external work done.

(i) If the heat is supplied at constant volume, the gas does not perform any external work. Kinetic energy of the gas molecules increases. Hence temperature (also internal energy) and pressure of the gas increase. Here all the heat supplied increases the internal energy of the gas.

Hence $Q = U_2 - U_1$

(ii) On the other hand, if the heat is supplied at constant pressure, the volume of the gas increases and also temperature rises. Now, the gas performs some external work. So in this case all the heat supplied does not increase the temperature. Here a part of the heat raises the temperature and the rest performs external work. So, $Q = (U_2 - U_1) + W$

So, we can say that the heat required to raise the temperature of 1g of a gas by 1°C at constant pressure is greater than the heat required for the gas for the same purpose at constant volume. Hence c_P is greater than c_V.

❂ 1.12. Difference between the two specific heats ❂

We consider one mole of the gas enclosed in a cylinder fitted with a piston. The initial pressure, volume and temperature of the gas are P, V and T respectively.

Now, ΔQ_1 heat is supplied to the gas (1 mole) at constant volume so that temperature rises by ΔT. Then, as change in volume is zero,

$$\Delta Q_1 = \Delta U = C_v\ \Delta T \quad \text{(i)}$$

Here C_v is the molar specific heat at constant volume.

Next, assume that ΔQ_2 heat has to be supplied to the enclosed gas to raise the temperature by ΔT at constant pressure $\Delta Q_2 = C_P \cdot \Delta T$. If the change in volume be ΔV then

$$\Delta Q_2 = C_P \cdot \Delta T = \Delta U + P\,\Delta V \quad \text{(ii)}$$

Now, for 1 mole of gas, $PV = RT$

$\therefore$ at constant pressure, $P\,\Delta V = R\,\Delta T$ .... (iii)

Hence from the equations (ii) and (iii), $C_p\ \Delta T = C_v\ \Delta T + R\ \Delta T$

$$\therefore \quad C_p - C_v = R \quad (1.6)$$

If R is expressed in work unit, $C_p - C_v = \dfrac{R}{J}$.

Also, $c_p - c_v = \dfrac{R}{MJ} = \dfrac{r}{J}$ [r = gas constant for unit mass]

Ratio of the two specific heats of gas :

The ratio of two specific heats of a gas is donated by γ.

So, $\gamma = \dfrac{c_P}{c_V} = \dfrac{c_p}{c_v}$ Such $C_p > C_v$, γ is always greater then 1.

○ 1.13. Importance of the ratio of two specific heats of a gas ○

The ratio of the specific heats of a gas at constant pressure and at constant volume $\left(\gamma = \dfrac{C_p}{C_v}\right)$ is very important in different ways.

(1) From the value of γ for a gas we can have an idea of the molecular constitution of a gas.

(i) **The value of γ for a monatomic gas :** From Kinetic theory we know, the total energy for one mole gas is $U = \frac{3}{2}RT$.

$\therefore$ Molar sp heat at constant volume, $C_v = \dfrac{dV}{dT} = \dfrac{3}{2}R$ and molar sp. heat at constant pressure,

$$C_P = C_v + R = \frac{5}{2}R$$

$$\therefore \text{ For monatomic gas } \gamma = \frac{C_P}{C_V} = \frac{\frac{5}{2}R}{\frac{3}{2}R} = \frac{5}{3} = 1{\cdot}67$$

(ii) **Value of γ for diatomic gas :** Total kinetic energy of one mole gas is $U = \frac{5}{2}RT$

$$C_v = \frac{dV}{dT} = \frac{5}{2}R \text{ and } C_P = C_v + R = \frac{7}{2}R \quad \therefore \gamma = \frac{C_P}{C_V} = \frac{7}{5} = 1{\cdot}4$$

(iii) For triatomic gas, γ = 1.3.

(iv) For polyatomic gas the value of γ lies between 1·3 and 1·1. Hence by determining γ, we can say whether the gas is monatomic or diatomic etc.

(2) The value of γ is necessary to find the velocity of sound in a gaseous medium.

(3) To steady adiabatic processes a knowledge of the value of γ is essential.

● **Example 1.6. For an ideal gas γ = 1·4, calculate the values of molar specific heats of the gas. R = 2 cal mol^{-1}. K^{-1}**

○ **Solution :** $\gamma = \dfrac{C_P}{C_V} \quad \therefore \; C_P = \gamma\, C_v$

Again, $C_P - C_V = R \quad \therefore \; C_V = (\gamma - 1)\,R$

$$\therefore \quad C_V = \frac{R}{\gamma - 1} = \frac{2}{1{\cdot}4 - 1} = 5 \text{ and } C_P = R + C_V = 5 + 2 = 7$$

$$\therefore \quad C_P = 7\,\text{cal mol}^{-1}\text{K}^{-1} \quad \text{and} \quad C_v = 5\,\text{cal mol}^{-1}\text{K}^{-1}$$

● **Ex. 1.7. Molar specific heat at constant volume for hydrogen is 4·87 cal mol^{-1}k^{-1} and its density at NTP is 0·09 g/ litre. Molecular mass of hydrogen is 2·016. Calculate specific heat at constant pressure of hydrogen.**

○ **Solution :** Molar gas constant is $R = \dfrac{PV}{T}$, Here $P = 1.013 \times 10^6$ dyne / cm^2

and molor volume, $V = \dfrac{2{\cdot}016}{0{\cdot}09 \times 10^{-3}} = 22{\cdot}4 \times 10^3\,\text{c.c}$; $T = 273\,\text{K}$

$$\therefore \quad R = \frac{1{\cdot}013 \times 10^6 \times 22{\cdot}4 \times 10^{-3}}{273} = 8{\cdot}31 \times 10^7 \;\; \text{erg mol}^{-1}\,\text{K}^{-1}$$

$$\therefore \quad C_P = C_v + \frac{R}{J} = 4{\cdot}87 + \frac{8{\cdot}31 \times 10^7}{4{\cdot}2 \times 10^7} = \mathbf{6{\cdot}85}\ \textbf{cal}\ \text{mol}^{-1}\text{K}^{-1}$$

● **Ex. 1.8. 1 mole gas is initially at 0°C. It is allowed to expand at constant pressure to double its volume. How much heat will be required ? Given, $C_V = 20{\cdot}7$ J mol^{-1}K^{-1} and R = 8·31 J mol^{-1}K^{-1}**

○ **Solution :** From Charles law : $\frac{V_1}{V_2} = \frac{T_1}{T_2}$ $\therefore$ $T_2 = \frac{V_2}{V_1} T_1 = 2\,T_1 = 2 \times 273 = 546$ K

Now, $C_p = C_v + R = 20{\cdot}7 + 8{\cdot}31 = 29{\cdot}01$ J mol^{-1} K^{-1}

$\therefore$ Amount of heat required, $Q = C_p\,(T_2 - T_1)$

$\therefore$ $Q = 29{\cdot}01 \times (546 - 273) =$ **7917 J.**

● **Ex. 1.9. A pressure of 10^6 dyne / cm^2 has to be applied on a gas to reduce its volume from 20 litres to 10 litres. How much heat will be produced ? J = 4·2 J / Cal. [H. S. 2000]**

○ **Solution :** Work done by the gas, $W = P\,(V_2 - V_1)$

$W = 10^6 \times (20 - 10) \times 10^3$ erg = **10^3 J**

$\therefore$ Heat produced, $H = \frac{W}{J} = \frac{10^3}{4{\cdot}2} =$ **240 cal**

✪ 1.14. Thermodynamic process ✪

A thermodynamic system is specified by the thermodynamic co-ordinates, viz, temperature, pressure and volume. Any change of any of these quantities produces a change in the state of the whole system. Such a change is called thermodynamic process or simply process. We consider the following processes.

(A) Isothermal process :

If the pressure and volume of a substance changes in such a way that its temperature does not change then the process called isothermal process. In an isothermal process, none of the quantities ΔQ, ΔW and ΔU in general zero. Of course, in the care of ideal gas ΔU = 0. Few examples of isothermal process are given below.

● (i) **Boiling :** When a liquid boils at constant pressure, its temperature remains constant. So the average translational kinetic energy of the molecules does not change during phase transformation. But internal energy increases.

● (ii) **Melting :** During melting at constant pressure the temperature remains constant. Here also average translational kinetic energy of the molecules does not change; but internal energy increases.

● **Explanation :** To explain the isothermal process, let a gas is enclosed in a metal cylinder fitted with a frictionless moveable piston, also made of metal. Now, let the gas is compressed slowly, work is done on the gas. Its internal energy and temperature tend to increase. The heat so produced is conducted to the surroundings through the metal cylinder. Thus the temperature and internal energy of the gas remain constant. It is called **isothermal compression.**

Similarly let us suppose that the gas is allowed to expand slowly. Work is done by the gas. Now, the internal energy and hence the temperature of the gas tends to decrease. But heat from the surroundings will be conducted through the metal cylinder into the gases and exactly compensate the loss of internal energy. So, the temperature of the gas remains constant. This process is called **isothermal expansion.**

Evidently isothermal process is a slow process. In this case either heat is removed from the system or heat is allowed to enter into the system. The following conditions must be satisfied to accomplish an isothermal process.

(i) Container of the gas must be good conductor of heat

(ii) Thermal capacity of the vessel should be high.

(iii) Compression or expansion of the gas should be very slow.

- **Relation between pressure and volume :** In the case of isothermal change of an ideal gas, its pressure and volume changes according to Boyle's law, PV = Constant. The (P – V) graphs at constant temperatures will be a rectangular hyperbola. [Fig 1.9]. These are called isothermals. The three graphs corresponding to the temperatures T_1, T_2 and T_3 are called **isothermals**.

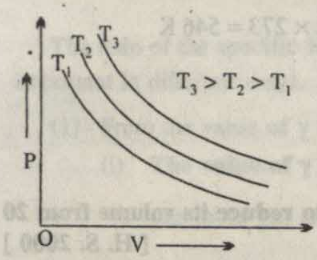


Fig. 1·9

- **Application of first law of thermodynamics :** During an isothermal change of a gas, temperature remains constant. So the internal energy of the gas remains unchanged during an isothermal process. So, change of internal energy of the gas $\Delta U = 0$. Hence from the first law of thermodynamics Q = W. So, during an isothermal process work done is equal to heat supplied.

- **Work done during isothermal process :** We consider 1 mole of an ideal gas. Its one isotherm on (P – V) graph is shown in fig 1.10. We shall calculate work done by the gas for isothermal expansion from A to B. Initial pressure and volume is (P_1, V_1) and final pressure and volume is (P_2, V_2) .

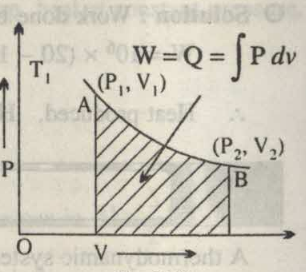


Fig. 1·10

Work done in the isothermal process is $W = \int_{V_1}^{V_2} PdV$

$$\therefore \quad W = \int_{V_1}^{V_2} \frac{RT}{V}\,dV = RT\ln\left(\frac{V_2}{V_1}\right) \quad \therefore \quad W = RT\ln\left(\frac{V_2}{V_1}\right) = RT\ln\left(\frac{P_1}{P_2}\right) \quad \ldots\ldots\ldots\ldots (1.7)$$

$\therefore$ Heat absorbed by the gas, $Q = W = RT\ln\left(\frac{P_1}{P_2}\right)$

(B) Adiabatic process :

It is a kind of thermodynamical process in which pressure and volume of the gas changes in such a way that no heat is allowed to enter and leave the system. The total heat content of a gas remains unchanged. In general any process that occurs under such a condition that no heat enters or leave the system, is called adiabatic process.

Suppose an enclosed gas is suddenly compressed. Some work is done and heat is produced. Again if the gas is allowed to expand suddenly work is done by the gas and the gas tends to cool. In the first case if no heat is removed from the gas or in the second case if no heat is supplied to the gas, the temperature of the gas does not remain constant.

As no heat leaves the system or no heat enters the system, the total heat content of the system does not change. Hence $\Delta Q = 0$. For a Adiabatic change the following conditions must be satisfied :

(i) The wall of the container should be non-conducting.
(ii) The thermal capacity of the vessel be low.
(iii) Change of pressure on the gas be very rapid.

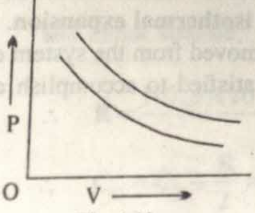


Fig. 1·11

During an adiabatic process the change of pressure and volume of the gas does not obey Boyle's law. In this case, the pressure and volume of the gas obeys the relation PV^{γ} = constant. γ is called the ratio of two specific heats of a gas. The other two adiabatic equations are :

$TV^{\gamma-1}$ = constant and $TP^{(1-\gamma)/\gamma}$ = constant. (P – V) diagram of a fixed mass of a gas undergoing adiabatic changes is shown in fig 1.11. These curves are known as adiabatics.

From the first law of thermodynamics $Q = U_2 - U_1 + W = \Delta U + W$.

In this case, $Q = 0$, Hence $W = -\Delta U$, So, in the case of adiabatic expansion work done by the gas is equal to loss of internal energy of the gas.

- **Work done under adiabatic process :** Let a gas expands adiabatically from volume V_1 to V_2. The expansion is represented by the adiabatic AB. Initial pressure, volume and temperature are P_1, V_1 and T_1 and the corresponding final quantities we P_2, V_2 and T_2.

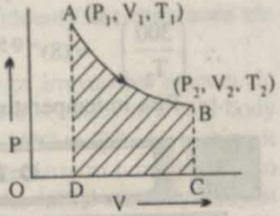


Fig. 1·12

Work done by the gas, $W = \int_{V_1}^{V_2} P dV$ = area ABCD [Fig 1.12]

$$\therefore \quad W = \int_{V_1}^{V_2} \frac{K}{V^{\gamma}} dV \quad [\because PV^{\gamma} = K, \text{ a constant }]$$

$$W = \frac{K}{1-\gamma}\left[V_2^{\,1-\gamma} - V_1^{\,1-\gamma}\right] = \frac{1}{1-\gamma}(P_2V_2 - P_1V_1)$$

$$= \frac{R}{1-\gamma}(T_2 - T_1) = C_V(T_1 - T_2) \left[\because C_V = \frac{R}{\gamma - 1}\right]$$

✪ 1.15. Slopes of isothermals and adiabatics ✪

In fig 1.13, AB is an isothermal and CD is an adiabatic force given mass of a gas. It can be shown that the slope of the adiabatic through any point is γ times the slope of the isothermal through the same point.

- (i) **For isothermal process :** In this case Boyle's law is applicable $\therefore PV = $ Constant.

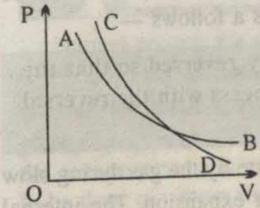


Fig. 1·13

By differentiating , $Pdv + Vdp = 0 \quad \therefore \quad \frac{dp}{dv} = -\frac{P}{V}$

So, the slope of the curve $= -\frac{P}{V}$

- (ii) **For adiabatic process :** In this case, $PV^{\gamma} = $ constant

On differentiation $\gamma P V^{\gamma-1} dv + dp.V^{\gamma} = 0$

$\therefore \quad \gamma P dv + V.dp = 0$ or, $\frac{dp}{dv} = -\left(\frac{P}{V}\right)\gamma$

$\therefore$ Slope of the curve $-\gamma\frac{P}{V}$

$\therefore$ slope of the adiabatics = γ × slope of the isothermals.

Since $\gamma > 1$, slope of the adiabatics > slope of the isothermals.

- **Example 1.10. A quantity of air at NTP is adiabatically compressed to $\frac{1}{5}$ th of its volume. Calculate the rise of its temperature. γ for air = 1·41**

○ **Solution :** We know $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1} \quad \therefore \quad \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$

By equation, $T_1 = 273\text{K}, \ \frac{V_1}{V_2} = 5, \ \gamma = 1{\cdot}41$

$\therefore \quad \frac{T_2}{273} = (5)^{1\cdot41-1} = (5)^{0\cdot41} \qquad \therefore \quad T_2 = 273 \times (5)^{0\cdot41} = 528{\cdot}2\text{ K} = 255{\cdot}2°\text{C}$

● **Ex. 1.11. A perfect gas at 27°C is suddenly compressed to 8 times its original pressure. Find its rise of temperature. $\gamma = 1{\cdot}5$.**

○ **Solution :** We know, $T_1^{\gamma} P_1^{1-\gamma} = T_2^{\gamma} P_2^{1-\gamma} \quad \therefore \quad \left(\frac{T_1}{T_2}\right)^{\gamma} = \left(\frac{P_2}{P_1}\right)^{1-\gamma}$

Here, $T_1 = 300\text{K}, \quad \frac{P_2}{P_1} = 8, \quad \gamma = 1{\cdot}5$

$$\therefore \left(\frac{300}{T_2}\right)^{1\cdot5} = (8)^{-0\cdot5} \quad \therefore \left(\frac{T_2}{300}\right)^{3} = 8 \quad \therefore \; T_2 = 2 \times 300 = 600 \text{ K}$$

$\therefore$ Rise in temperature = 600 – 300 = 300 K = **300°C**

○ 1.16. Reversible and Irreversible process ○

(A) Reversible process :

The concept of reversible process is very important in the field of thermodynamics.

Consider a gas contained in a cylinder fitted with a moveable piston. Let the gas absorbs some heat ΔQ and performs some work ΔW when it changes its state from A to B [Fig. 1.14]. Obviously in passing from state A to the final state B, the gas passes through a series of intermediate states.

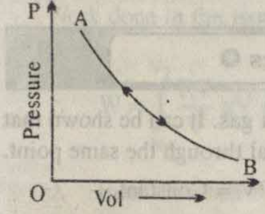


Fig. 1·14

We now force the system in reverse direction. Let the gas be brought back from state B to the state A by reversing the process i.e. instead of expanding the gas let us compress it. In the reverse process if the work done on the gas is ΔW and it passes through the same intermediate states with the release of heat ΔQ, we say that the process A → B and reverse process B → A are reversible. So, we may define the reversible process a follows —

From thermodynamical point of view, any process that can be exactly reversed so that the system passes through the same intermediate states as in the forward process with the reversal of sign in work done and heat is absorbed is called a reversible process.

A reversible process implies that there is no wastage of energy. The work done by the gas during slow expansion can be calculated and it is the same as that done by the gas during expansion. The internal energy, being a function of the state, is regained as the gas returns to its initial state. Thus, no change in the internal energy occurs in the process. Hence, from the first law of thermodynamics, the heat absorbed by the gas in the direct process (expansion) be equal to the heat rejected by the gas in the reverse process (compression).

Thus, all the requirements regarding heat and work are satisfied and the process is reversible. However, we have assumed that dissipative effects, such as friction are absent.

● **Conditions necessary for a reversible process :** Reversible process is an ideal concept and they cannot be realised in actual practice. Yet they are extremely important in thermodynamics because they can be handled analytically. To achieve reversibility the following condition must be fulfilled.

(i) Friction and other dissipative forces must be absent.

(ii) The process must be extremely slow.

(iii) The causes that bring about a change (i.e., pressure, or temperature different) should be extremely weak, so that the system remains in equilibrium with the surrounding.

(iv) There must not be any loss of heat due to conduction or radiation.

(B) Irreversible process :

All natural process are irreversible, because no natural process can fulfil the requirements of a reversible process. Actually reversible process is an idealisation and its importance lies in the fact that it can be analysed and yields useful information on the departures from reversibility.

Any process that can not be made to go in the reverse direction is called irreversible process. At the conclusion of the process the system can not be restored to the initial state when the process is reversed. A permanent change is left some where. All changes which occur suddenly like explosions etc may be considered as irreversible.

- **Examples of irreversible process :** There are innumerable examples of irreversible process. In fact all processes going around us are irreversible. (i) Conduction of heat from a hot body to a cold body is an irreversible process. Suppose water in a beaker (system) is heated from 15°C to 25°C by placing it on a hot plate. This process which is isobaric and isochoric, is irreversible because to bring it back to 15°C, it is to be kept in contact with cold body. So it can be brough back to the initial condition only at the cost of a change in internal energy of another body. Hence it is irreversible. (ii) Free expansion of a gas is an irreversible process. Free expansion means expansion against zero applied pressure. This process is irreversible because to bring the system to its initial volume considerable work has to be done.

(iii) Two solutions having different concentrations are mixed and the mixture acquire a uniform concentration due to diffusion. The reverse is not possible.

(iv) Gas at a higher pressure moves to a vessel at a lower pressure until the pressures are equalised. This process is unidirectional.

(v) Chemical reactions are irreversible.

(vi) Heat produced by friction, heat produced by electric current etc are all examples of irreversible processes.

1.17. Inadequacy of the first law

Natural process are spontaneous and unidirectional. All spontaneous process are irreversible. This fact is related to the second law, we consider the following example.

- **Volume expansion of a gas :** A confined gas always flows from high pressure region to low pressure region until the pressure is uniform. So the volume of the gas increases spontaneously. Evidently the reverse process is not possible. The gas itself can not go back to its initial condition. Some work has to be done to restore the gas to its initial state.

Let one mole of an ideal gas is enclosed in a cylinder fitted with a piston. The cylinder is placed on a source of heat at temperature T. The gas is allowed to expand isothermally and reversibly from the state (P_1, V_1, T) to the state (P_2, V_2, T) ($V_2 > V_1$). The work done is

$$\Delta W = \int_{V_1}^{V_2} P dv = RT \log_e \frac{V_2}{V_1} = RT \log_e \frac{P_1}{P_2}$$

For isothermal change of an ideal gas $\Delta U = 0$. Hence in this case $\Delta Q = \Delta W$. In this manner we can abstract heat from a source and convert it fulls to work. But at the same time, there will be change of state of the working substance. Hence, **whole of heat cannot be converted to work without producing a change in the state of the working substance.**

But to convert the internal energy of the same source we have to bring back the gas in the cylinder to its initial condition. For this some work has to be done on the gas. The cylinder should be kept in contact with heat source and the gas can be taken to its initial state by compressing the gas isothermally and reversibly. Work done on the gas in the reverse process is same as the work done by the gas during forward process. Now same amount of heat is rejected to the source. Evidently in this cyclic process no net work is obtained from the heat. So, it can be said **that continuous work cannot be obtained from a single heat source.** First law of thermodynamics does not oppose it. But if that be the case then the atmosphere and the ocean has a huge store of internal energy which could be utilised to do useful work.

Alternatively, the cylinder can be put in contact with source of hear at lower temperature T_1 $(T_1 < T)$ and by isothermal and reversible compresson the gas can be restored to its initial state. In this case the work done is $\Delta W_1 = RT_1 \log_e \frac{V_1}{V_2} = -RT_1 \log_e \frac{V_2}{V_1}$

So, heat rejected to the source at T_1 is $\Delta Q_1 = \Delta W_1$. Hence in the cyclic process, heat absorbed from the source of higher temperature is $\Delta Q = RT \log_e \left(\frac{V_2}{V_1}\right)$

$\therefore$ Amount of work done, $\Delta W' = R\,(T - T_1) \log_e \left(\frac{V_2}{V_1}\right)$

Hence, it is possible to abstract heat continuously from source at higher temperature, convert a part of the heat to a useful work. But a lower temperature body or sink is required where the rest of the heat will be rejected.

The fact is that it is not possible to construct an engine which can convert heat from a single reservoir called source into work, without rejecting some heat to a second reservoir called be sink at a lower temperature. This negative experience forms the basis of the second law of thermodynamics. This law asserts that it is futile to make any attempt to construct a machine which utilises heat from a single source.

Thus, the first law denies the possibility of destroying or creating energy and the second law denies the possibility of utilizing heat from a single body for conversion into work. The following condition must be fulfilled to utilize heat to useful work.

(i) A device called engine with a working substance is essential. (ii) The engine must operate in a cyclic process. (iii) The engine must operate between two temperatures. It will absorb heat from a hot body (called source), convert a part of it into useful work and reject the rest to a cold body (called sink).

✪ 1.18. Second law of Thermodynamics ✪

Second law of thermodynamics concerns with the direction in which energy transformation naturally takes place. It has been stated in several forms. There are two well known statements of second law— one given by Clausius and the other by Kelvin and Planck.

- **Clausius Statement :** It is impossible for a self acting machine, unaided by any external agency, to transfer heat from a body at a lower temperature to a body at a higher temperature, or heat can not by itself pass from a colder to a hotter body.

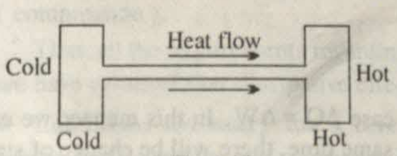


Fig. 1·15

The second law as stated above implies that heat will not flow spontaneously from a cold body to a hot body. Of course, refrigerator is a device which can transfer heat from cold body to a hot body. But for its opertion some work is done.

- **Kelvin-Planck statement** : It is impossible to construct a machine which, when it operates in a cycle, extracts heat from a source and converts it wholly into work.

This statement of the law implies that a continuous working device that draws heat energy from a high temperature source and converts it all into work is impossible. It has to reject some heat to the sink. For the success of a device known as heat engine at least two heat reservoirs are needed. One heat reservoir be at higher temperature from which heat is extracted and the other at low temperature where a part of the heat is rejected.

The second law thus forbids two processes to occur spontaneously, without other changes in the system, namely,

(i) Complete transformation of heat into work;

(ii) Flow of heat from lower to higher temperature.

The two statements of the second law are equivalent, and one can be derived from the other. Which one can be used in a particular case, is a matter of convenience or taste. The second law cannot be proved directly, as it is a **negative statement**. It is universally accepted as a true law, as no exception has yet been found. No other law of physics has been applied to so wide a range of phenomena as the second law of thermodynamics. The law has wide field of applications, and conclusions drawn from them, or predictions obtained from them have been experimentally verified. It has been applied to the study of change of state, solutions, osmotic pressures, electric cells, thermo electricity, chemical reactions, etc. In every case, the predictions of the law have been confirmed experimentally.

❂ 1.19. Significance of the law ❂

The second law of thermodynamics is actually a repetition of a universal natural law and based on our practical experiences. According to this natural law we see that (i) a body cannot move from a lower level to a higher level unless work is done on it by some external agency; (ii) Electric current cannot flow from a lower to a higher potential point unless work is done on it; (iii) Also heat cannot flow from a body at a lower temperature to a body at higher temperature unless work is done by some external agency.

Obviously, a machine is required for taking a body from a lower to a higher level or an electric machine is needed to drive current from lower potential to higher potential one. To transfer heat from colder body to hotter body, work has to be done as is done in refrigerator.

❂ 1.20. Difference between the first law and second law of thermodynamics ❂

It is important to note the distinction between the first law and the second law of thermodynamics. The first law asserts that heat and work are only different forms of energy and are mutually inter convertible. It does not tell us how and to what extent one can be converted into the other. It only tell that energy is conserved. But the first law does not tell the way of conversion of heat to work. The second law asserts that conversion of heat into work essentially requires a hot body and a cold body simultaneously and that when heat is converted into work, only a partial conversion is possible. The first law is concerned with the energy balance which must be satisfied in any process; the second law is concerned with the direction in which a given process may take place.

We summarise the differences as follows :

(i) According to the first law, heat and mechanical energy are the different forms of energy.

(ii) Mechanical energy can be converted to heat and vice versa.

(iii) First law does not mention how heat energy comes; when heat can be converted to mechanical work or whether the conversion is partial or complete.

(iv) From Second law we get the direction of flow of heat. Heat can not flow from a body at lower temperature to a body at higher temperature.

(v) Second law states that conversion of heat into work is partial and for the conversion a body at higher temperature and a body at lower temperature are required.

●● Short Answer Type Questions (with answers) ●●

● **Question 1. Can mechanical energy be converted continuously and completely into heat ? Is reverse also possible ?**

○ **Ans.** The mechanical energy can be completely and continuously converted into heat. But heat energy cannot be completely and continuously converted into work.

● **Q.2. A piece of metal is hammered. Does its internal energy increase ? Does heat enter from outside ?**

○ **Ans. 1st part :** Internal energy of the metal increases. Because : Kinetic energy of the hammer is converted to heat which is absorbed by the metal. As a result its temperature and internal energy increases.

2nd part : No heat enters from outside.

● **Q.3. What is the internal energy of one mole of monatomic gas at 0°C ? Does the value of this energy depend on pressure, volume and nature of the gas ? $R = 8{\cdot}31\ J\ mol^{-1}K^{-1}$**

○ **Ans.** The energy of 1 mole of an monatomic gas at TK is given by $U = \frac{3}{2}RT$

$$\therefore \quad U = \frac{3}{2} \times 8 \cdot 31 \times 273 = \mathbf{3403\ J}$$

2nd Part : In the case of ideal gas, there is no attraction or repulsion among the molecules. For this reason, the value of internal energy does not depend on volume or pressure of the gas. The value of internal energy depends only **on temperature of the gas.**

● **Q.4. A vessel of volume 1 m^3 is divided into two parts by a screen : On part is filled up with an ideal gas and the other is empty. The vessel is thermally isolated. Now, the screen is suddenly removed, will the temperature of the gas change ? [I. I. T. '93]**

○ **Ans.** The temperature of the gas will not change.

● **Reason:** Expansion of gas takes place against zero pressure. Hence no external work is done. Also internal energy will not change because the molecules have no attraction or repulsion along there.

Here no heat is supplied $\Delta Q = 0$ and workdone $Pdv = 0$

$\therefore$ change of internal energy $\Delta U = 0$ Hence there will be no change of temperature of the gas.

● **Q.5. The volume of a certain mass of a gas in reduced to half of its initial volume in two ways : (i) first slowly, (ii) then rapidly. In which case more work is done ? [J. E. E. '88; H. S. '98]**

○ **Ans.** More work is in the second case.

● **Explanation :** We know that change of pressure and volume of a gas can be shown in (P – V) diagram called indicator diagram and the area under the graph and the volume axis gives work done on the gas. (P–V) diagrams for the two cases are shown in fig [1.16]

Slow process is shown by the graph AC. So, it is also an isothermal process. The volume of the gas changes from V to V/2. The work due in the isothermal contraction is the area ACDE.

On the other hand, rapid change in volume from V to V/2 is shown by the graph AB. Evidently, it represents the adiabatic process. Final pressure is greater in the adiabatic process. The work done in the adiabatic contraction is given by the area ABDE.

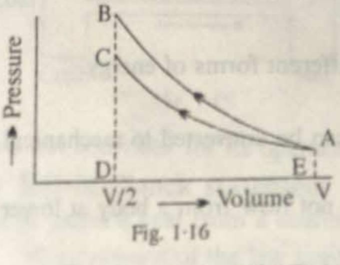


Fig. 1·16

● **Evidently : Area ABDE > area ACDE.**

Hence the work done in the rapid process is greating than in the slow process.

● **Q.6. A certain mass of a gas is allowed to expand to double its volume in two ways : (i) first slowly and (ii) then rapidly. In which case more work is done ?**

○ **Ans.** More work is done in the first case i.e., when expansion takes place slowly.

● **Explanation :** Slow expansion corresponds to isothermal process and the rapid expansion coresponds to adiabatic expansion.

In the fig 1.17 these two types of expansions are shown the curves AB and AC on the (P – V) diagram. Here the adiabatic curve is below the isothermal. Clearly, the work done in slow process (i.e. isothermal process) is equal to the area ABDE and that in the rapid process (i.e. adiabatic process) is equal to the area ACDE.

As the area ABDE > area ACDE, more work is done in the case when the gas expands slowly.

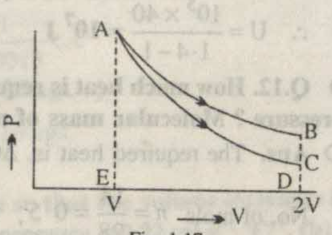


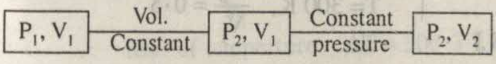
Fig. 1·17

● **Q.7. Why is the conversion of heat into work not possible without a sink at lower temperature?**

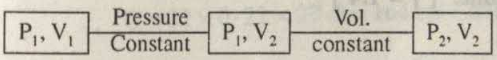
○ **Ans.** For converting heat into work continuously, a part of the heat energy absorbed from the source has to be rejected. The heat energy can be rejected only if there is a body whose temperature is less than that of the source. The body at low temperature is called sink.

● **Q.8. Some amount of gas is taken from state 1 to state 2 (higher pressure) in two methods [Fig. 1.18] The methods are : (a) first change of state takes place at constant volume and then the gas is taken to the final state at constant pressure. (b) first change of state takes place at constant pressure and then gas is taken to the final state at constant volume.**

In which method more heat is to be supplied to the gas ? Answer with proper reasons. [J. E. E. 1989]



1st case



2nd case

Fig. 1·18

○ **Ans.** If the initial pressure, volume and temperature are P_1, V_1 and T_1 respectively. Final values of there quantities are P_2, V_2 and T_2. Here $P_2 > P_1$. Assume that Q_1 be the heat given to the system in the first case. Then from the first law of thermodynamics, $Q_1 = U_2 - U_1 + P_2 (V_2 - V_1)$ . . . . . (i)

Similarly, the heat supplied at the second case is given by

$Q_2 = (U_2 - U_1) + P_1 (V_2 - V_1)$ . . . . (ii)

Since $P_2 > P_1$, from (i) and (ii) we see that $Q_1 > Q_2$.

So, more heat is supplied at the first case.

● **Q.9. In a water falls water is falling from a certain height on the ground. Why temperature of water at the bottom is slightly more that the temperature at the top ? [H. S. '98]**

○ **Ans.** At the top of the falls, all energy is potential energy. When the water falls from the top to the bottom potential energy is converted to kinetic energy. When the water falls on the ground, a part of the energy is converted to heat which raises the temperature of the water at the bottom of the falls.

● **Q.10. Does the gas performs any work in the adiabatic progress ? If yes, what is the source of that energy ?**

○ **Ans.** Yes, the gas performs work in the adiabatic process. The source of this energy is the internal energy of the gas itself.

● **Explanation :** From the first law of thermodynamics $\Delta Q = \Delta U + \Delta W$. As no heat is supplied, $\Delta Q = 0$. So, $\Delta W = - \Delta U$. Hence work done in the adiabatic expansion is equal to loss of internal energy of the gas.

● **Q.11. What is the internal energy of 40 m^3 air at normal pressure ?**

○ **Ans.** For the gas internal energy is, $U = \dfrac{PV}{\gamma - 1}$

As air is diatomic $g = 1\cdot4$, $P = 1 \text{ atm} = 10^5 \text{ N/m}^2$

$$\therefore \quad U = \frac{10^5 \times 40}{1\cdot4 - 1} = \mathbf{10^7 \text{ J}}$$

● **Q.12. How much heat is required to raise the temperature by 40°C of 14 g nitrogen gas at constant pressure ? Molecular mass of nitrogen = 28 and R = 8·3 J $mol^{-1}K^{-1}$.**

○ **Ans.** The required heat is, $\Delta Q = n.C_p \, \Delta T$

No. of mols $n = \dfrac{14}{28} = 0\cdot5$, $C_p = \dfrac{7}{2}R = \dfrac{7}{2} \times 8\cdot3 = 29\cdot05$, $\Delta T = 40°C$

$$\therefore \quad \Delta Q = 0\cdot5 \times 29\cdot05 \times 40 = \mathbf{581 \text{ Joule}}$$

● **Q.13. 3 moles of an ideal gas initially at 27°C is compressed isothermally to reduce its volume to half of its initial value. How much work is done ?**

○ **Ans.** Work done in isothermal process,

$n = 3$,

$R = 8\cdot31 \text{ cal mol}^{-1}\text{k}^{-1}$,

$T = 300 \text{ K}$, $\dfrac{V_2}{V_1} = 0\cdot5$

$$W = 2\cdot303 \, nRT \log_e \frac{V_2}{V_1},$$

$$W = 2\cdot303 \times 3 \times 8\cdot31 \times 273 \times \log(0\cdot5) = \mathbf{-5188 \text{ J}}$$

● **Q.14. 1 mole air at 27°C is compressed adiabatically to reduce its volume to half of its initial volume calculate (i) Final temperature and (ii) work done. [$\gamma = 1.4$]**

○ **Ans.** (i) The required relation is : $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$

$$T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{\gamma - 1} = 300\,(2)^{1\cdot4-1} = 396 \text{ K}$$

(ii) Work done, $W = \dfrac{R}{\gamma - 1}(T_1 - T_2) = \dfrac{8\cdot31}{1\cdot4 - 1}(300 - 396) = \mathbf{-1994 \text{ J}}$

●● Miscellaneous Examples ●●

● **Exercise 1.12. The pressure of 70 g of nitrogen gas at 25°C is 50 atm. It is allowed to expand isothermally against a pressure of 1 atm. If the nitrogen gas behaves as an ideal gas, calculate its change of internal energy. Also calculate work done by the gas. R = 2 cal $mol^{-1}K^{-1}$.**

○ **Solution :** Number of moles of the gas, $n = \dfrac{70}{28} = 2\cdot5$

Initial pressure, $P_1 = 50$ atm, final pressure $P_2 = 1$ atm.

As temperature remains constant, change of internal energy, $\Delta U = 0$.

$$\therefore \quad \text{Work done,} \quad \Delta W = P_2 (V_2 - V_1) = P_2 \left(\frac{nRT}{P_2} - \frac{nRT}{P_1} \right)$$

$$\therefore \quad \Delta W = \frac{nRT(P_1 - P_2)}{P_1} = \frac{2\cdot5 \times 2 \times 298\,(50-1)}{50} = 1\cdot46 \times 10^3 \text{ J}$$

● **Ex. 1.13. Sp heat of air at constant pressure is 0·22 cal g^{-1} °C^{-1} and sp. heat at constant volume is 0·16 cal g^{-1} °C^{-1}. If the density of air at N. T. P is 1·3 g / lit, calculate mechanical equivalent of heat.**

○ **Solution :** The required relation : $C_p - C_v = \frac{R}{J}$ and $PV = RT$

$$\therefore \quad J = \frac{PV}{T\left(C_p - C_v\right)}$$

$$\therefore \quad J = \frac{1\cdot013\times10^6}{273\times0\cdot0013\,(0\cdot22-0\cdot16)}$$

$$= \frac{1\cdot013\times10^6\times10^3}{273\times1\cdot3\times0\cdot06} = \mathbf{4\cdot76\times10^7\ erg\ /\ cal}$$

$P = 1\cdot013\times10^6$ dyne / cm^2

$V = \frac{1}{0\cdot0013}$ c.c

$C_p = 0\cdot22$ cgs

$C_v = 0\cdot16$ cgs.

● **Ex. 1.14. 1 mole oxygen gas at 0°C is heated at constant pressure so that the volume increases by 10%. (a) How much heat is required ? Sp heat of oxygen at constant pressure = 0.22 cal g^{-1} °C^{-1}. (b) If same quantity of heat is supplied at constant volume, what will be the final temperature ?**

○ **Solution :** (a) As the pressure is kept constant the required relation is

$$\frac{V_1}{V_2} = \frac{T_1}{T_2} \quad \Rightarrow T_2 = \frac{V_2}{V_1}.T_1$$

$$\therefore \quad T_2 = \frac{1\cdot1\,V_1}{V_1}\times273 = 300\cdot3\ K$$

$V_2 = V_1 + 10\%$ of $V_1 = 1\cdot1\,V_1$

$T_1 = 273$ K

$\therefore$ Rise of temperture, $\Delta T = 300\cdot3 - 273 = 27\cdot3$ K

Heat required for this rise of temperature, $Q = m\,C_v.\Delta T$

$\therefore$ $Q = 32\times0\cdot22\times27\cdot3 = 164$ cal $=$ **688·65 J**

(b) Specific heat at constant volume, $C_v = C_p - \frac{R}{M} = 0\cdot22 - \frac{2}{32} = 0\cdot1575$ cal g^{-1}°C^{-1}

Heat supplied, $Q = \frac{688\cdot65}{4\cdot2} = 164$ cal

$\therefore$ Rise of temperature, $\Delta T = \frac{Q}{m\,C_v} = \frac{164}{32\times0\cdot1575} = \mathbf{32\cdot54\ K}$ $\therefore$ Final temperature = **32·54°K**

● **Ex. 1.15. Initial pressure of one mole of an ideal gas an 10^5 N / m^2 and its volume 6 litre. If the gas is compressed adiabatically to final volume 2 litre. Calculate the work done. Given molar specific heat at constant volume $=\frac{3}{2}$R.** **[I. I. T.]**

○ **Solution :** Specific heat at constant pressure, $C_p = C_v + R$

$$\therefore \quad C_p = \frac{3}{2}R + R = \frac{5}{2}R \quad \therefore \quad \gamma = \frac{C_p}{C_v} = \frac{\frac{5}{2}R}{\frac{3}{2}R} = 1\cdot4$$

Now, for adiabatic process, $P_1V_1^{\gamma-1} = P_2V_2^{\gamma-1}$

$\therefore$ $10^5\times(6)^{1.4-1} = P_2\times(2)^{1.4-1}$ $\therefore$ $P_2 = 1\cdot55\times10^5$ Nm^{-2}

$\therefore$ Work done in adiabatic process is

$$W = \frac{P_1V_1 - P_2V_2}{\gamma-1} = \frac{10^5\times6\times10^{-3} - 1\cdot55\times10^5\times2\times10^{-3}}{1\cdot4-1} = 725\ J$$

$\therefore$ Work done, $W = \mathbf{725\ J}$

- **Ex. 1.16. The diameter of a hose pipe is 2 cm. Through the pipe 1 m^3 water flows in 20 sec and strikes a wall. If whole of kinetic energy of water is converted to heat, what will be the temperature rise of water ? J = 4.2 J cal^{-1}.**

○ **Solution :** Volume of water flowing per sec $= \frac{10^6}{20} = 5 \times 10^4$ c.c

Area of cross section of pipe, $a = \pi r^2 = 3 \cdot 14 \times (1)^2 = 3 \cdot 14 \text{ cm}^2$

$\therefore$ Velocity of water, $v = \frac{5 \times 10^4}{3 \cdot 14} = 1 \cdot 6 \times 10^4 \text{ cm/s}$

Kinetic energy of water $= \frac{1}{2} mv^2$ $\therefore$ Heat produced $= \frac{mv^2}{2J}$

$\therefore \frac{mv^2}{2J} = ms\theta \quad \therefore \theta = \frac{v^2}{2J} = \frac{\left(1 \cdot 6 \times 10^4\right)^2}{2 \times 4 \cdot 2 \times 10^7} = 3 \cdot 05°\text{C}$ $\therefore$ Rise of temperature = 3·05°C

- **Ex. 1.17. A stirrer in one litre of water revolves at 360 rpm in a circle of radius 5 cm, facing a resistance of 0·1N. Assuming no radiation loss, how much does the water temperature rise in 1 hour?** [J = 4·2 J / cal] **[J. E. E. 1996]**

○ **Solution :** Torque acting on the stirrer, $\tau = Fr = 0 \cdot 1 \times 5 \times 10^{-2} = 5 \times 10^{-3}$ N – m.

Angular velocity of the stirrer, $\omega = 2\pi n = 2\pi \frac{360}{60} = 12\,\pi$ rad / s

$\therefore$ Work done / sec = $5 \times 10^{-3} \times 12\,\pi = 6\pi \times 10^{-2}$ J/s

$\therefore$ Work done in 1 hr = $6\pi \times 10^{-2} \times 3600 = \mathbf{216\,\pi}$ J

$\therefore$ Heat produced, $H = \frac{W}{J} = \frac{216\,\pi}{4 \cdot 2}$ cal.

Let temperature rise of the water be t°C

$\therefore \quad 1000 \times 1 \times t = \frac{216\pi}{4 \cdot 2} \quad \therefore \quad t = 0 \cdot 161°\text{C}$

$\therefore$ rise of temperature = **0.161°C**

- **Ex. 1.18. An immersion type electric heater of 250 watts is immersed in 5 kg water contained in a bucket. In how much time will the temperature of water rise by 10°C ? Sp.heat capacity of water = 1000 cal kg^{-1} k^{-1} and J = 4.1 J cal^{-1}. Neglect heat capacity of the bucket.**

○ **Solution :** Let the required time be t sec.

Energy supplied in time t sec = 250 t Joule.

Heat absorbed by water to raise its temperature by 10°C is = 5000 × 10 × 4·1

$\therefore \quad 250\,t = 50 \times 4100 \quad \therefore \quad t = 820$ **sec**

- **Ex. 1.19. A cyclic process for 1 mol of an ideal monatonic gas ($C_v = 12 \cdot 5$ J $mol^{-1}K^{-1}$ is represented in fig. 1.19 . The temperatures at 1, 2, and 3 are 300 K, 600 K and 455 K respectively. Compute the values of ΔQ, ΔU and ΔW for each of the process. The process from 2 to 3 is adiabatic.**

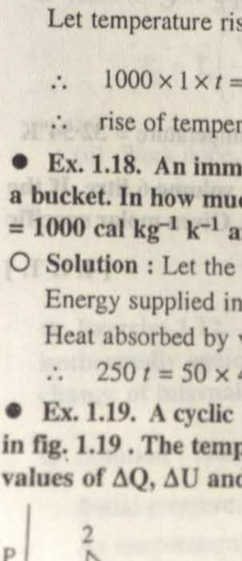


Fig. 1·19

○ **Solution :** In the process 1 to 2, volume remains constant.

So no work is done. $\therefore$ ΔW = 0

Heat supplied to the gas

$$\Delta Q = \int_{T_1}^{T_2} C_v dt = C_v(T_2 - T_1) = 12 \cdot 5 \times (600 - 300) = 3750 \text{ Joule}$$

By first law of thermodynamics, change of internal energy.

$\Delta U = \Delta Q - \Delta W = \Delta Q$ [AS $\Delta W = 0$] = **3750 J**.

Next, the process $2 \to 3$ is an adiabatic process. Here $\Delta Q = 0$

Work done in the process, $\Delta W = \dfrac{R}{\gamma - 1}(T_2 - T_3) = C_\nu(T_2 - T_1)$

$$= 12 \cdot 5\,(600 - 455) = 12 \cdot 5 \times 145 = \mathbf{18125\ J}$$

$\therefore$ Change in internal energy, $\Delta U = -\Delta W = -18125\ J$

Lastly, in the process $3 \to 1$, workdone $\Delta W = \displaystyle\int_{V_3}^{V_1} P dv = P\,(V_1 - V_3)$

$\Delta W = R\,(T_1 - T_3)$ [$\because$ PV = RT]

$= 8.31\ (\ 300 - 455\ ) = \mathbf{-1288\ J}$

Heat absorbed, $\Delta Q = \displaystyle\int_{T_3}^{T_1} C_\nu dt = C_\nu(T_1 - T_3) = 1 \cdot 67 \times 12 \cdot 5 \times (300 - 450) = -\ 3235 \cdot 6\ J$

$\therefore$ Change of internal energy, $\Delta U = \Delta Q - \Delta W$

$\therefore$ $\Delta U = -3235 \cdot 6 - (-1288) = 1989\ J$

● **Ex. 1.20. Three moles of an ideal gas $\left(C_p = \frac{7}{2}R\right)$ at pressure P_A and temperature T_A is isothermally expanded to twice the initial volume. It is then compressed at constant pressure to its original volume. Finally the gas is compressed at constant volume to its original pressure P_A. (a) Sketch P – V and P – T diagram, for the complete process. (b) Calculate the net work done by the gas and net heat supplied to the gas during complete process.** **[I. I. T. 1991]**

○ **Solution :** (a) (P – V) and (P – T) diagram is shown in fig 1.20.

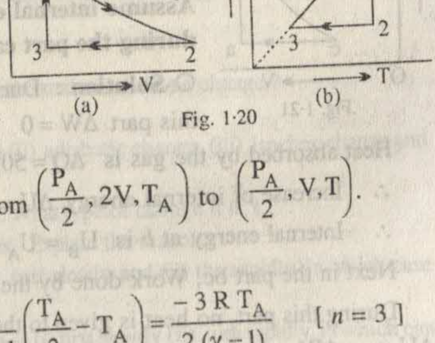


Fig. 1·20

(b) In the process $1 \to 2$, the change of state is (P_A, V, T_A) to $(P_2, 2V, T_A)$

Hence $P_A = 2P_2$ $\therefore$ $P_2 = \dfrac{1}{2}P_A$

As temperature remains constant, $\Delta U = 0$

$\therefore$ Work done, $\Delta W = \displaystyle\int_{V}^{2V} P dv = 3\,RT \ln 2$ $\therefore$ $\Delta Q = \Delta W$

Next in the process $2 \to 3$, the state of the gas changes from $\left(\dfrac{P_A}{2}, 2V, T_A\right)$ to $\left(\dfrac{P_A}{2}, V, T\right)$.

Hence $\dfrac{P_A}{2} \times \dfrac{2V}{T_A} = \dfrac{P_A}{2} \times \dfrac{V}{T_3}$ $\therefore$. $T_3 = \dfrac{T_A}{2}$

Hence change of internal energy $\Delta U = 3\,C_V \Delta T_A = 3.\dfrac{R}{\gamma - 1}\left(\dfrac{T_A}{2} - T_A\right) = \dfrac{-\,3\,R\,T_A}{2\,(\gamma - 1)}$ [$n = 3$]

$$\therefore\ \gamma = \frac{C_p}{C_\nu} = \frac{C_p}{C_p - R} = \frac{\frac{7}{2}R}{\frac{7}{2}R - R} = \frac{7}{5} = 1 \cdot 4$$

$$\therefore\ \Delta V = \frac{-\,3RT_A}{\left(\frac{7}{5} - 1\right) \times 2} = \frac{-15\,RT_A}{4}$$

Work done, $\Delta W = \int_{2V}^{V} P dv = \frac{P_A}{2}(V - 2V) = -\frac{P_A}{2}V = \frac{-3RT_A}{2}$

$\therefore$ Heat supplied $\Delta Q = \Delta U + \Delta W = -\frac{15}{4}RT_A - \frac{3}{2}RT_A = -\frac{21}{4}RT_A$

Lastly in the process, 3 → 1, the change of state takes place from

$\left(\frac{P_A}{2}, V, \frac{T_A}{2}\right)$ to (P_A, V, T)

$$\therefore \quad \frac{P_A}{2} \cdot \frac{V}{T_A/2} = \frac{P_A V}{T} \quad \therefore \quad T = T_A$$

$\therefore$ change of internal energy, $\Delta U = 3C_V\left(T_A - \frac{T_A}{2}\right) = \frac{3R}{\gamma - 1} \times \frac{T_A}{2}$

$\therefore \quad \Delta U = \frac{15}{4}RT_A \left[\text{Putting } \gamma = \frac{7}{5} \right]$ Work done $\Delta W = 0$

$\therefore$ Heat supplied, $\Delta Q = \Delta U = \frac{15}{4}RT_A$

Hence total workdone in the cyclic process,

$$\Delta W = 3RT_A \ln 2 - \frac{3}{2}RT_A + 0 = 3RT_A(\ln 2 - 0{\cdot}5)$$

$\therefore$ Total heat supplied, $\Delta Q = 3RT_A \ln 2 - \frac{21}{4}RT_A + \frac{15}{4}RT_A = 3RT_A\left(\ln 2 - \frac{1}{2}\right)$

- **Ex. 1.21. Some amount of an ideal gas is taken through a cyclic process abca [Fig 1.21] It absorbs 50J of heat during the part ab, no heat during bc and rejects 70 J of heat during ca. Work is done on the gas during the part bc. Work done during the process bc is 40 J. Calculate (i) internal energy of the gas at b and c. Assume internal energy at a 1500 J; (ii) Calculate the work done by the gas during the part ca.**

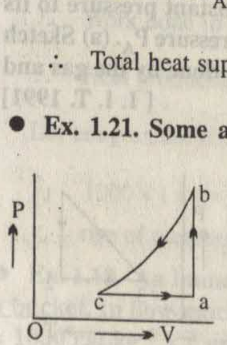


Fig. 1·21

○ **Solution :** During the process ab, Volume remains constant. So, work done in this part $\Delta W = 0$

Heat absorbed by the gas is $\Delta Q = 50$ J

$\therefore$ Increase of internal energy, $\Delta U = \Delta Q = \mathbf{50\ J}$

$\therefore$ Internal energy at *b* is $U_B = U_A + \Delta U = 1500 + 50 = \mathbf{1500\ J}$

Next in the part bc, Work done by the gas, $\Delta W = -40$ J

During this part, no heat is given to the system, Hence the increase of internal energyy from b to *c* is $\Delta U_1 = -\Delta W = \mathbf{40\ J}$

So, the internal energy at *c* is $U_C = U_B + \Delta U_1 = 1500 + 40 = \mathbf{1500\ J}$

(ii) Now, the change of internal energy from *c* to *a*

$$\Delta U_2 = 1300 - 1590 = -90 \text{ J}$$

The heat given to the system is $\Delta Q = -70$ J

$\therefore$ The work done by the gas during the part *ca*, $\Delta W = \Delta Q - \Delta U = -70 + 90 = \mathbf{20\ J}$.

● **Ex. 1.22. A Substance is taken through the process abc [Fig 1.22] If the internal energy of the substance increases by 5×10^3 J and a heat $26{\cdot}25 \times 10^2$ cal is given to the system. Calculate the value of mechanical equivalent of heat.**

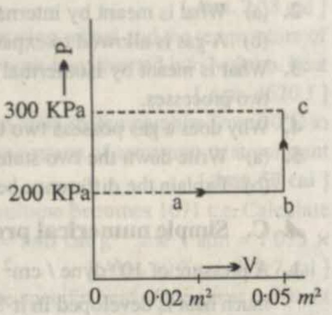


Fig. 1·22

○ **Solution :** Along *ab* work is done at constant pressure and no work is done along bc. So the work done in the path abc is

$$\Delta W = P\left(V_2 - V_1\right) = 200 \times 10^3 \times (0{\cdot}05 - 0{\cdot}02)\ \text{J} = \mathbf{6000\ J}$$

Heat supplied is $\Delta Q = 2625$ Cal $= 2625 \times$ J joule

$\therefore$ From first law of thermal dynamics, $\Delta Q = \Delta W + \Delta U$

$$\therefore \quad 2625\,\text{J} = 6000 + 5000 = 11000$$

$$\therefore \quad \text{J} = \frac{11000}{2625} = 4{\cdot}19$$

$\therefore$ Value of mechanical equivalent heat is **4·19 J / Cal.**

● E ◇ X ◇ E ◇ R ◇ C ◇ I ◇ S ◇ E ●

◢ A. Short answer type questions :

1. What is the condition of thermodynamic equilibrium ?
2. Why is the temperature at the bottom of a waterfall is slightly more than the temperature at the top of the fall ? **[H. S. 1996]**
3. A copper ball and an iron are dropped from the same height. Which one will be heated more ?
4. What is meant by internal energy of a gas ? On which factor its value depend for an ideal gas ?
5. Doe the internal energy of an ideal gas depend on its pressure and volume ? Explain your answer.
6. Two cylinders A and B of equal volume are connected by narrow tube provided with a stop cock. Cylinder A is filled up by an ideal gas and B is vacuum. The cylinders an thermally insulated. If the stop cock is suddenly open, answer the following questions. (i) pressure inside A or B; (ii) Change of internal energy of the gas (iii) change in temperature of the gas.
7. What is meant by isothermal and adiabatic changes ?
8. How does the internal energy of a gas change when it undergo isothermal of adiabatic change ?
9. State first law of thermodynamics.
10. What is the change of internal energy during (i) isothermal change (ii) adiabatic change (iii) isochric change and (iv) Cyclic change.
11. A gas is supplied some heat, but internal energy does not change. What type of change it is ?
12. During an adiabatic expansion, the temperature of a gas decreases. Explain the reason.
13. A gas is compressed to half of its initial volume in two ways (i) first slowly and (ii) the rapidly. In which case more work is done ? Show by indicator diagram.
14. A gas is allowed to expand to double its initial voulme in two ways— (i) first slowly (ii) turn rapidly. In which can more work is done on the gas ? Explain. **[H. S. '98]**
15. Draw two adiabatics and two isothermals on the same (P – V) diagram. Show that the slope of the adiabatic is γ - times more than that of an isothermal.
16. Define the terms (i) thermodynamic co-ordinates (ii) Thermodynamic states of a system. (iii) intensive and extensive property.
17. Explain reversible and irreversible process. Give examples.
18. How much work has to be done to raise the temperature of 1 g of water by 1°C ?
19. State second law of thermodynamics.

B. Essay type questions :

1. (a) What is meant by the mechanical equivalent of heat ? [H. S. 2000]
   (b) State and explain first law of thermodynamics. Write its general form also. [H. S. 2000]
2. (a) What is meant by internal energy of a gas ? On which factor internal energy of an ideal gas depends ?
   (b) A gas is allowed to expand at constant pressure. Deduce an expression for the work done by the gas.
3. What is meant by isothermal and adiabatic processes ? Deduce the expressions for the work done during these two processes.
4. Why does a gas possess two types of specific heats ? Obtain a relation between these two specific heats.
5. (a) Write down the two statements on second law of thermodynamics.
   (b) Explain the difference between the first law and the second law.

C. Simple numerical problems :

1. A pressure of 10^6 dyne / cm^2 is to be applied to bring down the volume of a gas from 20 litres to 10 litres. How much heat is developed in it ? [$J = 4 \cdot 2 \times 10^7$ erg / cal] **[Ans.** 240 cal **]** **[H. S. 2000]**
2. A block of ice at 0°C is dropped from a height and it melts completely due to impact with the ground. Find the height from which it is dropped assuming that 60% of its energy is converted into heat. **[Ans.** 34·28 km **] [H. S. 2003]**
3. Calculate the difference of temperature between the top and bottom of a water fall of height 400 m if 80 % of the heat produced is absorbed by water. **[Ans.** 0.747 °C **]** **[H. S. 1997]**
4. The difference of temperature between the top and bottom of a water fall is 0·49°C. If whole of the work done by the water in falling from top to the bottom of fall is converted into heat, calculate the height of the water fall. [$g = 980$ cm / s^2; $J = 4{\cdot}2 \times 10^7$ erg / cal] **[Ans.** 210 m **] [H. S. 1987 '04]**
5. A water fall is falling from a height of 50 m. If 75 % of the energy is converted into heat and absorbed by water, Calculate the rise of temperature of the fall. **[Ans.** 0·0875°C **]** **[H. S. '80]**
6. A meteorite of mass 42 kg while passing through earth's atmosphere loses its velocity from 15 km / sec to 5 km/ sec. Calculate the heat produced in calorie due to the change of kinetic energy $J = 4{\cdot}2 \times 10^7$ erg / cal. **[Ans.** 10^9 cal**]** **[H. S. 1987]**
7. A Kerosene lamp can emit as much light as a 15 watt lamp. Assuming that only 60% of the heat is converted into light how much kerosene is burnt in 5 hours ? (1 gm of kerosene produces about 45000 joule of heat) **[Ans.** 10gm**] [J.E.E 1992]**
8. From what height a piece of ice at 0°C be dropped so that it completely melts on hitting the ground if only 50% of the energy is converted into heat. Latent heat of ice = 80 Cal/g. **[Ans.** 68·57 km **]** **[J. E. E. '99]**
9. When a piece of metal is fall through a height of 50 m from rest, its temperature rises by 1·4°C. If it is given that only $\frac{2}{3}$ of the heat raises the temperature of the metal, Calculate mechanical equivalent of heat. Specific heat of metal = 0·1 cgs. **[Ans.** $4{\cdot}2 \times 10^7$ ergs / cal **] [H. S. '83]**
10. How much work has to be done to produce the amount of heat required to convert 50 g of ice at 0°C to water at 100°C ? Latent heat of ice = 80 cal / g **[Ans.** $37{\cdot}8 \times 10^3$ J **]**
11. A lead bullet moving with a velocity of $400 ms^{-1}$ strikes a rigid wall. If 25 % of the kinetic energy is converted to heat, what is the temperature rise of the bullet ? Sp. heat of lead = 0·03 cgs. **[Ans.** 158·73°C **]**
12. A block of mass 2 kg is drawn over a rough horizontal surface with a velocity of 2 ms^{-1}. If the coefficient of friction between the block and the surface be 0·2, calculate the heat produced is 5 sec [$J = 4{\cdot}2$ J / cal; $g = 9{\cdot}8$ ms^{-2}] **[Ans.** 9·33 cal **] [I. I. T]**
13. A copper calorimeter of mass 100 g contains oil of mass 1 kg. The oil is stirred by a rotating paddle which can exert a torque of 10^8 dyne-cm. If rise of temperature be 5°C after the paddle makes 220 revolutions, calculate the value of mechanical equivalent of heat. Given, sp. heat of copper = 0·1 and that of oil = 0·64 cgs. **[Ans.** 4·26 J / Cal **]**
14. A drill of power 0·25 HP is making a hole on a iron plate of mass 200 g. If 50 % of the energy spent is absorbed by iron plate what is the temperature rise after 1 min. 1 H. P. = 746 watt. Sp heat of iron = 0·11 cal g^{-1} $°C^{-1}$ **[Ans.** 60·55°C **]**
15. A steel drill making 180 revolutions per min is used to drill a hole in a block of steel. The mass of the steel block and the drill is 180 gm. If whole of mechanical work is converted to heat and the rate of rise of temperature of the

block is 0·5°C per sec, find (i) the rate of working of the drill in watt (ii) the torque required to drive the drill (sp. heat of steel = 0·1 cal g^{-1} $°C^{-1}$) **[Ans.** (i) 37·8 watts, (ii) 2 N-m **] [I. I. T.]**

16. A ball moving with a velocity of 4 m/ s collides with a stationary ball of same mass and they combine together. Calculate the heat produced. J = 4.2 J / Cal. **[Ans.** 2·38 cal **]**

17. A calorimeter of mass 500 g contains 500 g water at 18°C. It is stirred by a rotating wheel and the temperature of water rises by 20°C. Calculate the work done by the wheel. Assume, all the work is converted into heat. sp. heat of copper = 0·1 cal g^{-1} $°C^{-1}$ **[Ans.** 4620 J **]**

18. 70 Cal of heat is required to raise the temperature of 2 mole of an ideal gas to raise its temperature from 30°C to 35°C. How much heat will be required to raise the same gas through the same range of temperature at constant volume. [R = 2 cal mol^{-1} K^{-1}] **[Ans. 50 cal]**

19. 1 g water (vol 1 c.c) when converted to steam at atmospheric pressure, its volume becomes 1671 c.c. Calculate the work done and change of internal energy. Latent heat of vaporisation = 540 cal g^{-1} and 1 atm = 1·013 × 10^5 Nm^{-1} **[Ans.** 40·3 cal, 499·7 cal **]**

20. Specific heat of oxygen at constant volume is 0·155 cal g^{-1} $°C^{-1}$. Calculate the specific heat of the gas at constant pressure. Molecular mass of oxygen = 32 and R = 2 cal mol^{-1} K^{-1}. **[Ans.** 0·218 cal g^{-1} $°C^{-1}$ **] [J. E. E. 1997]**

21. A piece of ice at 0°C falls from certain height and on hitting the ground it melts completely. If 95% of the mechanical energy is converted into heat, Calculate the height of fall of the ice. Latent heat of ice = 80 cal g^{-1} $°C^{-1}$. **[Ans.** 3·61 × 10^4 m **] [J. E. E. 2001]**

22. The temperature of 5 g gas is raised at constant pressure form 28°C to 30°C. What is the change of internal energy of the gas ? sp. heat at constant pressure = 0·175 cal g^{-1} $°C^{-1}$ **[Ans.** 1·72 cal **]**

23. The temperature of a piece of aluminium is raised by 1°C. What will be the increase of energy of one atom of aluminium? Moleculer mass of aluminium = 27, Avogadro's number = 6 × 10^{23} and sp. heat of aluminium = 0·22 cgs. **[Ans.** 4·158 × 10^{-23} J **] [I. I. T.]**

24. In the boiler of a steam engine coal burns at the rate of 400 lb / hr. If calorific value of coal be 1.35 × 10^4 B. T. U / lb calculate the power of the engine. [1 B. T. U. = 780 ft - lb] **[Ans. 9.17 %]**

25. A drill of power 0·5Hp is used to made a hole in an iron block. If 70% of the energy spent raises the temperature of iron, what will be the rise of temperature of iron block in 30 sec. sp. heat of iron = 0·1 cgs. **[Ans.** 126.6°C **]**

26. Given, C_p = 6.85 cal mol^{-1} k^{-1}, C_v = 4·87 cal mol^{-1} k^{-1} and the value of gas constant R = 8·31 J mol^{-1} K^{-1}. Calculate mechanical equivalent of heat. **[Ans.** 4·18 J / Cal **]**

27. The volume of 2*g* oxygen at 27°C is expanded from initial volume from 1 litre to 2 litre at constant volume. How much work is done ? **[Ans.** 5·40 J **]**

28. At N.T. P mass of 1 litre of hydrogen is 0·0896 g. specific heat of hydrogen at constant pressure and at constant volume are 3·456 cal $g^{-1}$$°C^{-1}$ and 2·47 cal g^{-1} $°C^{-1}$ respectively. Calculate mechanical equivalent of heat. Given R = 8·31 J mol^{-1} K^{-1} **[Ans.** 4·2 J / cal **]**

29. One mole of an ideal gas is taken through a cyclic process ABCD [Fig 1.23]. What is the work done by the gas ? 1 atm = 10^6 dyne / cm^2 **[Ans.** 900 J **]**

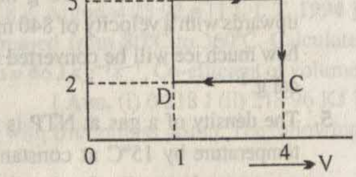


Fig. 1·23

30. A gas at temperature 127°C is at a pressure of 10^6 dyne / cm^2. It is allowed to expand to double its initial volume under isothermal condition. Next it is again allowed to expand adiabatically to double its volume again. What will be the final pressure ? Given γ = 1·4 **[Ans.** 1·895 × 10^5 dyne / cm^2**]**

31. A gas is compressed adibatically to reduce the volume to half of its initial volume. If the initial temperature be 27°C what will be the final temperature ? γ = 1·5 **[Ans.** 151·2 °C **]**

32. At 0°C the velocity of sound in carbon-di-oxide gas is 259 ms^{-1}. The specific heat capacity of carbon-di-oxide at constant pressure is 220 cal $kg^{-1}$$k^{-1}$ and γ = 1·31. Calculate the value of mechanical equivalent of heat.

[Hint : $v = \sqrt{\frac{\gamma P}{\rho}}; \frac{P}{\rho T} = r$ **]** **[Ans.** 3·6 J / cal **]**

33. 10 g air at 20°C is heated at constant volume to the final temperature 30°C. Calculate the change of internal energy of air? Given, $C_v = 0\cdot 17$ cal g^{-1} $°C^{-1}$ **[Ans.** 71·4 Joule **]**

34. One mole of gas initially at 127°C is expanded isothermally to double its volume. Calculate work done and heat absorbed by the gas. [**Ans.** $2{\cdot}32 \times 10^3$ J ; 552·3 J]

35. 1 m^3 of air at 27°C and at a pressure of 10^5 Nm^{-2} weighs 1·18 kg. Calculate the value of gas constant for 1 kg of the gas. Also calculate C_p of air if $C_v = 168$ cal $kg^{-1}K^{-1}$. [**Ans.** 282·5 J Kg^{-1} K^{-1}; 235·3 cal kg^{-1} k^{-1}]

36. 5 g ice at 0°C is converted to water at 0°C. If change of volume is negligible, calculate increase of internal energy of ice. [**Ans.** 1680 J]

37. A liquid of density 0·85 g / c.c is flowing through a horizontal tube at the rate of 8 c.c / sec. The water is being heated by an electric heater of power 255 watts. If the difference of temperature at the two ends of the tube be 15°C, Calculate the value of mechanical equivalent of heat. Given, specific heat of liquid = 0·6 cgs. [**Ans.** 4·16 J / cal]

38. 1 gm oxygen initially at N. T. P is allowed to expand adiabatically to double its volume. Calculate change in internal energy. Given $R = 2$ cal $mol^{-1}k^{-1}$, $\gamma = 1{\cdot}4$ [**Ans.** –43·38 J]

39. A tyre is full of air at a pressure of 6 atm and at temperature 15°C. It the tyre suddenly bursts what will be the final temperature ? $\gamma = 1{\cdot}4$ [**Ans.** – 94·4 °C]

40. Three moles of an ideal gas at 300 K are isothermally expanded to five times its volume and heated at this constant volume so that pressure is raised to its initial value before expansion. In the whole process 83·14 J heat is required calculate the ratio C_p / C_v of the gas. [**Ans.** 1·42]

◢ D. Harder numerical problems :

1. Two pieces of ice of equal mass while moving with equal velocity towards each other collide and with the heat generated they vapourise to steam. If the initial temperature of ice be –12°C and specific heat 0·5 cal $g^{-1}{}^{\circ}C^{-1}$, calculate the velocity with which the pieces of ice were moving. Latent heat of ice = 80 cal g^{-1} and latent heat of steam = 540 cal g^{-1}. [**Ans.** $246{\cdot}95 \times 10^3$ cm/s] [**J. E. E. 1987**]

2. In an X-ray tube 10^{18} electrons strike the target per sec with the velocity 2×10^8 ms^{-1}. The mass of the target metal is 200 g and its sp. heat is 0·12 cgs; Also mass of electron = $9{\cdot}1 \times 10^{-28}$ g. If 98 % of the energy is converted to heat, after what time, the temperature of the target rises by 95°C ? [**Ans.** 47 min]

3. The power of an engine is 10 KW and it burns 10 kg fuel in an hour. Calorific value of the fuel = 12,000 cal g^{-1}. If 25% of the heat produced due to combustion is lost, calculate : (a) what percent of heat produced is converted to work ? (b) If water equivalent of the engine be 8·0 kg, calculate the rate of temperature rise of the engine [**Ans.** (a) 23·81 % (b) 0·64 °C/ sec]

4. A lead bullet of mass 50 g having sp. heat 0·12 and at initial temperature 30°C is projected vertically upwards with a velocity of 840 ms^{-1}. On reaching the point of projection, it hits a block of ice. If the ice be at 0°C, how much ice will be converted to water at 0°C ? Assume that all heat is absorbed by ice. Latent heat of ice = 80 cal g^{-1} [**Ans.** 54·75 gm] [**Roorkee'88**]

5. The density of a gas at NTP is 0·00125 g/c.c and its volume 8 litre. It required 30 calorie of heat to raise its temperature by 15°C at constant pressure. Calculate the specific heats of the gas at constant pressure and at constant volume. R= 2 cal mol^{-1} k^{-1}. [**Ans.** 5·6 and 3·6 cal mol^{-1} k^{-1} [**J. E. E. 1981**]

6. The initial volume of a gas is 2 m^3. It is compressed adiabatically by applying a constant pressure of 4×10^5 N/ m^2 so that the volume of the gas is reduced to 0·5 m^3. What is the pressure now ? If the compression is effected at constant temperature what would be the final pressure. Calculate the work done in both the cases. Given $\gamma = 1{\cdot}4$. [**Ans.** (i) $2{\cdot}78 \times 10^6$ N m^{-2} (ii) $1{\cdot}6 \times 10^6$ N m^{-2} (iii) $-14{\cdot}75 \times 10^6$ J and $-11{\cdot}08 \times 10^5$ J]

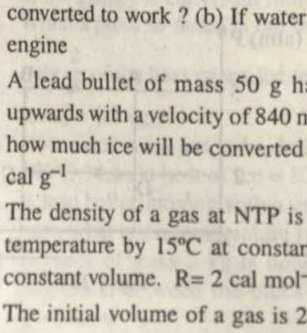


Fig. 1·24

7. A monatomic ideal gas is taken through a cyclic process ABCDA as shown in fig 1·24. What is the work done in the process? How much heat is supplied to the gas. [**Ans.** PV; Q = PV] [**I. I. T. 1983**]

8. An ideal gas has a molar specific heat at constant pressure equal to $\frac{5}{2}R$. It is kept in a closed vessel of volume $0{\cdot}0083\ \mathrm{m^3}$ at a temperature of 27°C and pressure of $1{\cdot}6 \times 10^6\ \mathrm{Nm^{-2}}$. An amount of heat energy equivalent to $2{\cdot}49 \times 10^4$ J is supplied to the gas. Calculate the final temperature and pressure of the gas.
[**Ans.** 675 K; $3{\cdot}584 \times 10^6\ \mathrm{Nm^{-2}}$] [**I. I. T. 1987**]

9. Initial temperature and volume of 2 g helium gas $\left(\gamma = \frac{5}{3}\right)$ are 27°C and 20 litre respectively. It is expanded at constant pressure until the volume is double. Then it is further expanded adiabatically so that its temperature becomes 27°C again.
(i) Show the processes on a P – V diagram.
(ii) What are the final volume and pressure ?
(iii) What is the work done by the gas ? [**Ans.** (ii) $113{\cdot}14 \times 10^{-3}\ \mathrm{m^3}$; $0{\cdot}44 \times 10^5\ \mathrm{Nm^{-2}}$ (iii) 12·44 KJ]
[**I. I. T. 1988**]

10. 4 mole of an ideal gas ($\gamma = 1{\cdot}67$) is mixed with 2 mole of another ideal gas ($\gamma = 1{\cdot}4$). Calculate the value γ of the mixture.
[**Ans.** $\gamma = 1{\cdot}54$]

11. Radius of the earth is 6400 km and the specific heat of earth's material is $0{\cdot}15\ \mathrm{cal\ g^{-1}\ {}^{\circ}C^{-1}}$. If whole of rotational kinetic energy of the earth is fulled converted to heat what will be the rise of temperature of the earth?
[**Ans.** 68·76°C]

12. One mole of ideal gas initially at 27°C is expanded isothermally to double its volume. Then it is compressed adiabatically so that its volume becomes equal to its initial volume. Calculate final temperature and work done by the gas. [$\gamma = 1{\cdot}4$ and $J = 8{\cdot}4\ \mathrm{J\ mol^{-1}\ k^{-1}}$]
[**Ans.** 830·6 K; 6302·74 J]

13. Some amount of gas is enclosed in a cylinder fitted with a frictionless piston. The initial pressure of the gas $3{\cdot}5 \times 10^5\ \mathrm{Nm^{-2}}$ and its temperature 366 K. The gas is expanded adiabatically so that its pressure becomes $0{\cdot}7 \times 10^5\ \mathrm{Nm^{-2}}$. Then the gas is heated to 366 K at constant volume and pressure rises to $1{\cdot}1 \times 10^5\ \mathrm{N\ m^{-2}}$. Calculate the specific heats of hydrogen.
[**Ans.** $C_p = 29{\cdot}58$; $C_v = 21{\cdot}28\ \mathrm{J\ mol^{-1}\ k^{-1}}$]

14. An ideal gas is taken through a cyclic thermodynamic process through four steps. The amount of heat involved in these steps are $Q_1 = 5960$ J; $Q_2 = -5585$ J ; $Q_3 = -2980$ J and $Q_4 = 3645$ J. respectively. The corresponding quantities of work involved are $W_1 = 2200$ J; $W_2 = -825$ J, $W_3 = -1100$ J and W4 respectively (i) Find the value of W_4, (ii) what is the efficiency of the cyclic.
[**I. I. T. '94**]
[**Hint :** Total change in internal energy, $\Delta U = 0$]
[**Ans.** (i) 765 J (ii) 0·108]

15. The volume of a closed room is $0{\cdot}02\ \mathrm{m^3}$. It contains a mixture of neon and argon at 27°C and at a pressure of $10^5\ \mathrm{Nm^{-2}}$. Mass of the mixture = 28 g. If the molar mass of neon and argon be 20 g and 40 g / mole, calculate the mass of the gases separately. Assume both the gases be ideal $R = 8{\cdot}31\ \mathrm{J\ mol^{-1}K^{-1}}$
[**Ans.** Neon = 4·28 g, Argon = 23·92 g] [**I. I. T. 1994**]

16. The mass of an iron cube of side 6 cm is 1·7 kg. Its temperature is increased from 80°C to 360°C. Calculate (i) external work and (ii) increase of internal energy. Given, sp. heat of iron = $46\ \mathrm{J\ kg^{-1}k^{-1}}$, Co-efficient of volume expansion = $3{\cdot}6 \times 10^{-5}\ \mathrm{k^{-1}}$ and atmospheric pressure = $10^5\ \mathrm{N/m^2}$.
[**Ans.** (i) 0·218 J (ii) 218·96 KJ]

17. 8 g oxygen at 27°C and at normal pressure is enclosed in a cylinder fitted with frictionless piston. The following changes are effected on the gas.
(a) temperature is raised to 127°C at constant pressure. (b) Then it is compressed isothermally so that the volumes becomes equal to the original volume. (c) Finally, it is cooled at constant volume to make its final temperature equal to the initial temperature.
Calculate : (i) the heat obserbed by the gas in the process (a)
(ii) work done in process (a) ; (iii) work done on the gas in the process (b);
(iv) heat rejected by the gas in the process (c)
[**Ans.** (i) 743·8 J, (ii) 207·8 J, (iii) – 239·1 J, (iv) 536 J]

18. A gas is taken through a cyclic process ABCA [Fig 1.25] Calculate the work done in different parts of the process.
[**Ans.** (i) workdone along AB, W_1 = area ABDE = 25 J ; (ii) Workdone along BC, $W_2 = 0$; iii) Work done along CA, $W_3 = -31{\cdot}25$ J]

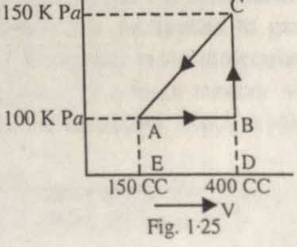


Fig. 1·25

19. The density of a gas at 27°C and at a pressure of 10^5 N / m^2 is 1·775 kg / m^3. Its specific heat at constant pressure = 846 J kg^{-1} k^{-1}, calculate the ratio of specific heats of the gas. [**Ans.** 1·29]

20. One mol of an ideal gas having volume 6 litre and pressure 10^5 N / m^2 is compressed adiabatically to reduce its volume to 2 litre. Calculate the work done. [**Ans.** 912 J] [**I. I. T 1982**]

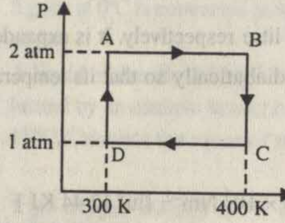


Fig. 1·26

21. Three mole of an ideal gas initially at 300 K is expanded isothermally to 5 times its initial volume. Then it is heated at constant volume when its pressure becomes equal to its initial pressure. If during the whole process 83·14 J heat is absorbed calculate the ratio of the two specific heats of the gas R = 8·31 J mol^{-1} K^{-1}.

[**Ans.** $\gamma = 1{\cdot}42$] [**Roorkee, 1990**]

22. 2 mole of an ideal gas is taken through the cyclic process ABCDA [Fig 1.26]. Calculate the following quantities in the process. (i)total change of heat (ii) total work done (iii) change of internal energy [**Ans.** (i) 1152 J, (ii) 1152 J (iii) $\Delta U = 0$] [**I. I. T. 1992**]

23. The pressure *m* monatonic gas is uniformly increased energy from 4 × 10^5 Pa to 8 × 10^5 Pa when its volume increases from 0·2 m^3 to 0·5 m^3 calculate the following :

(i) work done by the gas (ii) increase of internal energy

(iii) amount of heat supplied (iv) molar heat capacity of the gas [**Roorkee, 1995**]

[**Ans.** (i) 1·8 × 10^5 J (ii) 4·8 × 10^5 J (iii) 6.6 × 10^5 J (iii) 17 J mol^{-1} K^{-1}]

[**Hint :** (ii) $C_v = \frac{R}{\gamma - 1} = \frac{R}{2/n} = \frac{nR}{2}$ [n = no of degrees of freedom]

$$\Delta U = \frac{m}{M} C_v (T_2 - T_1) = C_v (P_2V_2 - P_1V_1)$$

(iv) $C_{moler} = \frac{Q}{T_2 - T_1} = \frac{QR}{P_2V_2 - P_1V_1}$]

24. The volume of two mole of a monatomic ideal gas at 27°C is V. It is expanded adiabatically to double its volume. Calculate (i) final temperature of the gas (ii) change of internal energy and (iii) work done by the gas.

[**Ans.** (i) 189 k, (ii) $\Delta U = -2767{\cdot}48$J (iii) 2767.48 J] [**I. I. T. 1996**]

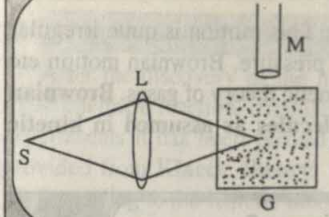
25. The temperature of 20 g of oxygen is increased from 20°C to 100°C. If this change takes place (i) at constant volume (ii) at constant pressure, calculate the following quantities in both the cases :

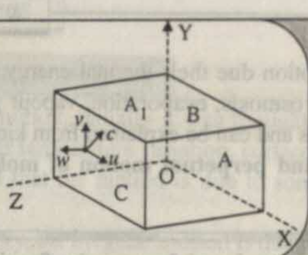
(a) Heat absorbed (b) change of internal energy and (c) work done by the gas.

Given R = 2 cal mol^{-1} K^{-1} and C_v of oxygen = 0·155 cal g^{-1} °C^{-1}

[**Ans.** (a) $\Delta Q = 248$ cal $\Delta W = 0$; $\Delta U = 248$ Cal (b) $\Delta Q = 347{\cdot}2$ Cal ; $\Delta U = 248$ Cal ; $\Delta W = 99{\cdot}2$ Cal]

26. 14 g nitrogen is enclosed in a vessel at 27°C. How much heat is to be supplied to the gas to double the r.m.s velocity of gas molecules; Given Avogadro's No = 6 × 10^{23} / mole, Boltzman constant K = 1·38 × 10^{-3} J k^{-1} and $C_v = \frac{5}{2}R$ [**Ans.** 9316 J]





CHAPTER 2

KINETIC THEORY OF GASES

● **TOPICS :** ○ Introduction; ○ Experimental evidence in favour of Kinetic theory of gases; ○ Brownian motion; ○ Basic assumptions of kinetic theory of gas; ○ Mean velocity and root mean square velocity of gas molecules; ○ Explanation of pressure of an ideal gas according to Kinetic theory of gas; ○ Pressure of an ideal gas according to Kinetic theory; ○ R. M. S. velocity, temperature and molecular weight; ○ Relation between pressure and Kinetic energy of a gas; ○ Kinetic energy of one mole ideal gas; ○ Kinetic interpretation of temperature; ○ Deduction of the perfect gas laws from the Kinetic theory; ○ Limitation of ideal gas law : Vander Waal's equation; ○ Mean free path; ○ Degree of freedom; ○ Principle of equipartition of energy; ○ Relation between degree of freedom and the ratio of two specific heats of a gas; ○ Short answer type questions (with answers); ○ Exercise.

✪ 2.1. Introduction ✪

Any matter— solid, liquid or gas – composed of large number of tiny particles called **molecules.** A molecule is a smallest unit having all the chemical properties of the substance. Molecules of a particular substance are all alike, but molecules of different substance are different.

Each molecule of a substance attracts other molecules towards it, and at the same time exerts a repulsive force to keep them at a fixed distance from itself. This mutual force of attraction or repulsion among the molecules of a substance are known as **intermolecular force**. From the observed differences in physical properties of solid, liquids and gases we can get an idea about the nature of intermolecular forces. Between any two adjacent molecules of a substance there is a gap called **intermolecular space**.

The molecules of a solid substance are very closely packed and intermolecular separation is very small. So in solids the molecules have strong cohesion among themselves. So, the solids have difinite shape and volume. The molecules of a liquid are further apart than in a solid and exerts less intermolecular forces. They cannot easily leave the liquid but are free to move about any where in the liquid. This explains why a liquid has a definite volume, but no definite shape of its own. In a gas, the molecules are far apart from one another, the intermolecular forces are very feeble and the gas molecules are free to move over the entire space available. For these reasons, gases have neither any shape or any volume of its own. They readily assume the volume and shape of the vessel in which they are kept.

A new theory was proposed which can adequately explain the thermal behaviour of molecules of a gas and be able to give a theoratical explanation of gas laws. This new theory is known as **Kinetic theory of gases**. This theory is bases on two fundamental hypothesis : (i) molecular structure of matter and (ii) identification of heat with energy.

The first hypothesis was established by Dalton, Avogadro and others, while the second was established by the experiments of Rutherford, Joule and Meyer etc. The law of mechanics can be applied to gas molecules. Since heat is a kind of energy, the thermal properties of a gas can be related to the molecular motions. The observed behaviour of a gas results from the detailed behaviour of a large number of molecules. The Kinetic theory of gases tries to develope a model of molecular behaviour which could explain the observed behaviour of an ideal gas.

Boyle, Bernoulli, Joule, Clausius, Van dar waals and Maxwell developed Kinetic theory. This theory is equally applicable to three states of matter. But here we shall discuss Kinetic theory of gases only.

The gas molecules are in continuous motion due their thermal energy. This motion is quite irregular and very fast. The phenomena of diffusion, osmosis, evaporation, vapour pressure, Brownian motion etc depends on molecular behaviour of molecules and can be explained from kinetic theory of gases. **Brownian motion is the direct prove of irregular and perpetual motion of molecules as assumed in kinetic theory of gases.**

✪ 2.2. Experimental evidences in favour of Kinetic theory of gases ✪

Various observations have revealed that the molecules of a gas are in the state of incessant motion. The motion of the molecules cannot be seen directly as the molecules are too small to be visible.

(i) If a drop of odourous oil falls in the corner of room, the smell at once spread throughout the room. It is due to diffusion of one gas into the other.

(ii) The phenomena of evaporation and vaporisation are intimately related with the motion of molecules. When a liquid is heated the molecules of the liquid move more rapidly. As the temperature rises the motion becomes so rapid that some of the molecules escape out of the liquid. This is vaparisation of liquid.

(iii) When a gas is placed in a vessel, the gas almost immediately occupies the volume of the vessel. This expansibility of gas proves that the molecules are in irregular and randam motion. Due to this motion of the molecules, the gas molecules spread out through out the vessel rapidly.

(iv) English Botanist Robert Brown first observed the irregular motion of suspended particles in a liquid. This motion of suspended particle in a liquid is called **Brownian motion.**

✪ 2.3. Brownian motion ✪

Brownian motion first observed by British Botanist Robert Brown is a phenomenon related to the irregular motion of suspended small particles (Pollen grains) in a liquid. If provides one of the most direct visual evidences of molecular motion.

Robert Brown discovered that when fine pollen grains suspended in water are viewed under a high power microscope, they appear to be continuously agitated and make a succession of quick jumps first one way and then another. This random motion of the particles goes on for ever and shows no sign of abatement. This is called Brownian motion. The possible path followed by a single particle is shown in fig. 2.1.

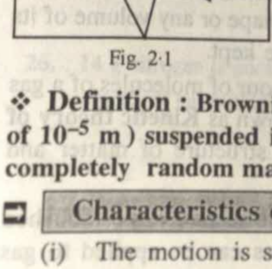


Fig. 2·1

Later it was found that the phenomenon is a general one. Any kind of fine particles suspended in both liquid and gases exhibit such motion.

❖ **Definition : Brownian motion is a phenomenon in which tiny particles (diameter of the order of 10^{-5} m) suspended in a liquid or in a gas are found to move continously and perpetually in a completely random manner making zig-zag straight line paths.**

Characteristics of Brownian motion :

(i) The motion is spontaneous, perpetual, completely random and irregular. In general any two particles do not possess identical motion.

(ii) The motion becomes more vigorous with the rise of temperature.

(iii) The motion is independent of the motion of the vessel.

(iv) Less the viscosity of the liquid, the more is the motion.

(v) Smaller the particle, the motion becomes more rapid.

(vi) The motion is not afflected by magnetic or electric field.

Explanation of Brownian motion :

After the discovery of the motion, initially it was thought that the reason behind the motion may be some chemical reaction, irregular temperature variation, surface tension of the liquid etc. But from different experiments it has been proved that the motion is due to some other reason. The correct explanation is provided from **Kinetic theory.**

According to the Kinetic theory, the irregular motion is due to the bombardment from different directions by the molecules of the surrounding gas or liquid. If the particle is fairly large, the forces due to the impact of the molecules will balance each other and the net force on the particle is zero. So the particle remains at rest. For this reason, a piece of wood floating on water does not exhibit Brownian motion. When the particle is very small, a net force due to molecular impacts acts on the particle. Since the impact is quite random, the motion of the suspended particle is completely irregular. Obviously, smaller the particles vigorous will be the motion. Also with the rise of temperature, the motion becomes very rapid.

Demonstration of the motion :

To observe the Brownian motion of suspended particles, the following simple experiment may be performed. The experimental arrangement is shown in fig 2.2. G is a small glass vessel in which a small amount of smoke is taken. The glass vessel is strongly illuminated from one side by focussing light by a convex lens from a bright source of light. In this way the smoke particles are illuminated and they are viewed from above by a high power microscope (M). The small smoke particles will appear as bright spicks of light executing completely random irregular motion.

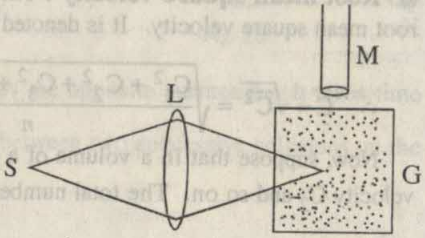


Fig. 2·2

Similarly, Brownian motion in liquid can also be observed. In that case we have to take colloidal solution in the vessel.

✪ 2.4. Basic assumptions of Kinetic theory of gas ✪

From the observation of Brownian motion, it is evident that the molecules of a gas are in incessant and irregular motion. The kinetic theory of gases attempts to explain the behaviour of gases on the basis of this picture. This theory is based on the following assumptions as regards to the motion of molecules and the nature of the gases.

(1) A gas is composed of large number of molecules. The molecules of a particular gas are identical in all respects but molecules of different gases are different.

(2) The molecules of a gas are in a state of continuous random motion in all directions and the magnitude of velocities may vary from zero to infinity.

(3) During motion, the molecules collide among themselves and with the walls of the container. Between any two collisions, the molecule travel along a straight line with uniform velocity. Between two successive collisions the distance travelled by a particle is called free path.

(4) These collisions are perfectly elastic. So both momentum and kinetic energy is conserved in such collisions.

(5) No force of attraction or repulsion act between the molecules. It means that energy of the gas is entirely kinetic.

(6) Though the molecules collide against each other and with the wall of the container, yet in the steady state the molecular density of the gas i.e. the number of molecules per unit volume of the gas remains constant.

It is to be noted that the gases for which the above assumptions are valid are called ideal gases.

○ 2.5. Mean velocity and root mean square velocity of gas molecules ○

- **Mean Velocity :** Let a closed vessel contains n number of molecules of a gas. If the velocities of the molecules be $C_1, C_2, \ldots\ldots, C_n$ respectively, the mean velocity is

$$\overline{C} = \frac{C_1 + C_2 + \ldots\ldots\ldots + C_n}{n}$$

- **Mean square velocity :** The mean square velocity is defined as the mean of the squares of all the molecular velocities. It is denoted by $\overline{C}^2$

$$\therefore \quad \overline{C}^2 = \frac{{C_1}^2 + {C_2}^2 + \ldots\ldots\ldots + {C_n}^2}{n}$$

- **Root mean square velocity :** The square root of the mean square velocity (r. m. s) is known as root mean square velocity. It is denoted by C

$$\therefore \quad C = \sqrt{\overline{C}^2} = \sqrt{\frac{{C_1}^2 + {C_2}^2 + {C_3}^2 + \ldots\ldots\ldots {C_n}^2}{n}}$$

Now, suppose that in a volume of a gas there are n_1 molecules with velocity C_1, n_2 molecules with velocity C_2 and so on. The total number n of the molecules is $n = n_1 + n_2 + \ldots\ldots\ldots + n$

Then the mean velocity $\overline{C}$ is defined by the relation $\overline{C} = \dfrac{n_1 C_1 + n_2 C_2 + \ldots\ldots\ldots}{n}$

The mean square velocity is defined as $\overline{C}^2 = \dfrac{n_1 {C_1}^2 + n_2 {C_2}^2 + \ldots\ldots\ldots}{n}$

and root mean square velocity $C = \sqrt{\overline{C}^2}$

It is to be noted that r-m.s velocity of gas molecules is not identical with the mean velocity. We take an example, we consider three molecules having velocities 15, – 20 and 30 m/s.

$$\therefore \quad \text{mean velocity} \quad \overline{C} = \frac{16 - 20 + 30}{3} = 11 \cdot 33 \text{ m/s}$$

$$\therefore \quad \text{r. m. s velocity} \quad C = \sqrt{\frac{16^2 + 20^2 + 30^2}{3}} = 22 \cdot 77 \text{ m/s}$$

○ 2.6. Explanation of pressure of an ideal gas according to Kinetic theory of gases ○

According to the kinetic theory of gases, the molecules of a gas in a vessel are in irregular motion. They Collide elastically with the walls of the vessel. After collision the molecules rebounds with the same velocity. After each collision there is change of momentum. This change of momentum of a molecule multiplied by collision frequency (i.e. number of collisions per sec) is the force exerted by the molecules on the wall. This force divided by the area of the wall gives the pressure exerted on the wall by the gas molecules.

Now, with the increase of temperature the molecular velocity and collision frequency both increases. As a result rate of change of momentum i.e. the force exerted on the wall also increases. So, the pressure exerted on the wall also increases. This is the explanation of pressure of a gas from kinetic theory.

2.7. Pressure of an ideal gas according to Kinetic theory

In 1848 Joule derived an expression for the pressure of an ideal gas on the basis of the assumptions of kinetic theory of gases.

We consider a cubical vessel of side l. It contains a gas whose number of molecules is N [Fig 2.3]. The molecules and the wall of the vessel is perfectly elastic. Out of all there molecules we consider a single molecule. Let at any instant the velocity of this particular molecule is C with components u, v and w along three axes. X–, Y– and Z– axis respectively. Hence $C^2 = u^2 + v^2 + w^2$

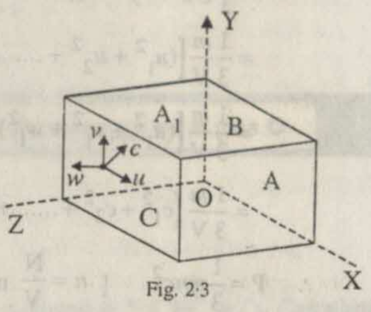


Fig. 2·3

The velocity component u is along X-axis and also perpendicular to the side A of the vessel. The corresponding component of momentum before collision is mu and after collision it is $-mu$ because the collision is elastic. So, the change of momentum after one collision $= -mu - (mu) = -2mu$.

After impact with the surface, it rebounds and move towards the opposite surface A_1. It takes time $t = \frac{2l}{u}$ to go from A to A_1 and again to A. So the time interval between two successive collisions on the surface A is $\frac{2l}{u}$ sec.

So the number of collisions per sec in $A = \frac{u}{2l}$

$$\therefore \quad \text{Change of momentum per sec} = -2mu\left(\frac{u}{2l}\right) = \frac{-2mu^2}{2l} = \frac{-mu^2}{l}$$

Now, the rate of change of momentum is the force. Therefore force exerted by the wall of the vessel on the gas molecule along X-axis is $F_x = -\frac{mu^2}{l}$

According to Newton's third law of motion, the molecules also exerts an equal but opposite force on the wall of the vessel. Therefore, the force exerted by the gas molecule on the walls of the container along X-axis is $F'_x = -F_x = \frac{2mu^2}{l}$

So, the pressure exerted by the gas molecule along X-axis on surface A is

$$p_x = \frac{F_x'}{A} = \frac{F_x'}{l^2} = \frac{mu^2}{l^3} \quad \dots\dots\dots\dots\dots\dots (1)$$

Next we consider N number of molecules of the gas. All molecules have different velocities and their velocity components along OX will also be different. Let X- components of the velocities of the molecules be $u_1, u_2, \dots\dots\dots u_n$ so the total pressure on the wall A along X-axis is

$$P_x = \frac{m}{l^3}\left(u_1^2 + u_2^2 + \dots\dots\dots u_n^2\right) = \frac{m}{V}\left(u_1^2 + u_2^2 + \dots\dots + u_n^2\right)\left[\because V = l^3\right]$$

Now, let Y- components of the velocities of the molecules be $v_1, v_2, \dots\dots v_n$

Total pressure on the wall B perpendicular to Y-axis is

$$P_y = \frac{m}{l^3}\left(v_1^2 + v_2^2 + \dots\dots\dots v_n^2\right) = \frac{m}{V}\left(v_1^2 + v_2^2 + \dots\dots + v_n^2\right)$$

Similarly, total pressure on the wall C perpendicular to Z-axis is

$$P_z = \frac{m}{l^3}\left(w_1{}^2 + w_2{}^2 + ...w_n{}^2\right) = \frac{m}{V}\left(w_1{}^2 + w_2{}^2 + ...+ w_n{}^2\right)$$

Since the gas exerts equal pressure in all directions, so $P_x = P_y = P_z = P$ (say)

$$\therefore \quad P = \frac{1}{3}\left(P_x + P_y + P_z\right)$$

$$= \frac{1}{3}\frac{m}{V}\left[(u_1{}^2 + u_2{}^2 + ... u_N{}^2) + (v_1{}^2 + v_2{}^2 + ...v_N{}^2) + (w_1{}^2 + w_2{}^2 + ...+ w_N{}^2)\right]$$

$$= \frac{1}{3}\frac{m}{V}\left[(u_1{}^2 + v_1{}^2 + w_1{}^2) + (u_2{}^2 + v_2{}^2 + w_2{}^2) + ...+ (u_N{}^2 + v_N{}^2 + w_N{}^2)\right]$$

$$= \frac{1}{3}\frac{m}{V}\left(c_1{}^2 + c_2{}^2 + ...+ c_n{}^2\right) = \frac{1}{2}\frac{m}{V} N C^2 \left[C = \text{r m s speed}\right]$$

$$\therefore \quad P = \frac{1}{3} mnc^2 \quad \left[\; n = \frac{N}{V} \text{ no. of molecules per unit volume } \right]$$

$$= \frac{1}{3}\frac{M}{V}c^2 = \frac{1}{3}\rho c^2 \quad [\, mn = M = \text{Total mass of gas and } \rho = \text{density of the gas} \,]$$

$$\therefore \quad \mathbf{P = \frac{1}{3}\rho c^2} \quad .... \quad (2.1)$$

- **Example 2.1. Calculate the r. m. s. velocity of nitrogen molecule at N T P. Given density of nitrogen at NTP = 1.25 × 10^{-3} g/c.c.**

○ **Solution :** The required relation is $C = \sqrt{\frac{3P}{\rho}}$

$$\therefore \quad C = \sqrt{\frac{3 \times 1{\cdot}013 \times 10^6}{0{\cdot}00125}} = 4{\cdot}93 \times 10^6 \text{ cm/s}$$

2.8. R. M. S. velocity, temperature and molecular weight

From kinetic theory of gases, the pressure of an ideal gas is $P = \frac{1}{3}\rho c^2$

$$\therefore \quad C = \sqrt{\frac{3P}{\rho}} \quad \text{Now, from ideal gas equation,} \quad \frac{P}{\rho} = \frac{RT}{M}$$

$$\therefore \quad C = \sqrt{\frac{3RT}{M}} \quad \therefore \; C \,\alpha\, \sqrt{T} \quad \text{and} \quad C \,\alpha\, \frac{1}{\sqrt{M}}$$

So, r. m. s. speed is directly proportional to the absolute temperature of the gas and it is inversely proportional to the square root of the molecular weight of the gas.

- **Ex. 2.2. At what temperature r. m. s. speed of nitrogen molecule be double the r. m. s. speed at 0°C ?**

○ **Solution :** If the r. m. s. be C then $C \,\alpha\, \sqrt{T}$

$$\therefore \quad \frac{C}{C_0} = \sqrt{\frac{T}{T_0}} \quad \therefore \quad \frac{2C_0}{C_0} = \sqrt{\frac{T}{T_0}} \quad \therefore \quad T = 4\,T_0$$

$\therefore \quad T = 273 \times 4 = 1092$ K $\qquad \therefore$ The required temperature is **t = 819° C**

● **Ex. 2.3. The r. m. s. speed of hydrogen molecule at a certain temperature is 5×10^5 cm / s. What will be the r. m. s. speed of oxygen molecule at the same temperature.**

○ **Solution :** r. m. s. $C \propto \frac{1}{\sqrt{M}} \quad \therefore \quad \frac{C_1}{C_2} = \sqrt{\frac{M_2}{M_1}}$

$$\therefore \quad C_2 = C_1\sqrt{\frac{M_1}{M_2}} = 5 \times 10^5 \times \sqrt{\frac{2}{32}} = \frac{5}{4} \times 10^5 = \mathbf{1 \cdot 25 \times 10^5\, cm / s}$$

$\therefore$ r. m. s. speed of the oxygen molecule = $1{\cdot}25 \times 10^5$ cm / s

❂ 2.9. Relation between pressure and Kinetic energy of a gas ❂

From the equation 2.1, $P = \frac{1}{3}\rho c^2 = \frac{2}{3} \times \frac{1}{2}\rho c^2 = \frac{2}{3}E$

$\therefore \quad P = \frac{2}{3} \times$ (average kinetic energy per unit volume)

● **Ex. 2.4. The density of a gas in 1.25×10^{-3} g/ c. c. and r. m. s. speed is 5×10^4 m / s. Calculate: (i) average kinetic energy of a molecule, (ii) pressure of the gas.**

○ **Solution :** (i) Kinetic energy per unit volume, $E = \frac{1}{2}\rho c^2$

$$\therefore \quad E = \frac{1}{2} \times 1{\cdot}25 \times 10^{-3} \times \left(5 \times 10^4\right)^2 = 15 \cdot 625 \times 10^5\,\text{erg / c.c.}$$

$\therefore$ (ii) Pressure of the gas, $P = \frac{2}{3}E = \frac{2}{3} \times 15 \cdot 625 \times 10^5$

$\therefore \quad P = \mathbf{10{\cdot}42 \times 10^5}$ dyne / cm^2

❂ 2.10. Kinetic energy of one mole ideal gas ❂

The number of molecules in the gas is N, where N is the Avogadro's number. Let mass of each molecule is *m* and their individual velocities be $C_1, C_2, \ldots\ldots\ldots C_N$. The kinetic energy of the molecule are respectively. $E_1 = \frac{1}{2}m\,C_1{}^2, \quad E_2 = \frac{1}{2}m\,C_2{}^2 \quad E_3 = \frac{1}{2}m\,C_3{}^2 \quad \ldots\ldots\ldots$ so on.

So the total kinetic energy of N molecules is

$$E = E_1 + E_2 + \ldots\ldots\ldots\ldots\ldots\ldots + E_N$$

$$\therefore \quad E = \frac{1}{2}m\left(C_1{}^2 + C_2{}^2 + \ldots\ldots\ldots\ldots + C_N{}^2\right)$$

$$= \frac{1}{2}mNC^2 \quad [\ C = \text{r. m. s. velocity of the molecules}\ ]$$

$$= \frac{1}{2}MC^2 \quad [\ M = \text{molecular mass}\ ]$$

$$\therefore \quad \mathbf{E = \frac{1}{2}M \times \frac{3RT}{M} = \frac{3}{2}RT} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.2) \quad [\ R = \text{molar gas constant}\ ]$$

$\therefore$ Average kinetic energy of single molecule,

$$\mathbf{E_1 = \frac{3}{2}\frac{R}{N}T = \frac{3}{2}KT} \ldots\ldots\ldots\ldots\ldots\ldots (2.3)$$

where K = Boltzmann constant.

- **Ex. 2.5. Calculate kinetic energy of 1 g of hydrogen gas at 50°C. Given, molecular mass of hydrogen = 2 and R = 8·3 J mol^{-1} K^{-1} erg $\text{mol}^{-1}\text{C}^{-1}$.**

○ **Solution :** Kinetic energy of one mole gas, $E = \frac{3}{2}RT$

∴ Kinetic energy 1 g hydrogen gas is $E' = \frac{1}{2}E = \frac{3}{4}RT$

∴ $E' = \frac{3}{4} \times 8{\cdot}31 \times 323 = \mathbf{2 \times 10^3\,J}$

- **Ex. 2.6. The average kinetic energy of one hydrogen molecule at 0°C is 5·64 × 10^{-14} erg. Calculate Avogadro's number. Given R = 8·32 × 10^7 erg $\text{mol}^{-1}\text{K}^{-1}$**

○ **Solution :** The required reaction is $E_1 = \frac{3}{2}\frac{R}{N}.T.$

∴ Avogadro's number, $N = \frac{3}{2}.\frac{RT}{E_1} = \frac{3}{2} \times \frac{8{\cdot}32 \times 10^7 \times 273}{5{\cdot}64 \times 10^{-14}}$

∴ N = **6·04 × 10^{23}**

2.11. Kinetic Interpretation of temperature

From the assumptions of kinetic theory of gases we know that the gas molecules are in incessant motion in all directions and there is no attraction or repulsion between the molecules of the gas. Also kinetic energy of one gas molecule is $E_1 = \frac{3}{2}KT$. So the kinetic energy of a molecule is proportional to absolute temperature of the gas.

As the gas molecules have no attraction between them, they have no potential energy. So total energy of gas is equal to kinetic energy of the molecules. Hence the kinetic energy of a gas molecule is proportional to the absolute temperature of the gas. Higher the temperature, higher is the kinetic energy. Temperature is the measure of kinetic energy of gas molecules. It is the kinetic interpretation of temperature.

When T = 0 K, kinetic energy is zero. At this temperature of the gas, molecules are motionless. This temperature is called absolute zero of temperature.

2.12. Deduction of the perfect gas laws from the Kinetic theory

(A) Boyle's law :

From kinetic theory $P = \frac{1}{3}mnc^2 = \frac{1}{3}\rho c^2$ ∴ $PV = \frac{1}{3}V\rho c^2 = \frac{1}{3}mc^2$ [m = mass of the gas]

Now, if the temperature is constant r. m. s. speed of gas molecules will remain constant. Hence at constant temperature. PV = Constant. ∴ $V \alpha \frac{1}{P}$ When T = Constant. This is Boyle's law.

(B) Charle's law :

From kinetic theory $PV = \frac{1}{3}mc^2$

Here m = mass of the gas and C = r. m. s. speed.

∴ $PV = \frac{1}{3}M.\frac{3RT}{M} = RT$ ∴ PV α T

Now, if pressure is constant, V α T. So, at constant pressure, volume of a given mass of any gas is proportional to its absolute temperature. This is Charle's law.

(C) Avogadro's hypothesis :

According to Avogadro's hypothesis, under the same condition of temperature and pressure, equal volumes of all gases contains equal number of molecules.

To obtain this hypothesis from kinetic theory of gases, we take two different gases. Their pressure, temperature and volume are same represented by P, T and V respectively. Let in the case of the first gas, its V volume contents n_1 number of molecules and mass of each molecule is m_1. The corresponding quantities for the second gas of volume V are n_2 and m_2. Then according to kinetic theory of gases.

$$PV = \frac{1}{3} m_1 n_1 c_1{}^2 = \frac{1}{3} m_2 n_2 c_2{}^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

C_1 and C_2 are r. m. s. velocities of two gases.

Now, both the gases are at same temperature. So, average kinetic energy of one molecule is same for both the gases

$$\therefore \quad \frac{1}{2} m_1 c_1{}^2 = \frac{1}{3} m_2 c_2{}^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(ii)}$$

Hence from equation (i) and (ii), $n_1 = n_2$. It is Avogadro's hypothesis.

✪ 2.13. Limitation of ideal gas law : Van der Waal's equation ✪

We know that the combination of Boyle's law and Charle's law gives the equation of state of ideal gas. For one mole ideal gas the equation of state is PV = RT.

An ideal gas obeys this equation at all temperatures. But from experiment it is found that the perfect gas equation is inadequate to describe the behaviour of real gases. They are found to obey the ideal gas equation at low pressure and at very high temperature.

Van der waal was the first to modify the equation of state by disregarding some of the simpling assumptions. He took into consideration two factors, namely : (i) mutual attraction between to molecules and (ii) finite size of the molecules. We shall see that the first factor leads to an increase over observed pressure and the second to a reduction of the effective volume.

● **(i) Correction due to intermolecular attraction :** In kinetic theory gases it is assumed that the gas molecules do not exert any force of attraction in each other. But actually, the molecule of real gas exert some force of attraction on each other. This force is supposed to be very small. When a molecule is inside the vessel it is attracted by other neighbouring molecules for all direction. The net force on the molecule is zero. But when the molecule is near the wall of the vessel, the molecule experiences a force towards the inner side of the vessel. So the speed of a molecule is slightly reduced before it hits the wall. As a result, the velocity of impact in the wall is slightly less. Due to presence of intermoleculer force, the pressure exerted by the molecules in the wall is smaller than predicted by ideal gas equation.

Vander waal proved that this decrease of pressure is $P' = a / V^2$

$\therefore$ Actual pressure due to the gas on the wall $= P + P' = P + a / V^2$

Here 'a' is a constant whose value depends on nature of the gas and V is the volume of the vessel.

● **(ii) Correction due to finite size of the molecules :** In Kinetic theory of gases it is assumed that gas molecules are point masses and their volume is negligible compared to the volume of the vessel. According to Vander waal, the volume of the gas molecules should be considered, because although the molecules are very small, but their volume is not negligible. So, the volume available to a gas molecule is less than V, where V is the actual volume of the vessel. If b be the decrease in volume, effect volume is $V' = (V - b)$

∴ modified gas equation given by Van-der-waal is $\left(P + \frac{a}{V^2}\right)(V - b) = RT$ . . . . (2.6)

It is for one mole of the gas.

❂ 2.14. Mean free path ❂

Kinetic theory assumes that the molecules of a gas are in random motion with in the vessel. They collide against each other and also against the wall. As there is no force of attraction between the molecules, the distance travelled by each molecule between two successive collisions is a straight line and the molecules travel along straight line between any two collisions. This distance is called free path.

❖ **Definition : The distance which a molecule in a gas travels between successive collisions its free paths.**

The free paths are, however, not all equal. The average value of free paths of all molecules in a given mass of gas is called the mean free path. If the total distance travelled by a molecule between n consecutive collision is 'd', then the **mean free path** (λ) of the molecule is given by $\lambda = \frac{d}{n}$.

The expression for the mean free path as established by clausius is $\lambda = \frac{1}{n\sigma^2 N}$ . . . . (i)

Where N = Number of molecules per unit volume of the gas and σ = molecular diameter.

Since $mN = \rho$, the density of the gas we can write, $\lambda = \frac{m}{\pi\sigma^2\rho}$ . . . . (ii)

Boltzmann establised the following expression for mean free path $\lambda = \frac{3}{4\pi\sigma^2 N} = \frac{0 \cdot 75m}{\pi\sigma^2\rho}$ . . . . . . (iii)

Later or, Maxwell determined a more exact expression for the mean free path. The expression is :

$$\lambda = \frac{1}{\sqrt{2}\pi\sigma^2 N} = \frac{0 \cdot 707}{\pi\sigma^2 N} = \frac{0 \cdot 707m}{\pi\sigma^2\rho} \quad \ldots\ldots\ldots\ldots \text{(iv)}$$

❂ 2.15. Degree of freedom ❂

The allowable independent motions, which a body can have may be termed its **degrees of freedom**. A degree of freedom of a dynamical system means a mode of its motion which the system can execute independently of all other possible modes. The number of degrees of freedom of such a system is the number of independent co-ordinates necessary for unique determination of the position of every particle of the system. Each degree of freedom is represented by one co-ordinate which can vary independent of the others. Hence,

The total number of co-ordinates or independent quantities required to completely specify the position or configuration of a dynamical system is called the degrees of freedom of the system.

The number of degrees of freedom depends on the geometrical shape of the body or system and on the number of constraints imposed on the motion of the body. It does not depend on the mass or energy of the system. We consider few examples—

(a) If a particle is constraints to move along a straight line, it has only one degree of freedom, as only one co-ordinate is necessary to specify its position. The particle can move in one direction only.

(b) If the particle in free to move in a plane, it has two degrees of freedom, for, two co-ordinate axes are necessary to specify its position. Here, two co-ordinates are required to specify the position of the particle.

(c) A particle moving in space has three degrees of freedom. Now, to specify the position of the particle in three dimensional space and three co-ordinates are required.

As all three motions are translational, we call them **translational degrees of freedom**.

(d) If, however, a rigid body is capable of rotation about three mutually perpendicular axes through a fixed point on it, the body will have three degrees of freedom of rotational motion. So, a rigid body capable of translational and rotatory motion simultaneously has, in total six degrees of freedom—three for translational and three for rotational motion.

The molecules of a monatomic gas consists of single atom. Therefore, the molecules of a monatomic gas have **three** degrees of freedom corresponding to translational motion. The molecules of a diatomic gas have **five** degrees of freedom—three for translational motion and two for rotational motion. A polyatomic molecule has six degrees of freedom including one of vibrational motion.

2.16. Principle of equipartition of energy

According to this principle, the total energy of a dynamical system in thermal equilibrium is partitioned equally among all the degrees of freedom, the energy per molecule per degree of freedom being equal to $\frac{1}{2}$ KT when K is Bolzmann constant and T is the absolute temperature of the gas.

This principle is known as Boltzmann law of equipartition of energy and it is true for all degrees of freedom—translational, rotational or vibratory. If a dynamical system has *f* degrees of freedom, the total energy of a molecule is $E = \frac{1}{2} f \text{ KT}$.

If we consider one gm-molecule of a gas, the average Kinetic energy per degree of freedom $= \frac{1}{2}\text{ KTN} = \frac{1}{2}\text{ RT}$.

When N = Avogadro's number and R = universal gas constant.

2.17. Relation between Degrees of freedom and the ratio of two specific heats of a gas

Assume that the number of degrees of freedom of each molecule of a gas is *f*. Then the Kinetic energy of one mole gas is

$$E = \frac{1}{2}\text{ KTN}f = \frac{1}{2} f \text{ RT} \quad [\text{ Energy per defree of freedom } = \frac{1}{2}\text{ KT }]$$

So, the molar sp. heat at constant volume is $C_v = \frac{dE}{dt} = \frac{1}{2}Rf$

$$\text{But } C_P - C_V = R \quad \therefore\ C_P = C_V + R = \frac{1}{2}Rf + R = R\left(1+\frac{f}{2}\right)$$

$$\therefore\ \gamma = \frac{C_P}{C_V} = \frac{1+\frac{f}{2}}{\frac{f}{2}} 1 + \frac{2}{f} \qquad \therefore\ \gamma = 1 + \frac{2}{f}$$

- **Values of γ for different gases :**

(a) In the case of monatonic gas, *f* = 3.

So the ratio of two specific heats, $\gamma = 1 + \frac{2}{3} = 1\cdot67$

(b) For diatomic gas *f* = 5. So, $\gamma = 1 + \frac{2}{5} = 1\cdot40$

● ● Numerical Examples ● ●

● **Example 2.7. The r. m. s. velocity of the molecules of an ideal gas at N T P is 0·5 km/s. Calculate the density of the gas. Normal atmospheric pressure = 10^5 Nm^{-2}. If the temperature of the gas is raised to 21°C at constant pressure, what will be its density ?** **[J. E. E. 1983]**

○ **Solution : 1st Part :** The required relation for r. m. s. velocity is $C_0 = \sqrt{\dfrac{3P}{\rho}}$

$\therefore\ \rho = \dfrac{3P}{{C_0}^2} = \dfrac{3\times10^5}{(0\cdot5\times10^{-3})^2} = 1\cdot2\ \text{kg.m}^{-3}$ $\therefore$ density of the gas at N. T. P. = **1.2 kg m^{-3}**

2nd Part : $C \propto \sqrt{T}$ $\quad\therefore\ \dfrac{C_1}{C_0} = \sqrt{\dfrac{T_1}{T_0}}$

or, $\dfrac{\rho_0}{\rho_1} = \sqrt{\dfrac{T_1}{T_0}}$ $\quad\therefore\ \rho_1 = \rho_0 \cdot \sqrt{\dfrac{T_0}{T_1}}$ $\quad\therefore\ \rho_1 = 1\cdot2\times\sqrt{\dfrac{273}{294}} = 1\cdot1\ \text{kg m}^{-3}$

● **Ex. 2.8. The mass of hydrogen molecule is 3.32 × 10^{-27} kg. If 10^{23} molecules strike over an area of 2 cm^2 of a wall at an angle of 45° with a velocity of 10^5 cm/sec. How much force is exerted on the wall ?**

○ **Solution :** Component of velocity of the molecule normal to the wall before collision = $v\cos\theta$ and the components of velocity of the molecule after collision in the same direction = $-\,v\cos\theta$

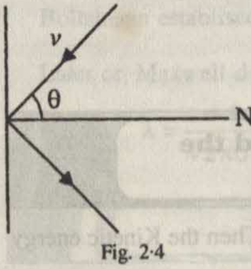


Fig. 2·4

$\therefore$ Change of velocity $= -\,v\cos\theta - (v\cos\theta) = -\,2v.\cos\theta$

$\therefore$ Change of momentum per collision $= -\,2mv\cos\theta$

$\therefore$ Rate of change of momentum i.e. the force exerted by the wall on the molecules $= 2\,mv\,N\cos\theta$ [N = Collision frequency]

$\therefore$ Force exerted by the molecules on the wall is

$$F = 2\,mv\,N.\cos\theta$$

$$= 2\times3\cdot32\times10^{-27}\times10^3\times10^{23}.\cos45^\circ$$

$$= 0\cdot468\ \text{N}$$

$m = 3\cdot32\times10^{-27}$

$v = 10^3$ m / s

$N = 10^{23}$

$\therefore$ Pressure exerted on the wall is $P = \dfrac{F}{A} = \dfrac{0\cdot468}{2\times10^{-4}} = \mathbf{2\cdot34\times10^3\,N/m^2}$

● **Ex. 2.9. At N. T. P. 1 m^3 of air contains 2·70 × 10^{25} number of molecules. Calculate the number of molecules in 1 m^3 air at a temperature of 223 K and pressure 1·33 × 10^{-4} N/m^2 .** **[1 Atm = 1·01 × 10^5 N/m^2]**

○ **Solution :** The pressure of a gas is given by $P = \dfrac{1}{3}mnC^2 = \dfrac{1}{3}mn\dfrac{3KT}{m}$

$\therefore\ P = n\,KT$

$\therefore\ \dfrac{P_1}{P_0} = \dfrac{n_1}{n_0}.\dfrac{T_1}{T_0}$ $\quad\therefore\ n_1 = \dfrac{n_0T_0}{T_1}\times\dfrac{P_1}{P_0}$

$n_0 = 2\cdot70\times10^{25}$; $T_0 = 273$

$T_1 = 223$; $P_0 = 1\cdot013\times10^5\ \text{Nm}^{-2}$

$P_1 = 1\cdot33\times10^{-4}\text{Nm}^{-2}$

$\therefore\ n_1 = \dfrac{2\cdot70\times10^{25}\times273\times1\cdot33\times10^{-4}}{223\times1\cdot013\times10^5} = \mathbf{4\cdot35\times10^{16}}$

$\therefore$ The required number of molecules = **4·35 × 10^6**

● **Ex. 2.10. Calculate r. m. s. velocity of smoke particles executing Browmian motion at N. T. P. Mass of a particle is 5×10^{-17}g, Boltzmann constant = $1{\cdot}38 \times 10^{-23}$ J K $^{-1}$** [Roorkee, '82]

○ **Solution :** We know $v_{\text{r.m.s}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3KT}{m}}$

$$\therefore \quad v_{\text{rms}} = \sqrt{\frac{3\times 1\cdot 38\times 10^{-23}\times 273}{5\times 10^{-17}}} = 15\times 10^{-3}\,\text{ms}^{-1} = 1\cdot 5\text{ cm s}^{-1}$$

● **Ex. 2.11. At what temperature r. m. s velocity of nitrogen molecule be equal to the escape velocity from earth ? Mass of nitrogen molecule is $4{\cdot}65 \times 10^{-23}$ gm, radius of the earth 6370 km and Boltzmann constant = $1{\cdot}37 \times 10^{-23}$ JK^{-1}.**

○ **Solution :** r. m. s velocity, $C = \sqrt{\frac{3KT}{m}}$ and escape velocity, $v_e = \sqrt{2gR}$

∴ By question, $\sqrt{\frac{3KT}{m}} = \sqrt{2gR} \quad \therefore \quad T = \frac{2gR\,m}{3K}$

$$\therefore \quad T = \frac{2\times 9\cdot 8\times 6370\times 10^{3}\times 4\cdot 65\times 10^{-26}}{3\times 1\cdot 37\times 10^{-23}} = 1\cdot 41\times 10^{5}\text{K}$$

∴ The required temperature = **$1{\cdot}41 \times 10^5$ K.**

● **Ex. 2.12. At what temperature r. m. s velocity of oxygen molecule the equal to r. m. s. velocity of hydrogen molecule at 150°C ?**

○ **Solution :** The required relation, $V = \sqrt{\frac{3RT}{M}} \quad \therefore \quad \frac{V_1}{V_2} = \sqrt{\frac{T_1}{T_2}}.\sqrt{\frac{M_2}{M_1}}$

In this case, $V_1 = V_2 \quad \therefore \quad \frac{T_1}{T_2} = \frac{M_1}{M_2} \quad \therefore \quad T_2 = \frac{M_2}{M_1}T_1$

$$\therefore \quad T_2 = \frac{32}{2}\times(273+150) = 6768\text{ K}$$

∴ The required temperature = **6495°C**

● **Ex. 2.13. The volume and pressure of two moles of an ideal gas in V and P respectively. Another 1 mol ideal gas having volume 2 V exerts the same pressure P. Molecular mass of the second gas is 16 times more than that of the first gas. Compare the r. m. s velocity of the two gases.**

○ **Solution :** r. m. s velocity $v = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3PV}{M}}$

Hence for the two gases, $\frac{v_1}{v_2} = \sqrt{\frac{V_1}{V_2}\times\frac{M_2}{M_1}} \quad [\because P_1 = P_2]$

Here, $V_1 = \frac{V}{2}, \quad V_2 = 2V, \quad M_2 = 16\,M_1$

$$\therefore \quad \frac{v_1}{v_2} = \sqrt{\frac{V/2}{2V}}\times\sqrt{16} = \sqrt{\frac{1}{4}}\times 4 = 2 \quad \therefore \quad \boldsymbol{v_1 = 2v_2}$$

● **Ex. 2.14. A gas is enclosed in a vessel. It temperature is 300 K and pressure 10^{-14} cm Hg. Calculate the number of molecules per unit volume of the gas. Boltzmann constant K = $1{\cdot}38 \times 10^{-23}$ JK^{-1}; Avogadro's No, N = $6{\cdot}023 \times 10^{23}$ and R = $8{\cdot}31$ J mol^{-1} K^{-1}.**

○ **Solution :** Pressure $P = nKT$ [n = number of molecules per unit volume]

$$\therefore \quad n = \frac{P}{KT}$$

$$\therefore \quad n = \frac{13\cdot33\times10^{-12}\times10^{23}}{1\cdot38\times300}$$

$$= \mathbf{3\cdot22\times10^{6}}$$

$$P = 10^{-14}\,\text{cm Hg}$$

$$= 10^{-16}\times13600\times9\cdot8 = 13\cdot33\times10^{-12}\,\text{N}/\text{m}^2$$

$$k = \frac{8\cdot31\times10^{-23}}{6\cdot023} = 1\cdot38\times10^{-23}$$

$$T = 300\ \text{K}$$

∴ No. of molecules per unit volume = $\mathbf{3{\cdot}22\times10^{6}}$

● **Ex. 2.15. The temperature of a planet is 427°C and density $5{\cdot}5\times10^{3}$ Kg m^{-3}. If oxygen gas can stay in the planet's atmosphere then what should be the minimum radius of the planet. Given, $G = 6{\cdot}67\times10^{-11}$ Nm^{2} Kg^{-2} and $R = 8{\cdot}31$ J mol^{-1} K^{-1}.**

○ **Solution :** Escape velocity from a planet of mass M and radius r is

$$v_e = \sqrt{\frac{2GM}{r}} = \sqrt{\frac{2G}{r}}\sqrt{\frac{4}{3}\pi r^3\rho} \quad [\ \rho = \text{density of the planet}\ ]$$

$$\therefore \quad v_e = \sqrt{\frac{8}{3}G\pi r^2\rho} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

Now, r. m. s velocity of a gas molecule, $C = \sqrt{\dfrac{3RT}{M_1}}$ ……………………… (ii)

Here M_1 = Molecular mass of the gas, T= absolute temperature of the gas.

Now, by question, $v_e = C$

$$\therefore \quad \sqrt{\frac{8}{3}G\pi r^2\rho} = \sqrt{\frac{3RT}{M_1}} \quad \therefore \quad r = \sqrt{\frac{9RT}{8G\pi\rho M_1}}$$

$$\therefore \quad r = \sqrt{\frac{9\times8\cdot13\times700}{8\times6\cdot67\times10^{-11}\times3\cdot14\times5\cdot5\times10^{3}\times32\times10^{-3}}}\ \text{metre} = 4.21\times10^{5}\ \text{m} = \mathbf{421\ km}$$

● **Ex. 2.16. A vessel of volume 4 litre contains gas mixture of 8 g oxygen, 14 g nitrogen and 22 g carbon dioxide. If the temperature of the mixture is 27°C, calculate the pressure of the gas. ($R = 8{\cdot}31$ J mol^{-1}K^{-1})**

○ **Solution :** We know, $P = \dfrac{m}{M}\cdot\dfrac{RT}{V}$

$$\therefore \quad \text{Total pressure, } P = P_{O_2} + P_{N_2} + P_{CO_2} = \frac{RT}{V}\left(\frac{m_0}{M_0} + \frac{m_N}{M_N} + \frac{m_C}{M_C}\right)$$

$$\therefore \quad P = \frac{RT}{V}\left(\frac{4}{32} + \frac{14}{28} + \frac{22}{44}\right) = \frac{5}{4}\frac{RT}{V} \quad \left[= \frac{5}{4}\times\frac{8\cdot31\times300}{4\times10^{-3}} = \mathbf{7\cdot79\times10^{5}\ N\text{-}m^{-2}}\right.$$

●● Short Answer Type Questions (with answers) ●●

● **Qustion 1. What is meant by 'molar gas constant' R= 8·31 J mol^{-1}K^{-1} ?**

○ **Ans.** Molar gas constant or universal gas constant means the amount of work done to raise the temperature of 1 mole gas by 1 K. Hence R = 8·31 J mol^{-1} K^{-1} means that to raise the temperture of 1 mole of an ideal gas by 1 K, 8·31 Joule of work has to be done.

● **Q. 2. The pressure of some amount of gas at 200K temperature is $1{\cdot}0 \times 10^{-3}$ atm. Calculate the number of molecules in 1 c.c. of the gas.**

○ **Ans.** No. of moles, $n = \frac{PV}{RT} = \frac{1{\cdot}013\times10^{2}\times10^{-6}}{8{\cdot}31\times200}$ $\left[\because P = 10^{-3}\text{atm} = 1{\cdot}013\times10^{2}\text{Nm}^{-2}\right]$

$\therefore\ n = 6{\cdot}1\times10^{-8}$ mole

$\therefore$ No. of molecules in 1 c.c. of the gas $= 6{\cdot}1 \times 10^{-8} \times 6{\cdot}023 \times 10^{23} = \mathbf{3{\cdot}68 \times 10^{18}}$.

● **Q. 3. Why real gases do not obey Boyles' law ?**

○ **Ans.** Only ideal gases obey Boyles' law. According to Kinetic theory ideal gas molecules do not exert any attraction or repulsion on each other and their volume is negligible compared to the volume of the vessel. These assumptions are not applicable to real gases. Because in the real gases, the molecules exert some force on each other and their volume is not negligible compared to the volume of the vessel. For these reason, real gases do not obey Boyles' law which is applicable to ideal gases.

● **Q. 4. What are the limitation of Boyles' law and Charles' law ?**

○ **Ans.** Only ideal gas strictly obeys Boyles' law and Charles' law always. Real gases do not obey Boyles' and Charles' law all the time. Only at a very low pressure and high temperature, real gases obey Boyles' law and Charles' law.

● **Q. 5. Why does the pressure of gas increases when it is heated ?**

○ **Ans.** When a gas is heat its temperature rises. Kinetic energy of the molecules also increases.

Now, pressure of a gas is given by $P = \frac{2}{3} \times$ Kinetic energy per unit volume.

Hence as kinetic energy increases, the pressure of the gas also increases.

● **Q. 6. If the absolute temperature of a gas is increased 4 times, what is the change in r. m. s. velocity of the gas molecules.**

○ **Ans.** We know that r. m. s. velocity of gas molecules is proportional to the square root of absolute temperature of the gas. So $C \propto \sqrt{T}$

$$\therefore \frac{C_2}{C_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{4} = 2 \quad \therefore \quad C_2 = 2\,C_1$$

So, if the absolute temperature is increased 4 time, r. m. s. velocity increases 2 times.

● **Q. 7. What is the Kinetic interpretation of temperature ?**

○ **Ans.** Vide Art. 2.11.

● **Q. 8. Different ideal gases are at the same temperature. Will their r. m. s. velocities be equal ?**

○ **Ans.** The r. m. s. velocities of different gases will not be equal, although their temperatures are same.

● **Reason** : r. m. s. velocity v is given by $v = \sqrt{\frac{3RT}{M}}$ so, if temperature is constant, $v \propto \frac{1}{\sqrt{M}}$.

So, at constant temperature r. m. s. velocity is inversely proportional to the molecular weight of the gas.

● **Q. 9. Why does the temperature of a gas rises if it is suddenly compressed ?**

○ **Ans.** When a gas is compressed, some work is done on the gas. Due to this work done kinetic energy of gas molecules increased. As temperature is directly proportional to the average kinetic energy of the molecules, so with increase of kinetic energy of the molecules, its temperature rises.

● **Q. 10. Equal masses of a monatomic gas and a diatomic gas are at same temperature. Now, they are given equal amount of heat, which one of the two gases will undergo greater temperature rise?**

○ **Ans.** Monatomic gas only possesses translation kinetic and have negligible rotational energy. So, the average kinetic energy of a monatomic gas is $E_k = \frac{3}{2}KT$, here K is Boltzman constant and T is the absolute temperature.

But a diatomic gas has both translational and rotational kinetic energy. So, total kinetic energy of a diatomic gas molecule is $E'_k = \frac{5}{2}KT$. Now, let the monatomic gas is given Q amount of heat at constant volume and the temperature rises by ΔT_1. Then $Q = \frac{3}{2}K.\Delta T_1$.

And if ΔT_2 be the temperature rise of diatomic gas at constant volume due to the same quantity of heat given to it, $Q = \frac{5}{2}K.\Delta T_2$

$$\therefore \quad \frac{3}{2}K.\Delta T_1 = \frac{5}{2}K.\Delta T_2 \qquad \therefore \frac{\Delta T_1}{\Delta T_2} = \frac{5}{3} \qquad \therefore \quad \Delta T_1 > \Delta T_2$$

∴ Rise of temperature will be more monatomic gas.

● **Q. 11. Why light gases like hydrogen and helium are rare in earth's atmosphere.**

○ **Ans.** Escape velocity from the earth's surface is 11·2 km/sec. Initially at the time of formation of the earth, temperature of earth was very high and r. m. s. velocities of the molecules of light gases like hydrogen and helium was also high. Although this r. m. s. velocity was less than escape velocity from the earth's surface, but some of the hydrogen molecules might have inidividual velocity more than escape velocity. These molecules have escaped from earth's atmosphere. For this reason light gases like hydrogen and helium are rarer in earth's atmosphere.

● **Q. 12. Three vessel of equal volume contains three different gases at the same temperature and pressure. First vessel contains monatomic neon gas, second vessel contain diatomic chlorine gas and the third vessel contain polyatomic uranium hexafloride.**

(i) Do the vessels contain equal to molecules ?

(ii) Are the r. m. s. velocities of the three gases of the vessel.

○ **Ans.** (i) All the three vessels contain equal number of molecules.

● **Reason :** By Avogadro's hypothesis we know that equal volume of all gases at the same contains of temperature and pressure contain equal number of molecules.

(ii) But the r. m. s. velocity of different gas molecules will be different. **Because :** r. m. s. velocity

$$C = \sqrt{\frac{3RT}{m}} \qquad \therefore \quad c \,\alpha\, \frac{1}{\sqrt{m}}$$

So, r. m. s. velocity depends on mass of the molecule. Heavier the mass less is the r. m. s. velocity at the same temperature. Of the given three gases neon is lightest. So, r. m. s. velocity of neon gas molecules will be greater.

• E ◇ X ◇ E ◇ R ◇ C ◇ I ◇ S ◇ E •

A. Short answer type questions :

1. What is an ideal gas from kinetic theory of gases ? **[J. E. E. 2002]**
2. What is meant by Brownian motion ? How it is explained for kinetic theory of gases ? **[H. S. 1998]**
3. What are the difference between ideal gases and real gases ? Under what condition real gases behave as ideal gases ?

4. Draw (PV–P) diagram for a ideal gases and for real gases.

5. The volumes and pressures of two gases are equal. What is the relation between the kinetic energies of a molecule of the two gases ?

6. Show that if tempertures of two gases are equal, the average kinetic energies of the molecules of the two gases will be equal.

7. Show that the average kinetic energy of a molecule of a gas does not depend on the nature of the gas.

8. Why do real gases behave as ideal gases at very low pressure ?

9. What is meant by average velocity and r. m. s. velocity of gas molecules ?

10. Why the ideal gas molecules do not possess potential energy ?

11. What is the interpretation of pressure of a gas from kinetic theory ? Why pressure of a gas increases with rise of temperature ?

12. What is the kinetic interpretation of temperature ?

13. What is Boltzmann constant ? Show that kinetic energy of ideal gas depends on temperature and not on the nature of the gas.

14. The molecules of different gases have same translational kinetic energy, will their temperatures be equal ? Explain from kinetic theory of gases.

15. Will the r. m. s. velocities of oxygen and hydrogen gas molecules be equal at equal tempertaure ? **[J. E. E. 1998]**

16. If the absolute temperature of a gas be increased four times, how do the following quantities change? (i) r. m. s. velocity of gas molecules (ii) total energy (iii) pressure of the gas.

17. At N. T. P. you are given 1 c.c. hydrogen and 1 c. c. oxygen which are will carry greater number of molecules ?

18. Show that, at the same temperature molecules of different gases will have same kinetic energy.

19. At what temperature r. m. s. velocity of a gas molecular be zero ?

20. With rise of temperature of an enclosed gas, pressures rises, but with the lowering of temperature pressure decreases. Explain this from kinetic theory.

21. Write down the Vander Waal real gas equation for one mole of the gas.

22. What is mean free path ? Write an expression for it.

◢ B. Essay type questions :

1. What is meant by Brownian motion ? What are the characterises of such motions ? Set up a domonstrative experiment on Brownian motion.

2. (a) What is meant by Kinetic theory of gases ? **[H. S. 2001]**
   (b) What are the assumptions of kinetic theory of gases ?

3. (a) What is the explanation of pressure of a gas from kinetic theory ?
   (b) Derive the expression for the pressure of an ideal gas ?

4. (a) Obtain an expression for the kinetic energy of one mole of ideal gas.
   (b) Hence give the explanation for temperature of a gas. **[H. S. 2000]**

5. Set up an experimental evidence in favour of Kinetic theory of gases. **[H. S. 1998]**

6. What is Brownian motion ? Give its explanation from kinetic theory.

7. Derive Boyles' law, Charles' law and Avogadro's hypothesis from kinetic theory.

8. (a) What are the limitations of ideal gas laws ?
   (b) What correction are applied in ideal gas equation by Vander Waal to derive the equation of state of a real gas ? Write down the equation.
   (c) What is meant by free path and mean free path ? Write the different expressions for mean free path.

◢ C. Simple numerical problems :

1. What is the volume of 1 mole of an ideal gas at N. T. P. ? $R = 8{\cdot}31$ J mol^{-1}K^{-1} **[Ans.** $2{\cdot}24 \times 10^{-2}$ m^3 **]**

2. Calculate the number of molecules contained in 1 c.c. of an ideal gas at N. T. P. **[Ans.** $2{\cdot}685 \times 10^{19}$ **]**

3. The density of an ideal gas at N. T. P. is $1{\cdot}25 \times 10^{-3}$ g/c.c. Calculate the molecular weight of the gas. **[Ans.** $28{\cdot}3$ g mol^{-1} **]**

4. An ideal gas of mass $0\cdot177$g occupies a volume of 1000 c.c. at N. T. P. What is the r. m. s. velocity of gas molecule ? [$R = 8\cdot31$ J $mol^{-1}K^{-1}$] [**Ans.** 1300 ms^{-1}]

5. At 0°C, the pressure of 1 c.c. ideal gas is 10^{-5} mm Hg. What is the number of molecules in it ? [**Ans.** $3\cdot53 \times 10^{11}$]

6. Two vessels of volume V and 2V contain air of equal mass. The temperature of the first vessel is 300 K and that of the second is 600 K. Calculate the ratio of the pressures in the two vessels. [**Ans.** 1 : 1]

7. Average kinetic energy of one molecule of air is $6\cdot4 \times 10^{-21}$ J. Calculate temperature of air. Boltzman constant $K = 1\cdot38 \times 10^{-23}$ J K^{-1} [**Ans.** 310 K]

8. A vessel contains 14 g hydrogen and 96 g oxygen at N. T. P. What is the volume of the vessel ? [1 Atm = $1\cdot0 \times 10^5$ Nm^{-2}, $R = 8\cdot31$ J mol^{-1} K^{-1}] [**Ans.** $0\cdot21$ m^3]

9. Calculate the kinetic energy of 1 mol of helium at 27°C. [$R = 8\cdot31$ J mol^{-1} K^{-1}] [**Ans.** 3735 J] [**H. S. '94**]

10. What is the kinetic energy of 20 g carbon-di-oxide at 27°C ? [$R = 8\cdot31$ J mol^{-1} K^{-1}] [**Ans.** 1699 J]

11. Calculate the increase of translational kinetic energy of one molecule of a gas due to a temperture rise of 5°C ? $R = 8\cdot31$ J mol^{-1} k^{-1} and $N = 6\cdot023 \times 10^{23}$. [**Ans.** $1\cdot035 \times 10^{-22}$ J]

12. Average kinetic energy of one molecule of hydrogen at 0°C is $5\cdot64 \times 10^{-21}$ J. Calculate Avogadro's number. $R = 8\cdot31$ J mol^{-1} k^{-1} [**Ans.** $6\cdot04 \times 10^{23}$]

13. A vessel of volume 1 litre contains $2\cdot6 \times 10^{23}$ number of nitrogen molecule. Mass of each nitrogen atom is $4\cdot65 \times 10^{-23}$ g and r. m. s. velocity 500 m/s. What is the pressure of the gas in the vessel ? [**Ans.** $9\cdot95 \times 10^5$ Nm^{-2}]

14. At 30°C and at a pressure of 1 cm Hg, the volume of an ideal gas is 4 c.c. At this temperature, average kinetic energy one molecule is 4×10^4 erg. Calculate the number of molecules in the gas. [**Ans.** $1\cdot9 \times 10^8$]

15. Calculate the internal energy of 2 g oxygen gas at N. T. P. Given $R = 8\cdot31$ J $mol^{-1}K^{-1}$ [**Ans.** $354\cdot5$ J]

[**Hint :** $U = m\,C_v T$, $C_v = \frac{5}{2}R$]

16. The density of a gas at 0°C and at a pressure of $0\cdot01$ atm is $1\cdot24 \times 10^{-2}$ kg / m^3. Calculate (i) r. m. s. velocity of nitrogen (ii) molecular mass. [**Ans.** (i) 495 ms^{-1} (ii) $27\cdot8$]

17. $0\cdot014$ kg nitrogen gas is enclosed in a vessel at 27°C. How much heat has to be supplied to the gas so that r. m. s. velocity of the gas molecules becomes double ? Boltzman constant $K = 1\cdot38 \times 10^{-23}$ 5 K^{-1} and molecular mass of nitrogen = 28 [**Ans.** 9315 J]

18. Calculate the r. m. s. velocity of oxygen molecule at 27°C. Given $R = 8\cdot31$ J mol^{-1} K^{-1}, atomic mass of oxygen = 16. [**Ans.** $4\cdot83 \times 10^4$ cm / s] [**H. S. '91**]

19. The average kinetic energy of a gas molecule at N. T. P. is $5\cdot6 \times 10^{-14}$ ergs. Calculate the number of molecules in 1 c.c. of that gas. Density of mercury = $13\cdot6$ g / c.c. [**Ans.** $3\cdot498 \times 10^{19}$]

[**Hints :** Pressure, $P = \frac{2}{3} \times$ kinetic energy per unit volume]

20. A vessel of volume 4 litre contains a mixture of 22 g carbon-di-oxide, 14 g nitrogen and 8 g oxygen gas. If the temperature of the mixture be 27°C, Calculate the pressure of the mixture of gases. [$R = 8\cdot31$ J mol^{-1} K^{-1}] [**Ans.** $7\cdot79 \times 10^5$ N / m^2]

21. A vessel of volume 1 litre contains 10^{25} number of oxygen molecule. Mass of each oxygen molecule is $2\cdot7 \times 10^{-25}$ g and its r. m. s. speed 4×10^4 cm / s. Calculate the pressure of the gas. [**Ans.** $1\cdot44 \times 10^5$ Nm^{-2}]

22. A vessel of volume 1 litre contains an ideal gas at normal temperature and pressure. Calculate (i) number of molecular in the gas (ii) At what temperature of the gas, r. m. s. velocity of the gas molecule be double ? [**Ans.** (i) 25×10^{21}, (ii) $1092\cdot6$ K]

23. A vessel is full of a gas at a pressure of 76 cm of Hg. At the same temperature more gas is added to the vessel so that the mass of the gas increases by 50 %. Calculate the pressure of the gas now. [**Ans.** 114 cm Hg]

24. If the temperature of an enclosed gas is raised by 1%, the pressure of the gas increases by $0\cdot4$ %. What was the initial temperature of the gas ? [**Ans.** 250 K]

25. The r. m. s. velocity of hydrogen molecule at NTP is 1600 m/s. What will be the r. m. s. velocity of oxygen molecule under the same condition ? Molecular weights of oxygen and hydrogen are 32 and 2 respectively. [**Ans.** 400 ms^{-1}]

26. The volume of 2 mols of hydrogen gas at a pressure P is $1{\cdot}25\ m^3$. 1 mol of oxygen gas of volume $2{\cdot}5\ m^3$ exerts the same pressure. Compare the r. m. s. velocities of hydrogen and oxygen. [**Ans.** 2 : 1]

27. A spherical vessel of radius 20 cm is full of 2 gm hydrogen gas and 3 g helium gas. If the temperature of the vessel be 22°C, calculate the pressure exerted on the wall of the vessel. [**Ans.** $1{\cdot}83 \times 10^6$ dynes / cm^2]

D. Harder numerical problems :

1. A vessel of volume 1 litre contains 10^{23} number of nitrogen molecules. Mass of a molecule is $4{\cdot}65 \times 10^{-23}$ g and r. m. s. velocity 5×10^4 cm/s. Calculate pressure and total kinetic energy of the gas.
[**Ans.** $9{\cdot}69 \times 10^5\ Nm^{-2}$; $1{\cdot}45 \times 10^3$ J] [**J. E. E. 1995**]

2. 1 mole oxygen at 23°C and 2 mole helium at 123°C is mixed together. What will be the temperature of the mixture ? [**Ans.** 45°C] [**I. I. T**]

3. The volume of an electric bulb is 250 c.c. It contains a gas at 27°C and at a pressure of 10^{-3} mm Hg. Calculate the number of molecules. [**Ans.** $8{\cdot}05 \times 10^{15}$]

4. A mixture neon and argon gas is contained in a vessel of volume $0{\cdot}02\ m^3$ at 27°C. Pressure of the gas mixture is $10^5\ Nm^{-2}$ and mass of the mixture is 28 g. The molecular weights of neon and argon are 20 and 40. Calculate the mass of each of the two gases. ($R = 8{\cdot}314\ J\ mol^{-1}\ K^{-1}$) [**Ans.** 4 g, 24 g] [**I. I. T. '74**]

5. One mole argon at 27°C is mixture with 1 mole helium at 57°C. Calculate the temperature of the mixture.
[**Ans.** 42°C]

6. At what temperature the average kinetic energy of a molecule be equal to the energy acquired by an electron while passing through a potential difference of 5 volt ? Boltzmann Constant, $K = 1{\cdot}38 \times 10^{-23}\ J\ K^{-1}$ and $1\ eV = 1{\cdot}6 \times 10^{-19} J$ [**Ans.** $38{\cdot}65 \times 10^5$ K]

7. A vessel contains 64 g of oxygen at a temperature of 127°C. What energy be given to it so that r. m. s. velocity of a molecule be double the initial r. m. s. velocity. [**Ans.** 49920 J]

8. The volume of a mixture of hydrogen and oxygen is 2 litres. Its temperature is 300 K, pressure $10^5\ Nm^{-2}$ and mass 0·76 g. Calculate the mass of oxygen and hydrogen in the mixture.
[**Ans.** Mass of hydrogen = 0·12 g and mass of oxygen = 0·64 g]

9. The length of a barometer tube is 1 m and area of cross section $2\ cm^2$. The reading of the barometer is 75 cm. When some oxygen is entered into the barometer, the mercury column drops by 5 cm. What is the mass of that oxygen ? Assume, temperature is 27°C, density of mercury 13600 kg / m^3 and $g = 10\ ms^{-2}$.
[**Ans.** $5{\cdot}24 \times 10^{-3}$ g]

10. A vessel of volume $8{\cdot}0 \times 10^{-3}\ m^3$ contains an ideal gas. The pressure of the gas is $2 \times 10^5\ Nm^{-2}$ and its temperature is 300K. When some gas escapes pressure drops to $1{\cdot}25 \times 10^5\ Nm^{-2}$. If the temperature remains constant, calculate the mass of the escaped gas. [**Ans.** 0·24 mole]

11. If the temperature of different layers of atmosphere be at same temperature, obtain the expression for the variation of pressure with height from the earth's surface. Average molecular weight of air is M.

[**Ans.** $P = P_0 e^{-Mgh/RT}$]

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26. The volume of 2 mols of hydrogen gas at a pressure P is 1·25 m^3. 1 mol of oxygen gas of volume 2·5 m^3 exerts the same pressure. Compare the r. m. s. velocities of hydrogen and oxygen. [Ans. 2 : 1]

27. A spherical vessel of radius 20 cm is full of 2 gm hydrogen gas and 3 g helium gas. If the temperature of the vessel be 22°C, calculate the pressure exerted on the wall of the vessel. [Ans. 1·83 × 10^6 dynes/cm^2]

D. Harder numerical problems :

1. A vessel of volume 1 litre contains 10^{23} number of nitrogen molecules. Mass of a molecule is 4·65 × 10^{-23} g and r. m. s. velocity 5 × 10^4 cm/s. Calculate pressure and total kinetic energy of the gas.

[Ans. 9·09 × 10^3 Nm^{-2}; 1·48 × 10^1 J] [J. E. E. 1995]

2. 1 mole oxygen at 23°C and 2 mole helium at 123°C is mixed together. What will be the temperature of the mixture ? [Ans. 45°C] [I. I. T.]

3. The volume of an electric bulb is 250 c.c. It contains a gas at 27°C and at a pressure of 10^{-3} mm Hg. Calculate the number of molecules. [Ans. 8·05 × 10^{15}]

4. A mixture neon and argon gas is contained in a vessel of volume 0·02 m^3 at 27°C. Pressure of the gas mixture is 10^5 Nm^{-2} and mass of the mixture is 28 g. The molecular weights of neon and argon are 20 and 40. Calculate the mass of each of the two gases. (R = 8·314 J mol^{-1} K^{-1}). [Ans. 4 g, 24 g] [I. I. T. '74]

5. One mole argon at 37°C is mixture with 1 mole helium at 57°C. Calculate the temperature of the mixture.

[Ans. 42°C]

6. At what temperature the average kinetic energy of a molecule be equal to the energy acquired by an electron while passing through a potential difference of 5 volt ? Boltzmann Constant, K = 1·38 × 10^{-23} J K^{-1} and 1 eV = 1·6 × 10^{-19}J [Ans. 38·65 × 10^3 K]

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[Ans. Mass of hydrogen = 0·12 g and mass of oxygen = 0·64 g]

9. The length of a barometer tube is 1 m and area of cross section 2 cm^2. The reading of the barometer is 75 cm. When some oxygen is entered into the barometer, the mercury column drops by 5 cm. What is the mass of that oxygen ? Assume, temperature is 27°C, density of mercury 13600 kg/m^3 and g = 10 ms^{-2}.

[Ans. 5·24 × 10^{-3} g]

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11. If the temperature of different layers of atmosphere be at same temperature, obtain the expression for the variation of pressure with height from the earth's surface. Average molecular weight of air is M.

[Ans. $P = P_0 e^{-Mgh/RT}$]

WAVES

• TOPICS DISCUSSED •

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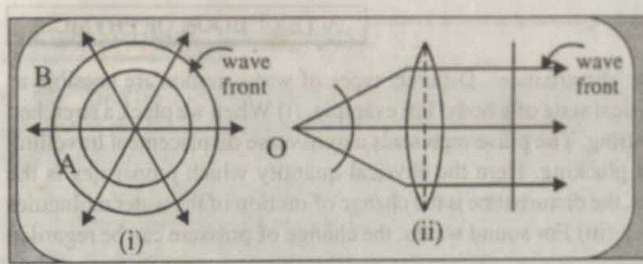


CHAPTER 1

WAVE MOTION

• **TOPICS :** ○ Introduction, ○ Wave motion; ○ Elastic wave properties; ○ Wave motion in stretched string ; ○ Generation of transverse wave in an elastic medium ; ○ Generation of longitudinal waves; ○ Comparison between transverse and longitudinal waves; ○ Progressive waves and their characteristics; ○ Some important definitions in connection with waves ; ○ Relation between wavelength (λ), frequency (n) and wave velocity (v); ○ Equation of progressive harmonic wave ; ○ Relation between particle velocity and wave velocity; particle acceleration; ○ Phase of a wave : Its variation with time and distance; ○ Graphical study of propagation of harmonic wave; ○ The velocity of progressive wave in different media ; ○ Some common properties of progressive waves; ○ Phase change due to reflection; ○ Reflection of waves; ○ Reflection of sound wave; ○ Practical application of reflection of sound; ○ Echo; ○ Practical application of echo ; ○ Refraction of waves ; ○ Refraction of sound; ○ Short answer type questions (with answers); ○ Exercise.

○ 1.1. Introduction ○

In different branches of physics wave related phenomena are observed. So it is essential to study the nature and properties of waves. Sound is a type of wave. In addition, there are water waves, heat waves, light waves, radio waves and other electromagnetic waves. The existence of some kind of wave is observed in subatomic particles like electrons, protons and neutrons. To explain the behaviour of these subatomic particles accurately, a new branch of physics named wave mechanics has been discovered. The wave phenomena are of fundamental importance to a physicist because any description of matter at atomic and subatomic scale without the concept of waves is simply not possible.

○ 1.2. Wave motion ○

There are essentially two ways of transporting energy from one place to another. The first involves actual transport of matter. For example, when a particle moves through space, it carries kinetic energy with itself. Wherever the particle goes, the associated energy goes with it. Hence the energy is transported from one region of space to the other together with the particle.

The second method by which energy can be transported is much more useful and important. It involves what we call a wave process. The waves are capable of transporting energy from one location to another, but in doing so they do not transport any matter. For example, (i) When a person beats a drum its sound is heard at distant points. The sound carries energy as it can move the diaphragm of the ear. (ii) When a stone is dropped in still water in a pond, water waves move steadily along the water surface. A floating cork on water is found to move up and down at its own location. It proves that the molecules of water do not move along with the wave. (iii) Electromagnetic radio waves carries energy from transmitter to receiver for which no material medium is required. Similar host of examples can be cited related to energy transfer by wave mechanism. Although these various process of transport of energy are different, yet they have a common feature which we call **wave motion**. We define a wave as follows :

A wave implies propagation of disturbance with some physical quantity of the medium from one place to another.

The key word related to wave motion is 'disturbance'. Different types of wave motion are possible as there are various ways of disturbing the physical state of a body. For example, (i) When we pluck a stretched string at some point, a pulse travels on the string. The pulse represents a transverse displacement travelling along the string originating at the point of plucking. Here the physical quantity which propagates is the displacement. (ii) In the case of water waves, the disturbance is the change of motion of the water molucules relative to the equilibrium state (flat surface). (iii) For sound waves, the change of pressure can be regarded as disturbance.

➤ *1.2.1. Classification of waves :*

(A) On the basis of physical quantity being propagated, the waves are classified into two groups :

(i) Mechanical Waves, (ii) Electromagnetic Waves.

- **Mechanical waves :** These waves are produced when an elastic medium is deformed due to some disturbance in the medium. In mechanical wave particles of the medium vibrate. A material medium is necessary for the propagation of such waves. This type of wave may also be called elastic wave because elastic property of the medium is involved in the propagation of the wave. Sound waves, water waves etc. are the examples of mechanical waves.

- **Electromagnetic waves :** A wave motion may take place without any material medium. Such waves are called electromagnetic waves. In electromagnetic waves electric and magnetic field vectors propagate in space. Light waves, radio-waves, microwaves are some examples of electromagnetic waves.

(B) On the basis of direction of vibration of the particles of the medium the waves are classified into two headings :

(i) Transverse waves, (ii) Longitudinal waves.

- **Transverse waves :** When the particles of the medium vibrate about their mean positions perpendicular to the direction of propagation of the disturbance, the wave is called transverse wave.

- **Example :** Wave produced due to transverse vibration of string and electromagnetic waves.

- **Longitudinal vibration :** When the particles of the medium vibrate about their mean position parallel to the direction of propagation of the disturbance, the wave is called longitudinal wave.

- **Example :** waves is spring, sound waves etc.

Transverse and longitudinal waves together are known as progressive waves, because these waves progress from one point to another through a medium. There are another type of waves, having opposite properties known as stationary waves which will be discussed later.

○ 1.3. Elastic Wave properties ○

Suppose a tuning fork (or any object) is vibrating. A disturbance is created in the medium. When a prong of the fork moves on one side, it pushes the layer of air in contact with it and the layer is compressed. When the prong goes in the opposite direction that layer of air is rarefied. Thus due to the vibration of the tuning fork the layers of air are alternately compressed and rarefied. Due to elasticity of air these compressions and rarefactions are transmitted from one layer to next layer. Thus disturbance is transmitted from one layer to another through compression and rarefaction. This type of wave is known as **elastic wave**.

The generation and propagation of elastic waves through a material medium (solid, liquid or gas) depends on the following three properties of the medium : **(i) elasticity (ii) inertia and (iii) cohesion.**

- (i) **Elasticity :** When a disturbance is created at any part of a continuous medium, that part is displaced from the mean position and elastic strain is produced. Due to elasticity the strained part try to return to its initial position of equilibrium.

● **(ii) Inertia :** During return motion the velocity of the particles gradually increases. The particles can not stop at the mean position, but due inertia of motion, they overshoots the mean position and goes in the opposite direction.

So, the particles of the medium in the disturbed part returns to their initial position due to elasticity and due to inertia they vibrate in a periodic motion.

● **(iii) Cohesion :** The adjacent particles of the medium attract each other. The attraction between the particles of the same medium is called force of cohesion. Due this cohesion when a part of a medium vibrate, the part adjacent to it also set into forced vibration. In this way forced vibration is transmitted from one layer to other layer in the medium. Here different layers in the medium execute identical forced vibration. But they are not disturbed simultaneous. For this reason, different particles are not in phase.

Hence the disturbance which travels through the medium due to **collective vibration of the particles of the medium is known as elastic waves.**

- **The characteristic of elastic waves may be summarised as follows :**

(i) A material medium is necessary for the propagation of elastic wave. It can not propagate through vacuum.

(ii) The elastic waves have definite velocity of propagation in a medium. This velocity depends on the property of elasticity of the medium.

(iii) Due to cohesive force one particle induces forced vibration to the next particle. The particles are not displaced from the mean position permanently. Only the waves propagate through the medium.

(iv) Any particle of the medium has some mechanical energy. This energy is transferred from one particle to another during propagation of elastic wave. Thus the energy of disturbance propagates through the medium.

✪ 1.4. Wave motion in stretched string ✪

A stretched string can be treated as an one dimensional continuous medium. Suppose one end of a string of uniform cross section is rigidly fixed and the other end is held in hand [Fig. 1.1]. A pulse is created by suddenly moving the hand upwards and it is brought back quickly to its original position. The pulse moves along the string. The particles of the string move up and down as the pulse passes with a definite velocity. Assume that the pulse is created at $t = 0$ and it is denoted by 1.

This pulse travels a distance x in time t when a second pulse denoted by 2 is produced by moving the hand downwards and bringing it back to its initial position. The second pulse travels the same distance x in time t. The velocity of each pulse is $v = x/t$. After a time $2t$ third pulse is produced and it will travel with same speed. A number of pulses produced in time $3t$ is shown in fig. 1.1.

Thus we see that energy is transmitted along with the waves in the string. The string does not move with the wave. The velocity of the wave depends on the nature of the medium.

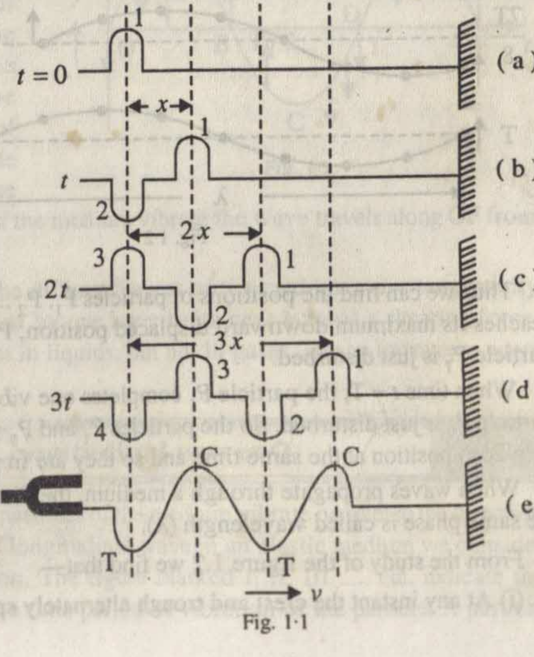


Fig. 1·1

So far pulse is produced in the string after a regular interval of time. Now continuous pulse can be created in the string. This can be done by tying the free end of the string to a prong of a tuning fork and setting it into harmonic oscillation. It is also shown in fig 1.1. The waves so produced are called harmonic waves.

⭘ 1.5. Generation of transverse wave in an elastic medium ⭘

If during propagation of a wave, the particles of the medium vibrate perpendicular to the direction of propagation of the wave then the wave is called **transverse wave.**

When a stone is dropped on water in a pond, transverse water wave is produced. When a stretched string is plucked at a point transverse wave is produced.

When a wave propagates through a medium, along the path of the wave the particles of the medium are not in phase.

To study the formation of transverse wave we consider a number of particles along a straight line [Fig. 1.2]. Let the particle be denoted by $P_1, P_2, P_3 \ldots\ldots, P_{10}$.

Assume that the particle P_1 is disturbed at $t = 0$ and it starts vibrating perpendicular to the line of the particles under consideration. If T be the time period, the particle will complete one oscillation in time T. As the particle P_1 moves up, the neighbouring particle is dragged on and it also starts vibrating.

In this way, disturbance travels from one particle to another. Let the next particle is just disturbed at time $t = T/8$.

At $t = T/4$, when the particle P_1 has attained its maximum upward displacement, the disturbance just reaches the next P_3 particle.

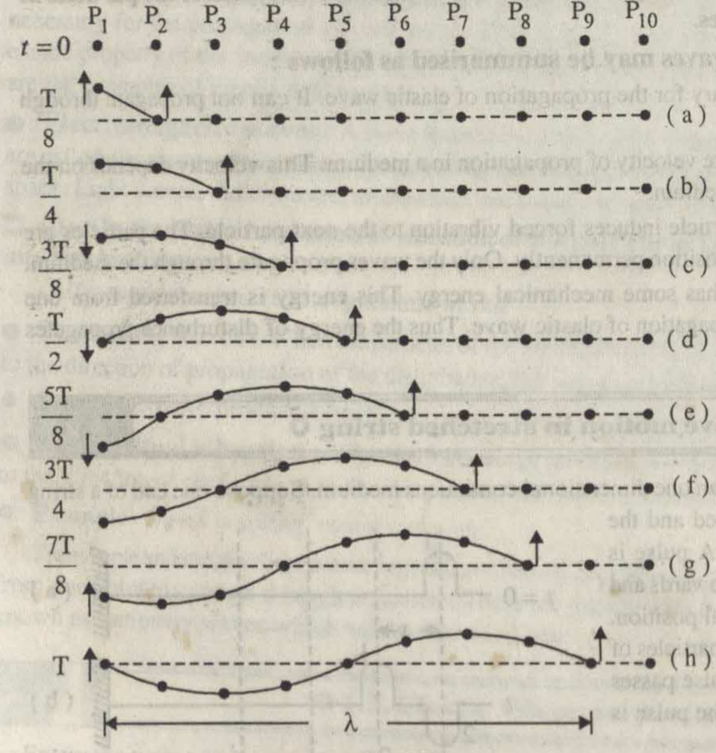


Fig. 1·2

Thus we can find the positions of particles P_1, P_2 ..... at different times. At time $t = 3T/4$, the particle P_1 reaches its maximum downward displaced position, P_5 attains its maximum upward displacement and the particle P_7 is just disturbed.

When time $t = T$, the particle P_1 completes one vibration and returns to its equilibrium position and the particle P_9 is just disturbed. So the particles P_1 and P_9 vibrate in step. Both P_1 and P_9 reaches the maximum displaced position at the same time and so they are in phase.

When waves propagate through a medium, the distance between the two nearest particles which are at the same phase is called wavelength (λ).

From the study of the figure 1.2 we find that—

(i) At any instant the crest and trough alternately spread.

(ii) The distance between two successive crests (or troughs) is λ and the distance between a crest and the next trough is $\lambda/2$.

- **Special Discussions :** (i) At time $t = 0$, the particle P_1 was at its equilibrium position [Fig. 1.2(a)].

So its equation of motion is $y(t) = A\cdot\sin\omega t = A\cdot\sin\frac{2\pi}{T}\cdot t$

(ii) At time $t = T/8$ the displacement of P_1 is $y\left(t=\frac{T}{8}\right)=A\cdot\sin\frac{2\pi}{T}\cdot\frac{T}{8}=\frac{A}{\sqrt{2}}$

(iii) At time $t = T/4$ the displacement of P_1 is $y\left(t=\frac{T}{4}\right)=A\cdot\sin\frac{2\pi}{T}\cdot\frac{T}{4}=A$

(iv) At time $t = T/2$ the displacement of P_1 is $y\left(t=\frac{T}{2}\right)=A\cdot\sin\frac{2\pi}{T}\cdot\frac{T}{2}=0$

During this time the particle completes half oscillation and moves downwards.

(v) At time $t = 5T/8$ the displacement of P_1 is $y\left(t=\frac{5T}{8}\right)=A\cdot\sin\frac{2\pi}{T}\cdot\frac{5T}{8}=-\frac{A}{\sqrt{2}}$

(vi) At time $t = T$, the particle P_1 completes one oscillation, the disturbance reaches the particle P_9 and it moves upwards.

Evidently at time $t = T$, $t = 2T$ and $t = 3T$, the particle P_1 completes 1, 2 and 3 complete oscillations. As the energy of disturbance flows along the wire, different particles of the medium execute identical simple harmonic motions. But they always have some phase difference with respect to each other. Here the particles P_1 and P_9 are at same phase and P_1 and P_5 are in opposite phase.

The displacement-time graph for the particles of the medium during propagation of the wave is shown in fig. 1.3. The graph will be either a sine or cosine graph. Here the direction of vibration of the particles is perpendicular to the direction of wave motion. The moment at which the particle O crosses its equilibrium position (OBF line) in the downward direction with maximum velocity, the direction of vibration of other particles are shown by small arrows. The point A at the maximum displacement in the positive direction is the **crest.** The point C at the maximum displacement in the negative direction is the trough. The state of motion of the two points A and C is opposite. So they are in opposite phase. But the points A and E are in same phase because they are in the same state of motion. As the particles of the medium vibrate the wave travels along OF from left to right.

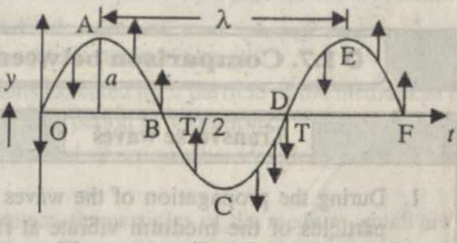


Fig. 1·3

- **Nature of the medium :** In a transverse wave, the different layers of the medium move tangentially with respect to each other and hence the forces exerted by one layer to its next layer is a shearing force. These forces can exist in solids and to a smaller extent in liquids, but not in gases. Hence transverse wave can not travel through a gas.

⊙ 1.6. Generation of Longitudinal waves ⊙

During the propagation of longitudinal waves, the particles of the medium vibrate parallel to the direction of propagation of the wave. To study the formation of longitudinal wave in an elastic medium we consider the particle 0, 1, 2, ... etc. on the line of propagation. The figure marked I, II, III ..... etc. indicate the position of particles at intervals of T/12 where T is the time period of vibration of the particle. A particle executes an SHM about the mean position of rest in the direction of wave propagation left to right. AB and AC represent the amplitude of the vibration. Any two neighbouring particles have a small relative phase difference.

● **Nature of the medium :** Longitudinal waves can propagate through any medium because all substances have elasticity and inertia. Also all substances are compressible to same extent. For these reasons longitudinal waves can pass through any medium — solid, liquid or gases.

● **Example** : We take a long spring AB whose force constant is small [Fig. 1.5]. Its one end B is rigidly fixed and the whole spring is placed on a frictionless surface. The end A is set into small vibration. The wave propagates along its length. The existence of wave is understood by observing the compression and rarefaction of the spring. At the part of compression the turns come closer and at the part of rarefaction the turns of the spring go away from each other as in fig.

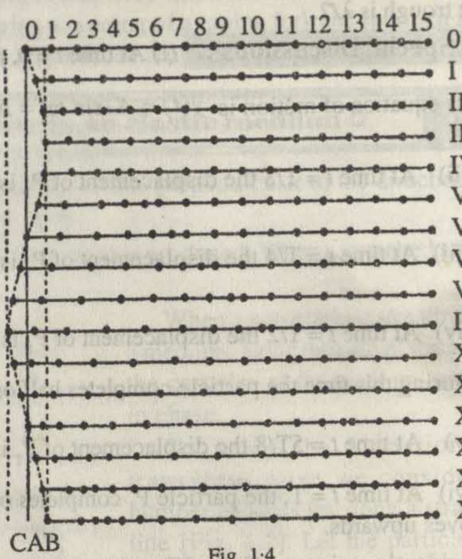


Fig. 1·4

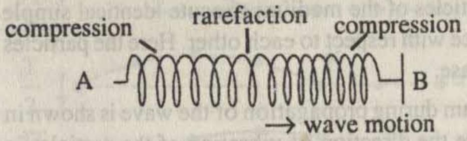


Fig. 1·5

During propagation of the wave alternate compression and rarefaction are produced. Evidently, in this case all the particles of the medium vibrate parallel to the length of the spring.

Longitudinal wave passes through a material medium through compression and rarefaction.

✪ 1.7. Comparison between transverse and longitudinal waves ✪

Transverse waves	Longitudinal waves
1. During the propagation of the waves the particles of the medium vibrate at right angles to the direction of wave propagation.	1. During propagation of the waves, the particles of the medium vibrate parallel to the direction of propagation of the waves.
2. Can be produced only the solids but not in gases or liquids.	2. Can be produced in solids, liquids and gases.
3. The distance between any two consecutive crests or consecutive troughs gives the wave length of a transverse wave.	3. The distance between the centres of consecutive compressions or rarefactions gives the wave length of the longitudinal wave.
4. Can be polarised.	4. Can not be polarised.

✪ 1.8. Progressive waves and their characteristics ✪

❖ **Definition : A progressive wave is a disturbance that passes through a medium with definite velocity and transfer energy from one point to another. In the process no physical transport of material takes place.**

- **We summarise the characteristics of progressive waves as follows :**
  - (i) Progressive waves are produced by continuous periodic vibrations of the particles of the medium. Such waves may be longitudinal or transverse.
  - (ii) Each particle of the medium executes the identical vibration about the equilibrium position with identical frequency and amplitude.
  - (iii) A small phase difference exists between the motions of successive particles. The phase difference is proportional to the distance between the particles along the direction of wave motion.
  - (iv) The medium as a whole does not move along the direction of wave motion.
  - (v) Progressive wave carries energy from one point to another without any transfer of matter.
  - (vi) The velocity of the wave is constant for a particular medium and is determined by the elasticity and density of the medium.

1.9. Some important definitions in connection with waves

- **(i) Amplitude of the wave :** When a wave passes through a medium, the maximum displacement of a particle of the medium is called amplitude of the wave. In fig. 1.6 'a' is the amplitude of the wave.

- **(ii) Wavelength :** When a wave passes through a medium, the minimum distance between any two particles of the medium which are in the same phase at any instant is called the wavelength of the wave. In fig. 1.6 the distance AE = BF is the wavelength λ.

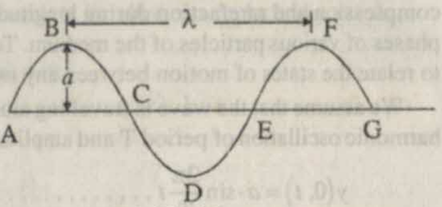


Fig. 1·6

Obviously, during this time a particle of the medium completes one oscillation, and the wave travels a distance λ.

- **(iii) Period :** When a wave passes through a medium, the time required by a particle of the medium to execute one complete oscillation is called the period of the wave.

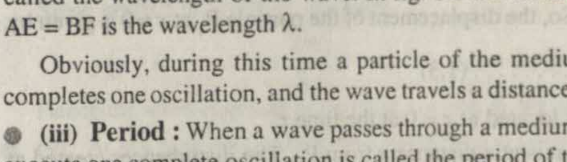
It is also the time taken by the wave to travel a distance equal to one wavelength.

- **(iv) Frequency :** It is the number of complete vibrations executed by a particle of the medium in the path of the wave in one second. So, frequency $n = 1/T$ (T = time period of vibration)

- **(v) Wave velocity :** It is the velocity with which the phase of vibration advances along a line of wave propagation.

- **(vi) Wave front :** When a wave travels through a medium, the particles of the medium which are at the same phase lie on a surface. This surface is called wave front. The surface produced by joining the crests or tronghs of transverse wave is wave front.

A wavefront does not remain stationary but advances through the medium with definite velocity called wave velocity or phase velocity.



(i) (ii)

Fig. 1·7

In a homogeneous medium, the disturbance produced at a point in it, spreads out in all directions will equal velocity. Particles equidistant from the source will vibrate in the same phase. Hence the wave front will be spherical. Such waves are called spherical waves. If the waves are confined to move on a plane, the wave fronts will be concentre circles.

If the source is at a large distance then a finite portion of the wave front coming from the source is practically a plane surface. It is called a plane wave front.

It must be remembered that energy of disturbance travels perpendicular to the wave front.

❂ 1.10. Relation between wavelength (λ), frequency (n) and wave velocity v ❂

The frequency of the wave is n. So, n number of waves are produced in one second and particles of the medium complete n vibrations. Now after one vibratrion of the particle, the wave travels a distance λ.

So, after n vibrations, the wave travels a distance $n\lambda$. Clearly, it is the velocity of the wave.

Hence, $v = n\lambda$ . . . . (1.1)

Wave velocity = frequency × wavelength

Again, if T be the time period then $n = 1/\mathrm{T}$

$\therefore \quad v = \lambda/\mathrm{T} \qquad \therefore \quad \lambda = v\mathrm{T}$ . . . . (1.2)

❂ 1.11. Equation of progressive Harmonic wave ❂

Wave motion (longitudinal or transverse) is the result of simple harmonic oscillations of the particles of the medium. Although every particle of the medium execute identical vibrations, they are not all in phase. The particles in the crests are in a different state of motion from those in the troughs. Similar is the case for compression and rarefaction during longitudinal wave propagation. Wave motion is actually the motion of phases of various particles of the medium. To describe wave motion in mathematical terms we must be able to relate the states of motion between any two particles in the path of the wave.

We assume that the wave is travelling along x-axis with velocity v. Let this wave is created at $x = 0$ due to harmonic oscillation of period T and amplitude a. So, the displacement of the particle P_1 at $x = 0$ is given by

$$y(0, t) = a \cdot \sin \frac{2\pi}{\mathrm{T}} t \quad \ldots\ldots\ldots\ldots \quad (1.3)$$

Here $y(0, t)$ means displacement of the particle located at $x = 0$ at the time t.

Now, we consider another particle P_2 on the x-axis at a distance x from P_1. The disturbances created at P_1 will reach P_2 after x/v sec. We now find the displacement $y(x, t)$ of P_2 located at x at any instant of time t. It is done as follows :

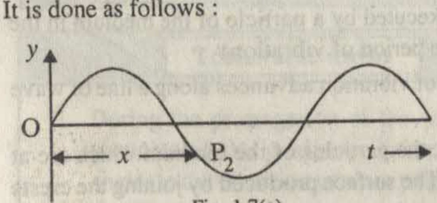


Fig. 1·7(a)

The displacement of particle P_2 located at x is equal to the displacement which the particle P_1 located at $x = 0$ had x/v sec earlier. Hence the displacement of particle P_2 at time t.

= displacement of particle P_1 at time $t' = t - \dfrac{x}{v}$

$$= a \cdot \sin \frac{2\pi}{\mathrm{T}} t' = a \cdot \sin \frac{2\pi}{\mathrm{T}} \left(t - \frac{x}{v} \right)$$

$$\therefore \quad y\,(x, t) = a \cdot \sin \frac{2\pi}{\mathrm{T}} \left(t - \frac{x}{v} \right) \quad \ldots\ldots\ldots\ldots \quad (1.4)$$

This is the equation of a progressive harmonic wave travelling along the positive x-direction.

For a wave travelling along the negative x-direction, the equation is

$$y(x, t) = a \cdot \sin \frac{2\pi}{\mathrm{T}} \left(t + \frac{x}{v} \right) \quad \ldots\ldots\ldots\ldots \quad (1.5)$$

- **A discussion :**

(i) The equation of progressive wave may also be expressed in cosine form. In that case, the equation will be

$$y = a \cdot \cos \frac{2\pi}{\mathrm{T}} \left(t - \frac{x}{v} \right)$$

- **(ii)** We can express the equation (1.4) in alternative forms also.

$$y = a\sin\frac{2\pi}{\mathrm{T}v}(vt-x)$$

$$= a\sin\frac{2\pi}{\lambda}(vt-x)\,\ldots\ldots\ldots\ldots(1.5)\quad \left[v\mathrm{T}=\frac{v}{n}=\lambda\text{, wave length}\right]$$

$$= a\sin\left(\frac{2\pi}{\lambda}vt-\frac{2\pi}{\lambda}x\right)=a\sin\left(2\pi nt-\frac{2\pi}{\lambda}x\right)\ [n=\text{frequency}]$$

$$= a\sin\left(\omega t-\frac{2\pi}{\lambda}x\right)\ [\omega=\text{angular frequency}]$$

- **(iii)** In deriving the above equations it has been assumed that the amplitude of oscillation remains unchanged and there is no loss of energy. But actually there is a loss of energy mainly due to viscosity of the medium. As a result the amplitude of oscillation gradually decreases. Considering loss of energy, the equation of progressive wave will be

$$y = ae^{-bx}\sin\frac{2\pi}{\lambda}(vt-x)\text{, } b \text{ is called damping factor.}$$

- **(iv)** To prove that the wave equation is periodic in x, let us evalute it, when x is replaced by $x+\lambda$.

$$y(x+\lambda,\,t)=a\sin\frac{2\pi}{\lambda}\,[vt-(x+\lambda)]=a\,\sin\left[\frac{2\pi}{\lambda}(vt-x)+2\pi\right]$$

Since $\sin(\theta\pm 2\pi)=\sin\theta$, we have

$$y(x+\lambda,\,t)=a\cdot\sin\frac{2\pi}{\lambda}(vt-x)\quad \therefore\ y(x+\lambda,\,t)=y(x,\,t)$$

Hence the wave equation is periodic is x and its wave length is λ.

- **(v)** At any position (x fixed), the displacement again becomes same after time T, where T is the time – period. Substituting $(t+\mathrm{T})$ for t in equation (1.5), we have,

$$y=a\,\sin\left[\frac{2\pi}{\lambda}v\,(t+\mathrm{T})-x\,2\pi\right]=a\,\sin\left[\frac{2\pi}{\lambda}(vt-x)+2\pi\right]=a\sin\frac{2\pi}{\lambda}(vt-x)$$

It tells that time period of vibrating particle is T.

1.12. Relation between particle velocity and wave velocity ; particle acceleration

When a disturbance is created at any point of a medium, the particles of the medium vibrate about their positions of equilibrium and the energy of disturbance travels from one part to other part of the medium in the form of wave. The velocity with which wave or energy of disturbance travels is called **wave velocity**. The velocity of the wave depends on elasticity and inertia of the medium.

On the other hand, the velocity with which the particles vibration during the propagation of the wave is called **particle velocity**. It is maximum at the equilibrium position and zero at the two extreme ends.

To obtain a relation between these two velocities we consider the equation of progressive waves :

$$y=a\,\sin\frac{2\pi}{\mathrm{T}v}(vt-x)$$

- Here v is the **wave velocity** of the wave motion. So, $v=n\lambda$
- **Particle velocity** of the above equation is obtained by differentiating the displacement y [eq. 1.3] with respect to time.

$\therefore$ Particle velocity $v_P = \dfrac{dy}{dt} = a\dfrac{2\pi}{\lambda}\cdot v\cos\dfrac{2\pi}{\lambda}(vt-x)$ .... (i)

Again $\dfrac{dy}{dx} = -\dfrac{2\pi}{\lambda}a\cdot\cos\dfrac{2\pi}{\lambda}(vt-x)$ .... (ii)

From equation (i) and (ii), $v_P = -v\cdot\dfrac{dy}{dx}$ .... (iii)

$\therefore$ Particle velocity = – wave velocity × slope of displacement graph.

- **Particle acceleration :**

If the displacement at time t be y, its acceleration, $f = \dfrac{d^2y}{dt^2}$

$$\therefore\ f = \frac{d}{dt}(v_p) = \frac{d}{dt}\left[\frac{2\pi\, a\, v}{\lambda}\cdot\cos\frac{2\pi}{\lambda}(vt-x)\right]$$

$$= \frac{-4\pi^2}{\lambda^2}av^2\sin\frac{2\pi}{\lambda}(vt-x) \qquad \therefore\ f = -\left[\frac{4\pi^2}{\lambda^2}av^2\right]y$$

1.13. Phase of a wave : Its variation with time and distance

The quantity by which the motion of the wave can be completely determined is called phase of the wave. From its value the state of motion of the particles of the medium can be ascertained. In fig. (1.3) the points O and D are at the same phase of vibration. The points O and B are the opposite phase. Now the equation of progressive wave is

$$y = a\sin\frac{2\pi}{\lambda}(vt-x) = a\sin\theta$$

where $\theta = \dfrac{2\pi}{\lambda}(vt-x)$ .... (1.6)

This quantity θ is called **phase** of the wave. It follws that the phase of the wave is funciton of both t and x i.e. it varies both with time and position.

- (i) **Phase change with time :** At any point of the medium in the path of the wave motion (x = constant), the phase changes with time. Let t increases to $t + \Delta t$. Then θ increases to Δθ. Then from equation 1.6

$$\Delta\theta = \frac{2\pi}{\lambda}v\cdot\Delta t = \frac{2\pi}{T}\Delta t$$

$\therefore$ When $\Delta t = T$, $\Delta\theta = 2\pi$. It means that after a time T, the phase change of vibrating particle is 2π i.e. the particle returns to initial phase.

- (ii) **Phase change with distance :** At any instant (t = constant), the phase of the particle decreases with distance, when x changes to $x + \Delta x$ phase changes from θ to $\theta + \Delta\theta$

$$\therefore\ \Delta\theta = -\frac{2\pi}{\lambda}\Delta x$$

The negative sign indicates that for a progressive wave travelling along x-axis, the further away the particle is situated along x-axis, the more it lags behind in phase.

It $\Delta x = \lambda$, $\Delta\theta = -2\pi$.

Phase difference 2π means that two particles are at equal phase.

So, λ is the minimum distance between two particles which are at same phase.

- **(iii) Phase difference :** The phase difference at any instant between two vibrating particles located at two positions is equal to the difference in phases of the two particles. Let at any instant the particle are at distances x_1 and x_2 from the origin. Then the path difference between the particles $\Delta x = x_2 - x_1$

Now, phase of first particle at time t, $\theta_1 = \frac{2\pi}{\lambda}(vt - x_1)$ and phase of second particle at the same time,

$$\theta_2 = \frac{2\pi}{\lambda}(vt - x_2)$$

$$\therefore \quad \text{Phase difference, } \theta_1 - \theta_2 = \frac{2\pi}{\lambda}(x_2 - x_1)$$

$$\therefore \quad \text{Phase difference} = \frac{2\pi}{\lambda} \times \text{ path difference.}$$

Hence (a) when path difference be $0, \lambda, 2\lambda, \ldots\ldots$, the phase difference will be $0, 2\pi, 4\pi, \ldots\ldots$ All the particles are at same phase.

(b) Again, if the path difference be $\frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \ldots\ldots$ the corresponding phase difference is $\pi, 3\pi, 5\pi, \ldots\ldots$ Here the particles are in opposite phase.

- **A Discussion :**

- **How does a wave travels through a medium ?**

The equation for the displacement of a particle during the propagation of longitudinal wave is

$$y = a \cdot \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) \ldots\ldots\ldots\ldots (1.7)$$

If v be the velocity of the wave, in time Δt the wave travels a distance $v \cdot \Delta t$. At time $t + \Delta t$, if the displacement of a particle at a distance $(x + v \cdot \Delta t)$ be y' then

$$y' = a \cdot \sin 2\pi\left(\frac{t + \Delta t}{T} - \frac{x + v \cdot \Delta t}{\lambda}\right)$$

$$\text{Now, } \frac{v \cdot \Delta t}{\lambda} = \frac{n\lambda \cdot \Delta t}{\lambda} = n\ \Delta t = \frac{\Delta t}{T}$$

$$\therefore \quad y' = a \cdot \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right). \text{ So, } y = y'$$

It means that whatever be the displacement of a particle at a certain time, the same displacement occurs after a time Δt which is $v\ \Delta t$ distance ahead of the first particle.

1.14. Graphical study of propagation of harmonic wave

We can study the variation of harmonic progressive wave graphically. From the equation of harmonic wave [Equ. 1.7] we see that the equation contains three variables — (i) particle displacement, y (ii) particle location, x and (iii) time, t. Then the graph of y when x and t are both variables will be a three dimentional graph. A graph in two dimensions can be obtained, if one of the two independent variable (x and t) is kept constant and the other is varied.

Here we shall draw $(y - x)$ graph for different values of t. We can write the equation (1.7) in the following way.

$$y(x, t) = a \cdot \sin\frac{2\pi}{T}\left(t - \frac{x}{v}\right) = a \sin\frac{2\pi}{\lambda}(vt - x)$$

(i) At time $t = 0$, $y(x, 0) = a \sin \frac{2\pi}{T} \cdot \frac{x}{v} = -a \sin \frac{2\pi}{\lambda} x$

$x \to 0$	0	$\lambda/4$	$\lambda/2$	$3\lambda/4$	λ	$5\lambda/4$	$3\lambda/2$	$7\lambda/4$	2λ
$y(x, 0) \to$	0	$-a$	0	a	0	$-a$	0	a	0

The curvee (a) of the fig. 1.8 shows the plot of $y(x, 0)$ against x.

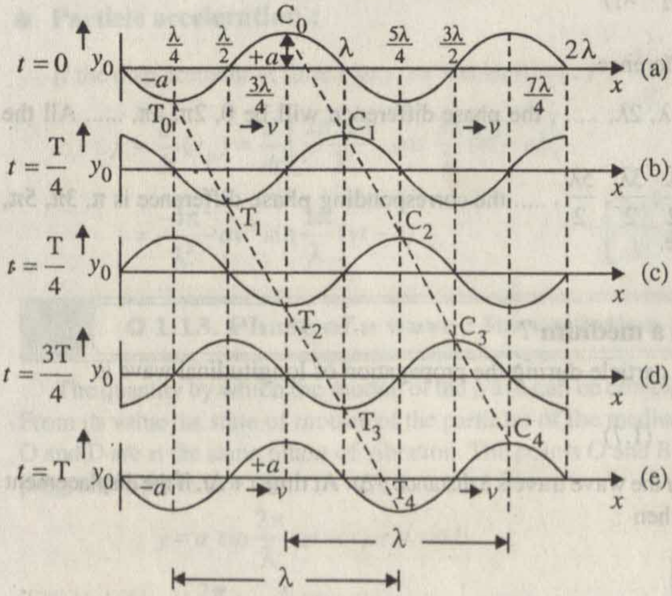


Fig. 1·8

(ii) When time $t = \frac{T}{4}$,

$$y\left(x, \frac{T}{4}\right) = a \sin\left(\frac{\pi}{2} - \frac{2\pi}{\lambda} x\right)$$

$$\therefore \quad y\left(x, \frac{T}{4}\right) = a \cos \frac{2\pi}{\lambda} x$$

Curve (b) of fig. 1.7 represents $y\left(x, \frac{T}{4}\right)$ against x

(iii) At time $t = \frac{T}{2}$,

$$y\left(x, \frac{T}{2}\right) = a \cdot \sin \frac{2\pi x}{\lambda}$$

The graph 1.8(c) shows the graph of $y\left(x, \frac{T}{2}\right)$ against x.

(iv) At $t = \frac{3T}{4}$, $y\left(x, \frac{3T}{4}\right) = -a \cdot \cos \frac{2\pi x}{\lambda}$

The graph of this equation is shown in fig. 1.8(d)

(v) When $t = T$, $y(x, T) = -a \cdot \sin \frac{2\pi x}{\lambda}$ The graph of this equation in shown in fig. 1.8(e).

Evidently the fig. 1.8 shows a progressive wave (whose amplitude is a, time period T and wavelength λ) which is generated at $x = 0$ and travels with a velocity v towards right.

The situation for longitudinal wave is quite different. For a longitudinal wave in a gas there is a crowding of particles in a compression when density is high and in a rarefaction it is low. The fig. 1.9(a) shows the picture of a longitudinal wave. Evidently it is not very convenient representation. We can represent a longitudinal wave by a graph which shows the variation of particle density as in fig. 1.9(b). The horizontal line represents the normal density. All points above this line represent density larger the normal. The points below represent less than normal density.

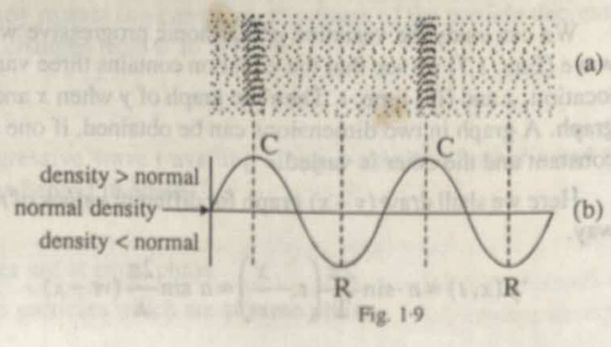


Fig. 1·9

1.15. The velocity of progressive wave in different media

(i) The velocity of transverse wave in a stretched string, $V = \sqrt{\frac{T}{m}}$

where T = tension in the string, m = mass per unit length of the string.

(ii) The velocity of longitudinal wave in a stretched string, $V = \sqrt{\frac{Y}{\rho}}$

Here Y = Young's modulus of the material of the wire, ρ = density of material.

(iii) The velocity of longitudinal wave in a solid, $V = \sqrt{\frac{Y}{\rho}}$

(iv) The velocity of longitudinal wave in a liquid or gas is $V = \sqrt{\frac{K}{\rho}}$

K = bulk modulus of the medium, ρ = density.

(v) The velocity of sound in gaseous medium, $V = \sqrt{\frac{\gamma P}{\rho}}$

P = pressure of the gas, γ = ratio of two specific heats of a gas.

(vi) The velocity of electromagnetic wave, $V = \frac{1}{\sqrt{\mu K}}$

When μ = permeability of the medium and K = permittivity of the medium.

●● Numerical Examples ●●

● **Example 1.1. The equation of a progressive wave is $y = 15\sin(660\,\pi t - 0{\cdot}02\,\pi x)$ cm. Find velocity of wave.** **[H. S. 2003]**

○ **Solution :** The general equation of a progressive wave is $y = a\cdot\sin\left(2\pi nt - \frac{2\pi}{\lambda}x\right)$

Comparing this equation with the given equation, we get $2\pi nt = 660\,\pi t$

$\therefore$ frequency of the given wave, $n = 330$ Hz.

and $\frac{2\pi}{\lambda}x = 0{\cdot}02\,\pi x$ $\quad\therefore$ Wavelength $\lambda = \frac{1}{0{\cdot}02} = 100$ Hz

$\therefore$ velocity of the wave, $v = n\lambda = 330\times100 = \mathbf{330\ m/s}$

● **Ex. 1.2. The frequency of a tuning force 400 Hz and velocity of sound in air is 320 m/s. Calculate the distance travelled by the sound wave during the time the fork completes 30 vibrations.** **[H. S. 1983]**

○ **Solution :** $v = 320$ m/s and frequency $n = 400$ Hz

$\therefore$ Wavelength of the wave $\lambda = \frac{320\times100}{400} = 80$ cm

$\therefore$ After 1 vibration, the sound travels a distance 80 cm

$\therefore$ after 30 vibrations the sound travels a distance $= 80\times30 = 2400$ cm $= \mathbf{24\ m}$.

● **Ex. 1.3. The frequency of a tuning fork 512 Hz. The sound travels 20 m in air during the time the force completes 30 vibrations. Calculate wave length and velocity of sound in air.**

○ **Solution :** In 1 sec. the fork completes 512 vibrations.

∴ To complete 30 vibration it takes $\frac{30}{512}=\frac{15}{256}$ sec .

Evidently, in time $\frac{15}{256}$ sec. sound travels 20 m the air

∴ Velocity of sound in air, $V=20\times\frac{256}{15}=\mathbf{341\cdot 33\ m}$

∴ Wavelength of sound in air, $\lambda=\frac{V}{n}=\frac{341\cdot 33}{512}=\mathbf{66\cdot 67\ cm}$

● **Ex. 1.4. A cork is floating on water in a pond. When wave travels through water surface the cork executes oscillatery motion in a vertical plane. Calculate the maximum velocity of the cork when the velocity of the wave is 0·2 m/s, wavelength of the wave 1·5 cm and amplitude of the wave 5 mm.**

○ **Solution :** When the wave passes through the water surface, the amplitude and frequency of the cork will be equal to the amplitude and frequency of the wave.

Now, wave velocity $v=n\lambda$

∴ $n=\frac{v}{\lambda}=\frac{0\cdot 2}{0\cdot 015}=13\cdot 33$ Hz [Here $v=0{\cdot}2$ m/s, $\lambda=0{\cdot}015$ m]

∴ Amplitude of vibration, $a=5$ mm $=0{\cdot}005$ m

∴ Maximum velocity of the cork, $V_{max}=a\omega=2\pi na$

∴ $V_{max}=2\pi\times 13\cdot 33\times 0\cdot 005=\mathbf{0\cdot 42\ ms^{-1}}$

● **Ex. 1.5. A spring is suspended from a support such that lower portion of the spring is immersed under water. The spring completes 120 vibrations per min. As a result a wave is created on the water surface. It is found that the distance between 11 consecutive crests is 20 cm. Calculate the velocity of the wave.**

○ **Solution :** Frequency of the wave is equal to frequency of vibration of the spring.

∴ frequency of the wave, $n=\frac{120}{60}=2$ Hz

The distance between two consecutive crests is wave length λ.
11 consecutive crests include 10 waves.

∴ lengths of 10 waves = 20 cm. $\therefore \lambda=\frac{20}{10}=2$ cm

∴ Velocity of the wave, $v=n\lambda=2\times 2=4$ cm / s

● **Ex. 1.6. When a bat flies over a lake it creates ultrasonics of frequency 100 KHz. When this sound falls on the water of the lake, a part is reflected and the rest is refracted. If the velocity of sound is air be 340 m/s and that in water 1450 m/s, calculate the wavelengths of reflected and refracted waves.**

○ **Solution :** The frequency of sound emitted by the bat is 10^5 Hz

Velocity of sound in air, $V_a=340\ ms^{-1}$ and in water $V_w=1450\ ms^{-1}$

As the reflected wave travels in air, the wavelength of reflected wave.

$$\lambda_a=\frac{V_a}{n}=\frac{340\times 100}{10^5}=\mathbf{0\cdot 34\ cm}$$

The refracted wave travels in water. So, the wavelength of refracted wave is

$$\lambda_w=\frac{1450\times 100}{10^5}=\mathbf{1\cdot 45\ cm}$$

● **Ex. 1.7. The equation of a progressive wave is $y = 5\sin 2\pi\left\{\frac{t}{0\cdot004}-\frac{x}{136}\right\}$ cm.**

Calculate : (i) Amplitude of the wave, (ii) Wavelength, (iii) frequency, (iv) velocity of the wave and (v) maximum particle velocity.

○ **Solution :** General equation of a progressive wave, $y = a\cdot\sin 2\pi\left(\frac{t}{T}-\frac{x}{\lambda}\right)$ comparing this equation with given equation we get,

(i) Amplitude of the wave, $a = 5$ cm. (ii) Wavelength $\lambda = 136$ cm.

(iii) Time period = 0·004 sec. $\therefore$ frequency $n = \frac{1}{0\cdot004} = 250\,\text{Hz}$

(iv) Wave velocity, $v = n\lambda = 250\times136 = 340\cdot0\ \text{ms}^{-1}$

(v) Particle velocity at any instant is $v_P = \frac{dy}{dt} = 5\times\frac{2\pi}{0\cdot004}\cos 2\pi\left\{\frac{t}{0\cdot004}-\frac{x}{136}\right\}$

$\therefore$ maximum particle velocity, $V_P = \frac{10\pi}{0\cdot004} = 7850\ \text{cms}^{-1} = \mathbf{78\cdot5\ ms^{-1}}$

● **Ex. 1.8. The bulk modulus and density of steel are 80 times and 8 times those of water. Calculate the velocity of sound in steel. Given the velocity of sound in water = 1493 m/s. [J.E.E. 1995]**

○ **Solution :** Velocity of sound in elastic medium, $V = \sqrt{\frac{E}{\rho}}$

Here, E = modulus of elasticity of the medium and ρ = density of the medium

$\therefore$ Velocity of sound in water, $V_w = \sqrt{\frac{E_w}{\rho_w}}$ and in steel, $V_s = \sqrt{\frac{E_s}{\rho_s}}$

$$\therefore\quad \frac{V_s}{V_w} = \sqrt{\frac{E_s}{E_w}\times\frac{\rho_w}{\rho_s}} = \sqrt{80\times\frac{1}{8}} = \sqrt{10} = 3\cdot16 \quad \therefore\quad V_s = 3\cdot16\times1493 = \mathbf{4721\ ms^{-1}}$$

● **Ex. 1.9. The equation of progressive wave travelling along X-axis on the string is $y = 3\cdot\sin(314\lambda - 314t)$ cm. Find :**

(a) Find the maximum velocity of a particle of the string.

(b) Find the acceleration of a particle at $x = 6\cdot0$ cm at time $t = 0\cdot11$ sec.

○ **Solution :** Here $y = -3\cdot\sin(314t - 3\cdot14x)$ cm

(a) The velocity of the particle at x at time t is

$$v_p = \frac{dy}{dt} = -3\times314\cdot\cos(314t-3\cdot14x) = -942\cos(314t-3\cdot14x)$$

$\therefore$ Maximum particle velocity, $v_p = 942$ cm/s $= 9\cdot42$ m/s

(b) The acceleration of the particle at x at time t is

$$a = 942\times314\cdot\sin(314t-3\cdot14x)$$

The acceleration of the particle at $x = 6\cdot0$ cm and at time $t = 0\cdot11$ sec. is

$$a = 942\times314\cdot\sin(11\pi-6\pi) = 0.\quad [\pi = 3\cdot14]$$

$\therefore$ Acceleration at $x = 6$ cm and $t = 0\cdot11$ sec. is zero.

- **Ex. 1.10. The equation of progressive wave $y = 0\cdot1\sin\left(400\,\pi - 1\cdot25\pi x\right)$**
Calculate (i) frequency (ii) wavelength (iii) velocity (iv) phase difference between two points located at $x = 0\cdot2$ m and $x = 1\cdot0$ m. Here y is expressed in mm unit and x is in m unit.

○ **Solution :** The general equation of a progressive wave is $y = a\cdot\sin\,2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$ comparing this equation with the given equation we get

(i) $\frac{2\pi}{T} = 400\,\pi$ $\therefore$ frequency $n = \frac{1}{T} = \mathbf{200\ Hz}$

(ii) $\frac{2\pi}{\lambda} = 1\cdot25\,\pi$ $\therefore$ wavelength $\lambda = \frac{2}{1\cdot25} = \mathbf{1\cdot6\ m}$

(iii) wave velocity, $v = n\lambda = 200\times1\cdot6 = 320\ \text{m/s}$

(iv) phase, $\phi = 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$ $\therefore$ $\Delta\phi = -\frac{2\pi}{\lambda}\Delta x$

Here $\Delta x = 1\cdot0 - 0\cdot2 = 0\cdot8$ m

$\therefore$ Phase difference between the two points is $\Delta\phi = -\frac{2\pi}{1\cdot6}\times0\cdot8 = -\pi$.

- **Ex. 1.11. When a progressive wave travels through a medium, the equation of motion of a particle is $y(x, t) = 0\cdot01\sin\,2\pi(2t - 0\cdot01x)$; Here y and x are in metres and t is in sec. Calculate : (i) frequency and velocity of the wave (ii) Phase change at a point during a time interval of 0·25 sec (iii) the phase difference at a given time between two particles at a separation of 50 m.**

○ **Solution :** The general equation of a progressive wave is $y\,(x, t) = a\,\sin\frac{2\pi}{\lambda}(vt - x)$

(i) Comparing this equation with the given equation we get wavelength of the wave, $\lambda = 100$ m, wave velocity, $v = 200$ m/s.

$\therefore$ frequency of the wave, $n = \frac{v}{\lambda} = \frac{200}{100} = 2$ Hz

(ii) Phase difference of a particle in time interval Δt is

$$\Delta\phi = \frac{2\pi}{T}\Delta t = 2\pi n\cdot\Delta t = 2\pi\times2\times0\cdot25 = \pi$$

(iii) The phase difference between two particles at a separation of 50 m at a particular time is

$$\Delta\phi = -\frac{2\pi}{\lambda}\Delta x = -\frac{2\pi}{100}\times50 = -\pi.$$

- **Ex. 1.12. The equation of a progressive wave is $y = 2\cdot0\cos2\pi\left(10t - 0\cdot008x\right)$. Here x and y are expressed in cm and t is in sec. Calculate the difference of phase between two points which are at a separation of (i) 0·4 m and (ii) $\frac{3\lambda}{4}$.**

○ **Solution :** The phase difference, $|\Delta\phi| = \frac{2\pi}{\lambda}\Delta x$

Here $\frac{2\pi}{\lambda} = 2\pi\times0\cdot008$ $\therefore$ $\lambda = \frac{1}{0\cdot008} = 125\ \text{cm} = \mathbf{1\cdot25\ m}$

(i) $\Delta x = 0\cdot4\,\text{m}$ then $\Delta\phi = \frac{2\pi}{1\cdot25}\times0\cdot4 = \mathbf{0\cdot64\pi\ rad}$

(ii) $\Delta x = \frac{3\lambda}{4}$ $\therefore \Delta\phi = \frac{2\pi}{\lambda}\cdot\frac{3\lambda}{4} = \frac{3\pi}{2}$ rad.

● **Ex. 1.13. One end ($x = 0$) of long horizontal wire is producing is progressive wave by a tuning fork of frequency 500 Hz. The amplitude of the wave 0·01 m. At any instant the displacement of the particle located at $x = 0{\cdot}1$ m is – 0·005 m and the displacement of the particle located at $x = 0{\cdot}2$ m is + 0·005 m. Calculate wavelength and wave velocity.**

If the particle moves along negative x-axis obtain the equation of the progressive wave. Assume that at $t = 0$, the end of the string is in equilibrium at $x = 0$.

○ **Solution :** Let the equation of the progressive wave be $y = a\sin\frac{2\pi}{\lambda}(vt - x)$

$$\therefore \quad -0{\cdot}005 = 0{\cdot}01\sin\frac{2\pi}{\lambda}(vt - x_1)\ \ [x_1 = 0{\cdot}1\text{m}]$$

$$\therefore \quad \sin\frac{2\pi}{\lambda}(vt - x_1) = -\frac{1}{2}$$

$$\therefore \quad \text{Phase } \phi_1 = \frac{2\pi}{\lambda}(vt - x_1) = \frac{7\pi}{6} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(i)}$$

$$\text{Again, } 0{\cdot}005 = 0{\cdot}01\sin\frac{2\pi}{\lambda}(vt - x_2) \quad \therefore \sin\frac{2\pi}{\lambda}(vt - x_2) = \frac{1}{2}$$

$$\therefore \quad \phi_2 = \frac{2\pi}{\lambda}(vt - x_2) = \frac{\pi}{6} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(ii)}$$

$$\therefore \quad \text{Phase difference } \Delta\phi = \phi_1 - \phi_2 = \frac{7\pi}{6} - \frac{\pi}{6} = \pi$$

$$\text{Now, } \Delta\phi = -\frac{2\pi}{\lambda}\Delta x \quad \text{or, } \pi = -\frac{2\pi}{\lambda}\cdot(x_1 - x_2)$$

$$\pi = -\frac{2\pi}{\lambda}(0{\cdot}1 - 0{\cdot}2) \quad \text{or, } \boldsymbol{\lambda = 0{\cdot}2\ \text{m}}$$

frequency of the wave, $n = 500$ Hz

∴ Velocity of the wave, $v = n\lambda = 500 \times 0{\cdot}2 = 100\ \text{ms}^{-1}$

$$\therefore \quad \text{The equation of the wave, } y = 0{\cdot}01\sin\frac{2\pi}{0{\cdot}2}(100t - x) = 0{\cdot}01\sin 10\pi(100t - x)$$

Here y and x are in metre unit.

● **Ex. 1.14. The equation of simple harmonic motion of a source is $y = 6\sin 300\,\pi t$ cm. Calculate displacement, velocity and acceleration of a particle at a distance of 150 cm from the source at time $t = 0{\cdot}01$ sec after the source starts vibrating ; velocity of wave = 300 ms^{-1}.**

○ **Solution :** The equation of vibration of the wave at any time t is $y = 6\sin 300\,\pi t$.

The equation of displacement of a particle at the same time at a distance 150 cm from the source is

$$y = 6{\cdot}0\sin\left(300\pi t - \frac{2\pi}{\lambda}x\right) \quad [\lambda = \text{wavelength}]$$

At this instant, the equation of velocity of the particle is

$$v = \frac{dy}{dt} = 6 \times 300\,\pi\cdot\cos\left(300\,\pi t - \frac{2\pi}{\lambda}x\right)$$

$$\text{and equation of acceleration, } f = \frac{dv}{dt} = -6(300\pi)^2\sin\left(300\,\pi t - \frac{2\pi}{\lambda}x\right)$$

$$\text{Now, } \omega = 300\pi \quad \therefore \quad n = 150\ \text{Hz and } \lambda = \frac{\text{V}}{n} = \frac{30000}{150} = 200\ \text{cm}$$

Hence the displacement of the particle at time $t = 0{\cdot}01$ sec at a distance of 150 cm from the source is

$$y = 6\sin\left(300\pi t - \frac{2\pi}{\lambda}x\right) = 6\cdot\sin\left(300\pi\times{\cdot}01 - \frac{2\pi}{200}\times 150\right)$$

$$= 6\sin(3\pi - 1{\cdot}5\pi) = 6\sin\frac{3\pi}{2} \quad \therefore y = -6 \text{ cm}$$

velocity at this instant, $v = 1800\cos\frac{3\pi}{2} = 0$

and acceleration, $f = -6(300\pi)^2\cdot\sin\frac{3\pi}{2} = \mathbf{-5{\cdot}33\times10^6\ cm/s^2}$

- **Ex. 1.15. The equation of a progressive wave is $y = 2{\cdot}5\sin\left(400t - 0{\cdot}68x + \frac{\pi}{2}\right)$. Calculate : (i) the phase of the particle at $x = 0$ at time $t = 0$, (ii) phase difference between two points lying along x-axis at a separation of 20 cm, (iii) change of phase during time interval 4×10^{-4} sec at any point.**

○ **Solution :** Phase of the wave, $\phi = 400t - 0{\cdot}68x + \frac{\pi}{2}$

(i) At the point $x = 0$ at time $t = 0$, $\phi = \frac{\pi}{2}$

(ii) Phase difference in distance, $\Delta\phi = -0{\cdot}68(x_2 - x_1) = -0{\cdot}68\times 20 = -13{\cdot}6$ rad

Phase change in time $\Delta t = 400\cdot\Delta t = 400\times4\times10^{-4} = 0{\cdot}16$ rad .

1.16. Some common properties of progressive waves

Progressive waves (transverse or longitudinal) has some common properties. These are :

- (i) **Reflection :** A progressive wave while travelling through a medium is incident on a second medium, a part of the wave returns to the first medium. This is known as reflection of wave. The energy of the reflected wave depends on the nature of the surface of separation.
- (ii) **Refraction :** When a progressive wave falls on the boundary of two media, a part of the incident wave enters into a second medium. It then moves in a different direction and with different velocity. This change of direction of wave is called refraction.
- (iii) **Interference :** When two progressive waves of equal frequency and having a constant phase relation cross each other, they may at some point reinforce each other, while at other point they many destroy each other. The two waves are then said to interfere.
- (iv) **Diffraction :** When a wave passes over an obstacle, its direction of propagation changes. It is found to bend round corners. This phenomenon of bending of light round corners is called diffraction.
- (v) **Polarisation :** We know that during the propagation of progressive wave, the particles of the medium vibrate at right angle to the direction of propagation of the wave. Usually the plane of vibration may be any plane. If the vibration of the particles is confined in one plane only then the wave is said to be **polarised.**

In the case of longitudinal waves, the particles of the medium vibrate parallel to the direction of propagation. So such wave can not be polarised.

Hence transverse wave can only be polarised. For example, light waves being transverse, can be polarised.

1.17. Phase change due to reflection

In some cases, a change of phase takes place during reflection of light. If the surface of the reflector be rigid, the phase of the reflected wave will be opposite i.e. the phase difference between the reflected and incident wave will be 180°. On the other hand, if the wave falls on the flexible boundary or if the density of the second medium be less than the density of the first medium, there will no phase change due to reflection.

If the compression of the longitudinal wave is reflected as rarefaction then the phase change is 180°. Also in the case of transverse wave the crest is reflected as trough.

✪ 1.18. Reflection of waves ✪

When a progressive wave (transverse or longitudinal) falls at the separation of two homogeneous media, (i) a part of the wave returns to the first medium as reflected wave and (ii) the rest part enters the second medium and travels in different direction with different velocity. It is called refraction. It is to be noted that the velocity and wavelength of the reflected wave remain unchanged. But both of velocity and wavelength of refracted wave change. However, in both the cases the frequency of the wave remain constant.

To discuss the phenomenon of reflection of wave, we make use of concept of rays. In fig. 1.10, a wave travelling along AO in incident obliquely on the reflecting surface PQ. AO is called the **incident ray**. After reflection the wave travels along OB. So, OB is called **reflected ray**. ON is the normal at O on PQ. $\angle AON = i$ is the angle of incidence and $\angle BON = r$ is the angle of reflection.

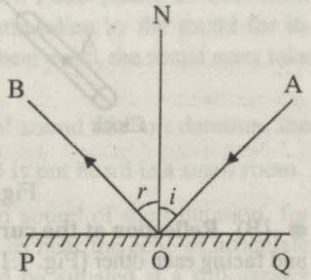


Fig. 1·10

- **Laws of reflection :** The reflected waves obey the following two laws of reflection.

(i) The incident ray, reflected ray and the normal at the point of incidence all lie in one plane.
(ii) The angle of incidence is equal to the angle of reflection i..e. $\angle i = \angle r$

➧ More informations about the reflected ray :

(i) Due to reflection, wavelength of the wave and velocity of the wave remain unchanged.
(ii) If the reflector be rigid and fixed the reflected wave will be of opposite phase with respect to the incident wave i.e. the reflected wave will be 180° out of phase. So, in the case of longitudinal wave, the compression will be reflected as rarefaction. Similarly in the case of transverse wave, the crest is reflected as trough.
(iii) If the second medium is rarer than the first medium, there will no phase change on reflection. So, now the compression of the longitudinal wave will be reflected as compression and the crest of the transverse wave will be reflected as crest.

✪ 1.19. Reflection of sound wave ✪

Sound travels through a medium in the form of a wave. So, similar to light wave sound wave can also be reflected. Now, wave length of sound wave is much longer than wavelength of light. Wavelengths of audiable sound ranges from 1·5 cm to 16 m. But wavelengths of visible light wave lie within the ranges from 4000 A° to 8000 A°. For this reason, for the refleciton of sound wave much larger surface is required than for reflection of light wave. Also very smooth surface is required for regular reflection of light wave than the case of sound wave.

➡ Demonstration experiment :

- **(A) Reflection at a plane surface :** A plane reflector M is placed vertically on a harizontal table [Fig. 1.11]. Two hollow tubes A and B are placed horizontally in front of the reflector. The axis of the tubes meet at O on the reflector. The tube B can be turned horizontally about the point O. A wooden screens in placed between the two tubes.

A source of sound (say, a table clock) is placed at the mouth of A and the ear is placed at the mouth of B to hear the ticking sound of the watch. The tube is turned slowly until the sound in heard loudly and clearly. Due to the presence of the screen the sound of the clock is not heard directly. The sound is reflected at O and reaches the ear of the observer. It is found that the sound is not heard if the tube is slightly displaced or slightly raised from the table.

It means that the sound waves obey the law of reflection. It is found by actual measurement that the angle-which the incident ray makes with the normal at O on the reflector is equal to the angle which the reflected ray makes with the normal.

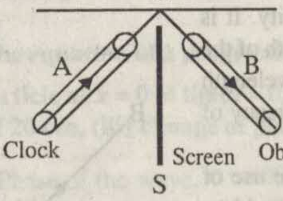


Fig. 1·11

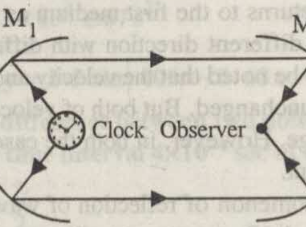


Fig. 1·12

- **(B) Reflection at the curved surface :** Two large concave reflectors M_1 and M_2 are placed co-axially and facing each other [Fig. 1.12]. A source of sound (say, a watch) is placed at the focus of the reflector M_1. The ticking sound of the watch after reflection by both the mirrors is clearly heard when the ear is placed at the focus of the reflector M_2. If the position of either of the reflector or the ear is shifted the sound is not heard.

- **Explanation :** As the watch is placed at the focus of the reflector M_1, the sound waves from the watch after reflection by M_1 will proceed as a parallel beam towards M_2. After reflection by M_2 the sound waves will converge towards the focus of M_2. So the ticking sound is heard distinctly at the focus of M_2.

❂ 1.20. Practical application of reflection of sound ❂

- **(i) Speaking tube :** The speaking tube is actually a metal tube [Fig. 1.13]. The action of the tube depends on multiple reflection of sound. When words are spoken at one end, it is heard clearly at the other end. The sound reaches from one end to the other end by multiple reflection. Evidently, the energy of the wave remains confined within the tube.

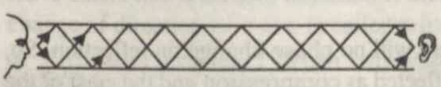


Fig. 1·13

- **(ii)** The stethoscope used by doctors to examine the chest of a patient also works on the principle of multiple reflection.

- **(iii)** When we speak in a room, due to multiple reflection at the walls and floor and roof, the intensity of sound increases.

Similarly when one speaks in a large hall, sound is heard for a longer time due to multiple reflection. It is known as reverberation.

❂ 1.21. Echo ❂

The phenomenon of echo is a familar example of reflection of sound by an obstacle. When sound returns back after reflection from an obstacle and is heard distinctly as repetition of the original sound, it is called **echo.** So echo is the reflected sound.

Distant extended surface such as wall, tall building, cliffs, hillsides, a row of close trees etc. can act as a reflector for the production of echoes. For hearing echoes the reflector must be at a certain minimum distance.

Minimum distance of the reflector :

Reflected sound forms echo. But all reflected sound can not be heard as echo. There must be a minimum time interval between the hearing of original sound and reaching the reflected sound from the reflector to the ear.

- **(i) Sound of short duration :** When a sound is heard, its impression persists in our hearing mechanism for about 0·1 sec. This time intervals is called persistence of hearning. Within this time interval if similar sound enters into our ears we can not hear it separately. Hence to hear the echo of a sound distinctly, the time interval between the original sound and the reflected sound must be at least 0·1 sec. Hence the minimum distance between the observer and the reflector should be such that the time taken by the sound for its journey to the reflector and back to the observer is at least 0.1 sec. Hence to hear echo, the sound must take at least 1/20 sec. to go from observer to the reflector.

Now, the velocity of sound in air is 340 m/s. Hence to hear the echo of sound of short duration, the minimum distance of the reflector is $\frac{1}{20} \times 340 = 17$ m. For this reason, echo is not heard in a small room.

- **(ii) Articulate sound :** In our previous discussion we have considered sound of short duration, for example : sound of clapping, sound of cracker, sound from a gun etc. But to hear the echo of articulate sound i.e. the sound of conversation of human beings, the reflector should be situated at a still greater distance to hear its echo. It is so because a person can not pronounce more than 5 syllables distinctly in one second, and ear can not recognise them if more than 5 syllables are pronounced in one second. If a person utters a single syllable (say, a) he will be able to hear the echo distinctly, if the reflected sound returns after 1/5 sec. So, the reflector must be situated at the minimum distance of 34 m (because in 1/10 sec. sound travels 34 m]. Similarly to hear the echo of disyllabic sound (say, ab) or to hear the echo of tri-syllabic sound (say, abc) the minimum distance of the reflector should be 68 m or 102 m.

- **Multiple echo :** Let two reflectors are situated at a separation at least 34 metres. If an observer remains standing at a point between the reflectors, he will be able to hear a series of echoes. Here sound reflected by one reflector is again reflected by other reflecter and so on. Due to multiple reflection of a single sound series of echoes are heard. However, due to energy absorption at each reflection, the successive echoes will be more and more feeble.

✪ 1.22. Practical application of echo ✪

- **(i) Determination of distance :** The distance of an object (e.g. a hill) situated at a large distance can be determined by the echo method. Obviously the distance should be greater than the minimum distance to hear echo.

The observer makes a sharp sound (say, by firing a pistol) and immediately he starts a stop watch. On hearing the echo from the distant hill, he stops the watch.

Let the time interval between the original sound and the reflected sound be *t* and velocity of sound in air be V. Then the distance of the hill is

$$x = \frac{1}{2} \mathrm{V}t \ldots\ldots\ldots\ldots\ldots\ldots (i)$$

- **Determination of velocity of sound :** We can also determine the velocity of sound by the equation (i).

Here $\mathrm{V} = \frac{2x}{t}$

So, if the distance of the hill or reflector is known, the velocity of sound in air can be determined fairly accurately.

- **(ii) Echo depth sounding :** Echo has been used to measure the depth of a sea.

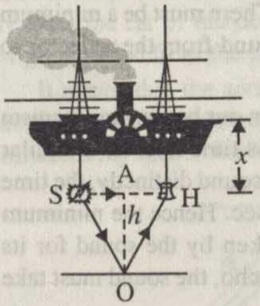


Fig. 1·14

To measure the depth of a sea, a hydrophone H is kept under water at the end of a ship [Fig. 1.14]. A small amount of explosive is kept at S under water at the same level from the other end of the ship. After the explosion the hydrophone receives two sounds, because there are two possible ways of the sound reaching H from S. One is direct from S to H and the other is from S to O at the bed of the sea and then to H, by reflection.

Let after explosion at S, the hydrophone receives two sounds after the times t_1 and t_2. The time t_1 and t_2 are measured electrically. Let v be the velocity of sound in water.

$$\mathrm{SH} = vt_1 \text{ and } \mathrm{SO} + \mathrm{OH} = vt_2 \quad \text{or, } \mathrm{SO} = \frac{1}{2}vt_2 \;[\because \mathrm{SO} = \mathrm{OH}]$$

$$\therefore \quad h = \mathrm{OA} = \sqrt{(\mathrm{SO})^2 - (\mathrm{SA})^2} = \frac{v}{2}\sqrt{t_1^2 - t_1^2} \left[\because \ \mathrm{SA} = \frac{1}{2}\mathrm{SH}\right]$$

If the explosive or the hydrophone be at a depth x, then the depth of the ocean is

$$\mathrm{D} = x + h = x + \frac{v}{2}\sqrt{t_1^2 - t_1^2}$$

- **(iii) Determination of height of an aeroplane from the ground :**

The height of a flying aeroplane can also be determined by the echo method.

Let an aeroplane is flying horizontally at a height h from the ground. It is moving along the path AB with a velocity v. When the plane is at A, a sound is produced and its echo is received at B after a time t. [Fig. 1.15]

Evidently, the time taken by the plane to travel ACB is the same as the sound takes to travel the path AOB.

$$\therefore \quad \mathrm{AO} + \mathrm{OB} = \mathrm{V}t \quad \therefore \quad \mathrm{AO} = \mathrm{OB} = \frac{1}{2}\mathrm{V}t \quad [\mathrm{V} = \text{velocity of sound in air}]$$

$$\text{Again } \mathrm{AB} = vt \quad \therefore \quad \mathrm{AC} = \mathrm{CB} = \frac{1}{2}\mathrm{AB} = \frac{1}{2}vt \quad [v = \text{velocity of the plane}]$$

$$\therefore \quad h = \mathrm{OC} = \sqrt{\mathrm{AO}^2 - \mathrm{AC}^2} = \frac{1}{2}\sqrt{\mathrm{V}^2 - v^2}$$

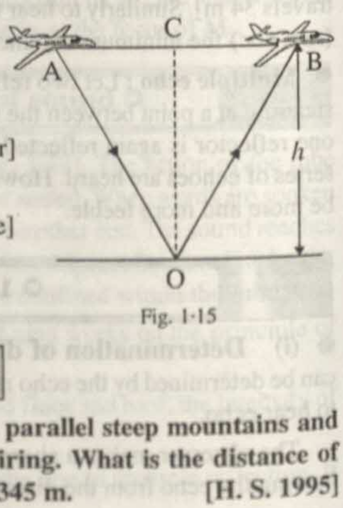


Fig. 1·15

●● Numerical Examples ●●

- **Example 1.16. A person fires a gun from a position between two parallel steep mountains and successively hears two echos one 1·5 sec and the other 2·5 sec after firing. What is the distance of separation between two mountains ? Given, velocity of sound in air = 345 m.** **[H. S. 1995]**

○ **Solution :** Let the distance of one mountain from the person be x metre and the distance of the other mountain from him = y metre.

The man hears the first echo after 1·5 sec.

$$\therefore \quad \frac{2x}{\mathrm{V}} = 1{\cdot}5 \qquad \therefore \quad x = \frac{3\mathrm{V}}{4} = \frac{3\times345}{4} = \mathbf{258{\cdot}75\ m}$$

The sound is also reflected from the second mountain. The second echo is heard after 2·5 sec.

$$\therefore \quad \frac{2y}{\mathrm{V}} = \frac{5}{2} \qquad \therefore \quad y = \frac{5\mathrm{V}}{4} = \frac{3\times345}{4} = \mathbf{431{\cdot}25\ m}$$

∴ The distance between the two mountains = 258·75 + 451·25 = **690 m.**

● **Ex. 1.17. A person while running towards a cliff with a speed of 4 m/s, fires a gun when he is at a distance 2·4 km from the cliff. When and where he will hear the echo, if the velocity of sound be 330 m/s.**

○ **Solution :** Let the person fires the gun at A. So AO = 2400 m.

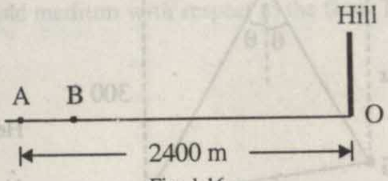


Fig. 1·16

Let the person hears the echo after t sec. at B.

$\therefore \quad AB = vt = 4t$ m $\quad \therefore \;\; BO = (2400 - vt)$

If the velocity of sound be V ms^{-1}

$2400 + 2400 - vt = Vt$

or, $\quad 4800 - 4t = 330t \quad [v = 4\ ms^{-1}, V = 330\ ms^{-1}]$

$\therefore \; t = \dfrac{4800}{334} = 14\cdot37$ sec and $BO = 2400 - 4\times14\cdot37 = \mathbf{2342\cdot5\ m}$

● **Ex. 1.18. A sharp sound is made in front of hill and echo is heard after 3·2 sec. On approaching further by 66 m, another sharp sound is made and echo is heard after 2·8 sec. Calculate velocity of sound and distance of the hill.**

○ **Solution :** Let velocity of sound = V ms^{-1} and distance of the hill = d metre.

In the first case, echo is heard after 3·2 sec.

$2d = 3{\cdot}2\ V$ .... (i)

in the second case, distance of the hill = $(d - 66)$ m

$\therefore \;\; 2(d - 66) = 2{\cdot}8V$ .... (ii)

$\therefore \;\; \dfrac{d-66}{d} = \dfrac{2\cdot8}{3\cdot2} = \dfrac{7}{8} \qquad \therefore \; d = 528$

$\therefore$ distance of the hill = 528 m and velocity of sound, $V = \dfrac{2\times528}{3\cdot2} = \mathbf{330\ ms^{-1}}$

● **Ex. 1.19. A road runs midway between two long parallel rows of buildings. A man in a car moving with a velocity of 36 km/hr sounds the horn. He hears the echo 1 sec after he horns. Find the distance between the two rows of building. When will he hear the second sound ? Velocity of sound = 330 ms^{-1}.**

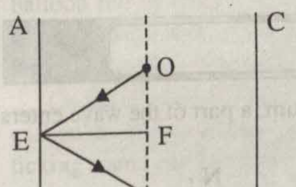


Fig. 1·17

○ **Solution :** In fig. 1.17 AB and CD are the two rows of buildings. The man gives the horn at O. After 1 sec, when the car is at O_1 the echo is heard. Speed of the car is 10 ms^{-1}.

$\therefore \quad OO_1 = 10$ m and $OF = 5$ m

Also OE = distance travelled by sound in 1/2 sec. = 165 m.

$\therefore \quad EF = \sqrt{OE^2 - OF^2} = \sqrt{(165)^2 - 5^2} = \sqrt{27200} = 164\cdot9$

$\therefore$ Distance between the two rows of building = $2\times164\cdot9 = 329\cdot8$ m

The second echo will be heard 2 sec. after the man sounds the horn.

● **Ex. 1.20. A gun is fired on a seashore in front of a line of cliffs. A man standing 300 ft away from the gun and equidistant from the cliffs notices that the echo takes twice as long to reach him as does the direct report. Find the distance of the gun from the cliffs. [J.E.E. 1999]**

○ **Solution :** In fig. 1.18, AB is the row of cliffs. G and M are respectively the position of gun and the man. So, GM = BM = 300 ft. The reflected sound takes double the time as required by the direct sound to reach the man from the gun.

$\therefore \quad GO + OM = 2 \cdot GM = 600$ ft

From the triangle GOM,

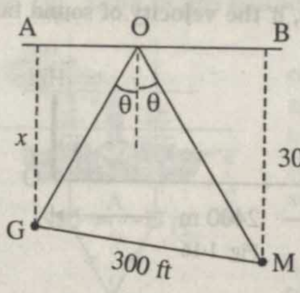


Fig. 1·18

$$GM^2 = GO^2 + OM^2 - 2\cdot GO\cdot MO\cdot \cos 2\theta$$

$$\therefore \quad GM^2 = (GO+OM)^2 - 2\cdot GO\cdot OM\cdot(1+\cos 2\theta)$$

$$= (GO+OM)^2 - 4\cdot GO\cdot OM\cdot \cos^2\theta$$

$$= (GO+OM)^2 - 4(GO\cos\theta)\,(OM\cdot\cos\theta)$$

Here $GO\cdot\cos\theta = x$ and $OM\cdot\cos\theta = 300$

$$\therefore \quad GM^2 = (600)^2 - 4x\times 30$$

$$\therefore \quad GM^2 = (600)^2 - 4\cdot x\cdot 300$$

$$\therefore \quad (300)^2 = (600)^2 - 1200x \quad \therefore\ x = 225\,ft.$$

- **Ex. 1.21. A man is beating a drum at a certain distance from a hill. He finds that if the beating rate be 40 per minute echo cannot be heard distinctly. He moves towards the hill by a distance of 90 m. Now echo cannot be heard when the beating rate is 60 per minute. Calculate (i) initial distance of the man from the hill and (ii) velocity of sound.** **[I.I.T.]**

○ **Solution :** If the sound of beating the drum coincides with the reflected sound, then the echo will not be heard.

Let initial distance of the man in the first case from the hill = x metre and velocity of sound = V m/s

So, in the first case, time interval between two successive sound is $t_1 = \dfrac{60}{40} = 1{\cdot}5\,\text{sec}$

when the echo is not heard clearly, $2x = 1{\cdot}5\ V$ . . . . (i)

In the second case, the time interval between two successive sound is $t_2 = \dfrac{60}{60} = 1$ sec .

$\therefore \quad 2(x-90) = V\times 1 = V$ . . . . (ii)

from equation (i) and (ii), $\dfrac{2(x-90)}{2x} = \dfrac{2}{3}$ or, $x =$ **270 m**

From equation (i), $2\times 270 = \dfrac{3}{2}V \quad \therefore$ **V = 360 m / s**

✪ 1.23. Refraction of waves ✪

When a progressive wave falls on the interface of two homogeneous medium, a part of the wave enters into the second medium when its velocity and wavelength change. If the wave is incident obliquely, its direction of motion also changes in the second medium. This phenomenon is called refraction. However, the frequency of the wave does not change due to refraction.

In fig. 1.19, PQ represents the surface of separation of the two media. The ray AO is the direction of the incident wave. It is incident at O on the refracting surface PQ at angle of incidence i.

The ray OB gives the direction of refracted wave where the angle of refraction is r. So, OB is the refracted ray.

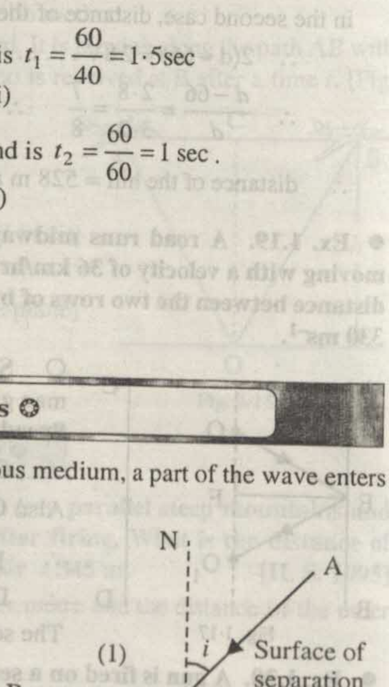


Fig. 1·19

- **Laws of refraction :** The phenomenon of refraction of wave obeys the following two laws.

(i) The incident ray, the refracted ray and the normal at the point of incidence lie on the same plane.

(ii) The ratio of the sine of angle of incidence and the sine of the angle of refraction is a constant. So, $\frac{\sin i}{\sin r} = \mu$ (a constant) μ is called the refractive index of the second medium with respect to the first. The second law is also known as Snell's law of refraction.

$$\text{Again, } \mu = \frac{\text{velocity of the wave in the first medium}}{\text{velocity of the wave in the second medium}} = \frac{v_1}{v_2}$$

$$\therefore \quad \frac{\sin i}{\sin r} = \frac{v_1}{v_2}$$

(a) If $v_1 > v_2$ i.e. if the velocity of the wave in the first medium be greater than that in the second medium, then $\frac{\sin i}{\sin r} > 1$ or, $i > r$

So, the refracted ray will bent towards the normal.

(b) If $v_2 > v_1$ i.e. if the velocity of the wave in the second medium is larger than that in the first medium then,

$\sin i < \sin r$ or, $i < r$

So now the ray goes away from the normal.

1.24. Refraction of sound

In a homogeneous medium sound travels with constant velocity. But the velocity of sound is different in different medium. When sound wave falls at the interface of two media its direction of motion in general changes in the second medium due to refraction. The refraction of sound wave can be demonstrated by the following experiment.

We can show the phenomenon of refraction of sound by the following method. Here we show the refraction of sound wave by a balloon full of carbon-di-oxide gas. Behaviour of balloon full of carbon-di-oxide placed in air is similar to that of convex lens of glass in air. It is due to the fact that density of carbon-di-oxide of greater than air.

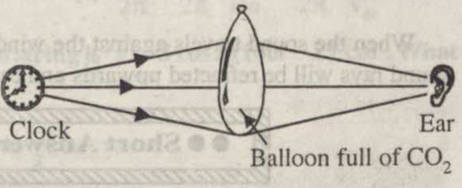


Fig. 1·20

On the left side of the balloon a table clock is placed. Sound waves emitted by the clock are refracted by carbon-di-oxide and converge to a point on the left (Fig. 1.20). If ear is placed at the point of convergence ticking sound can be heard.

Refraction of sound, however, has got very little important applications in our daily life.

Atmospheric refraction of sound :

When sound wave passes through a medium, it sametimes gets refracted under different conditions. When temperature of the atmosphere changes with altitude, sound waves are refracted in air. Similarly due to variation of density of air layers, the sound waves will be refracted.

● (i) **Effect of temperature variation :** The velocity of sound in a medium depends on density of the medium. Lower the density higher is the velocity of sound in the medium. During day time, the lower layers of air are at heigher temperature than those at higher levels. Hence density of air gradually increases with height. Consequently, sound wave travels faster near the ground than in upper layers. So, the sound waves will be refracted upwards i.e. the sound rays will be directed away fromt the ground [Fig. 1.21(a)]. Similarly during night when the upper atmosphere is hotter the sound rays will be reflected downwards i.e. towards the ground [Fig. 1.21(b)].

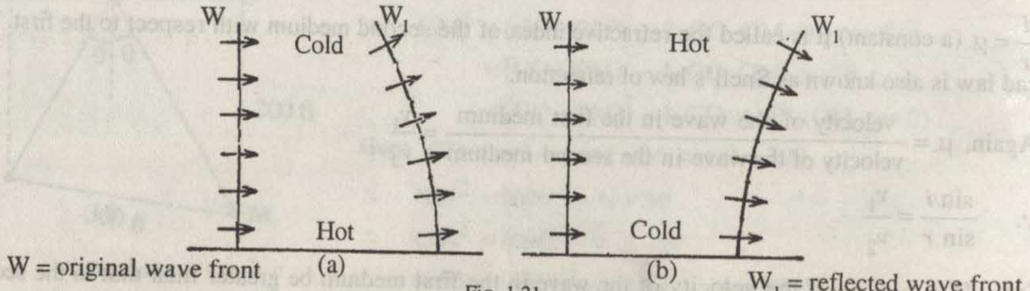


Fig. 1.21

So, at night sound from a longer distance will be heard much more clearly than during day time.

- **(ii) Effect of wind blow :** The velocity of wind in upper layers of air is greater than that at the bottom layers. The wave front of the sound waves will be bent towards the direction of the wind. So, when the sound propagates in the direction of wind, the wave front bends downwards as shown in fig. 1.22(a). Hence there will be concentration of sound waves near the surface.

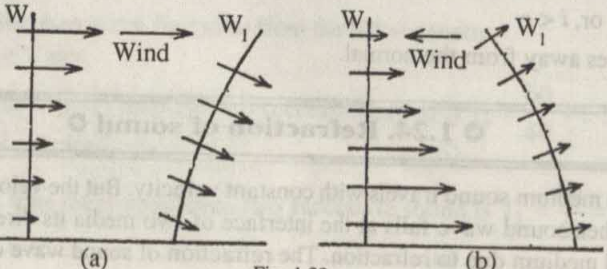


Fig. 1.22

When the sound travels against the wind, the state of affairs will be as shown in fig. 1.22(b). Here the sound rays will be refracted upwards and cannot reach sufficient distance on the earth's surface.

●● Short Answer Type Questions (with answers) ●●

- **Question 1. Tranverse waves can pass only through solids but longitudinal wave can pass through any medium. Explain.**

○ **Ans.** For the propagation of transverse wave the medium must have the property to resist the shearing stress. So it must have rigidity. Only solids have this property. For this reason transverse wave can only pass through solids.

On the other hand, for the propagtation of longitudinal waves, the medium must have (i) elasticity (ii) inertia and (iii) cohesion. Now any material medium have these three properties. Hence longitudinal wave can pass through any medium — solid, liquid or gas.

- **Q. 2. A sound wave passes from air to water. The angle of incidence at air and water interface is α_1 and angle of refraction in water is α_2. Snell's law is applicable here. Explain to show whether α_1 is either greater or less than α_2.** **[J.E.E. 1989]**

○ **Ans.** From Snell's law, $\frac{\sin \alpha_1}{\sin \alpha_2} = \frac{v_1}{v_2}$, v_1 = velocity of sound in the first medium (air) and v_2 = velocity of light in the second medium (water). It is found that $v_2 > v_1$.

Hence $\sin\alpha_1 < \sin\alpha_2 \quad \therefore \alpha_1 < \alpha_2$

It means that angle of incidence is smaller angle of refraction.

● **Q. 3. Prove that the equation $y = 5\sin\frac{2\pi}{15}(75t - x)$ cm represents an equation of a progressive wave.**

○ **Ans.** The velocity of the progressive wave in the medium is $v = 75$ cm/s.

If the displacement of the particle be y, at time $(t + 1)$ sec at a distance $(x + 75)$ m then,

$$y_1 = 5\sin\frac{2\pi}{15}[75(t+1) - (x+75)] = 5\sin\frac{2\pi}{15}[75t - x] = y$$

Evidently, the motion of a particle at a distance x cm at any time is same as the motion of another particle 1 sec. later at a distance $(x + 75)$ cm. In this case disturbance of the medium travels 75 cm after 1 sec.

So, the given equation represents the equation of a progressive wave.

● **Q. 4. When a progressive wave passes through a medium, the maximum velocity of a particle is V_m and maximum acceleration f_m. Calculate amplitude and frequency of vibration of the particle.**

○ **Ans.** Assume that the equation of progressive wave be, $y = a\sin(\omega t - x)$

$\therefore$ Particle velocity, $V = \frac{dy}{dt} = \omega a\cos(\omega t - x)$

$\therefore$ maximum particle velocity, $V_m = \omega a$ .... (i)

Also acceleration of the particle, $f = \frac{dv}{dt} = -\omega^2 a\sin(\omega t - x)$

$\therefore$ maximum particle acceleration, $f_m = \omega^2 a$

$\therefore$ Amplitude of vibration is $a = \frac{(\omega a)^2}{\omega^2 a} = \frac{V_m^2}{f_m}$ and frequency, $n = \frac{\omega}{2\pi} = \frac{1}{2\pi}\cdot\frac{\omega^2 a}{\omega a} = \frac{1}{2\pi}\cdot\frac{f_m}{V_m}$

● **Q. 5. The equation of a wave flowing through a stretched string is $y = 3\cos\pi(100t - x)$ cm. What is the wavelength ?**

○ **Ans.** The general equation of progressive wave is $y = a\cos\frac{2\pi}{\lambda}(vt - x)$

Comparing this equation with the given equation we get

$$\frac{2\pi}{\lambda}x = \pi x \quad \therefore \lambda = 2 \text{ cm}$$

$\therefore$ This wavelength of the wave = 2 cm.

● **Q. 6. Regular reflection of sound can occurs on a rough extended surface, but it does not occur in the case of light wave. Again, regular reflection of light can take palce from a small smooth reflector but sound is not reflected on such a small reflector. Explain.**

○ **Ans.** For reflecting a wave, the size of the reflector should be large compared to wavelength of the wave. Range of wavelength of audiable sound is from 1·5 cm to 16 m. But the range of wavelength of visible light is 4×10^{-5} cm to 8×10^{-5} cm. Evidently, the wavelength of sound is much greater than the wavelength of light. For this reason, small reflector can reflect light wave, but can not reflect sound wave.

Again, for regular reflection to occur, the irregularities of the surface should be smaller than the wavelength of the wave. As the wavelength of light is very small, the surface should be very smooth for regular reflection of light wave.

But wavelength of sound wave is very large. So regular reflection may occur even on a rough surface.

● **Q. 7. A tuning fork of frequency, 480 Hz produces sound in air. What is the length of a compression during propagation of the wave ? Give velocity of sound in air = 340 m/s.**

○ **Ans.** The length of a compression is equal to half of the wavelength.

Velocity of the wave, V = 34000 cm/s and frequency, $n = 480$ Hz.

$$\therefore \quad \text{Wavelength, } \lambda = \frac{V}{n} = \frac{34000}{480} = 70 \cdot 83 \text{ cm}$$

$$\therefore \quad \text{length of a compression } = \frac{\lambda}{2} = \frac{70 \cdot 83}{2} = \mathbf{35 \cdot 42\ cm}$$

● **Q. 8. The wavelength of a wave is 2 m. What is the phase difference between two particles situated 1 m apart ?**

Ans. Phase difference, $\phi = \frac{2\pi}{\lambda}$ (Path difference) $= \frac{2\pi}{\lambda} \times x = \frac{2\pi}{2} \times 1 = \pi$ rad

● **Q. 9. What is harmonic function ?**

○ **Ans.** The funciton which can be expressed in terms of sine or cosine functions is called harmonic functions.

● **Q. 10. The amplitude of a wave is 5 cm. What is the displacement of a particle at a distance $x = \lambda/6$ from the source at time $t = T/3$?**

○ **Ans.** Displacement $y = a \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) = 5 \cdot \sin 2\pi\left(\frac{T}{3} \times \frac{1}{T} - \frac{\lambda}{6} \cdot \frac{1}{\lambda}\right) = 5 \sin \frac{\pi}{3} = 4 \cdot 375$ cm

∴ Required displacement = **4·325 cm.**

● **Q. 11. The displacement of a particle at a distance of 5 m from a vibrating source of time $t = T/6$ is half of the amplitude of vibration. What is the wavelength of the wave ?**

○ **Ans.** Let the equation of the wave be $y = a \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$

$$\therefore \quad \frac{a}{2} = a \sin 2\pi\left(\frac{T}{6T} - \frac{5}{\lambda}\right) \quad \therefore \quad \frac{1}{2} = \sin 2\pi\left(\frac{1}{6} - \frac{5}{\lambda}\right)$$

$$\therefore \quad 2\pi\left(\frac{1}{6} - \frac{5}{\lambda}\right) = \frac{\pi}{6} \quad \therefore \lambda = 60 \text{ cm}$$. It is the wavelength.

● **Q. 12. The time period of vibration of a source is 0·02 sec and the velocity of the wave is 400 m/s. What is the phase difference between two points which are at a distance of 10 cm and 15 cm respectively ?**

○ **Ans.** The phase of a progressive wave is given by $\phi_2 = 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$

$$\therefore \quad \phi_1 = 2\pi\left(\frac{t}{T} - \frac{10}{\lambda}\right) \text{ and } \phi_2 = 2\pi\left(\frac{t}{T} - \frac{15}{\lambda}\right)$$

$$\therefore \quad \text{Phase difference, } \Delta\phi = \phi_1 - \phi_2 = 2\pi \times \frac{5}{\lambda} = \frac{10\pi}{\lambda} = \frac{10\pi}{VT} = \frac{10\pi}{400 \times 0 \cdot 02} = \frac{5}{4}\pi \text{ rad}$$

● **Q. 13. The equation of a transverse wave is $y = y_0 \sin 2\pi\left(ft - \frac{x}{\lambda}\right)$. If the maximum particle velocity be four times the wave velocity show that $\lambda = \frac{1}{2}\pi\, y_0$.** **[I. I. T. 1984]**

○ **Ans.** The given equation is $y = y_0 \sin 2\pi\left(ft - \frac{x}{\lambda}\right)$

$$\therefore \quad y = y_0 \sin \frac{2\pi}{\lambda}(\lambda\, f\, t - x)$$

Now, the general equation of a transverse wave, $y = a \cdot \sin \dfrac{2\pi}{\lambda}(vt - x)$

Comparing these two equations : $a = y_0$ and wave velocity, $v = \lambda f$

$\therefore$ Particle velocity, $v_P = \dfrac{dy}{dt} = 2\pi\ f\ y_0 \cdot \cos 2\pi\left(ft - \dfrac{x}{\lambda}\right)$

$\therefore$ Maximum particle velocity, $V_P = 2\pi f y_0$

$\therefore$ By question, $2\pi f y_0 = 4\lambda f$ or, $\lambda = \dfrac{\pi y_0}{2}$

● E◇X◇E◇R◇C◇I◇S◇E ●

A. Short answer type questions :

1. What is a wave ? What are the different types of waves ? Give examples.

2. What should be the nature of the medium suitable for the production of longitudinal wave and transverse wave ?

3. In a gaseous medium longitudinal wave can propagate but a transverse wave cannot. Explain with reasons.

4. What are the characteristics of progressive waves ?

5. What is the nature of sound waves ? Can it propagate through vacuum ? Give reasons.

6. When a wave enters from air to water, its wavelength changes but frequency remains unchanged. Why ?

7. Longitudinal wave can travel through any medium, but transverse wave can travel only through solids. Explain.

8. Regular reflection of light can take place on a brick wall but not in the case of light wave. Explain the reason.

9. Light is reflected from a small mirror, but sound is not reflected. Why ?

10. What is echo ? What should be the minimum distance of the reflector to distinctly hear the echo of a sharp sound ? Given that the velocity of sound is 1120 ft/s.

11. What property of waves proves that light waves are transverse ?

12. What is echo ? How is it produced ?

13. What is the difference between longitudinal and transverse waves ? Why do sound waves not undergo polarisation ?

14. The ratio of the wavelength of incident wave and the wavelength of refracted wave in medium is equal to refractive index of that medium. Justify the statement. **[J.E.E. 1985]**

15. What is meant by phase of a wave ?

16. The equation of a progressive wave is $y = a\ \sin(wt - \theta)$. What is the maximum velocity and acceleration of the particle ?

17. What is the phase difference between (i) two consecutive crests (ii) one crest and the next trough ? **[Ans.** 0°, 180°**]**

18. For refraction of sound wave water is considered as a rarer medium than air. Why ?

19. A sound ray passes from air to water. Angle of incidence at the interface of air and water is α_1 and angle of refraction in water is α_2. If the Snell's law applicable here, then α_1 is greater or less than α_2. Explain. **[J.E.E. 1989]**

20. What is the minimum distance of the reflector to heat districtly the echo of disyllabic sound ?

21. A transverse wave is travelling along a wire from left to right. The shape of the string at any instant is an shown in fig. A, B, C, D, E, F, G and H are the points on the string. In which points the velocity is (i) upwards (ii) downwards (iii) zero (iv) maximum. **[Ans.** (i) D, E, F (ii) A, B, H (iii) C, G (iv) A, E, H**]**

22. Distance sound can be heard very distinctly at night than during day time. Explain.

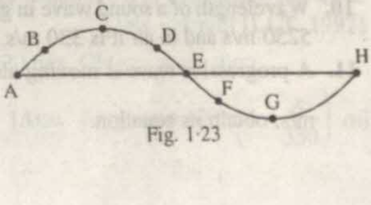


Fig. 1·23

23. For refraction of sound rays balloon filled with carbon-di-oxide and a balloon filled with water behaves as what type of lens ?

24. What is reverberation of sound ?

25. Will total reflection of sound rays take phase when it passes from water to air ?

◢ B. Essay type questions :

1. (a) What is wave ? How it is produced ? What we mean be longitudinal wave and transverse wave ?

(b) What is elastic wave ? Give examples.

2. What is progressive wave ? Extablish the equation of progressive wave ?

3. (a) What are the characteristics of progressive waves ?

(b) Define the following quantities related to a wave : (i) Amplitude, (ii) frequency, (iii) velocity, (iv) phase, (iv) wave front.

4. What is a wave ? Deduce the relation between the frequency, velocity and wavelength of the wave. What is the differencle between longitudinal and transverse waves ? **[H.S. 2000]**

5. (a) What is echo ? What minimum distance is to be maintained between the source of sound and the reflector to hear the echo of mono-syllabic sound ? (Velocity of sound in air = 332 m/s)

6. (a) What is meant by refraction of wave ?

(b) Describe an experiment to show the refraction of sound wave.

(c) Why sound from a distant source can be heard at night more districtly than during day time.

7. Describe a method to determine the velocity of sound in air ?

8. Describe how we can determine the depth of a sea and the height of an aeroplane.

◢ C. Simple numerical problems :

1. The wavelength of a progressive wave is 1·5 m and time period 1/200 sec. Calculate its velocity. [**Ans.** 300 m/s]

2. The equation of a progressive wave is $y = 15\sin(660\pi t - 0\cdot 02\pi x)$ cm. Find the velocity of the wave.
[**Ans.** 330 m/s.]

3. The wavelength of a wave is 5000Å and its velocity in a medium is 3×10^8 m/s. Calculate its frequency. How many waves can be accomodated in a length of 1·5 cm. [**Ans.** 6×10^{14} Hz ; 3×10^4]

4. The frequency of two tuning forks are 150 Hz and 450 Hz. Calculate the ratio of the wavelengths of the waves produced by these forks in air. [**Ans.** 3 : 1]

5. Two vibrating tuning forks of frequencies 50 Hz and 100 Hz touch the surface of water, and produce waves of wavelengths 0·6 cm and 0·36 cm respectively in water. Compare the velocities of the two surface waves.
[**Ans.** 5 : 6] **[J.E.E. 1998]**

6. A body vibrating with constant frequency produces a wave of wavelength 10 cm in medium A and a wave of wavelength 15 cm in medium B. If the velocity of the wave in medium A be 90 cm/s. What will be the velocity of the wave in medium B ?
[**Ans.** 135 cm/s]

7. The frequency of a tuning fork is 512 Hz. The time during which the fork makes 30 oscillations, the sound in air travels 20 m. Calculate the velocity and wavelength of the wave in air. [**Ans.** 341·33 $\mathrm{ms^{-1}}$; 0·67 m]

8. The wavelength of a wave on water surface is 3·7 m. The velocity of the wave in water and in air are respectively 1480 m/s and 350 m/s. Calculate frequency and wavelength of the wave the air. [**Ans.** 400 Hz, 87·5 cm]

9. The velocity of a light wave is 3×10^8 m/s and wavelength 5893 A°

(i) What will be the velocity of light in water. Refractive index of water is 1·33.

(ii) What will be the frequency of light in water ? [**Ans.** $2\cdot25\times10^8$ m/s, (ii) $5\cdot09\times10^{14}$ Hz]

10. Wavelength of a sound wave in glass is 175 cm. What will be its wavelength in air ? Velocity of sound in glass is 5250 m/s and in air it is 330 m/s.
[**Ans.** 11 cm]

11. A progressive wave is moving along –*ve* X-axis. If its amplitude be 0·01 m, frequency 550 Hz and velocity 330 m/s, obtain its equation.
[**Ans.** $y = 0\cdot01\sin 10\pi\left(110t + \dfrac{x}{3}\right)$ m]

12. The wavelength of a wave in 50 cm. What is the phase difference between two points whose separation is 10 cm ? **[Ans.** $0{\cdot}4\pi$ rad**]**

13. The equation of a progressive wave is $y = 15\sin(660\pi t - 0{\cdot}02\pi x)$ cm. Calculate the frequency and velocity of the wave. **[Ans.** 330 Hz ; 330 m/s**] [J.E.E. 1986]**

14. The equation of a progressive wave is $y = a\sin\left(2000\pi t - \dfrac{\pi x}{17}\right)$. Here t is in sec ; y and x are in cm units. Calculate (i) Wave length, (ii) velocity, (iii) frequency, (iv) phase difference between two points 1·8 m apart. **[Ans.** (i) 34 m, (ii) 340 m, (iii) 1000 Hz, (iv) $10\pi/17$ rad**]**

15. The equation of transverse wave is $y = y_0 \sin 2\pi\left(ft - \dfrac{x}{\lambda}\right)$. If the maximum particle velocity be 4 times the wave velocity. Show that $\lambda = \dfrac{\pi\, y_0}{2}$. **[I.I.T. 1984]**

16. A wave is propagating along negative X-axis with the vibration along Y-axis. Its amplitude, frequency and wavelength are respectively 10 cm, 500 Hz and 100 cm. What is the equation of the wave. **[Ans.** $y = 10\sin 2\pi\left(500t + \dfrac{x}{100}\right)$ cm**] [J.E.E. 1990]**

17. The equation of a wave is $y = 0{\cdot}5\sin\pi(0{\cdot}01x - 3t)$ where y and x are in metre units and time t is in sec. Calculate wave velocity and maximum particle velocity. **[Ans.** 300 m/s ; $1{\cdot}5\pi$ m/s**]**

18. Obtain the equation of the progressive wave which is travelling along positive x-axis. Given its amplitude 0·05 m, velocity 200 m/s and frequency 50 Hz. **[Ans.** $y = 0{\cdot}05\sin 2\pi(50t - 0{\cdot}25x)$ m**]**

19. The equation of a progressive wave is $y = 8\sin 2\pi(0{\cdot}628x - 12{\cdot}56t)$. Here y and x are in metre and t is in sec. Calculate : (i) amplitude, (ii) frequency, (iii) wave velocity, (iv) phase difference between two points which are at a separation of 2·0 cm. **[Ans.** (i) 8 cm, (ii) 2 Hz, (iii) 20 cm/s, (iv) 72°**]**

20. A wave is travelling along X-axis with a velocity of 30 cm/s. Its amplitude is 3 cm and wavelength 15 cm. If the time is counted when $x = 0$, obtain the equation of the wave. **[Ans.** $y = 3\sin 2\pi\left(2t - \dfrac{x}{15}\right)$ cm**]**

21. The equation of the two progressive waves travelling along X-axis are $y = 10\sin 2\pi(6t - 0{\cdot}25x)$; $y = 10\cos 2\pi(6t - 0{\cdot}25x)$, Calculate : (i) frequency, (ii) velocity, (iii) phase difference between the two waves. **[Ans.** (i) 6 Hz, (ii) 24 m/s, (iii) $\pi/2$**]**

22. What should be the minimum distance between the source of sound and the reflector so that the echo of a trisyllabic sound can be heard distinctly ? Velocity of sound in air is 330 m/s. **[Ans.** 99m**] [H.S. 2002]**

23. The equation of a progressive wave is $y = 2{\cdot}5\sin(800t - 0{\cdot}82x)$. Calculate : (i) phase difference between two points which are at a separation of 20 cm along X-axis, (ii) change of phase at a point during a time interval $0{\cdot}3\times10^{-3}$ sec. **[Ans.** (i) 16·4 rad, (ii) 0·24 rad**]**

24. The distance between two points of a stretched string is 10 cm. The frequency and velocity of a wave travelling along the wire are 500 Hz and 100 m/s. Calculate the phase difference between these two points. **[Ans.** π rad**]**

25. The equation a travelling wave is $y = 20\sqrt{3}\sin\left(\omega t - \dfrac{2\pi}{\lambda}x\right)$. Calculate the displacement of the particle at $x = \dfrac{\lambda}{6}$ at time $t = T/3$. **[Ans.** 30 cm**]**

26. A progressive wave is travelling along negative X-axis. Its frequency is 25 Hz, amplitude $2{\cdot}5\times10^{-5}$ m and initial phase zero. Calculate the phase difference at any time between two points which are 6 m apart along the wave propagation. **[Ans.** = π rad**] [I.I.T. 1997]**

27. The amplitude of a progressive is 10 cm, frequency 20 Hz and velocity 330 m/s. Obtain its equation. **[Ans.** $y = 0{\cdot}1\sin 400\pi\left(t - \dfrac{x}{330}\right)$ m**]**

28. The amplitude of a wave is 0·01 m, wavelength = 0·1 m and time period 0·01 sec. Obtain the equation of the wave. [**Ans.** $y = 0 \cdot 01 \sin 2\pi\left(100t - \frac{x}{10}\right)$ m]

29. Frequency of a tuning fork is 200 Hz. The velocity of the wave produced by the fork in the medium is 320 m/s. Calculate its wavelength and period. What is the phase difference between two points which are at a separation of 40 cm along the path of the wave ? [**Ans.** 160 cm, 0·005 sec ; $\pi/2$]

30. The equation of travelling wave is $y = 2 \sin\frac{\pi}{2}\left(2t + \frac{x}{8}\right)$ cm. Calculate : (i) change of phase of a particle at a time interval of 0·5 sec, (ii) phase difference between two points at any time which are at a separation of 16 cm. [**Ans.** (i) 90°, (ii) 180°]

31. A sound is produced at some distance from the hill. The echo is heard after 1 sec. If the velocity of sound in air be 340 m, Calculate the distance of the hill. [**Ans.** 170 m]

32. What is the minimum distance between an observer and the reflector to hear the echo of a sound of 5 syllables ? Velocity of sound = 1120 ft/sec. [**Ans.** 560 m]

33. Two persons A and B are standing on a line which is perpendicular to the length of the hill. A is near the hill and B is at a distance of 825 ft from A. If A fires a gun, he hears the echo after 1 sec. But if B fires a gun, A hears the echo after 1·75 sec. Calculate the velocity of sound in air and the distance of A from the hill. [**Ans.** (i) 1100 ft/sec ; (ii) 550 ft]

34. A man fires a gun standing at a point in between the two parallel cliffs. He hears two successive echos after 3/2 sec. and 5/2 sec. Calculate the separation between the hills. After what time he will hear the third echo ? Velocity of sound = 350 m/s. [**Ans.** 700 m ; 4 sec.]

35. The driver of a car while moving towards a hill fire a gun when he is at a distance of 3 km from hill. The speed of the car is 5 m/s and that of sound in air = 340 m/s. When and at what distance from the hill will the driver hear the echo ? [**Ans.** After 17·4 sec ; at a distance 2·913 km]

36. Some explosives are kept at a depth x from the surface of water in a sea. At the same level a hydrophone is placed at some distance from the position of the explosives. Two sounds are recorded by the hydrophone at time t_1 and t_2. Show that the depth of the ocean is $x + \frac{v}{2}\sqrt{t_2^2 - t_1^2}$ where v = velocity of sound in water.

◢ D. Harder numerical problems :

1. The equation of a progressive wave is $y = 0 \cdot 25 \times 10^{-2} \cdot \sin(500t - 0 \cdot 025x)$. Here y and x are in cm and t is in sec. What are its frequency and wavelength ? Calculate also the maximum velocity and acceleration of a particle of the medium. [**Ans.** 79·6 Hz ; 251·2 cm ; 1·25 cm/s ; 625 cm/s^2]

2. A man is running towards a hill at a speed of 4 m/s. When he is at a distance of 2 km from the hill he fires a gun. If the velocity of sound in air be 330 m/s, when and where he will hear the echo. [**Ans.** 11·98 sec ; 1·952 km] **[J.E.E. 1986]**

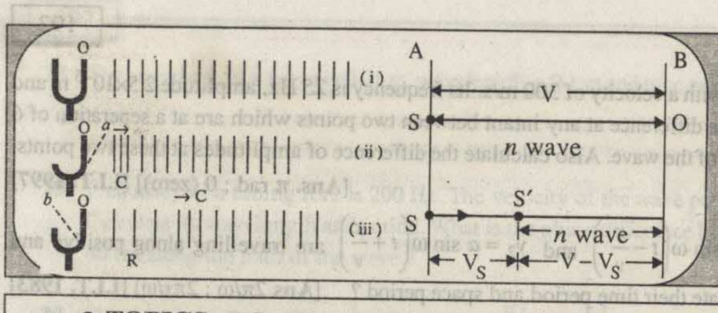
3. The equation of a progressive wave is $y = 0 \cdot 1 \sin\left(200\pi t - \frac{200\pi x}{17}\right)$. Here x is the distance from the origin. What is the phase difference between two points situated at a distance of 25 cm and 110 cm from the origin ? Obtain the equation of another progressive wave whose amplitude and frequency are double of the first wave. Also the wave is travelling in opposite direction of the first wave. [**Ans.** π ; $y = 0 \cdot 2 \sin\left(400\pi t + 200\pi\frac{x}{17}\right)$]

4. Bulk modulus and density of iron are 80 times and 5 times the corresponding quantities of water. Calculate velocity of sound in steel. Given, velocity of sound in water in 1493 m/s. [**Ans.** 5932 m/s] **[J.E.E. 1995]**

5. A wave is propagating along X-axis with a velocity of 300 m/s. Its frequency is 25 Hz, amplitude $2{\cdot}5\times10^{-5}$ m and initial phase zero. Calculate the phase difference at any intant between two points which are at a seperation of 6 m along the direction of propagation of the wave. Also calculate the difference of amplitudes at these two points. **[Ans.** π **rad ; 0 (zero)] [I.I.T. 1997]**

6. The two progressive waves $y_1 = a \sin \omega\left(t - \frac{x}{v}\right)$ and $y_2 = a \sin \omega\left(t + \frac{x}{v}\right)$ are travelling along positive and negative X-axis respectively. Calculate their time period and space period ? **[Ans** $2\pi/\omega$ **;** $2\pi x/\omega$**] [I.I.T. 1983]**

7. A string 25 m long has mass of 0·5 kg. A 10 m long specimen of the string is stretched by 5 kg-wt force. How long will the transverse wave take to travel the length of the string ? **[Ans. 0·2 sec.]**



CHAPTER 2

SOUND WAVES

• TOPICS : ○ Introduction, ○ Sound is an elastic wave, ○ Mechanism of propagation of sound waves, ○ Velocity of sound wave, ○ Velocity of sound in gases, ○ The effect of various factors on velocity of sound in air, ○ Doppler effect, ○ Doppler effect in the case of light waves, ○ Short answer type questions (with answers), ○ Exercise.

2.1. Introduction

When a body vibrates with suitable frequency and amplitude, longitudinal waves are produced in air. If the frequency of this wave lies in the range of 20 Hz – 30 KHz, it is audiable. It produces sensation of hearing, when falls on the ear. So, sound is a mechanical form of energy generated by the vibrating body. Sound energy is not an independent form of energy like heat energy or electric energy. The longitudinal waves having frequencies below and above the audiable range are called **infrasonic** and **ultrasonic** waves respectively. Although these waves do not produce sensation of hearing still they are identical to the audiable sound waves. Any vibrating body may behave as a source of sound.

The specialised study of sound waves is called **acoustics**. Acoustics is a large field of study and includes such topics as noise, musical scale, acousties of building, pitch and loudness, Doppler effect etc.

2.2. Sound is an elastic wave

Sound is a form of energy. It is produced by a vibrating body and propagates through a material medium in the form of waves which on reaching our ear produces sensation of hearing.

From the study of various properties of sound we conclude that sound propagates through a medium in the form of waves. Now sound waves can travel through a gaseous medium like air. As only longitudinal waves can travel through a gaseous medium and not transverse waves, it follows that sound waves are longitudinal.

A source of sound is always in **a state of vibration**. A vibrating body may be solid, liquid or gas. If the vibration stops, emission of sound also stops immediately. For example, when a tuning fork is struck, sound is produced. If we now touch the fork we feel the vibration and no further sound is produced. So only vibrating body can produce sound.

Evidences to show that sound propagates by wave motion :

(i) For producing waves, a disturbance must be created in the medium. This is also true in the case of sound. Sound is produced only when a body vibrates.

(ii) For the propagation of sound from one point to another, a medium is necessary. In the process, the medium is not displaced. Such is the property of wave motion.

(iii) Like any other waves, sound travels through a medium with definite velocity. This velocity is greater in solids and liquids than in gases.

(iv) Like all waves sound exhibits reflection, refraction, interference and diffraction.

(v) Sound can travel through a gas. This property and the fact that sound can not be polarised show that sound waves are longitudinal.

(vi) Sound waves can be photographed.

We, therefore, conclude that sound propagates through a material medium by longitudinal wave motion.

❂ 2.3. Mechanism of propagation of sound waves ❂

A vibrating body is a source of sound. Sound waves travels through the surrounding medium as longitudinal waves. Alternate compression and rarefaction of the medium takes place during propagation of sound waves. So, **the medium must be elastic**. For this reason, sound waves are called **elastic waves.**

Moreover, for the propagation of waves the particles of the medium must have the property of **inertia.** Suppose, any part of a medium is compressed due to increase of pressure. With the release of pressure, due to elastic- property, the particles of the medium return to their initial position. But due to inertia, they will overshoot the initial position and go equal distance in the opposite direction. Now, there will be a rarefaction. To understand the mechanism of propagation of sound in air we consider a vibrating turning fork as a source of sound.

Fig. 2.1 shows how a vibrating tuning fork sends out a sound waves. As the right prong 'O', moves periodically to and fro the column of air in front of the prong is imagined to be divided into equisposed layers [Fig. 2.1(i)]. When the prong moves forward [Fig. 2.1(ii)], it compresses the layer of air in front of it. This disturbance is transmitted from layer to layer through air till the prong reaches '*a*'. The compression passes through the medium with definite velocity. This velocity depends on elasticity and density of the medium.

During the reverse movement of the prong 'O' partial vacuum will be produced behind it. Air being elastic the layer in contact with the prong rushes behind to fill up the vacuum and a rarefaction is thus produced among the layers in contact with the prong. So a pulse of rarefaction is created during this half of vibration of the prong [fig. 2.1(iii)].

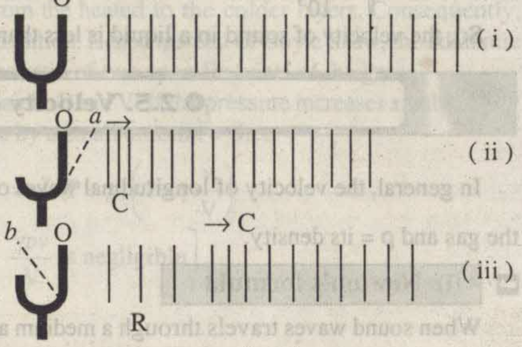


Fig. 2·1

Hence, during a complete vibration of the prong, a compression and a rarefaction are created, whose lengths are equal because each is created in half the vibration. As long as the fork vibrates,pulses of compression and rarefaction move outward through the air, keeping their respective position unchanged. When such longitudinal wave consisting of compression and rarefactions reach the ear, one hears the sound.

❂ 2.4. Velocity of sound wave ❂

Sound waves travels through a medium (solid, liquid or gas) with a definite velocity. The velocity of sound varies greatly from medium to medium. Sound travels faster is solids and liquids than in gases. Also the velocity of sound in any material medium is much smaller than the velocity of light. For example, the sound of thunder is heard after few seconds since the flash of lightning is actually seen.

From theoretical consideration, Newton had shown that the velocity of logitudinal elastic wave in a homogeneous elastic medium is given by

$$V=\sqrt{\frac{E}{\rho}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2.1)$$

when E = elastic modulus of the medium and ρ = density of the medium.

(i) When longitudinal waves travels through a thin rod or wire, the effective modulus of elasticity is Young's modulus. Hence the velocity of sound in a wire is given by

$$V=\sqrt{\frac{Y}{\rho}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2.2)$$ [Y = Young's modules of the material of the rod or wire.]

For example, for steel $Y = 2\times10^{11}\ \mathrm{N/m^2}$, $\rho = 7\cdot6\times10^3\ \mathrm{kg/m^3}$

$$\therefore\quad V = \sqrt{\frac{2\times10^{11}}{7\cdot6\times10^3}} = \mathbf{5130\ m/s}$$

This velocity is about 15 times more than the velocity of sound in air.

(ii) In case of liquid and gas bulk modulus K of the material has to be used. So, velocity of sound in a fluid is given by

$$V = \sqrt{\frac{K}{\rho}}$$

For example, for water $K = 2\cdot2\times10^9\ \mathrm{N/m^2}$, $\rho = 10^3\ \mathrm{kg/m^3}$

$$\therefore\quad V = \sqrt{\frac{2\cdot2\times10^9}{10^3}} = \mathbf{1483\ m/s}$$

So, the velocity of sound in a liquid is less than velocity of sound is solid.

❂ 2.5. Velocity of sound in gases ❂

In general, the velocity of longitudinal waves or sound waves in gas is $V = \sqrt{\frac{K}{\rho}}$; K = bulk modulus of the gas and ρ = its density.

(i) Newton's formula :

When sound waves travels through a medium a train of compressions and rarefactions flow through the medium. During compression, temperature of the layer rises and at the rarefaction the temperature of the layer decreases. Newton assumed that compression and rarefaction of layers takes place slowly so that the heat generated in a layer due to compression is transferred to the adjoining layers before the next rarefaction starts in the layer. Similarly for the cold produced in a rarefaction is fully compensated by heat flowing in from the surrounding volume of the gas. Consequently the temperature of the layers remains unaltered. According to Newton, therefore, propagation of sound through gaseous medium takes place under **isothermal condition**. So pressure-volume change obey Boyle's law.

Under isothermal condition the bulk modulus of the gas equals its pressure.

- **Proof :** Let us consider a given mass of a gas under a pressure P and having a volume V. Let the pressure of this mass of air be increased by a small amount p and let the consequent diminution in volume be v. According to Boyle's law

$$PV = (P + p)(V - v) = PV + pV - Pv - pv$$

$$\therefore\quad pV = Pv \quad [\because pv \text{ is negligible}]$$

$$\therefore\quad P = \frac{p}{v/V} = \frac{\text{Volume stress}}{\text{Volume strain}} = K$$

- **Alternative proof using calculus :** Boyle's law gives PV = constant. Differentiating $P\,dv + V\,dp = 0$.

$$\therefore\quad P = -\frac{dP}{dV/V} = \frac{\text{Volume stress}}{\text{Volume strain}} = K$$

The negative sign shows that with increase of pressure, volume decreases. Hence the velocity of sound is gaseous medium is

$$V = \sqrt{\frac{P}{\rho}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(2.3)}$$

- **Velocity of sound in air at NTP by Newton's formula (equ. 2.3).**

Normal pressure, $P = 76 \text{ cm of Hg} = 76 \times 13\cdot6 \times 980 \text{ dyne/cm}^2 = 1\cdot013 \times 10^5 \text{ N/m}^2$ and density of air $\rho = 0\cdot001293 \text{ g/c.c.}$

$$\therefore \quad V = \sqrt{\frac{1\cdot013 \times 10^6}{0\cdot001293}} = 28000 \text{ cm/s} = \mathbf{280 \text{ m/s}}$$

The calculated value of velocity of sound in air differ widely from the experimental value which is nearly 332 m/s.

(ii) Laplace's correction of Newton's formula :

The difference between the calculated value of velocity of sound in air and the experimentally observed value was first explained by Laplace. He pointed out that during propagation of sound waves compression and rarefaction take place rapidly. As air is a bad conductor of heat, rise or fall of temperature during compression or rarefaction, heat cannot flow quickly from the heated to the colder layers. Consequently, propagation of sound waves takes place under adiabatic condition. Hence instead of Boyle's law, the adiabatic equation $PV^\gamma =$ constant should be applied, where γ is the ratio of two specific heats of the gas.

Let the initial pressure and volume of a certain mass of air be P and V. If the pressure increases adiabatically by a small amount p and consequently volume decreases by a small amount v then

$$PV^\gamma = (P+p)\,(V-v)^\gamma = (P+p)\,V^\gamma\left(1-\frac{v}{V}\right)^\gamma = v^\gamma\,(P+p)\left(1-\gamma\frac{v}{V}\right)$$

$$\therefore \quad P = P + p - \gamma\frac{Pv}{V} - \gamma\frac{pv}{V} \qquad \therefore\ p = \gamma\frac{Pv}{V} \quad \left[\because\ \frac{\gamma pv}{V} \text{ is negligible}\right]$$

$$\gamma P = \frac{p}{v/V} = K$$

∴ Bulk modulus K = pressure of the gas × ratio of two specific heats.

- **Alternative proof using calculus :** $PV^\gamma =$ constant.

Differentiating, $V^\gamma\,dp + P\gamma\,v^{\gamma-1}\,dv = 0$

$$\therefore \quad V\,dp + \gamma p \cdot dv = 0 \qquad \therefore\ \gamma P = \frac{-dp}{dv/V} = K$$

Hence the expression for the velocity of sound in air after Laplace's correction is

$$V = \sqrt{\frac{\gamma P}{\rho}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.4)$$

According to this formula, the velocity of sound in air is

$$V = \sqrt{\gamma}\,\sqrt{\frac{P}{\rho}} = \sqrt{1\cdot4} \times 280 \text{ m/s} \quad [\gamma = 1\cdot4 \text{ for air}]$$

$$\therefore \quad V = \mathbf{332\ ms^{-1}}$$

This value of velocity of sound agrees with experimentally determined value and therefore justifies Laplace assumption.

2.6. The effect of various factors on velocity of sound in air

All factors which affect the ratio of pressure and density of air also affect the velocity of sound in air.

(i) Effect of pressure :

If the temperature remains unchanged, a change of pressure does not change the velocity of sound, because at constant temperature P/ρ is constant, density being proportional to pressure. So, the velocity of sound in a gas is thus independent of pressure.

(ii) Effect of temperature :

When the temperature of a gas changes, its density also changes. Consequently, the velocity of sound in a gas will change. With increase of temperature, the density of a gas decreases ; hence the velocity of sound increases. Similarly, with decrease of temperature, the velocity of sound in air decreases.

It can be proved that the velocity of sound in a gaseous medium is proportional to the square root of its absolute temperature.

$\therefore \quad V \alpha \sqrt{T}$ [T = absolute temperature]

- **Proof :** From ideal gas equation $PV = nRT = \frac{m}{M}RT$

$$\therefore \quad \frac{PV}{m} = \frac{RT}{M} \quad \text{or,} \quad \frac{P}{m/V} = \frac{RT}{M} \quad \therefore \quad \frac{P}{\rho} = \frac{RT}{M}$$

[M = molecular mass and m = mass of the gas taken]

$$\therefore \quad \text{Velocity of sound } V = \sqrt{\frac{\gamma RT}{M}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2.5)$$

$$\therefore \quad V \alpha \sqrt{T}$$

Let corresponding to the temperatures 0°C and t°C the velocities of sound in air be V and V_0. Then

$$\frac{V}{V_0} = \sqrt{\frac{T}{T_0}}$$

Now $T_0 = 0°C = 273$ K and $T = t°C = (273 + t)$ K

$$\therefore \quad \frac{V}{V_0} = \sqrt{\frac{273+t}{273}} = \left(1+\frac{t}{273}\right)^{1/2}$$

$$\therefore \quad V = V_0\left(1+\frac{1}{2}\,\frac{t}{273}\right) = V_0(1+0\cdot 00183t)$$

$$= 332(1+0\cdot 00183t) \text{ [Putting, } V_0 = 332 \text{ m/s]}$$

$$= (332 + 0\cdot 61\ t) \text{ m/s}$$

Hence the velocity of sound in air increases by about 0·61 m/s or 61 cm/sec. for each degree Celcius rise of temperature.

(iii) Density :

Consider two gases having densities D_1 and D_2. Pressure and temperature of the gases are the same and they are P and T respectively. If the velocities of sound in the gases be V_1 and V_2 respectively then,

$$V_1 = \sqrt{\frac{\gamma P}{D_1}},\ V_2 = \sqrt{\frac{\gamma P}{D_2}} \quad \therefore \frac{V_1}{V_2} = \sqrt{\frac{D_2}{D_1}} \quad \text{[Assuming the value of } \gamma \text{ be same]}$$

So, the velocity of sound in a gas is inversely proportional the square root of density of the gas. For example if we want to compare the velocities of sound in oxygen and hydrogen

$$\frac{V_1}{V_2} = \sqrt{\frac{16}{1}} = 4$$

[as oxygen is 16 time denser than hydrogen under the same condition of temperature and pressure]

Hence, under identical condition of temperature and pressure, sound travels four times faster in hydrogen than in oxygen.

(iv) Humidity :

With increase of humidity of air, the velocity of sound in it increases. The reason is that at the same temperature and pressure the density of water is about 0·625 the density of dry air. So, due to less density of moist air, the velocity of sound in it increases.

Let V_d be the velocity of sound in dry air at t°C and at a pressure P and V_m be the velocity of sound in moist air at the same temperature and pressure. It can be proved that

$$V_d = V_m\sqrt{\frac{P-0\cdot378f}{P}} \quad [f = \text{SVP at } t^{\circ}\text{C}]$$

●● Numerical Examples ●●

● **Example 2.1 A stone is dropped into a well 44·1 m deep. Find after how much time will the sound of the splash be heard ? Velocity of sound in air 340 m/s.**

○ **Solution :** Time taken by the stone to reach the surface of water be t_1. Then

$$h=\frac{1}{2}gt^2 \quad \therefore \quad t_1=\sqrt{\frac{2h}{g}}=\sqrt{\frac{2\times44\cdot1}{9\cdot8}}=\mathbf{3\ sec}$$

Next sound of splash will take time t_2 to the observer above the wall. So

$$t_2=\frac{44\cdot1}{340}=0\cdot13\text{ sec.}$$

$\therefore$ Total time = 3 + 0·13 = **3·13 sec.**

● **Ex. 2.2. At 50°C the velocity of sound in a gas is 340 ms^{-1}. If the pressure is doubled and the temperature is 125°C, what will be the velocity of sound ?**

○ **Solution :** The velocity of sound in a gaseous medium does not depend on pressure of the gas. But velocity will increase with rise of temperature.

In this case $V \propto \sqrt{T}$ $\quad \therefore \quad \frac{V_2}{V_1}=\sqrt{\frac{T_2}{T_1}} \quad \therefore \quad V_2 = 340\sqrt{\frac{398}{323}} = \mathbf{377\cdot41\ ms^{-1}}$

$\therefore$ The required velocity of sound = **377·41 ms^{-1}**

● **Ex. 2.3. A stone is dropped from the top of a tower and the sound of hitting the ground is heard after 4 sec. Obtain the height of the tower. Given the velocity of sound in air = 340 ms^{-1} and g = 9·8 ms^{-2}.**

○ **Solution :** Let height of the tower = h metre.

If the stone takes t sec to reach the bottom of the tower then,

$$h=\frac{1}{2}gt^2=4\cdot9\,t^2\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

So, the sound of hitting the ground will take $(4-t)$ sec to reach the top of the tower. Then

$$h=(4-t)\,340=1360-340\,t\ \ldots\ldots\ldots\ldots\ldots\ldots \text{(ii)}$$

From the equation (i) and (ii), $4.9\,t^2 = 1360 - 340\,t$

$\therefore \quad 4{\cdot}9\,t^2 + 340\,t - 1360 = 0.$

$$\therefore \quad t=\frac{-340\pm\sqrt{(340)^2+4\times4\cdot9\times1360}}{9\cdot8}=\mathbf{3\cdot79\ sec}$$

$\therefore$ Height of the tower, $h=4\cdot9\,t^2=4\cdot9\times(3\cdot79)^2=\mathbf{70\cdot3\ m}$

● **Ex. 2.4. Calculate the velocity of sound in copper. Given, density of copper is 8·6 g/c.c. and its Young's modulus 11·8×10^{11} dyne/cm^2.**

○ **Solution :** The velocity of sound in solid medium, $V=\sqrt{\frac{Y}{\rho}}$

$\therefore \quad V=\sqrt{\frac{11\cdot 8\times 10^{11}}{8\cdot 6}}=3\cdot 704\times 10^{5}\ \text{cms}^{-1}=\mathbf{3{\cdot}704\times 10^{3}\ ms^{-1}}$

$\therefore$ The required velocity of the sound in copper = **$3{\cdot}704\times10^3$ ms^{-1}**

- **Ex. 2.5. A person corrected his watch at 27°C by hearing the sound of firing at a fort at a particular time. He found that watch goes slow by 10 sec. If the velocity of sound at 0°C is 330 ms^{-1}, calculate the distance of the person from the fort.**

○ **Solution :** The velocity of sound at t°C is $V = 330\ (1 + 0{\cdot}00183\ t)$

$\therefore \quad V = 330\ (1 + 0{\cdot}00183 \times 27) = 346{\cdot}305$ m/s

$\therefore$ Distance travelled by sound in 10 sec, s = 346·305 × 10 = **3463·05 m**

The watch is slow by 10 sec. It means that sound has taken 10 sec. to reach the person from the fort.

$\therefore$ Distance of the person from the fort is 3463·05 m = **3·463 km**

- **Ex. 2.6. The sound produced at the surface of sea water takes 4·5 sec to reach the surface again after reflection at the bottom of the sea. What is the depth of the sea ? Given, compressibility of sea water = $4{\cdot}6\times10^{-11}$ cm^2/dyne and density = 1·03 g/c.c.**

○ **Solution :** Evidently the sound produced at the surface of sea water takes 2·25 sec to reach the bottom of the sea. So the depth of the sea will be equal to the distance travelled by the sound in sea water during 2·25 sec.

Now, velocity of sound in water,

$$V=\sqrt{\frac{K}{\rho}}\ ;\ \text{here}\ K=\frac{1}{4\cdot 6}\times 10^{11}\ \text{dynes/cm}^2\ \text{and}\ \rho=1{\cdot}03\ \text{g/c.c.}$$

$$\therefore \quad V=\sqrt{\frac{10^{11}}{4\cdot 6\times 1\cdot 03}}=1\cdot 452\times 10^{5}\ \text{cm/s}$$

$\therefore$ Depth of the sea = $1{\cdot}452\times10^5\times2{\cdot}25$ = **3267 m**

- **Ex. 2.7. At what temperature the velocity of sound in air be double of the velocity of sound in air at 0°C ?**

○ **Solution :** We know $V \propto \sqrt{T}$ $\quad \therefore \frac{V_2}{V_1}=\sqrt{\frac{T_2}{T_1}}$

$\therefore \quad 2=\sqrt{\frac{T_2}{T_1}} \qquad \therefore\ T_2=4T_1=4\times 273=1092\ \text{K}$ = **819 °C**. It is the required temperature.

- **Ex. 2.8. At what temperature, the velocity of sound in air be 1% more than the velocity of sound at 27°C in air.**

○ **Solution :** If V_1 be the velocity sound in air at 27°C, then the velocity at the required temperature is

$$V_2=V_1+\frac{V_1}{100}=\frac{101}{100}V_1 \quad \therefore\ \frac{V_2}{V_1}=\frac{101}{100}$$

Now $V \propto \sqrt{T}$ or, $\frac{V_2}{V_1}=\sqrt{\frac{T_2}{T_1}} \quad \therefore\ \sqrt{\frac{T_2}{300}}=\frac{101}{100} \quad \therefore\ T_2=300\times(1\cdot 01)^2$ = **306 K**

$\therefore$ The required temperature = 306 K = **33°C**

- **Ex. 2.9. The density of hydrogen at NTP is 0·089 g/lit and γ = 1·41. Calculate the velocity of sound in hydrogen at 90°C.**

○ **Solution :** If V_0 be the velocity of sound in hydrogen at 0°C.

$$V_0 = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{1\cdot41\times1\cdot013\times10^6}{0\cdot089\times10^{-3}}} = 1266\cdot8\ \text{ms}^{-1}$$

∴ Velocity of sound in hydrogen at 90°C is

$$V = 1266{\cdot}8\ (1 + 0{\cdot}00183\ t) = 1267\ (1+0{\cdot}00183\times90) = \mathbf{1475\ ms^{-1}}$$

● **Ex. 2.10. A man being at a distance of 1730 m from a gun hears the light and report of firing at an interval of 5 sec at 29°C. It atmospheric pressure be 76 cm Hg and the ratio of two specific heats be 1·41, calculate the density of air.**

○ **Solution :** Velocity of sound at 29°C is $V = \frac{1750}{5} = 350\ \text{m/s}$

Now, $V = \sqrt{\frac{\gamma P}{\rho}}$ ∴ density of air, $\rho = \frac{\gamma P}{V^2}$

$$\therefore\quad \rho = \frac{1\cdot41\times1\cdot013\times10^6}{(35000)^2} = 1\cdot17\times10^{-3}\ \text{g/c.c.}\left[P = 1\cdot013\times10^6\ \text{dyne/cm}^2\right]$$

● **Ex. 2.11. At –173°C the velocity of sound in helium gas is 582 ms^{-1}. Molecular weight of helium is 4, calculate its γ. [R = 8·31×10^7 erg/mol°C] [J.E.E. 1998, 2000]**

○ **Solution :** The velocity of sound in a gas is, $V = \sqrt{\frac{\gamma RT}{M}}$

$$\therefore\quad \gamma = \frac{V^2 M}{RT} \qquad [\ V = 58200\ \text{cm/s},\ M = 4,\ R = 8{\cdot}31\times10^7\ \text{ergs/(mol°C)},\ T = 100\ \text{K}\ ]$$

$$= \frac{(58200)^2\times4}{8\cdot31\times10^7\times100} = 1{\cdot}63$$

● **Ex. 2.12. The mass of one litre of hydrogen and 1 litre of air are 0·896 gm and 1·293 gm respectively at the same temperature and pressure. If the velocity of sound in air at that temperature is 330 m/s, what will be the corresponding velocity of sound in hydrogen ? [H.S. 1998]**

○ **Solution :** We know $V \propto \frac{1}{\sqrt{\rho}}$

$$\frac{V_H}{V_A} = \sqrt{\frac{\rho_A}{\rho_H}} \quad \therefore\ V_H = V_A\sqrt{\frac{\rho_A}{\rho_H}} \quad [\ V_A = 330\ \text{m/s},\ \rho_A = 1.293\ \text{g/lit.},\ \rho_H = 0{\cdot}0896\ \text{g/lit.}]$$

$$\therefore\quad V_H = 330\times\sqrt{\frac{1\cdot293}{0\cdot0896}} = \mathbf{1253\cdot6\ m/s}$$

∴ The required velocity of sound = **1253·6 ms^{-1}**

● **Ex. 2.13. If the temperature of air is raised 10°C to 20°C, what is the percentage change in velocity of sound in it ? [J.E.E. 1997]**

○ **Solution :** $V \propto \sqrt{T}$ $\quad\therefore \frac{V_2}{V_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{293}{283}} = 1\cdot0175 = 1+0\cdot0175$

$$\therefore\quad \frac{V_2 - V_1}{V_1}\times100 = 0\cdot0175\times100 = \mathbf{1{\cdot}75\%}$$

∴ Increase of velocity of sound = 1·75%.

● **Ex. 2.14. The velocity of sound in hydrogen is 1270 ms^{-1}. When hydrogen and oxygen gas are mixed in the ratio 4 : 1, what will be the velocity in the mixture. Molecular weight of oxygen is 32 and that of hydrogen 2.**

○ **Solution :** Let volume of oxygen = V and its density 32 ρ

Then the volume of hydrogen = 4V and its density = 2ρ

$\therefore$ Total volume of the mixture $V_1 = 5V$ and total mass $= 32\rho V + 8\rho V = 40\rho V$

So, the density of the mixture, $\rho_1 = \frac{40\rho V}{5V} = 8\rho$ Now, velocity of sound is $V \propto \frac{1}{\sqrt{\rho}}$

$$\therefore \quad \frac{V_m}{V_H} = \sqrt{\frac{\rho_H}{\rho_m}} = \sqrt{\frac{2\rho}{8\rho}} = \frac{1}{2} \qquad \therefore \quad V_m = \frac{V_H}{2} = \frac{1270}{2} = 635$$

$\therefore$ Velocity of sound in the mixture = 635 ms^{-1}

● **Ex. 2.15. Calculate the velocity of sound at 27°C in air saturated with water vapour. Given, the velocity of sound in dry air at 0°C is 332 m/s and saturated vapour pressure at 27°C is 11.2 mm Hg.**

○ **Solution :** Let V_d = velocity of sound in dry air and V_m the velocity of sound in moist air.

$$\text{Now,}\ V_d = V_m \cdot \sqrt{1 - 0\cdot378\frac{f}{P}} \qquad [\ f = 11\cdot2 \text{ mm Hg},\ P = 760 \text{ mm Hg}\ ]$$

$$\therefore \quad V_d = V_m\left(1 - 0\cdot378 \times \frac{11\cdot2}{760}\right)^{1/2} = V_m\left(1 - \frac{1}{2} \times 0\cdot378 \times \frac{11\cdot2}{760}\right) = \mathbf{0\cdot95\ V_m}$$

Now, the velocity of sound in dry air at 27°C is $V_d = V_0\sqrt{\frac{273+27}{273}}$

$$\therefore \quad V_d = 332\sqrt{\frac{300}{273}} = 348 \text{ m/s} \quad \therefore V_m = \frac{V_d}{0\cdot95} = \frac{348}{0\cdot95} = \mathbf{366\cdot3\ m/s}$$

● **Ex. 2.16. A vibrating tuning fork having frequency 67·5 cm. can produce a sound of wavelength 67·5 cm in air at 17°C. Density of air at NTP = 1·293 g/lit. Calculate the ratio of two specific heats.**

○ **Solution :** Velocity of sound in air at 17°C, $V = n\lambda = 512 \times 67\cdot5$ m/sec

If ρ_0 and ρ be the densities of air at 0°C and 17°C respectively.

$$\therefore \quad \frac{\rho}{\rho_0} = \frac{T_0}{T_1} \qquad \therefore \quad \rho = \rho_0\frac{T_0}{T} = 1\cdot293 \times \frac{273}{373}$$

$$\therefore \quad \rho = 1\cdot205 \text{ g/lit} = 1\cdot205 \times 10^{-3} \text{ g/c.c.} \quad \text{Now, } V = \sqrt{\frac{\gamma P}{\rho}} \quad \therefore \gamma = \frac{\rho V^2}{P}$$

$$\therefore \quad \gamma = \frac{(512 \times 67\cdot5)^2 \times 1\cdot205 \times 10^{-3}}{1\cdot013 \times 10^5} = 1\cdot42$$

$\therefore$ the ratio of two sp. heats of air = **1·42**

● **Ex. 2.17. The velocity of sound in a liquid is 1400 m/s and density of the liquid 0·78 g/c.c. Calculate compressibility of the liquid.**

○ **Solution :** We know, $V = \sqrt{\frac{K}{\rho}}$

$$\therefore \quad \text{Compressibility } \frac{1}{K} = \frac{1}{V^2\rho} \left[V = 14 \times 10^4 \text{ cm/s}\right]$$

$$\therefore \quad \frac{1}{K} = \frac{10^{-8}}{(14)^2 \times 0\cdot78} = 6\cdot54 \times 10^{-11} \text{ cm}^2 \text{ dyne}^{-1}$$

❂ 2.7. Doppler effect ❂

It is an effect associated with waves when the source or the observer is moving. When the source emitting waves and the observer receiving them, are in relative motion with respect to the medium in which the waves propagate, the frequency of the waves received by the observer is different from the frequency of the vibrating source. This phenomenon was first observed by C. J. Doppler in sound waves and is known as **Doppler Effect.**

❖ (a) **Definition : When there is a relative motion between the source of sound waves and the observer receiving them, the apparent change in frequency of the sound as perceived by the observer is known as Doppler effect.**

This phenomenon is exhibited by all types of waves, including light waves which do not require a medium for their propagation. We shall discuss **Doppler effect in sound waves**. We shall consider the motion along the line, joining the source and the observer.

The Doppler effect in sound waves may be readily observed by a person at a railway platform. When fast moving train blowing a whistle approaches the observer, the pitch (frequency) of the whistle appears to rise and it suddenly appears to drop when the engine has passed by.

● (b) **Explanation :** (i) Let a source of sound (say, a tuning fork) is emitting a sound of frequency n. The observer at a certain distance will hear a sound of same frequency n when the source and the observer are stationary. Now the observer is receiving n number of waves per second as emitted by the source per second.

(ii) If there is a relative motion between the source and the observer. (e.g. source moving or observer moving or both moving simultaneously) then the frequency of sound heard by the observer will be different from actual frequency. Now, the observer will receive different number of waves in a second than emitted by the source. When the observer approaches the source, the apparent frequency will increase because now the observer receives more waves in a second. Again when the observer moves away from the source, the apparent frequency will decrease because now the observer receives less number of waves in a second. Similar situation arises when the source is in motion with respect to the observer.

For waves which require a medium for propagation, the apparent change of frequency depends on which of the three — the source, the observer and the medium is moving. We shall compute the change of frequency in the following cases :

➡ (1) Source is in uniform motion – observer stationary :

Let V be the velocity of sound in air and V_s is the velocity of the source (S) and now the observer is stationary at O. Air is not blowing i.e. the medium is stationary.

If the frequency of emitted sound be n, the source will produce n waves in one second. If the source is stationary, these waves will occupy a distance V. This length is shown as AB in fig. 2.2. Hence $n = \dfrac{V}{\lambda}$ (λ = actual wavelength).

Now, let the source is moving towards the observer with a speed V_s ($V_s < V$). The source will come to the position S′. So, $SS' = V_s$. As the velocity of sound with respect to the source remains constant, all n waves emitted per second will be contained within the length ($V - V_s$). So the waves are now compressed and wavelength will decrease. The changed wavelength is

$$\lambda' = \frac{V - V_s}{n}$$

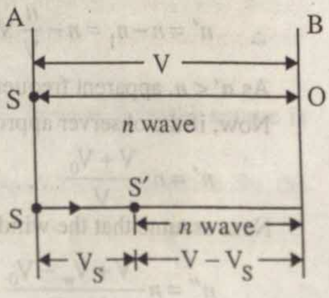


Fig. 2·2

Hence the apparent frequency as heard by the observer is $n' = \frac{V}{\lambda'}$

$$\therefore \quad n' = n \cdot \frac{V}{V - V_s} \ \ldots\ldots\ldots\ldots (2.6)$$

$$\therefore \quad \text{Increase in frequency } \Delta n = n' - n = n\frac{V}{V - V_s} - n = \frac{n\ V_s}{V - V_s} \ \ldots\ldots\ldots (2.7)$$

On the other hand, if the source is moving away from the observer, the sign of V_S will be negative. Now, the apparent frequency is $n' = n\frac{V}{V + V_s}$ ....(2.8)

So, now, $n' < n$ i.e. the apparent frequency will decrease and decrease of frequency is

$$\Delta n = n - n' = \frac{n\ V_s}{V +.V_s}$$

- If wind blows with velocity V_w in the direction of sound, then effective velocity of sound is $V + V_w$. Then by equation (2.6)

$$n' = n \cdot \frac{V + V_w}{V + V_w - V_s} \ \ldots\ldots\ldots\ldots (2.9)$$

If the wind velocity be opposite to sound velocity then

$$n' = n \cdot \frac{V - V_w}{V.- V_w - V_s} \ \ldots\ldots\ldots\ldots (2.10)$$

(2) Observer is in uniform motion—source stationary :

If the source is stationary n waves will be contained within a distance V (V = velocity of sound). So, length of the waves (λ) will not change. So, $\lambda = \frac{V}{n}$.

Now assume that the observe (O) is moving away from the source with a velocity V_0. If the observer be stationary at B then n waves will reach the observer per second. But if the observer moves away by a distance V_0 in one second, all the waves produced in 1 sec. will not reach the observer. The number of waves contained in AA′ will not be heard by the observer.

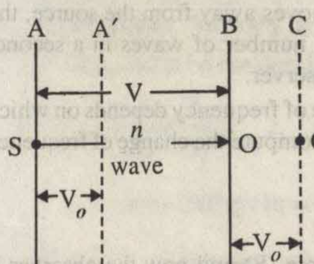


Fig. 2·3

Now, the number of waves contained in AA′ is $n_1 = \frac{n}{V} \cdot V_0$

Hence, the number of waves received by the observer per second i.e. the apparent frequency is

$$\therefore \quad n' = n - n_1 = n - \frac{n}{V} V_0 \qquad \therefore \quad n' = n\frac{V - V_0}{V} \ \ldots\ldots\ldots\ldots (2.11)$$

As $n' < n$, apparent frequency or pitch is less than actual frequency.

Now, if the observer approaches the source then the apparent frequency is

$$n' = n \cdot \frac{V + V_0}{V} \ \ldots\ldots\ldots\ldots (2.12)$$

Next assume that the wind is blowing in the direction of velocity of sound then apparent frequency is

$$n'' = n \cdot \frac{V + V_w - V_0}{V + V_w} \quad \text{[from equation 2.11]}$$

If the wind blows opposite to the sound then apparent frequency

$$n' = n\cdot\frac{V - V_w - V_0}{V - V_w.}$$

(3) Source and observer both in uniform motion :

- **Source approaching a receding observer :**

Let the source (speed, V_s) and observer (speed, V_0) are moving along the same straight line in the same direction. The medium is stationary.

Due to the motion of the source wavelength will change and due to the motion of the observer effective velocity of sound will change.

If observer is stationary, changed wavelength due to the motion of the source is $\lambda' = \frac{V - V_s}{n}$

The apparent frequency is $n' = \frac{V}{\lambda'} = n\cdot\frac{V}{V - V_s}$

This frequency will again change due to the motion of the observer. Considering the motion of the source and observer the final apparent frequency is $n'' = n'\cdot\frac{V - V_0}{V} = n\frac{V}{V - V_s}\cdot\frac{V - V_0}{V}$

$$\therefore \quad n'' = n\cdot\frac{V - V_0}{V - V_s} \quad \text{where } n = \frac{V}{\lambda} \quad \ldots\ldots (2.13)$$

- **Both the source and observer receding from each other :**

The apparent frequency in this case is obtained by reversing the sign of V_s.

$$\therefore \quad n'' = n\cdot\frac{V - V_0}{V + V_s}$$

- **Both the source and the observer approaching each other :**

Reversing the sign of V_0 the apparent frequency in this case $n'' = n\cdot\frac{V + V_0}{V - V_s}$

- **Observer approaching a receding source :**

In this case the signs of V_0 and V_s are reversed and the apparent frequency is $n'' = n\cdot\frac{V + V_0}{V + V_s}$

Hence in general, apparent frequency is $n'' = n\cdot\frac{V \pm V_0}{V \pm V_s}$ .... (2.14)

Now, we consider an example – A car A is moving with a velocity of 90 ms^{-1}. Another car B in front of A is moving with velocity of 30 m/s. Car A is emitting a sound of frequency 1000 Hz. What is the apparent frequency as heard by B ?

First, we substitute different values in equation (2.14) without considering the signs

$n'' = 1000\times\frac{332 \pm 30}{332 \pm 90}$ [Here V = 332 m/s, V_0 = 30 m/s and V_s = 90 m/s]

Next we consider the signs of V_0 and V_s.

(i) Car A (source) is approaching the car B (observer). So when only the motion of the source is considered the denominator of the fraction will be (332–90).

(ii) Car B (observer) is moving away from the source (car A). So the frequency will decrease. So, the numerator of the fraction will be 332–30.

Hence, the apparent frequency, $n'' = 1000\times\frac{332 - 30}{332 - 90} = \mathbf{1289\ Hz}$

2.8. Doppler effect in the case of light waves

All types of waves show Doppler effect. So the apparent change in frequency can also be detected in the case of light. As the velocity of light is extremely large, it is not possible to detect the change of frequency by any terrestrial experiment. We have to consider celestrial or heavenly bodies as sources which are moving with a high speed.

Again frequency of light is extremely large. So, it is not possible to measure the change in frequency of light. But change in wavelength can be measured comparatively easily.

When light emitted by a source is observed through a spectroscope the spectrum shows a number of lines on a dark background. Each line corresponds to a particular wavelength. If there is a change in wavelength, the line shifts from the original position in the spectrum.

When the source approaches the observer, the wavelength decreases and the line shift towards the **violet end** of the spectrum. Similarly, when the source goes away from the observer, the wavelength increases and the line will be shifted towards the **red side** of the spectrum.

By measuring the amount of shift of the line, the change in wavelength can be measured. Hence the velocity of the source of light can be determined.

Application of Doppler effect in light :

Some of the applications of Doppler effect in light are given below :

(i) Doppler effect is applied to measure the speed of the vehicles by the traffic police. When electromagnetic waves of definite frequency emitted by a source are reflected by a vehicle in motion, a shift of wavelength of waves takes place. From the shift, the velocity of the vehicles can be found.

(ii) In Radar, Doppler effect is applied to detect the enemy's plane. The microwave emitted by the transmitter of the radar, after reflection from the enemy plane are picked up by the receiver of the radar. If the aeroplane is approaching the radar, there will be an increase in frequency or decrease in wavelength of the waves. On the other hand, wavelength of the wave reflected by the aeroplane increases when the aeroplane moves away from the radar. By measuring, change in frequency or wavelength, the velocity of the plane can be determined.

The same principle is applied in the case of SONAR in the sea for the detection of submarines and rocks under water.

(iii) We know about the other galaxies of the universe by the Doppler effect. It is found that the wavelength of the spectral lines from the galaxy shows a shift towards the red. But measuring the red shift, the distance and velocities of other galaxies can be predicted. Red shift due to Doppler effect confirms the hypothesis that the universe is expanding.

- **Example 2.18. A train moving with a speed of 80 km/hr crosses a man standing by the side of the railway line and it is continuously blowing its whistle. Compute the ratio the apparent frequencies of the sound heard by the man (i) when the train approaches him and (ii) when the train goes away from him. Velocity of sound is 330 m/s.**

○ **Solution :** Velocity of the source $V_s = 80$ km/hr $= \frac{200}{9} \text{ m/s} = 22\cdot22 \text{ m/s}$

General equation for computing apparent frequency is $n' = n\cdot\frac{V \pm V_0}{V \pm V_s}$ [from equ. 2.14]

1st Case : The source (train) is approaching the observer. So, the apparent frequency will increase and we have to take negative sign for V_s. Now, $V_0 = 0$ as the observer is stationary.

$$\therefore \quad n' = n\cdot\frac{V}{V - V_s} = n\cdot\frac{330}{330 - 22\cdot22} \cdots\cdots\cdots\cdots\cdots\cdots (i)$$

2nd case : Now, the apparent frequency will be $n'' = n \cdot \dfrac{V}{V+V_s}$ [Now, frequency will decrease]

$$\therefore \quad n'' = n \cdot \frac{330}{330+22{\cdot}22} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(ii)}$$

$$\therefore \quad \frac{n'}{n''} = \frac{330+22{\cdot}22}{330-22{\cdot}22} = 1{\cdot}14$$

- **Ex. 2.19. An engine is approaching a hill with a speed of 30 miles per hour and it whistles emitting a sound of frequency 224 Hz. Calculate the apparent frequency of the reflected sound as heard by the driver of the engine. Velocity of sound in air = 1080 ft/s**

○ **Solution :** We know $n' = n \cdot \dfrac{V \pm V_0}{V \pm V_s}$

Here the observer (driver of the train) is approaching the hill and hears the sound reflected from the hill. As the engine approaches the hill, he hears the echo earlier. So, it is a case as if the source and the observer approaches each other. So, the apparent change in frequency is $n' = n \cdot \dfrac{V+V_0}{V-V_s}$

$$\therefore \quad n' = 224 \times \frac{1080+44}{1080-44} \qquad [V = 1080 \text{ ft/s}, V_0 = V_s = 44 \text{ ft/s}, n = 224]$$

$$= \mathbf{243\ Hz}$$

- **Ex. 2.20. Two sources give out identical notes of frequency 1360 Hz. As observer half-way between them moving from each other hears 4 beats per sec. At what speed is he moving ? [Given, the velocity of sound = 340 m/s]**

○ **Solution :** Let the man is moving towards the first source and moves away from the second source.

So, the apparent frequency of sound emitted by the first source is $n' = n \cdot \dfrac{V+V_0}{V}$

and the apparent frequency of sound emitted by the second source as heard by the man is

$$n'' = n \cdot \frac{V-V_0}{V}$$

∴ Difference in apparent frequencies

$$\Delta n = n' = n'' = \frac{n}{V}\left[V+V_0-V+V_0\right] = \frac{n}{V}(2V_0)$$

$$\therefore \quad \text{By question, } 4 = \frac{1360}{340} \times 2 \times V_0 \text{ or, } V_0 = \frac{1}{2}\text{ m/s}$$

$$\therefore \quad V_0 = \frac{1}{2} \times \frac{18}{5} = \mathbf{1{\cdot}8\ km/hr}$$

- **Ex. 2.21. A car travels perpendicular to a high wall with a speed of 26 km/hr. The driver sounds a horn of frequency 200 cps. What is the frequency of sound heard by the driver coming after reflection from the wall ? Velocity of sound = 340 m/s**

○ **Solution :** $n' = n \cdot \dfrac{V+V_0}{V-V_s} = 200 \times \dfrac{340+10}{340-10} \left[V_0 = 36 \times \dfrac{5}{18} = 10 \text{ m/s} \right]$

$$= 200 \times \frac{350}{330} = \mathbf{212{\cdot}12\ Hz}$$

- **Ex. 2.22. Two observers A and B have sources of sound of frequency 500 Hz. If A remains stationary while B moves away with a velocity 10.0 m/s, find the number of beats heard by A and B. (Velocity of sound = 332 m/s)**

○ **Solution : Beats heard by A :** In this case, the observer (A) is stationary and the source (B) is moving away. So

$$n' = n\frac{V}{V+V_s} = 500\times\frac{332}{332+10} = 485{\cdot}4 \text{ Hz}$$

$\therefore$ Frequency of beats = 500 – 485·4 = 14.6 Hz

Beats heard by B : Here the source (A) is stationary and the observer (B) is moving away from the source.

$$\therefore \quad n' = n\cdot\frac{V-V_0}{V} = 500\times\frac{332-10}{332} = 485 \text{ Hz}$$

$\therefore$ frequency of Beats heard by B = 500 – 485 = **15 Hz.**

●● Short Answer Type Questions (with answers) ●●

● **Question 1. Velocity of sound is much higher in solids than in gases. What is the cause of this difference ?** **[J.E.E., 1991, 1996]**

○ **Ans.** The velocity of sound in solids is given by $V = \sqrt{\frac{Y}{\rho_s}}$

Here Y = Young's modulus of the solid and ρ_s is its density,

and the velocity of sound in gases is $V_g = \sqrt{\frac{K}{\rho_g}}$

where K = bulk modulus of the gas and ρ_g = density of the gas.

It is found that $\frac{Y}{\rho_s} > \frac{K}{\rho_g}$. For this reason, the velocity of sound in solid is greater than in gases.

● **Q. 2. What is the fundamental difference between sound wave and light waves ?**

○ **Ans.** The fundamental differences between sound waves and light waves are :

(i) Sound waves are longitudinal elastic waves. But light waves are transverse electromagnetic waves.

(ii) Sound wave cannot be polarised but light wave can be polarised.

(iii) For the propagation of sound wave a material medium is necessary. But light waves do not require any medium for its propagation.

(iv) Velocity of light wave is much greater than the velocity of sound wave.

● **Q. 3. Why a medium is necessary for the propagation of sound waves ? What conditions must the medium fulfil ?** **[J.E.E. 1997]**

○ **Ans. 1st part :** Sound wave is an elastic wave. The mechanical energy of the vibrating body causes disturbance in the medium. This energy of disturbance produces sound wave which passes through the medium through compression and rarefaction. For this reason, without a material medium sound waves can not propagate.

2nd Part : The medium must fulfil the following conditions for transmission of sound waves : (i) The medium must be elastic (ii) Particles of the medium must have inertia and (iii) Force of cohesion must be effective between the molecules.

● **Q. 4. Explain why velocity of sound in air is higher is rainy season than in winter ?**

○ **Ans.** The temperature and moisture content in air is higher is rainy season than in winter. Now, velocity of sound in air is proportional to the square root of absolute temperature of air. i.e. $V \alpha \sqrt{T}$. Velocity of sound increases with temperature of the medium.

Again, moisture content or humidity in air is higher in rainy reason and density of the medium is less. Now $V \alpha \frac{1}{\sqrt{\rho}}$. So, with decrease of density, velocity of sound in air increases.

For these two reasons velocity of sound in air is greater in rainy season than in winter.

● **Q. 5. What is meant by supersonic speed ? What is mac number ?**

○ **Ans.** If the velocity of an object is greater than the velocity of sound in air, the speed is called supersonic speed.

Mac No. : The ratio of the velocity of an object to the velocity of sound in air is called mac number. When mac no is greater than 1, the speed is supersonic.

● **Q. 6. What is ultrasonic ?**

○ **Ans.** The longitudinal waves (i.e. sound waves) whose frequency is higher than audiable frequency (i.e., above 30 KHz) are called ultrasonic.

● **Q. 7. Compare the velocities of sound in hydrogen ($\gamma = 1{\cdot}4$) and in helium gas ($\gamma = 1{\cdot}67$) at the same temperature.**

○ **Ans.** Velocity of sound in a gas, $V = \sqrt{\frac{\gamma RT}{M}}$

If V_1 and V_2 be the velocities of sound in hydrogen and in helium then

$$\frac{V_1}{V_2} = \sqrt{\frac{\gamma_1}{\gamma_2} \times \frac{M_2}{M_1}} = \sqrt{\frac{1{\cdot}4}{1{\cdot}67} \times \frac{4}{2}} = 1{\cdot}3$$

● **Q. 8. Show that the velocity of longitudinal wave in gaseous medium is given by $V = \sqrt{\frac{1}{3}\gamma V_{rms}}$ where V_{rms} = r.m.s. velocity of gas molecule.**

○ **Ans.** The velocity of sound in a gaseous medium, $V = \sqrt{\frac{\gamma P}{\rho}}$

Now, from kinetic theory of gases, $P = \frac{1}{3}\rho V_{rms}^2$

$$\therefore \quad \frac{P}{\rho} = \frac{1}{3}V_{rms}^2 \qquad \therefore \quad V = \sqrt{\frac{1}{3}\gamma V_{rms}^2}$$

● **Q. 9. Prove that according to Doppler principle the change of frequency is greater when the source approaches the observer than when the observer approaches the source with the same speed.**

○ **Ans.** When the source approaches the stationary observer, the apparent frequency is $n' = n \cdot \frac{V}{V - V_s}$

and when the observer approaches the stationary source the apparent frequency is

$$n'' = n \cdot \frac{V + V_s}{V} \quad [\text{As } V_0 = V_s]$$

$$\therefore \quad \frac{n'}{n''} = \frac{V^2}{V^2 - V_s^2} > 1 \quad \therefore n' > n''$$

● E◇X◇E◇R◇C◇I◇S◇E ●

A. Short answer type questions :

1. Sound wave is an elastic wave. Explain.
2. Is sound wave longitudinal or transverse ? Explain.
3. Sound waves can travel through any material medium. Why ?
4. What is ultrasonic ? Mention some of its applications.
5. Explain why a medium is necessary for the propagation of sound. What conditions must the medium fulfil ? [J.E.E. 1997]
6. What are the points of similarities and dissimilarities between sound and light waves ?
7. Which property of the wave proves that the sound waves are logitudinal ?
8. The velocity of sound is generally greater in solid than in gases at NTP. – Why ? [J.E.E. 1991, 1996]
9. Show that, under isothermal condition, bulk modulus of the gas is equal to its pressure.
10. Show that, under adiabatic condition, bulk modulus of a gas is the product of pressure and the ratio of the two specific heats of the gas.
11. Higher the density of a gas, lower is the velocity of sound in that gas. But velocity of sound is higher in solids although the density of solid is greater than that of a gas. Explain.
12. Will the velocity of sound in two gases be equal if the ratio P/ρ be same for both gases ?
13. Why do the astronauts on the moon have to talk to each other over radio ? [J.E.E. 1998]
14. Why is the velocity of sound in air is greater in rainy season than in winter ?
15. Between oxygen and hydrogen in which gas, the velocity of sound is greater ?
16. At what temperature, the velocity of sound in air is double the velocity at 0°C ?
17. When one end of a long metal tube is hammered two sounds are heard at the other end. Why ?
18. What is Doppler effect ?
19. The value of γ of a gas is 1·41 ; show that the velocity of sound in the gas is $v = 0{\cdot}68C$ when C is r.m.s. of the gas molecules.
20. Prove that if the observer moves away from the source with a velocity half of the velocity of sound in air, the apparent frequency will be half of the actual frequency.
21. Is Doppler effect observed in light waves ?
22. Are diffraction and polarisation possible in the case of sound waves ? [J.E.E. 2001]

B. Essay type questions :

1. (a) Mention the evidences to prove that sound waves are longitudinal waves.
   (b) Explain why a material medium is necessary for the propagation of sound waves ?
2. State and explain Newton's formula for the velocity of sound waves in a gas. What is Laplace's correction of Newton's formula ?
3. What are the effect of (i) pressure, (ii) temperature, (iii) density and (iv) humidity on the velocity of sound in air ?
4. Discuss the effect of temperature and pressure on the velocity of sound in a gas. [J.E.E. 1996]
5. What is Doppler effect ?

   Obtain the expressions for the apparent frequency in the following cases :

   (i) Source moving and observer stationary.
   (ii) Observer moving and source stationary.
   (iii) Source and observer both are moving.
6. Show that under isothermal condition bulk modluus of the gas is equal to its pressure.

C. Simple numerical problems :

1. The density of a solid medium is 8000 kg/m^3 and its Young's modulus is 1·62×10^{11} Nm^{-2}. Calculate the velocity of sound in the solid. **[Ans.** 4500 ms^{-1}]

2. Calculate the increase of velocity of sound in air where its temperature is raised from 0°C to 1°C. **[Ans.** 0·61 ms^{-1}]

3. What is the velocity of sound in a diatomic gas at 0°C ? $\left[\textbf{Ans. } \sqrt{\frac{382\,R}{M}}\right]$

4. At the same temperature and pressure the velocity of sound in oxygen is V_0 and in hydrogen the velocity is V_H. Obtain the relation between V_0 and V_H. [Given, molecular weight of oxygen = 32 and of hydrogen = 2] **[Ans.** $V_H = 4V_0$]

5. The velocity of sound in a gas at 0°C is 1260 m/s. Its molecular weight 2. Find its γ. R = 8·31 J mol^{-1} K^{-1}. **[Ans.** 1·4]

6. At what temperature velocity of sound in air be equal to half the velocity at 0°C ? **[Ans.** –204·75°C]

7. The velocity of sound in air at 0°C is 332 m/s. At what temperature the velocity be 350 m/s ? **[Ans.** 30·4°C]

8. Find the velocity of sound in hydrogen gas at NTP. The density of hydrogen at NTP = 0·09 g/litre. γ = 1·4 **[Ans.** 1255·25 m/s] **[H.S. 1988]**

9. Determine the percentage change in velocity of sound when the temperature changes from 10°C to 20°C. **[Ans.** 1·77%] **[J.E.E. 1997]**

10. The temperature of air falls from 27°C to 7°C. What is the percentage change in velocity of sound in air ? **[Ans.** –3·4%]

11. Calculate the speed of sound in steel, given speed of sound in water = 1493 m/s, and bulk modulus and density of steel 80 and 8 times respectively of those of water. **[Ans.** 4721·3 m/s] **[J.E.E., 1995]**

12. The masses of one litre of hydorgen and one litre of air are 0·089 and 1·293 gm respectively at the same temperature and pressure. If the velocity of sound in air at the temperature be 330 m/s, what will be the corresponding velocity in hydrogen ? **[Ans.** 396 ms^{-1}] **[H.S. 1998]**

13. A stone is dropped into a well 44·1 m deep. Find after how much time will the sound of the splash be heard. Velocity of sound in air = 340 m/s. **[Ans.** 3·13 sec.]

14. If the velocity of sound in oxygen under normal conditions be 317 ms^{-1}, what should be its velocity in hydrogen at 30°C and 374 mm pressure. **[Ans.** 1335·85 m/s]

15. When one end of metal tube of length 1 km is hammered, the time interval between two sounds heard at the other end is 2·8 sec. If the velocity of sound in air be 333 m/s. calculate the velocity of sound in the metal. **[Ans.** 4930 m/s]

16. A tuning fork of frequency 512 Hz emits a tone of wavelength 67·5 cm in air at 27°C. Calculate the value of γ of air. Given density of air at NTP is 1·293×10^{-3} g/c.c. **[Ans.** γ = 1·39]

17. Calculate the velocity of sound in carbon-di-oxide gas at 30°C and 760 mm of Hg pressure. Given 11·1 litre of carbon-di-oxide gas at NTP weighs 22 g and the ratio of two specific heats of the gas is 1·26. **[Ans.** 267·3 ms^{-1}]

18. The velocity of sound in a gas at 16°C is 340 m/s. If the pressure is double and temperature is 168°C what will be the velocity in the gas ? **[Ans.** 420 ms^{-1}]

19. What is the velocity of sound at 27°C in air saturated with water vapour ? Assume that velocity of sound in dry air at 0°C is 332 m/s and saturation vapour pressure at 27°C is 18·2 mm Hg. **[Ans.** 365·2 m/s]

20. Young's modulus of brass is 9×10^{10} N/m^2 and its density 8500 kg/m^3. The corresponding quantities for steel are 2×10^{11} N/m^2 and 7800 kg/m^3. Obtain the ratio of the velocities of sound in brass and steel. **[Ans.** 0·64]

21. For helium γ = 1·67 and at N.T.P. 22.4 litres of helium weighs 4g. Find the velocity of helium gas at 0°C and at normal pressure. **[Ans.** 973·3 ms^{-1}]

22. At what temperature the velocity of sound in nitrogen gas is exactly equal to the velocity of sound in oxygen gas. Given, atomic weight of nitrogen is 14 and that of oxygen is 16. [Ans. –21°C]

23. A man hears the sound of thunder 4 sec after he sees the flash of lightning. If the air temperature be 4°C, calculate the distance of the man from the place of thunder. Gives the velocity of sound in air at 0°C is 332·5 m/s. **[Ans.** 1378·8 m]

24. The velocity of sound in a gas at 50°C is 340 ms^{-1}. If the pressure is double and temperature be 125°C, what will be the velocity of sound now ? **[Ans.** 377·41 m/s] **[H.S. 2002]**

25. The ratio of the densities of oxygen at nitrogen is 16 : 14. At what temperature velocity of sound in oxygen be equal to the velocity of sound in nitrogen gas ? **[Ans.** 56·1°C]

26. A train while blowing a horn, is approaching a railway platform with a velocity of 90 km/hr. The frequency of emitted sound is 600 Hz and velocity of sound is 325 m/s. What is the apparent frequency of sound as heard by a person standing on the platform ? **[Ans.** 650 Hz] **[J.E.E. 1986]**

27. A train is approaching a stationary observer with a velocity of 118·8 km/hr and it is blowing a horn continuously at frequency 500 Hz. The observer is approaching the train with the same speed. Calculate the apparent frequency of sound as heard by the observer. Velocity of sound in air = 330 m/s. **[Ans.** 610 Hz]

28. A source of sound having frequency 256 Hz is moving towards a wall with a velocity of 5 m/s. How many beat per second will be heard if sound travels at a speed of 330 m/s. The source of sound is situated between the listener and the wall. **[Ans.** 8 Hz] **[I.I.T.]**

◢ D. Harder numerical problems :

1. The velocity of sound in a gas at 20°C is 1305 ms^{-1}. If γ of the gas be 1·4, calculate its molecular weight. R = 8·31 J mol^{-1} K^{-1}. **[Ans.** 2]

2. The velocity of sound in helium gas at –173°C is 582 m/s. If the molecular wt of helium be 4, calculate the value of γ of the gas. [R = 8·31 J mol^{-1} K^{-1}] **[Ans.** 1·63] **[J.E.E. 1998, 2000]**

3. A vibrating tuning fork of frequency 512 Hz at 15°C emits a sound of wavelength 66 cm in air. If the density of air at NTP be 1·29 g/lit, find the ratio of the two specific heats of air. **[Ans.** 1·38]

4. The velocity of sound in hydrogen gas at 0°C is 1200 m/s. When some oxygen is mixed with hydrogen, the velocity of sound in the mixture is 500 m/s. Calculate the ratio of hydrogen and oxygen in the mixture by volume. Given density of oxygen is 16 time more than that of hydrogen. **[Ans.** 2·15] **[J.E.E. 1982]**

5. An aluminium rod of length 90 cm is clamped at the middle and set into longitudinal vibration by resin cloth. If the rod emits its fundamental frequency, calculate : (i) velocity of sound in aluminium, (ii) wavelength of emitted sound, (iii) frequency, (iv) wavelength of sound in air. Given density of aluminium 2600 kg/m^3, its Young's modulus. $Y = 7{\cdot}8\times10^{10}$ N/m^2. Velocity of sound in air = 340 m/s.

[Ans. (i) 5480 ms^{-1}, (ii) 180 cm, (iii) 3050 Hz, (iv) 11·1 cm]

6. The velocity of sound in hydrogen is 1270 m/s. When oxygen and hydrogen are mixed in the ratio 1 : 4 by volume, calculate the velocity of sound in the mixture. **[Ans.** 635 m/s]

7. The velocity of sound in hydrogen at 0°C is 4200 ft/sec. If hydrogen and oxygen are mixed in the ratio 2 : 1, what will be the velocity in mixture at 0°C. **[Ans.** 1714·3 ft/s]

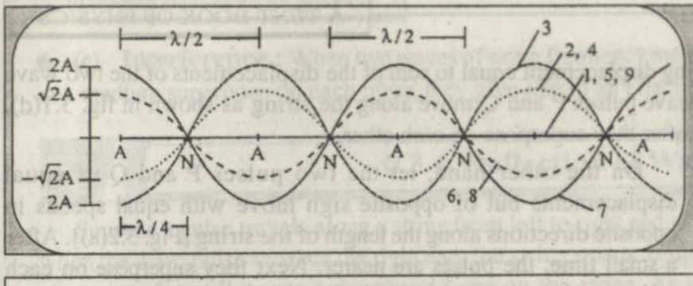
8. The tension in a sonometre 64 N. When the string is set into vibration, its fundamental is at resonance with a tuning fork. The length of the sonometer wire is 10 cm and mass 1g. The vibrating tuning fork is removed with a certain speed and one beat per second is heard. Calculate the velocity with which the tuning fork is removed. The velocity of sound in air is 300 m/s. **[Ans.** 0·752 ms^{-1}] **[I.I.T. 1983]**

9. The velocity of sound in air saturated with water vapour at 30°C in 350 m/s. Calculate the velocity of sound in dry air and at normal pressure. Mass of 1 c.c. of dry air at NTP 0·001293 gm. Vapour density of water vapour is 0·62 and barometer reading 760 mm Hg. Aqueous tension at 30°C = 31·5 mm Hg. **[Ans.** 329·59 ms^{-1}]

10. The ratio of the frequencies of sound as received by a stationary observer when the source moves towards away from the observer is 9 : 8. If the velocity of source of sound be 20 m/s, calculate the velocity of sound in air.

[Ans. 340 m/s]

11. Two tuning forks of frequencies of 340 Hz each are in motion relative to a stationary observer. One fork moves away from the observer, while the other moves towards him at the same speed. The observer hears beats of frequency 3 Hz. Find the speed of the tuning forks. **[Ans.** 1·5 m/s] **[I.I.T. 1986]**



CHAPTER 3

SUPERPOSITION OF WAVES

● **TOPICS :** ○ Introduction; ○ Superposition of Waves; ○ Principle of Superposition; ○ Reflection of waves; ○ Refraction of waves; ○ Stationary waves; ○ Explanation of formation of stationary waves by Analytical method; ○ Explanation of formation of stationary waves by Graphical method ; ○ Characteristics of stationary waves; ○ Distinction between progressive waves and stationary waves; ○ Transverse vibration of string ; ○ Fundamental frequency of transverse vibration of a string; ○ Laws of transverse vibrations of stretched strings; ○ Different modes of vibration of a stretched string ; ○ Analytical treatment of vibration of a string fixed at both ends; ○ Stringed instruments; ○ Comparison of velocities of longitudinal and transverse waves in stretched string; ○ Longitudinal vibration of a rod ; ○ Vibration of air column; ○ Frequencies of fundamental in closed and open pipes of same length; ○ Effect of temperature and humidity on the frequency of air column ; ○ Determination of velocity of sound or frequency of a tunning fork by resonance air column method ; ○ Beats; ○ Condition of perception of beats; ○ Practical application of beats; ○ Short answer type questions (with answers); ○ Exercise.

✪ 3.1. Introduction ✪

Two or more than two waves can travel through the same medium simultaneously. Each proceeds without being affected by the others. That different waves can pass through a medium independently can be understood from the following examples.

(i) We can hear quite distinctly the sounds of several independent conservations of persons gathered in a room. (ii) If two stones are dropped into a pond, each set of ripples passes through the other without any change of shape, direction or speed. (iii) A clear photograph is obtained with a camera. It shows that lights from different objects have entered through the shutter opening. The light waves do not affect one another.

✪ 3.2. Superposition of Waves ✪

In our previous discussions we have considered the passage of a single wave through a medium. We shall now consider the effect when two or more waves proceed simultaneously through the same medium.

When two or more wave trains travel in a medium at a time, each train of wave proceeds independently, as if the other trains of waves are absent. The displacements of a particle of the medium at any instant due to these waves are independent of one another and is obtained by the vector sum of individual displacements. **Such process by which different trains of waves travelling through a medium simultaneously overlap one another without losing their individual nature or shape is called superposition of waves.**

We consider two similar wave pulses P and Q having displacement of same signs (both upwards). These pulses travel along the length of the string with equal speeds but in opposite direction [Fig. 3.1(a)]. After a small time interval, the position of the pulses are shown in fig. 3.1(b). Next fig. 3.1(c) shows the effect at any time t, when the pulses superpose.

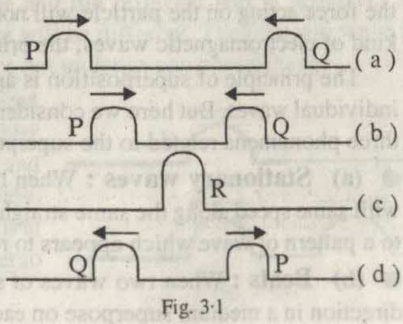


Fig. 3·1

It is found that a wave pulse R having displacement equal to sum of the displacements of the two wave pulses is produced. After a while the wave pulses P and Q move along the string as shown in fig. 3.1(d). Evidently the pulses retain their shape after they superpose on each other.

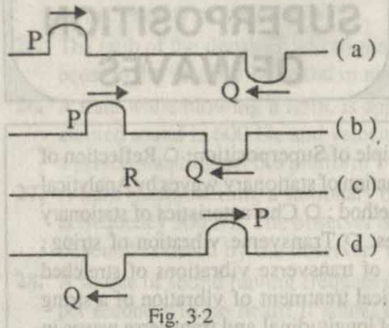


Fig. 3·2

On the other hand, let the two pulses P and Q of equal displacements but of opposite sign move with equal speeds in opposite directions along the length of the string [Fig. 3.2(a)]. After a small time, the pulses are nearer. Next they superpose on each other producing a resultant pulse R of zero displacement as shown in fig. 3.2(c). After crossing each other they continue to move retaining their shape [Fig. 3.2(d)].

In both the cases the resultant waveform has been determined by adding the displacement of the two wave pulses by the law of vector addition.

❂ 3.3. Principle of Superposition ❂

When two or more waves are passing simultaneously through the same medium, the resultant displacement of each particle of the medium at any instant is equal to the vector sum of the displacements produced by the waves separately. This principle is known as the principle of superposition.

In general, if $\vec{y_1}, \vec{y_2}, \vec{y_3}, \ldots \vec{y_x}$ are the displacements of the particle of the medium, when n waves superpose on each other, then the resultant displacement is

$$\vec{y_1} = \vec{y_1} + \vec{y_2} + \ldots \vec{y_n}$$

(i) If we consider two wave pulses which produce equal displacements in the same direction then the resultant displacement of the particle will be

$$y = y_1 + y_2 = 2a \text{ [Putting } y_1 = y_2 = a]$$

It is the same result as obtained graphically in fig. 3.1(c). (ii) On the other hand, if the wave pulses produce equal and opposite displacements of the particle of the medium, the resultant displacement will be

$$y = y_1 - y_2 = a - a = 0$$

Again, it is same resultant as obtained graphically in fig. 3.2(c).

The principle of superposition is applicable to all types of waves provided the amplitudes of the waves are not too large. For waves travelling through a material medium, like water waves, sound waves etc. the principle of superposition does not hold if the amplitude is large. Reason is – if the amplitude is not small, the force acting on the particle will not be proportional to the displacement. But for light waves and other kind of electromagnetic waves, the principle holds rigorously.

The principle of superposition is applicable regardless of frequency or direction of propagation of the individual waves. But here we consider waves of equal or nearly equal frequencies. Here we small consider three phonomena related to the superposition of sound waves :

- **(a) Stationary waves :** When two identical waves having same frequency and amplitude moving with same speed along the same straight time but in opposite direction, superpose each other, they give rise to a pattern of wave which appears to remain stationary in space. It is called **stationary wave.**
- **(b) Beats :** When two waves of slightly different frequencies moving with same speed in the same direction in a medium superpose on each other, they give rise to **beats**.

- **(c) Interference :** When two waves of same frequency moving with same speed in the same direction in a medium superpose on each other, they give rise to an effect called interference of waves.

3.4. Reflection of Waves

Suppose a pulse travels along a string from left to right end which is rigidly fixed on a wall [Fig. 3.3(a)] when the wave pulse reaches the right end, the string exerts an upward force on the wall. By Newton's law of motion, the wall exerts a downward force on the string. As a result the string is accelerated in the downward direction and a reflected pulse is created. It has negative displacement and travels in the opposite direction [Fig. 3.3(c)]. So crest is reflected as trough.

Now, let a harmonic wave instead of a pulse be incident on a wall. then reflected harmonic wave will travel in opposite direction in which the particle displacement is reversed. Let the incident wave (which travels in + x-axis) be represented as

$$y_i = a \sin \frac{2\pi}{\lambda}(vt - x) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.1)$$

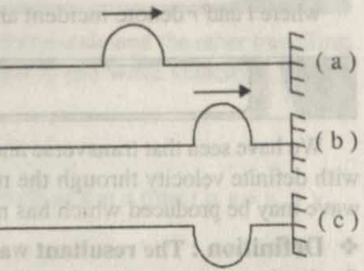


Fig. 3·3

Then the reflected wave travelling in $-ve$ x-axis is

$$\text{Now, } y_r = -a \sin \frac{2\pi}{\lambda}(vt + x)$$

As the reflected waves travels in the same medium as the incident wave, its speed, wavelength and frequency will remain unchanged.

$$\text{Now, } -a \sin \frac{2\pi}{\lambda}(vt + x) = a \sin\left\{\frac{2\pi}{\lambda}(vt + x) \pm \pi\right\} \ldots\ldots\ldots(3.2)$$

So, the reflected wave suffers a phase change of π radians with respect to the incident wave. It is true for sound waves, light waves or any other types of waves.

3.5. Refraction of waves

In the last article we have discussed the reflection of a wave pulse in a stretched string at a rigid boundary i.e. infinitely massive wall. Now let a positive pulse (i.e. having displacement in the upward direction) starts at the free end of a light string. A and meets a heavier string B which is joined to the other end of the light string A [Fig. 3.4(a)]. Here both the strings are under the same tension and mass per unit length of B is higher than that of A. When the pulse reaches the point of contact of the strings, it is partly transmitted and partly reflected. The transmitted or refracted pulse is positive and the reflected part is negative. The velocity of the refracted wave will be different. The reflected wave has suffered 180°, phase difference, while the transmitted pulse does not undergo any phase change.

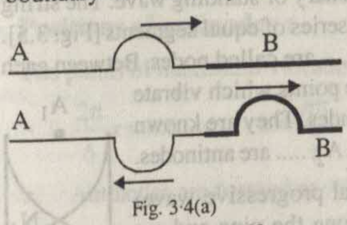


Fig. 3·4(a)

Now, we consider the reverse situation. The pulse is produced in the heavier string B and falls at the point of contact of the lighter thin string A [Fig. 3.4(b)]. Here also the pulse is partly reflected and partly transmitted. But the reflected wave is not inverted in this case. It proved that when a wave travelling along a denser medium falls at the boundary of rarer medium, the reflected wave does not undergo any phase change.

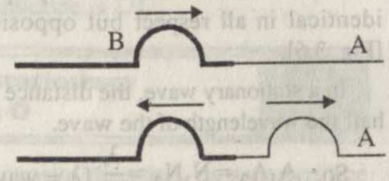


Fig. 3·4(b)

If a wave, instead of pulse, falls at the interface of two media, a part will be refracted in the second medium and the rest is reflected back to the first medium.

- **A point to note :** The wavelength and velocity of the refracted wave are different from those of the incident wave. But the frequency of the refracted wave will remain unchanged. The refracted wave obeys Snell's law of refraction. So the frequency of both incident and refracted waves is

$$n=\frac{v_i}{\lambda_i}=\frac{v_r}{\lambda_r}$$

where *i* and *r* denote incident and refracted wave.

❂ 3.6. Stationary waves ❂

We have seen that transverse and longitudinal waves are **progressive waves** because these waves move with definite velocity through the medium. But by superposition of two progressive waves, a new kind of wave may be produced which has no forward motion but remains steady in space.

❖ **Definition : The resultant wave produced by the superposition of two similar progressive waves (i.e. having same wavelength and amplitude) travelling in opposite directions is called stationary waves.**

In stationary waves there are certain points, where the amplitude is zero and strain is maximum. These points are called **nodes**. Midway between these nodes, there are another category of points, where amplitude is maximum and strain is zero. These points are called antinodes. In between nodes and antinodes, the amplitude lies between zero and maximum.

Like progressive waves, stationary waves may be transverse or longitudinal in nature. Transverse stationary waves are produced by the superposition of two progressive transverse waves e.g., stationary waves are produced in the vibrating string of a sonometer.

Similarly, longitudinal stationary waves are formed due to superposition of two progressive longitudinal waves. e.g., the stationary waves formed in the vibrating air columns in the pipes, Kundts tube and resonance apparatus.

- **Demonstration :** A long uniform string is kept horizontally by fixing its two ends to rigid supports [Fig. 3.5]. It is plucked at the middle. The resulting transverse waves travels along the string to both ends and are reflected. These reflected waves travel along the string in opposite directions with the same velocity.

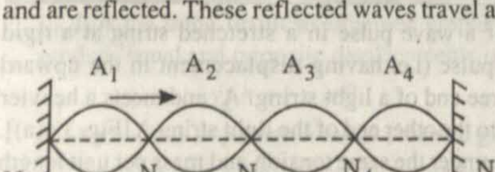


Fig. 3·5

These waves being similar in all respect, combine to form a transverse stationary or standing wave. The string is found to vibrate in a series of equal segments [Fig. 3.5]. The points N_1, N_2, N_3 ..... are called nodes. Between each pair of nodes, there are points which vibrate with maximum amplitudes. They are known as antinodes. In fig. A_1, A_2, ... are antinodes.

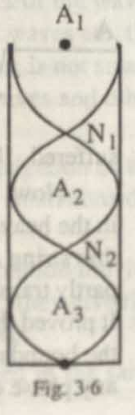


Fig. 3·6

Similarly when a vibrating tuning fork (which produces longitudinal progressive wave) is held at the mouth of an origin pipe, the longitudinal waves travel along the pipe and are reflected from the other end. Superposition of the direct and the reflected waves which are identical in all respect but oppositely moving produce longitudinal stationary waves [Fig. 3.6].

In a stationary wave, the distance between two consecutive nodes or antinodes is equal to half the wavelength of the wave.

So, $A_1A_2 = N_1N_2 = \frac{\lambda}{2}$ [λ = wavelength of the wave]

- It may be mentioned that all sounds emitted from musical instruments are the result of stationary waves. In stringed instruments like sitar, esraj etc. stationary waves are produced by the superposition of transverse waves, while in wind instruments like flute, organ etc., they are produced by the superposition of longitudinal waves.

3.7. Explanation of formation of stationary waves by Analytical method

Here we consider two progressive waves – one travelling along positive x-axis and the other travelling along the negative x-axis. Both the waves have amplitude 'a' wavelength λ and wave velocity v. So the superposing progressive waves are

$$y_1 = a \sin \frac{2\pi}{\lambda}(vt - x) \text{ and } y_2 = a \sin \frac{2\pi}{\lambda}(vt + x)$$

By the principle of superposition, the resultant displacement of a particle at x at a time t is $y = y_1 + y_2$

$$\therefore \quad y = a \sin \frac{2\pi}{\lambda}(vt - x) + a \sin \frac{2\pi}{\lambda}(vt + x)$$

$$= 2a \cdot \sin \frac{2\pi}{\lambda} vt \cdot \cos \frac{2\pi}{\lambda} x = \mathrm{A} \cdot \sin \frac{2\pi}{\lambda} vt \quad \left[\mathrm{A} = 2a \cdot \cos \frac{2\pi}{\lambda} x\right] \ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

This equation represents a simple harmonic vibration of same wavelength λ as the superposing waves. The amplitude $\mathrm{A} = 2a \cos \frac{2\pi}{\lambda} x$ is not a constant. For different values of x, the amplitude has different values. It must be mentioned that this simple harmonic vibration does not represent a progressive wave because its phase does not contain term like $(vt - x)$. So the above equation represents a stationary wave.

- **Positions of Nodes :**

The nodes will be formed at the points where $\cos \frac{2\pi}{\lambda} x = 0$

$$\therefore \quad \cos \frac{2\pi}{\lambda} x = \cos (2n+1) \frac{\pi}{2} \quad \text{or,} \quad x = (2n+1) \frac{\pi}{4} \quad [n = 0, 1, 2, \ldots.. \text{ etc.}]$$

$\therefore$ Nodes will be formed at $x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots..$ the particles at these points will be at rest. Evidently, the distance between two consecutive nodes is $\lambda/2$.

- **Positions of antinodes :**

The points of maximum vibration or antinodes will be formed at those points for which $\cos \frac{2\pi}{\lambda} x = \pm 1$

$$\therefore \quad \frac{2\pi}{\lambda} x = n\pi \quad \text{or,} \quad x = \frac{n\pi}{2} \quad [n = 0, 1, 2, 3 \ldots..]$$

$\therefore$ Antinodes are formed at $x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \ldots..$

The amplitude of vibration at these points is $\pm 2a$. Also the distance between any two consecutive antinodes is $\lambda/2$. Evidently, the distance between one node and next antinode is $\lambda/4$.

3.8. Explanation of formation of stationary waves by Graphical method

We know that two identical progressive waves travelling from opposite direction, on superposition produce stationary waves.

The formation of stationary waves by graphical method is explained below.

One progressive wave (shown by continuous curve) is travelling through the medium from left towards right. Another identical progressive wave (shown by dotted curve) proceeds from right to left along the same line [Fig. 3.7].

(i) Let us reckon the time from the moment the two waves meet in opposite phase at a point as shown in fig. 3.7(a). Now, the crest of one coincides with the trough of the other so that the resultant displacement at all points will be zero because the displacement of the individual waves are equal and opposite. So, the resultant curve is a straight line which is shown by thick line [Fig. 3.7(a)].

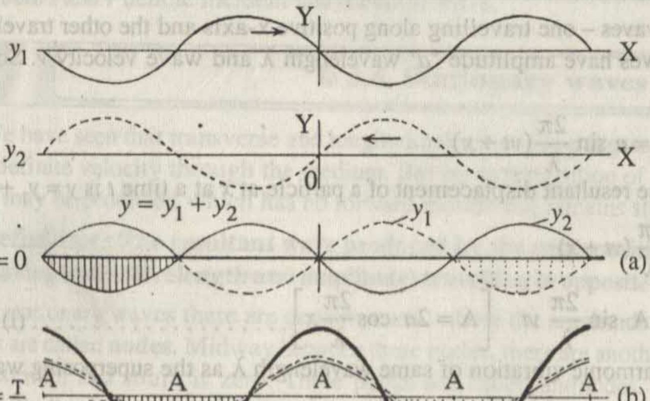


Fig. 3·7

(ii) One quarter of a period later i.e. at $t = T/4$, first wave advances $\lambda/4$ towards right and the second wave by $\lambda/4$ towards left. Now the waves meet in same phase. The crest of one wave falls on the crest of the other wave [Fig. 3.7(b)]. At points marked A, the displacement is maximum. Total displacement at these points is sum of the displacements produced by the two waves. Again at the points marked N, displacement is zero. The particles at these points are at rest. The resultant wave has been shown by thick curve [Fig. 3.7(b)].

(iii) At time $t = T/2$, each wave has travelled a distance $\lambda/4$ further in their respective directions. It means that both the waves has moved a distance equal to $\lambda/2$ in opposite directions so that crest of one falls on the trough of the other. All the particles have zero displacement and the resultant curve is again a straight line [Fig. 3.7(c)].

(iv) At $t = \frac{3T}{4}$, the waves move through a distance $\frac{3\lambda}{4}$. The resultant displacement of all the points is now same as at T/4 but in opposite direction [Fig. 3.7(d)]. Here also the points marked A has maximum displacements and the points marked N have zero displacements.

(v) When $t = T$ i.e. after one complete time period, the initial state of affairs as in fig 3.7(a) is repeated [Fig. 3.7(e)].

The following results are obtained from our previous discussions.

(a) At points marked A, the particles always have maximum displacements i.e. the particles at these points vibrate with maximum displacements. These points are called antinodes.

These antinodes are situated at $x = 0, \frac{\lambda}{2}, \frac{3\lambda}{2}, ...$

Evidently the distance between two antinodes is $\lambda/2$.

(b) At points marked N, the displacement of the particles is zero. The particles at these points are at rest. These are called nodes.

The nodes are situated at $x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, ...$

Here also the distance between two nodes is $\lambda/2$. Again the distance between one node to next antinode is $\lambda/4$.

In fig 3.8 we show only the resultant wave formed by the superposition of the identical progressive waves. Here we can have better idea regarding the formation of nodes and antinodes. The superposing waves are not shown. Each wave is formed after a time interval T/4.

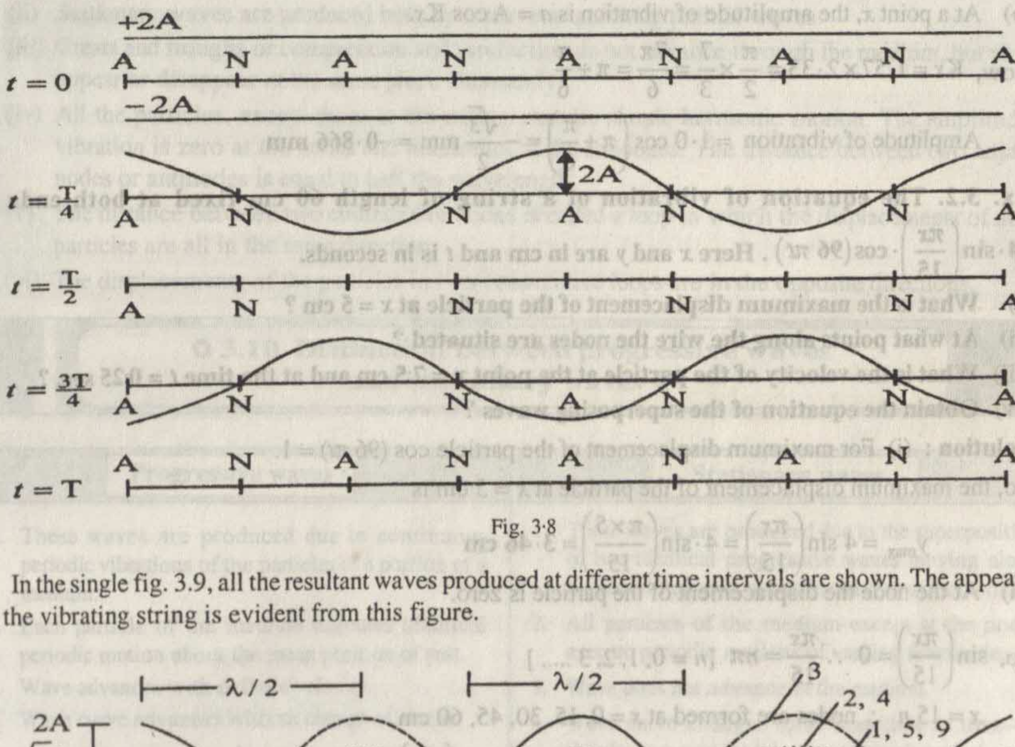


Fig. 3·8

In the single fig. 3.9, all the resultant waves produced at different time intervals are shown. The appearance of the vibrating string is evident from this figure.

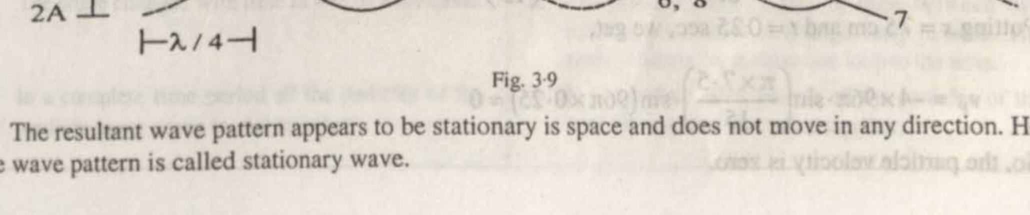


Fig. 3·9

The resultant wave pattern appears to be stationary is space and does not move in any direction. Hence the wave pattern is called stationary wave.

●● Numerical Examples ●●

● **Example 3.1. The equation of a stationary wave formed by the superposition of two identical progressive waves travelling in opposite direction is given by :**

$$y = A\cos Kx\cdot\sin wt\ \left[K = \frac{2\pi}{\lambda}\right]\ A = 1{\cdot}0\text{ mm},\ K = 1{\cdot}57\text{ cm}^{-1}\text{ and }\omega = 78{\cdot}5\text{ rad/s}$$

Calculate (a) the velocity of the superposing waves and (b) The amplitude of vibration of the wave at $x = 2{\cdot}33$ cm.

○ **Solution :** (a) The equations of the superposing waves are

$$y_1 = \frac{A}{2}\cdot\sin(\omega t - Kx)\text{ and }y_2 = \frac{A}{2}\cdot\sin(\omega t + Kx)$$

So the wave velocity, $v = \frac{\omega}{K} = \frac{78{\cdot}5}{1{\cdot}57} = \mathbf{50\ cm/s.}$

(b) At a point x, the amplitude of vibration is $a = A\cdot\cos Kx$.

Now, $Kx = 1{\cdot}57\times 2{\cdot}33 = \frac{\pi}{2}\times\frac{7}{3} = \frac{7\pi}{6} = \pi + \frac{\pi}{6}$

$\therefore$ Amplitude of vibration $= 1{\cdot}0\cos\left(\pi + \frac{\pi}{6}\right) = -\frac{\sqrt{3}}{2}\text{ mm} = \mathbf{-0{\cdot}866\ mm}$

● **Ex. 3.2. The equation of vibration of a string of length 60 cm fixed at both ends is $y = 4\cdot\sin\left(\frac{\pi x}{15}\right)\cdot\cos(96\,\pi t)$. Here x and y are in cm and t is in seconds.**

(i) What is the maximum displacement of the particle at $x = 5$ cm ?

(ii) At what points along the wire the nodes are situated ?

(iii) What is the velocity of the particle at the point $x = 7{\cdot}5$ cm and at the time $t = 0{\cdot}25$ sec. ?

(iv) Obtain the equation of the superposing waves ?

○ **Solution :** (i) For maximum displacement of the particle $\cos(96\,\pi t) = 1$.

So, the maximum displacement of the particle at $x = 5$ cm is

$$y_{\max} = 4\sin\left(\frac{\pi x}{15}\right) = 4\cdot\sin\left(\frac{\pi\times 5}{15}\right) = \mathbf{3{\cdot}46\ cm}$$

(ii) At the node the displacement of the particle is zero.

So, $\sin\left(\frac{\pi x}{15}\right) = 0\ \therefore \frac{\pi x}{15} = n\pi\ \ [n = 0, 1, 2, 3\ .....\ ]$

$\therefore$ $x = 15\,n$ $\therefore$ nodes are formed at $x = 0, 15, 30, 45, 60$ cm.

(iii) Here, the displacement of the particle is $y = 4\cdot\sin\left(\frac{\pi x}{15}\right)\cdot\cos(96\,\pi t)$

$\therefore$ Particle velocity, $v_P = \frac{dy}{dt} = -4\,(96\pi)\cdot\sin\left(\frac{\pi x}{15}\right)\cdot\sin(96\,\pi t)$

Putting $x = 7{\cdot}5$ cm and $t = 0{\cdot}25$ sec, we get,

$$v_P = -4\times 96\pi\cdot\sin\left(\frac{\pi\times 7{\cdot}5}{15}\right)\cdot\sin(96\pi\times 0{\cdot}25) = 0$$

So, the particle velocity is zero.

(iv) We put A = 2 cm, $\alpha = \frac{\pi x}{15}$ and $\beta = 96\,\pi t$ in the given equation. We get,

$$y = 2\text{ A} \sin \alpha \cdot \cos \beta = \text{A} \sin(\alpha + \beta) + \text{A} \sin(\alpha - \beta)$$

$$= 2 \sin\left(\frac{\pi x}{15} + 96\pi t\right) + 2 \cdot \sin\left(\frac{\pi x}{15} - 96\pi t\right) = y_1 + y_2$$

Here the component waves are : $y_1 = 2\sin\left(\frac{\pi x}{15} + 96\,\pi t\right)$ and $y_2 = 2\sin\left(\frac{\pi x}{15} - 96\,\pi t\right)$

❂ 3.9. Characteristics of stationary waves ❂

The stationary waves have the following characteristics :

(i) Stationary waves are produced when two indentical progressive waves travelling along the same straight line but in opposite direction are superposed.

(ii) Stationary waves are produced both by transverse and longitudinal waves.

(iii) Crests and troughs or compression and rarefaction do not advance through the medium, but simply appear or disappear at the same place alternately.

(iv) All the particles, except those at the nodes, execute simple harmonic motion. The amplitude of vibration is zero at the nodes and maximum at the antinodes. The distance between two adjacent nodes or antinodes is equal to half the wavelength.

(v) The distance between two consecutive nodes is called a loop in which the displacements of all the particles are all in the same direction.

(vi) The displacements of the particles in two consecutive loops are in the opposite directions.

❂ 3.10. Distinction between progressive waves and stationary waves ❂

Progressive waves	Stationary waves
1. These waves are produced due to continuous periodic vibrations of the particles of a portion of a medium.	1. These waves are produced due to the superposition of two identical progressive waves moving along the same line but in opposite directions in a medium.
2. Each particle of the medium executes identical periodic motion about the mean position of rest.	2. All particles of the medium except at the nodes execute periodic motions of varying amplitudes.
3. Wave advances with definite velocity.	3. Wave does not advance in the medium.
4. Wave curve advances without change of shape.	4. Wave curve changes form from time to time—it shrinks to a straight line twice in each period.
5. The amplitude of each particle is same.	5. The amplitude varies continuously from point to point – it is maximum at the antinodes and minimum at the nodes.
6. The phase changes with time as well as with space.	6. The phase at any instant in same between two consecutive nodes. If changes only in time. The phase changes by π from one loop to the next.
7. In a complete time period all the particles of the medium never come to rest together.	7. In a complete time period, all the particles of the medium come to rest twice together.

✪ 3.11. Transverse vibration of string ✪

We know that for vibration or for wave motion the vibrating body must have the property of inertia and elasticity. The characteristics of a wire is that it can act as a vibrating body as well as a medium for wave propagation.

In different musical instruments strings are used. For example, sitar and esraj are stringed instruments. In these instruments wires are kept under tension and when these strings are plucked at a point musical sounds of different frequencies are emitted. When a wire fixed at both ends is plucked at a point, the wire starts vibrating. At the plucked point two identical waves are produced which move in opposite directions along the wire [Fig. 3.10]. These two progressive waves get reflected at the two ends.

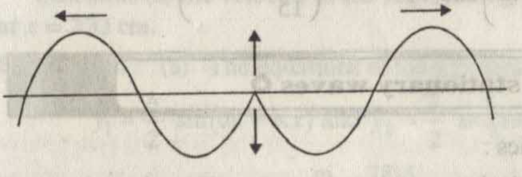


Fig. 3·10

These reflected waves superpose on each other and the wire vibrates in one or in a member of segments. Thus different modes of vibration are produced and they have definite frequencies. The resultant wave is a transverse stationary wave.

A stretched string can also be set into longitudinal vibration by pulling the wire along its length by leather sheet.

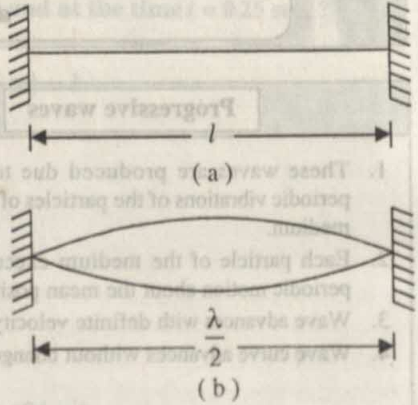
✪ 3.12. Fundamental frequency of transverse vibration of a string ✪

Consider a wire of length l stretched under a tension T between two fixed points. If the string be plucked or struck or bowed and then released, the transverse wave travels along the string and is reflected at the ends. A stationary wave is set up in the string.

The two ends of the string is fixed, they are nodes. In the simplest mode of vibration one antinode is formed at the middle i.e. the string vibrates in single segment [Fig. 3.11]. This vibration is called **fundamental** vibration and the sound emitted by the string is called **fundamental tone**. As the distance between two consecutive nodes is half the wavelength then, $l = \lambda/2$

If n be the fundamental frequency and V be the velocity of the wave then, $V = n\lambda = 2l.n$

$$\therefore \quad n = \frac{V}{2l} = \frac{1}{2l}\sqrt{\frac{T}{m}} \quad \left[\because V = \sqrt{\frac{T}{m}}\right] \quad \ldots\ldots\ldots (3.4)$$



(a)

(b)

Fig. 3·11

Where T is the tension in the string and m is the mass per unit length of the string.

Again if r be the radius of cross-section of the wire and ρ be the density of the material of the wire then, $m = \pi r^2 \rho$

$$\therefore \quad n = \frac{1}{2l}\sqrt{\frac{T}{\pi r^2 \rho}} = \frac{1}{2lr}\sqrt{\frac{T}{\pi\rho}} \quad \ldots\ldots\ldots\ldots (3.5)$$

3.13. Laws of transverse vibrations of stretched strings

The laws relating to the fundamental frequency of transverse vibration of a stratched string are as follows :

- (i) **Law of length :** The frequency of transverse vibration of a stretched string is inversely proportional to its length provided the tension T and mass per unit length m are kept constant.

  i.e. $n \alpha 1/l$ if T and m are constant.

- (ii) **Law of tension :** The frequency of transverse of a stretched string is directly proportional to the square root of tension of the string provided the length and mass per unit length are kept constant.

  i.e., $n \alpha 1/\sqrt{T}$ if l and m are constant.

- (iii) **Law of mass :** The frequency of transverse vibration of a stretched string is inversely proportional to the square root of mass per unit length provided tension and length of the string are kept constant.

  i.e. $n \alpha 1/\sqrt{m}$ if T and l are kept constant.

3.14. Different modes of vibration of a stretched string

We have seen in fig. 3.11 that when a stretched string vibrates in one segment, two nodes are formed at the two ends and an antinode at the middle. It is the simplest mode of vibration of lowest frequency n given by $n = \frac{1}{2l}\sqrt{\frac{T}{m}}$

This mode is called fundamental mode [Fig. 3.12(a)] and the frequency is called fundamental frequency. The emitted tone is called **Fundamental tone.**

The modes of vibration of the stretched string can be changed. The string may be made to vibrated in two, three, four etc. segment as shown in fig. 3.12.

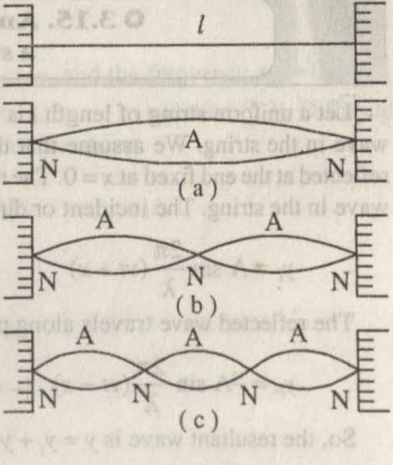


Fig. 3·12

(i) First overtone or second harmonic :

When the string is touched at the middle point of the string and bowed or plucked half way between one fixed end and the midpoint, the string begins to vibrate in two segments [Fig. 3.12(b)]. Now, $l = \lambda_1$ (wavelength of the wave). So its frequency

$$\therefore \quad n_1 = \frac{1}{\lambda_1}\sqrt{\frac{T}{m}} = \frac{1}{l}\sqrt{\frac{T}{m}} = \frac{2}{2l}\sqrt{\frac{T}{m}} = 2n$$

It is called first overtone or second harmonic.

(ii) 2nd overtone or third harmonic :

When the string is touched lightly at one third of its length from one end and plucked half way between the fixed end and the touched point, the string begins to vibrate in three segments [Fig. 3.12(c)]. Here

$$l = \frac{3\lambda_2}{2}\text{; and frequency, } n_2 = \frac{3}{2l}\sqrt{\frac{T}{m}} = 3n$$

It is the second overtone or third harmonic.

In this way, by producing nodes at different points of the string and plucking it at suitable points, the string may be made to vibrate in four, five etc. segments.

In general, if the stretched wire vibrates in p segments

$$n_p = \frac{p}{2l}\sqrt{\frac{T}{m}} = pn \quad [p = 1, 2, 3, ...]$$

Overtone and Harmonics :

It is possible to set a string vibrate with its fundamental and several of its higher modes at the same time. This is accomplished by plucking or bowing the string vigorously. All modes of vibration except the fundamental, are called overtones. Fundamental is the lowest frequency tone.

Those modes of vibration whose frequencies are integral multiple of the fundamental frequency are called harmonics. So, all harmonics are overtones but all overtones are not harmonics.

It may be pointed out that the musical value of a note is enhanced by the presence of greater number of overtones. In case of sound emitted by a vibrating string, the possible overtones present is determined by the location of the point of excitation. For example—

- (i) If a string is plucked at the middle, the point will be an antinode. Hence, all harmonics which have antinodes at the middle will be present and other harmonics will be absent. Fig. 3.12 shows that 1st, 3rd, 5th harmonics have antinodes at the middle. So these harmonics will be present in the note while 2nd, 4th, 6th i.e. all even harmonics will be absent. So, in general, **those overtones which have a node at the point of excitation will not be present in the sound produced by a string.** Musicians control the quality of sound emitted by stringed instrument like sitar etc. by plucking the string at suitable places.
- (ii) On the other hand, if a vibrating string is touched lightly at any point, a node will be formed at that point. Consequently, all overtones except those which have nodes at that particular point will be suppressed. Thus, if a vibrating string is touched at 1/3rd of its length, 3rd, 6th, 9th etc. harmonics having nodes at the point will be emitted and other are suppressed while playing the stringed instruments ; musicians suppress the undesirable harmonics by touching the string at suitable points with their fingers.

3.15. Analytical treatment of vibration of a string fixed at both ends

Let a uniform string of length l is fixed at $x = 0$ and $n = l$. It is plucked at a point to produce transverse wave in the string. We assume that the wave travelled along the string in the negative x-direction and is reflected at the end fixed at $x = 0$. The reflected wave and the direct wave on superposition produce stationary wave in the string. The incident or direct wave travels along negative x-axis. So its equation is

$$y_i = A \sin \frac{2\pi}{\lambda}(vt + x)$$

The reflected wave travels along positive x-axis. So its equation is

$$y_r = -A \sin \frac{2\pi}{\lambda}(vt - x) \quad [-\textit{ve} \text{ sign takes care of phase change at the rigid support}]$$

So, the resultant wave is $y = y_i + y_r$

$$\therefore \quad y = A\left[\sin \frac{2\pi}{\lambda}(vt + x) - \sin \frac{2\pi}{\lambda}(vt - x)\right]$$

$$= 2A \,.\sin \frac{2\pi\, x}{\lambda}\cdot \cos \frac{2\pi}{\lambda} vt = 2A \sin .\frac{2\pi\, x}{\lambda}\cdot \cos \frac{2\pi\, t}{T} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.6)$$

Here $T = \dfrac{\lambda}{v}$ is the period of each wave.

The equation (3.6) represents a stationary wave. It is not a progressive wave because only functions such as $f(vt - x)$ or $g(vt + x)$ represents a travelling wave.

$$\therefore \quad y = R\cdot\cos\omega t \left[R = 2A\cdot\sin\frac{2\pi x}{\lambda} \text{ and } \omega = \frac{2\pi}{T} \right] \ldots\ldots\ldots\ldots\ldots\ldots (3.7)$$

Evidently R is constant for a fixed value of x. Equation (3.7) is an equation of simple harmonic motion whose amplitude varies with x. Different particles have different amplitudes, but their time periods are equal. Such simple harmonic motions of the particles of a medium are called **normal modes**.

● Different normal modes :

Here the string is fixed at both ends. Hence the two boundary conditions which must be satisfied are

(i) At $x = 0$, $y = 0$ (ii) At $x = l$, $y = 0$.

The first boundary condition that y must vanish at $x = 0$ is automatically satisfied by equation (3.6). Again at $x = l$, the displacement of the particle must be zero for all value of t.

$$2\cdot A \sin\frac{2\pi l}{\lambda}\cdot\cos\frac{2\pi vt}{\lambda} = 0$$

The requirement is satisfied for all values of t only if

$$\sin\frac{2\pi l}{\lambda} = 0 \quad \therefore \frac{2\pi l}{\lambda} = n\pi \qquad n = 0, 1, 2, 3 \ldots\ldots$$

For each value of n there is a corresponding value of λ. So we write

$$\frac{2\pi l}{\lambda_n} = n\pi \quad \text{or, } \lambda_n = \frac{2l}{n} \ldots\ldots\ldots\ldots\ldots\ldots (3.8)$$

The frequency of vibration of the string is $v_n = \dfrac{V}{\lambda_n}$

$\therefore \quad v_n = \dfrac{n}{2l}\sqrt{\dfrac{T}{m}}$ [Putting the values of V and λ_n]

It is to be noted that the value $n = 0$ is unphysical since then $\lambda_0 \to \infty$ and the frequency $v_0 = 0$.

Notice that λ can take only discrete values corresponding to $n = 1, 2, 3, \ldots\ldots$ in equation (3.8). Different values of n give wavelengths of different normal modes of vibration of the string.

Now, $y = 2A \sin\dfrac{n\pi x}{l} \cos 2\pi v_n t$ [by equations (3.6) and (3.8)]

● (i) Fundamental tone or first harmonic : [$n = 1$]

Putting $n = 1$ in equation (3.8), $\lambda_1 = 2l \ \therefore \ v_1 = \dfrac{1}{2l}\sqrt{\dfrac{T}{m}}$

and $y = 2A \sin\dfrac{\pi x}{l}\cdot\cos 2\pi v_1 t = R \cos 2\pi v_1 t$

Here $R = 2A\cdot\sin\dfrac{\pi x}{l}$ is the amplitude of the fundamental tone.

So, when $x = 0$ and $x = l$, $R = 0$ and $y = 0$.

y is not zero for any other value of x. Also at $x = l/2$, the value of R is maximum. $R_{max} = 2A$.

Hence **when $n = 1$, nodes are formed at the two ends and one anitnode is formed at the middle [Fig. 3.12(a)]. This simple mode of vibration is called fundamental or first harmonic.**

● **(ii) First overtone or second harmonic : [$n = 2$]**

If we put $n = 2$ in equation (5.8), $\lambda_2 = l$ and frequency $\nu_2 = \frac{1}{l}\sqrt{\frac{T}{m}}$

Also $y = 2A \cdot \sin\frac{2\pi x}{l} \cdot \cos 2\pi\nu_2 t$ .... (3.9)

$\therefore \quad y = 0$, at $x = 0$, $l/2$ and l.

and amplitude $R = 2\ A\ \sin\left(\frac{2\pi x}{l}\right)$ is maximum at $x = l/4$ and $x = \frac{3l}{4}$

Hence in this case three nodes (at $x = 0$, $l/2$ and l) and two antinodes (at $x = l/4$ and $x = \frac{3l}{4}$) are formed along the string [Fig. 3.12 (b)]

So, the string vibrate in two segment in the case.

● **(iii) Second overtone or third harmonic : [$n = 3$]**

When $x = 3$, the string vibrate in three segments.

Its wavelength and frequency are $\lambda_3 = \frac{2l}{3}$ and $\nu_3 = \frac{3}{2l}\sqrt{\frac{T}{m}}$

We can also calculate the positions of different nodes and antinodes as in previous cases.

So, $\nu_1 : \nu_2 : \nu_3 : ..... = 1 : 2 : 3 : .....$

❂ 3.16. Stringed instruments ❂

Transverse vibrations of stretched strings find their application in the production of musical sound in stringed instruments. Various stringed instruments are employed to obtain musical sounds. As the sound emitted by a string is feeble the strings are mounted on a hollow wooden box known as sound chest, which acts as a resonant body. The vibration of the string induces forced vibration in the sound chest and the air inside it which intensifies the sound.

In these instruments, a number of metallic or catgut strings having different lengths and thickness are stretched over the wooden box. The frequencies emitted by these strings depend upon the tension, the mass per unit length of the strings as well as the point of plucking or bowing.

Stringed instruments are classified into three distinct categories depending on the manner in which the string is set into vibration. (i) In instrument like sitar, esraj, veana, guitar etc. the strings are set into vibration by plucking. (ii) In musical instrument like violin the string is excited by bowing. (iii) In the piano, however, the strings are of thin steel wire and they are set into vibration by striking with soft felt covered hammer.

● ● Numerical Examples ● ●

● **Example 3.3. A steel wire of length 40 cm and area of cross-section 0·2 mm^2 is stretched by a load of 4 kg-wt. If the string is set into transverse vibration, what is the frequency of the fundamental emitted by the string ? [density of steel = 7·7 g/c.c., g = 980 cm s^{-2}]**

○ **Solution :** The frequency of the fundamental, $n = \frac{1}{2l}\sqrt{\frac{T}{m}}$

mass per unit length, $m = \pi r^2\rho = 0{\cdot}002 \times 7{\cdot}7 = 0{\cdot}0154\ \text{g/cm}\ \left[\pi r^2 = 0{\cdot}002\ \text{cm}^2\right]$

and Tension $T = 4 \times 1000 \times 980 = 3{\cdot}92 \times 10^6$ dyne, $l = 40$ cm

$$\therefore \quad n = \frac{1}{2 \times 40}\sqrt{\frac{3{\cdot}92 \times 10^6}{0{\cdot}0154}} = \mathbf{199{\cdot}43\ Hz}$$

● **Ex. 3.4. A tuning fork is in unison with stretched string of length 75 cm. If the length of the string is reduced by 3 cm, the frequency of the string increases by 6 Hz. Calculate the frequency of the tuning fork.**

○ **Solution :** Let the frequency of the tuning fork be n. Then the fundamental frequency of the stretched string of length 75 cm is also n.

$$\therefore \quad n=\frac{1}{2l}\sqrt{\frac{T}{m}}=\frac{1}{2\times 75}\sqrt{\frac{T}{m}}$$

also $n+6=\dfrac{1}{2\times 72}\sqrt{\dfrac{T}{m}} \quad \therefore \dfrac{n}{n+6}=\dfrac{144}{150} \quad \therefore \boldsymbol{n=144}$

$\therefore$ The frequency of the turning fork = **144 Hz**

● **Ex. 3.5. A wire of length 100 cm is stretched by 0·05 cm when a tension is applied to it. Calculate the fundamental frequency of the stretched wire. Given, density of wire = 9·0 g/c.c. and $Y = 9\times 10^{11}$ dyne/cm².**

○ **Solution :** $n=\dfrac{1}{2l}\sqrt{\dfrac{T}{m}}=\dfrac{1}{2l}\sqrt{\dfrac{T}{A\rho}}$

Here $l = 100$ cm, $\dfrac{T}{A}$ = strain × Young's modulus $=\dfrac{0\cdot 05}{100}\times 9\times 10^{11}=45\times 10^{7}$ dyne/cm²

$$\therefore \quad n=\frac{1}{200}\sqrt{\frac{45\times 10^{7}}{9}}=\mathbf{35\cdot 35\ Hz}$$

● **Ex. 3.6. A wire of length 50 cm and mass 0·5 g is stretched by 9·0 kg-wt. (i) Calculate the velocity of transverse wave in the string and (ii) What will be the frequency of the fundamental, first and second overtones ?** **[C.U.P. 1987]**

○ **Solution :** (i) The velocity of transverse wave, $V=\sqrt{\dfrac{T}{m}}$

$$\therefore \quad V=\sqrt{\frac{9\times 9\cdot 8\times 10^{5}}{0\cdot 01}}=3\cdot 0\times 10^{4}\ \text{cm/s}$$

$$T = 9000\times 980\ \text{dyne}$$

$$m=\frac{0\cdot 5}{50}=0\cdot 01\ \text{g/cm}$$

(ii) Frequency of the fundamental, $n=\dfrac{V}{2l}=\dfrac{3\cdot 0\times 10^{4}}{2\times 50}=300$ Hz

$\therefore$ Frequency of the first overtone = 300 × 2 = 600 Hz

and that of the second overtone = 300 × 3 = 900 Hz

● **Ex. 3.7. A vertical tube of length 200 cm is filled with water and a vibrating tuning fork of frequency 256 Hz is held at the upper open end. Now the water in the tube is gradually drained out from the lower end of the tube. Calculate the position of the water level in the tube at first and second resonance. Neglect end error. Velocity of sound in air = 320 m/s.**

○ **Solution :** If l_1 and l_2 be the length of the air column at the first and second resonance then

$$n=\frac{V}{4l_1} \text{ and } n=\frac{3V}{4l_2} \quad \therefore l_2=3l_1$$

$$\therefore \quad l_1=\frac{V}{4n}=\frac{320\times 100}{4\times 100}=31\cdot 25\ \text{cm} \text{ and } l_2=31\cdot 25\times 3=93\cdot 75\ \text{cm}.$$

∴ Heights of the water column at the first and second resonance are

= (200 – 31·25) cm = **168·7 cm** and (200 – 93·75) cm = **106·25 cm**

✪ 3.17. Comparison of velocities of longitudinal and transverse waves in stretched string ✪

Both longitudinal and transverse waves can be produced in a string under tension. These waves move with different velocities along the wire.

If a stretched string is stroked along its length with the help of a piece of resin-soaked leather, longitudinal vibrations are set up in the wire.

Transverse waves can also be set up in a stretched string by plucking or bowing the string.

Now, the velocity of transverse wave in the string is given by $V_t = \sqrt{\frac{T}{m}} = \sqrt{\frac{T}{A\rho}}$

where T = applied tension, ρ = density of the material of the string and A = area of cross section of the wire.

Also velocity of longitudinal wave in the string is

$$V_l = \sqrt{\frac{Y}{\rho}} \quad [\text{Y = Young's modulus of the string material}]$$

$$\therefore \quad \frac{V_l}{V_t} = \sqrt{\frac{Y}{\rho} \times \frac{A\rho}{T}} = \sqrt{\frac{YA}{T}} = \sqrt{\frac{Y}{T/A}} \qquad \therefore \quad \frac{V_l}{V_t} = \sqrt{\frac{Y}{\text{Stress}}} = \frac{1}{\sqrt{\text{Strain}}} > 1$$

∴ In a stretched string the velocity of longitudinal wave is greater that of the transverse wave.

✪ 3.18. Longitudinal vibration of a rod ✪

Assume that a rod of length l = AB is rigidly fixed at the mid-point 'O' [Fig. 3.12(a). If the rod is stroked along its length, a stationary longitudinal wave is set in the rod. Node is formed at the middle and two antinodes are formed at the two free ends. So the length l of the rod is equal to half the wavelength. $l = \frac{\lambda}{2}$.

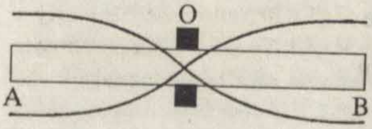


Fig. 3·12(a)

So the frequency of note emitted by the rod, $n = \frac{V}{\lambda} = \frac{V}{2l}$

where V = velocity of longitudinal wave in the string

$$\therefore \quad n = \frac{1}{2l}\sqrt{\frac{Y}{\rho}} \qquad \left[\because \; V = \sqrt{\frac{Y}{\rho}}\right]$$

- **Example 3.8. The diameter of a stretched wire is 1·2 mm. If the velocity of transverse wave in the string be 50 m/s, calculate the tension in the string. Density of steel = 7·7 g/c.c.**

○ **Solution :** The velocity of transverse wave is $V = \sqrt{\frac{T}{A\rho}}$

$\therefore \quad T = V^2 A\rho$

$\therefore \quad T = (50)^2 \times 36\pi \times 10^{-8} \times 7700$

$= 21{\cdot}76$ N

$= 2{\cdot}22$ kg-wt

$V = 0$ m/s

$A = \pi(6 \times 10^{-4})^2$

$= 36\,\pi \times 10^{-8}\ \text{m}^2$

$\rho = 7700\ \text{kg/m}^3$

- **Ex. 3.9. A tension of 245 N is applied to a uniform rope having mass 0·05 kg/m. Its length is 30 m. A transverse pulse is created at the free end. After what time it reaches the other end ?**

○ **Solution :** The velocity of transverse wave in the string, $V_t = \sqrt{\dfrac{T}{m}}$

$$\therefore \quad V_t = \sqrt{\frac{245}{0\cdot05}} = 70 \text{ m/s}$$

The pulse will reach the other end in time t where $t = \dfrac{30}{70} = 0\cdot43$ **sec**

- **Ex. 3.10. A wire is stretched to produce elongation of 0·04%. Compare the velocities of longitudinal and transverse wave in the string.**

○ **Solution :** The velocity of transverse wave in the string, $V_t = \sqrt{\dfrac{T}{m}} = \sqrt{\dfrac{T}{A\rho}}$

and the velocity of longitudinal wave, $V_l = \sqrt{\dfrac{Y}{\rho}}$ [Y = Young's modules]

Now, $Y = \dfrac{T/A}{l/L} = \dfrac{TL}{lA}$

$$\therefore \quad \frac{V_l}{V_t} = \sqrt{\frac{Y}{\rho}} \times \sqrt{\frac{A\rho}{T}} = \sqrt{\frac{TL}{lA} \times \frac{A\rho}{T}}$$

$$\therefore \quad \frac{V_l}{V_t} = \sqrt{\frac{1}{l/L}} = \frac{1}{\sqrt{\text{Strain}}} = \sqrt{\frac{100}{0\cdot04}} = 50 \qquad \therefore \; V_l : V_t = 50 : 1$$

✪ 3.19. Vibration of air column ✪

Air column is a volume of air enclosed in a cylindrical tube. So it is bounded volume of air. We have seen that a stretched vibrating string is a source of musical sound in **stringed instruments**. Similarly a vibrating air column enclosed in a pipe or tube is also source of musical sound in different **wind instruments** (e.g. flute, organ etc.). A detailed study of the vibration of a bounded volume of air is essential for the understanding of wind instruments.

Two types of pipes or tubes are used in wind instruments :

- (i) **Closed pipe :** The pipe is called closed when it is open at one end and closed at the other end.
- (ii) **Open pipe :** It is a pipe which is open at both ends.

When the air column is set into vibration, stationary wave pattern is formed within the pipe just as in the case of a vibrating string. Difference is that in this case stationary waves formed are longitudinal in nature.

➤ 3.19.1. *Stationary waves in a closed pipe and various modes of vibration :*

The closed pipe of length l is placed in the horizontal position along x-axis so that its closed end is at $x = 0$ and open end is at $x = l$. A sound wave is send along the pipe. It is represented as

$$y_1 = a \sin \frac{2\pi}{\lambda}(Vt - x) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

The direct wave meets the rigid boundary at the closed end ($x = 0$) and is reflected back. It is represented as

$$y_2 = -a \sin \frac{2\pi}{\lambda}(Vt + x) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(ii)}$$

The direct and reflected wave superpose giving rise to the standing longitudinal wave in the air column. The resultant displacement of the particle at x at time t is

$$y = y_1 + y_2 = a\left[\sin\frac{2\pi}{\lambda}(\mathrm{V}t - x) - \sin\frac{2\pi}{\lambda}(\mathrm{V}t + x)\right]$$

$$= 2a\cdot\cos\frac{2\pi}{\lambda}\,\mathrm{V}t\cdot\sin\frac{2\pi}{\lambda}\,x \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(iii)}$$

where $\mathrm{V} = \sqrt{\frac{\gamma \mathrm{P}}{\rho}}$ is the velocity of sound wave.

Evidently at $x = 0$, $y = 0$. So, the closed end is a **node**. The resultant displacement at $x = l$ will be maximum (positive or negative) since the open end is free. This condition is satisfied if

$$\sin\frac{2\pi l}{\lambda} = \pm 1$$

$$\therefore \quad \frac{2\pi l}{\lambda} = (2p+1)\frac{\pi}{2} \ldots\ldots\ldots\ldots \text{(iv) where } p = 0, 1, 2, \ldots..$$

- **First mode of vibrations :** Corresponding to $p = 0$, the equation (iv) gives the **first mode** of vibration in case of a closed pipe.

If the wavelength of the standing wave be λ_1 for $p = 0$ then

$$\frac{2\pi l}{\lambda_1} = \frac{\pi}{2} \qquad \therefore \lambda_1 = 4l$$

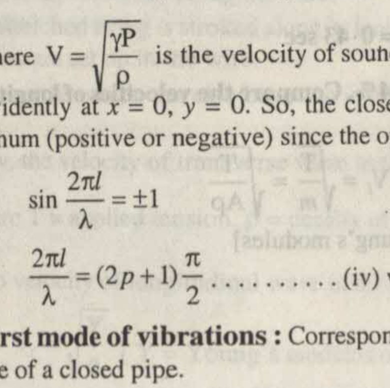

A
N
$l = \frac{\lambda_1}{4}$
A
N
A
N
$l = \frac{3\lambda_2}{4}$
A
N
A
N
A
N
$l = \frac{5\lambda_3}{4}$

Fig. 3·13

So, for the first mode, $l = \frac{\lambda_1}{4}$. The corresponding frequency of the wave is $n_1 = \frac{\mathrm{V}}{\lambda_1} = \frac{\mathrm{V}}{4l}$

It is called the **fundamental mode or first harmonic.**

- **Second mode of vibration :** Corresponding to $p = 1$, the equation (iv) gives the second mode of vibration.

If λ_2 be the wavelength of the waves produced in the pipe for $p = 1$ then

$$\frac{2\pi l}{\lambda_2} = \frac{3\pi}{2} \qquad \therefore \lambda_2 = \frac{4l}{3}$$

and the frequency of the wave is $n_2 = \frac{\mathrm{V}}{\lambda_2} = \frac{3\mathrm{V}}{4l} = 3n_1$

It is called **first overtone** or **third harmonic.**

- **Third mode of vibration :** The equation (iv) gives the third mode of vibration when we put $p = 2$. If λ_3 be the wavelength of the wave corresponding to $p = 2$, we get

$$\frac{2\pi l}{\lambda_3} = \frac{5\pi}{2} \qquad \therefore \lambda_3 = \frac{4l}{5}$$

and the frequency of the waves, $n_3 = \frac{\mathrm{V}}{\lambda_3} = \frac{5\mathrm{V}}{4l} = 5n_1$

It is called the **second overtone** or **fifth harmonics**.

It follows that the frequency of the nth mode of vibration is

$$n_p = (2p+1)\frac{\mathrm{V}}{4l} = (2p+1)n_1$$

It is called **$(p + 1)$ th overtone** or **$(2p + 1)$ harmonic.**

Thus in a closed pipe, the frequencies of the normal modes of vibration of the air column are in the ratio 1 : 3 : 5 : ..... etc.

Hence **in the closed pipe only odd harmonics are present.**

➤ *3.19.2. Stationary waves in an open pipe and various modes of vibration :*

We assume that the pipe is held along x-axis, one end being at $x = 0$ and the other end at $x = l$.

The incident direct wave is $y_1 = a \sin \frac{2\pi}{\lambda}(\mathrm{V}t - x)$ [No phase change during reflection]

When it reaches the other end of the tube it is reflected back.

Since the air outside the tube has more freedom of movement than that in the interior, we can treat them to be different media. The open end acts as the interface where reflection takes place. Effectively, here reflection occurs from denser to rarer medium. Now, the reflected wave is given by

$$y_2 = a \sin \frac{2\pi}{\lambda}(\mathrm{V}t + x)$$

The resultant stationary wave pattern is $y = y_1 + y_2$

$$\therefore \quad y = a\left[\sin \frac{2\pi}{\lambda}(\mathrm{V}t - x) + \sin \frac{2\pi}{\lambda}(\mathrm{V}t + x)\right] = 2a \cdot \sin \frac{2\pi}{\lambda}\mathrm{V}t \cdot \cos \frac{2\pi}{\lambda}x$$

Here the displacement is maximum at $x = 0$ and $x = l$.

At $x = 0$, $\cos \frac{2\pi}{\lambda}x = 1$ So the first open end is an antinode.

The other end ($x = l$) will be an antimode if $\cos \frac{2\pi l}{\lambda} = \pm 1$

$$\therefore \quad \frac{2\pi l}{\lambda} = p\pi \text{ where } p = 1, 2, 3, \ldots\ldots\ldots\ldots \text{(i)}$$

- **First mode of vibration :** $p = 1$ in equation (i) gives the simplest first mode of vibration. Its wavelength is λ_1, where

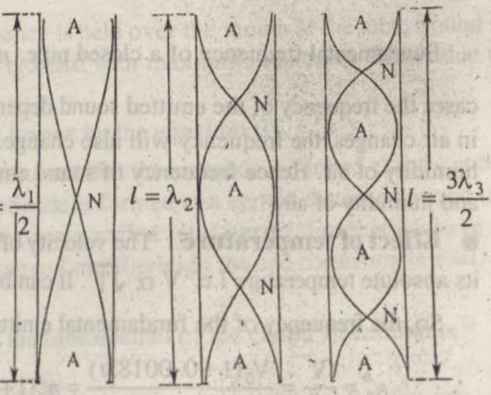
$$\frac{2\pi l}{\lambda_1} = \pi \quad \therefore \lambda_1 = 2l \quad \therefore l = \frac{\lambda_1}{2}$$

and the frequency is $n_1 = \frac{\mathrm{V}}{\lambda_1} = \frac{\mathrm{V}}{2l}$

It is called **fundamental frequency** or **first harmonic**.

- **Second mode of vibration :** Corresponding to $p = 2$, the equation (i) given the second mode of vibration. The wavelength of the wave is

$$\frac{2\pi l}{\lambda_2} = 2\pi \quad \therefore \lambda_2 = l$$



(a) (b) (c)

Fig. 3·14

and the frequency is $n_2 = \frac{\mathrm{V}}{\lambda_2} = \frac{\mathrm{V}}{l} = \frac{2\mathrm{V}}{2l} = 2n_1$

It is called the **first overtone** or **second harmonic.**

- **Third mode of vibration :** If λ_3 be the wavelength of stationary wave for $p = 3$ then the equation (i) gives

$$\frac{2\pi l}{\lambda_3} = 3\pi \quad \therefore \quad l = \frac{3\lambda_3}{2} \quad \therefore \quad \lambda_3 = \frac{2l}{3}$$

∴ The frequency, $n_3 = \frac{V}{\lambda_3} = \frac{3V}{2l} = 3n_1$

It is the **second overtone** or **third harmonic**

So, the frequency of the p th mode of vibration $n_p = p\frac{V}{2l} = pn_1$

It is called $(n-1)$ th overtone or n th harmonic.

Thus, **in an open pipe, all harmonics (even as well as odd) are present.**

- So, the sound produced by an open pipe is more musical than a closed pipe. This is due to the fact that in the sound emitted by an open pipe all harmonic (even as well as odd) are present. But a closed pipe can produce a sound containing only the odd harmonics.

3.20. Frequencies of fundamental in closed and open pipes of same length

Let the length of both the closed and open pipe be l.

Then the frequency of the fundamental of closed pipe is $n_c = \frac{V}{4l}$

where V = velocity of sound in air

Again, the frequency of the fundamental of open pipe is $n_0 = \frac{V}{2l} = 2\frac{V}{4l} = 2n_c$

So, the frequency of the fundamental of the open pipe is twice the frequency of the fundamental of closed pipe when their lengths are equal. It means that open pipe fundamental frequency is one octave higher than the closed pipe fundamental frequency.

3.21. Effect of temperature and humidity on the frequency of air column

Fundamental frequency of a closed pipe, $n_c = \frac{V}{4l_c}$ and that of an open pipe, $n_0 = \frac{V}{2l_0}$. So in both the cases the frequency of the emitted sound depends on velocity of sound in air. Thus if the velocity of sound in air changes, the frequency will also change. Now, velocity of sound in air depends on temperature and humidity of air. Hence frequency of sound emitted by a closed pipe or open pipe depends on temperature and humidity of air.

- **Effect of temperature :** The velocity of sound in a gaseous medium is proportional to square root of its absolute temperature i.e. $V \propto \sqrt{T}$. It can be shown that $V = V_0(1 + 0{\cdot}00183\,t)$

So, the frequency of the fundamental emitted by a closed pipe at t°C is

$$n''_c = \frac{V}{4l_c} = \frac{V_0(1+0{\cdot}00183t)}{4l_c} = n'_c(1+0{\cdot}00183t)$$

where n'_c = frequency of the fundamental at 0°C.

So, frequency of sound increases with temperature.

Similarly in the case of open pipe, the frequency of the fundamental at t°C is

$$n''_0 = \frac{V}{2l_0} = \frac{V_0(1+0{\cdot}00183t)}{2l_0} = n'_0(1+0{\cdot}00183t)$$

Effect of humidity : The velocity of sound in air also depends on its humidity. It is found that velocity of sound increases with humidity of air. So, in this case also the frequency of sound emitted by closed pipe or open pipe increases with humidity of air.

3.22. Determination of velocity of sound or frequency of a tuning fork by resonance air column method

Theory : The air column in a closed pipe has its natural frequency of vibration. This natural frequency depends on the length of the air column. When a vibrating tuning fork is held at the open end of a closed pipe the air column is set into forced vibration. If the frequency of the fork is different from the natural frequency of the air column, the air column will vibrate at the frequency of the fork but with smaller amplitude. As a result intensity of sound produced by the air column will be feeble. But if the two frequencies (natural frequency of the air column and frequency of the fork) are equal, resonance will take place and the air column now vibrates with larger amplitude. The sound emitted by the pipe will be louder. This air column is called **"resonance air column"**.

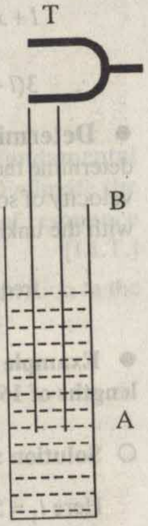


Fig. 3·15

If the resonance is obtained at the minimum length of the air column, it means that the frequency of the fundamental of the air column is equal to the frequency of the fork. Under this condition, from the theory of closed pipe we know, $\lambda = 4l$.

Here λ = wavelength of the fundamental of the air column.

l = length of the air column.

Then the velocity of sound is $V = n\lambda = 4nl$ .... (i) [n = frequency of the fork]

Apparatus : Simple sketch of the apparatus required for determining the velocity of sound in air by resonance air column method is shown in fig. 5.15. A is a long cylindrical glass jar containing water. B is a uniform glass tube open at both ends. The tube can be raised or lowered into the water of the glass jar and length of the air column can be adjusted.

Procedure : A vibrating tuning fork of known frequency is held over the mouth of the tube. Sound is emitted. The tube is gradually raised. At one time sound is heard with maximum loudness. This is due to resonance. The length of the air column 'l' in measured.

So, we can measure the velocity of sound 'V' if 'n' is known by the equation (i).

End error and its elimination : So far, we have assumed that the antinode is formed exactly at the month of the tube. Strictly speaking this is not so. The antinode is formed at a certain distance beyond the open end. The distance of the antinodal point beyond the open is called **'end-error'**. For this we have to apply a correction. It is called **'end-correction'**. According to Lord Rayleigh the end correction is $x = 0{\cdot}6r$ where r is the radius of the tube.

So, if we apply the end correction, the wavelength of the fundamental of the closed air column is

$$\lambda = 4(l + 0{\cdot}6\,r) \ldots\ldots\ldots\ldots\ldots\ldots \text{(ii)}$$

$$\therefore \quad \text{Velocity of sound is } V = 4n\,(l + 0{\cdot}6\,r) \ldots\ldots\ldots \text{(iii)}$$

Elimination of end correction : To eliminate the end correction, the tube is raised further until second resonance is obtained. This occurs when the frequency of the first overtone (or third harmonic) produced by the new length of the air column is equal to that of the tuning fork. If this new resonant length be l_1 then,

$$\frac{3\lambda}{4} = l_1 + 0{\cdot}6r \ldots\ldots\ldots\ldots\ldots\ldots \text{(iv)}$$

From (ii) and (iv), $\frac{\lambda}{2} = l_1 - l \quad \therefore \lambda = 2\,(l_1 - l)$ . . . . (v)

$\therefore$ Velocity of sound is $V = 2n\,(l_1 - l)$ . . . . (3.16)

- **Determination of end correction :** End correction is $x = 0{\cdot}6\,r$.

Hence x can be determined by measuring the internal diameter (d) of the resonant tube by slide callipers. So, $\boldsymbol{x = 0{\cdot}3\,d}$.

Alternatively, end correction x can also be determined by measuring the first and second resonant length of the air column in the closed tube. If these lengths be l and l_1 then,

$$l + x = \frac{\lambda}{4} \text{ and } l_1 + x = \frac{3\lambda}{4}$$

$$\therefore \quad 3(l + x) = l_1 + x \quad \text{or,} \quad x = \frac{l_1 - 3l}{2}$$

- **Determination of frequency of a unknown tuning fork :** We can employ the equation (3.16) to determine the frequency of the fork. For this we take a tuning fork of known frequency and determine the velocity of sound (V) in the air column as described earlier. Then the two resonant lengths of the air column with the unknown fork is determined. If l_2 and l_3 be the two resonant lengths then $\lambda = 2\,(l_3 - l_2)$

$$\therefore \quad \text{frequency, } n = \frac{V}{\lambda} = \frac{V}{2\,(l_3 - l_2)}$$

- **Example 3.11. A tuning fork of frequency 480 Hz produces first and second resonance with the lengths of 16·5 cm and 52·5 cm of an open pipe. Calculate velocity of sound and end correction.**

○ **Solution :** The difference between the second and first resonant length is $l_1 - l = \frac{\lambda}{2}$

Here $l_1 = 52{\cdot}5$ cm, $l = 16{\cdot}5$ cm

$\therefore \quad \lambda = 2\,(l_1 - l) = 2\,(52{\cdot}5 - 16{\cdot}5) = 72$ cm.

$\therefore$ Velocity of sound, $V = n\lambda = 480 \times 720$ cm $= 345{\cdot}6$ m

Next for the first resonance, $l + x = \frac{\lambda}{4} = \frac{72}{4} = 18$

$\therefore \quad x = 18 - 16{\cdot}5 = \mathbf{1{\cdot}5}$ **cm.**

- **Ex. 3.12. The fundamental frequency of an open pipe of length 100 cm is in resonance with a sonometer wire of length 45 cm. Calculate the tension in the string if its mass per unit length be 0·1 g/cm and velocity of sound in air be 332 m/s.**

○ **Solution :** The frequency of the fundamental of the open pipe is

$$n_1 = \frac{V}{2l_1} = \frac{33200}{2 \times 100} = 166 \text{ cm}$$

and the frequency of the fundamental of transverse wave in the string is

$$n_2 = \frac{1}{2l}\sqrt{\frac{T}{m}} = \frac{1}{2 \times 45}\sqrt{\frac{T}{0{\cdot}1}} \text{ Hz}$$

$\therefore$ By question, $\frac{1}{90}\sqrt{\frac{T}{0{\cdot}1}} = 166 \quad \therefore \quad T = (90 \times 166)^2 \times 0{\cdot}1$ dyne

$\therefore$ the required tension $= 22{\cdot}77$ kg

- **Ex. 3.13. The length of an open organ pipe is twice the length of a closed organ pipe. If the fundamental frequency of the open pipe be 100 Hz, what is the frequency of the third harmonic of the closed pipe ?**

○ **Solution :** In the case of open pipe, its fundamental frequency is $n = \frac{V}{2l_0}$ and the frequency of the third harmonic of a closed pipe is $n_1 = \frac{3V}{4l_c}$

$$\therefore \; n_1 = \frac{3}{4l_c} \times 2nl_0 = \frac{3nl_0}{2l_c}$$

Here $n = 100$ Hz and $l_0 = 2l_c$ $\quad \therefore \; \frac{l_0}{l_c} = 2$

$\therefore \; n_1 = \frac{3}{2} \times 100 \times 2 = \mathbf{300\ Hz}$ $\quad \therefore$ The required frequency = **300 Hz.**

- **Ex. 3.14. A tube of uniform diameter and length 48 cm is open at both ends. Its fundamental frequency of reasonance is found to be 320 Hz. The velocity of sound in air is 320 ms^{-1}. Estimate the diameter of the tube. One end of the tube is now closed. Calculate the lowest frequency of resonance for the tube. [I.I.T.]**

○ **Solution :** If x be the end correction, then the wave length of the fundamental mode of vibration in the open pipe is

$$l + 2x = \frac{\lambda}{2} \qquad \therefore \; l + 0{\cdot}6\, d = \frac{\lambda}{2}$$

Now, wavelength of the fundamental, $\lambda = \frac{320}{320}$ m = 1m = 100 cm

$$\therefore \quad 48 + 0{\cdot}6\, d = \frac{100}{2} \qquad \therefore \; d = \mathbf{3{\cdot}33\ cm}$$

2nd Part : If one end is closed, the tube now behaves as a closed pipe. So for its fundamental mode of vibration

$$\frac{\lambda}{4} = l + x \qquad \therefore \; \lambda = 4 \times (48 + 0{\cdot}3 \times 3{\cdot}33) = \mathbf{196\ cm}$$

So, the lowest frequency of resonance is $n = \frac{V}{\lambda} = \frac{320 \times 100}{196} = \mathbf{163{\cdot}3\ Hz}$

✪ 3.23. Beats ✪

There is another class of phenomenon related to the principle of superposition of two progressive waves having slight difference in frequencies. The phenomenon is called beats.

❖ **Definition of beats : When two progressive harmonic waves having nearly equal frequencies travelling in a medium along the same direction superpose on each other, the intensity of the resultant sound at a point on the medium rises and falls regularly with time. These periodic variations in the intensity of sound due to superposition of two sound waves of slightly different frequencies are called beats.**

A beat consists of one rise and one fall of intensity of the wave. The time interval between two successive beats is called beat period and the number of beats per second is called beat frequency. It can be proved that the number of beats per second is equal to the difference of frequencies of the two component waves.

➤ **Formation of beats :** The formation of beats can be studied both graphically and analytically as given below.

● **(A) Graphical method :** We consider two trains of harmonic waves of slightly different frequencies. These waves are travelling in a medium with equal velocity. For better understanding, we consider two wave trains of equal amplitude and having frequencies 10 Hz and 8 Hz respectively. In 1 sec the two waves will cover the same distance as their velocities are equal. But in this distance, the number of waves due to the first and the second wave trains will be 10 and 8 respectively as shown in fig. 3.16(a) and (b). When the two waves superpose on each other the resultant displacement at each point is obtained from the principle of superposition by adding the displacements of the two waves by algebraic addition at each point. The resultant wave form will be as shown in fig. 3.16(c). The amplitude and hence its intensity becomes maximum at A, C and E and minimum at the points B and D. So two beats are formed in one sec – one between A and C and the other between C and E.

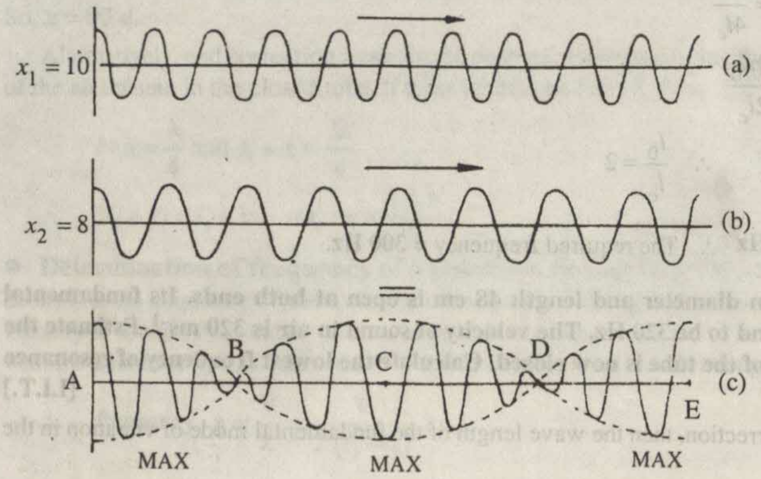


Fig. 3·16

● **(B) Analytical Method :** By actual calculation we can show that the beat frequency i.e. the number of beats per second is equal to the difference between the superposing frequencies.

We consider two harmonic waves of frequencies n_1 and n_2 $(n_2 > n_1)$ travelling in the $+x$ direction in a medium. the waves reaches the point x of the medium simultaneously. The displacements of the particle due to the two waves at any instant will be given by

$$y_1 = a \sin \frac{2\pi}{\lambda_1}(\mathrm{V}t - x) \text{ and } y_2 = b \sin \frac{2\pi}{\lambda_2}(\mathrm{V}t - x) \ [\mathrm{V} = \text{velocity}]$$

Here the waves have same velocity. But frequencies and wavelengths are different. To simplify the calculation we assume that the observation point be located at $x = 0$. Then the above two equations reduces to

$$y_1 = a \sin 2\pi n_1 t \left(n_1 = \frac{\mathrm{V}}{\lambda_1} \right)$$

$$\text{and } \ y_2 = b \sin 2\pi n_2 t \left(n_2 = \frac{\mathrm{V}}{\lambda_2} \right)$$

Using the principle of superposition, the resultant displacement is given by $y = y_1 + y_2$

$$\therefore \quad y = a \sin 2\pi n_1 t + b \sin 2\pi(n_1 + n)t \ [\text{ Putting } n_2 = n_1 + n \ ]$$

$$= \sin 2\pi n_1 t\,(b \cos 2\pi nt + a) + b \cos 2\pi n_1 t \cdot \sin 2\pi nt$$

Let $b \cos 2\pi nt + a = \mathrm{R} \cos \phi$ and $b \sin 2\pi nt = \mathrm{R} \sin \phi$

$$\therefore \quad y = \mathrm{R} \sin 2\pi n_1 t \cdot \cos \phi + \mathrm{R} \cos 2\pi n_1 t \cdot \sin \phi$$

$$= \mathrm{R} \sin(2\pi n_1 t + \phi) \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (\text{i})$$

Here $\mathrm{R}^2 = (b \cos 2\pi\, nt + a)^2 + (b \cdot \sin 2\pi\, nt)^2$

$= a^2 + b^2 + 2ab \cos 2\pi\, nt \ldots\ldots\ldots\ldots\ldots\ldots\ldots$ (ii)

and $\tan \phi = \dfrac{b \sin 2\pi\, nt}{a + b \cos 2\pi nt}$

So we see that the resultant wave is a simple harmonic wave with its amplitude R changing with time [Eqn. (ii)].

Minimum and Maximum Value of R :

(i) R is minimum when $t = \frac{1}{2n}, \frac{3}{2n}, \frac{5}{2n} \ldots\ldots$ then $\cos 2\pi\, nt = -1$

and $\mathrm{R} = \sqrt{a^2 + b^2 - 2ab} = (a - b)$ (Minimum)

(ii) R is maximum when $t = \frac{1}{n}, \frac{2}{n}, \frac{3}{n}, \frac{4}{n}, \ldots\ldots$ then $\cos 2\pi\, nt = 1$

and $\mathrm{R} = \sqrt{a^2 + b^2 + 2ab} = a + b$ (maximum)

Hence the minimum intensity of the resultant wave is $\mathrm{I}_{\min} = (a - b)^2$

and the maximum intensity is $\mathrm{I}_{\max} = (a + b)^2$

Now, if the amplitudes of the superposing waves be same, $a = b$. then $\mathrm{I}_{\min} = 0$ and $\mathrm{I}_{\max} = 4a^2$

So the intensity of the sound fluctuates between maximum and minimum values. So throbbing sound will be heard.

Time interval between two maximum intensities, $t_1 = 1/n$ sec

and time interval between two minimum intensities, $t_2 = 1/n$ sec.

As $t_1 = t_2$, the time interval between two maximum intensivities and two minimum intensities in $1/n$ sec.

So, the beat period is $\mathrm{T} = 1/n$ sec. with beat frequency is n

where $n = n_2 - n_1$. So the beat frequency is the difference between the two superposing frequencies.

The fig. __ shows how the intensity (i.e., R^2) of the resulting wave varies with time.

So, the intensity or loudness varies periodically with time.

Time interval between two maximum intensities, $t_1 = 1/n$ sec

and time interval between two minimum intensities, $t_2 = 1/n$ sec.

As $t_1 = t_2$, the time interval between two maximum intensities and two minimum intensities in $1/n$ sec.

So, the beat period is $\mathrm{T} = 1/n$ sec. with beat frequency is n

where $n = n_2 - n_1$. So the beat frequency is the difference between the two superposing frequencies.

The fig. 3.17 shows how the intensity (i.e., R^2) of the resulting wave varies with time.

So, the intensity or loudness varies periodically with time.

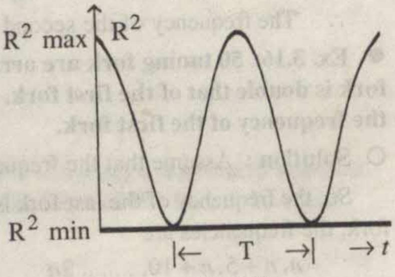


Fig. 3·17

✪ 3.24. Condition of perception of beats ✪

Beats formed due to the superposition of two sound waves may be heard clearly if the frequencies of the two waves do not differ by more than 10 Hz. If the beat frequency is more than this value, due to persistence of hearing the ear can not distinguish the variation of loudness of sound. In that case continuous note is heard.

Also, the difference of amplitudes of the two waves should not be large. Otherwise beats cannot be clearly recognised. As loudness of sound depends on amplitude, if the difference of amplitudes of the two waves be very large then the fluctuation of loudness due to formation of beats may not be clearly heard.

✪ 3.25. Practical application of beats ✪

(A) Determination of Unknown frequency : The phenomenon of beats can be applied to determine the unknown frequency of source of sound. For this we have to take another tuning fork whose frequency is known and its frequency is nearly equal to the unknown frequency so that when the two forks are sounded together, beats can be recognised. The number of beats per second is counted. If the beat frequency is n then $n = n_1 \sim n_2$

When n_1 is the unknown frequency and n_2 is the frequency of the known fork. Next to decide whether n_1 is greater or smaller than n_2, one prong of the unknown fork is loaded with some wax. As a result its frequency will decrease. If now, the number of beats per second increases, it means that $n_1 < n_2$. On the other hand if the number of beats per second decreases, $n_1 > n_2$. Hence n_1 can be determined.

(B) Detection of poisonous gases in mines : With the help of the phenomenon of beats, the existence of poisonous gases in mines can be detected. It is done in the following way :

Two identical organ pipes are taken. Now, one is filled with pure dry air and the other with the air of the mine. The pipes are then blown together. If the mine air be pure, no beats are heard. But if it is impure, the velocity of sound in it changes and so the frequency also changes while the wavelength remain unchanged.

- **Example 3.15. The frequency of a tuning fork is 255 Hz. It produces 5 beats per second with another tuning fork. When some wax is attached with one prong of the second tuning fork, 8 beats per second it are produced. Calculate the frequency of the second fork.**

○ **Solution :** The frequency of the first fork is $n_1 = 255$ and number of beats is 5. So, if the frequency of the second fork be n_2 then, $n_2 = n_1 \pm 5 = 255 \pm 5$

So, $n_2 = 260$ or 250 Hz.

Next it is given that if the second fork is loaded with wax beat frequency increases to 8. It means the difference of frequency increases. Hence the frequency of the second fork is less than that of the first fork as second fork frequency decreases when loaded with wax.

$\therefore$ The frequency of the second tuning fork is 250 Hz.

- **Ex. 3.16. 50 tuning fork are arranged in order of increasing frequency. The frequency of the last fork is double that of the first fork. Any two consecutive forks produce 5 beats per second. Calculate the frequency of the first fork.**

○ **Solution :** Assume that the frequency of the first fork is n

So, the frequency of the last fork is $2n$. As 5 beats per second is produced between any two consecutive fork, the frequencies are

$n, n+5, n+10, \ldots\ldots\ldots 2n$

$\therefore \quad n + (50-1)5 = 2n \quad \therefore \quad n = \mathbf{245\ Hz}$

So the frequency of the first fork = **245 Hz**

- **Ex. 3.17. When sounded together two tuning forks produce 8 beats per sec. The first fork is in resonance with a closed pipe of length 32 cm. The second fork can produce resonance with the same air column if its length is increased by 1 cm. Calculate the frequencies of the two forks.**

○ **Solution :** Let the frequencies of the two tuning forks be n_1 and n_2.

Then either $n_1 - n_2 = 8$ or $n_2 - n_1 = 8$

The first fork produces resonance with a closed pipe of length 32 cm. and the second fork with the closed pipe of length 32 + 1 = 33 cm.

$$\therefore \quad n_1 = \frac{V}{4\times 32} \quad \text{and} \quad n_2 = \frac{V}{4\times 33} \ [V = \text{velocity of sound in air}]$$

$$\therefore \quad \frac{n_1}{n_2} = \frac{33}{32} \qquad \therefore \quad n_1 > n_2 \quad \text{Hence } n_1 - n_2 = 8$$

$$\therefore \quad \frac{33}{32} n_2 - n_2 = 8 \qquad \therefore \quad n_2 = 256 \text{ Hz and } n_1 = 256 + 8 = 264 \text{ Hz}$$

So, the required frequencies are **256 Hz** and **264 Hz.**

- **Ex. 3.18. In any gaseous medium two waves on superposition produce 6 beats per second. The wave lengths of the two waves are 50 cm and 50·5 cm repectively. Calculate the velocity of sound in the medium.**

○ **Solution :** We know, velocity of sound in a gaseous medium, $V = n\lambda$

Where n is the frequency and λ is the wavelength of the wave,

If n_1 and n_2 be the frequencies of the two waves then, $n_1 = \frac{V}{50}$ and $n_1 = \frac{V}{50\cdot 5}$

$$\text{So, } n_1 > n_2 \qquad \therefore \quad n_1 - n_2 = 6.$$

$$\therefore \quad \frac{V}{50} - \frac{V}{50\cdot 5} = 6 \quad \text{or,} \quad \frac{0\cdot 5\ V}{2525} = 6 \quad \therefore V = 30300 \text{ cm/s}$$

$\therefore$ Velocity of sound in the gaseous medium = **303 ms^{-1}**.

- **Ex. 3.19. The length of a glass cylinder 50 cm and volume 400 c.c. A vibrating tuning fork of frequency 256 is held at the mouth of the cylinder. Now water is slowly poured into the cylinder. When the volume of water poured into the cylinder is 130 c.c. resonance is obtained. Calculate the velocity of sound in air.**

○ **Solution :** When resonance takes palce, the volume of air in the cylinder is = 400 − 130 = 270 c.c.

$$\therefore \quad \text{The length of the air column, } l = \frac{270}{400} \times 50 = \frac{135}{4} \text{ cm}$$

Now, in the case of closed pipe frequency of the fundamental, $n = \frac{V}{4l}$

$$\therefore \quad V = 4ln = 4 \times \frac{135}{4} \times 256 = 34560 \text{ cm/s.} = 345{\cdot}6 \text{ m/s}$$

So the velocity of sound wave in air = **345·6 ms^{-1}**

- **Ex. 3.20. The frequencies of three tuning fork are 400, 401 and 402 Hz. They are sounded together. Calculate the number of beats per second. Assume amplitudes of vibration of the fork to be equal.**

○ **Solution :** Let the equations of the three waves be

$$y_1 = a \cdot \sin 2\pi \cdot 400 \cdot t = a \sin 800\ \pi t$$
$$y_2 = a \cdot \sin 2\pi \cdot 401 \cdot t = a \sin 802\ \pi t$$
$$y_3 = a \cdot \sin 2\pi \cdot 402 \cdot t = a \sin 804\ \pi t$$

By the principle of superposition, the resulant wave is

$$y = y_1 + y_2 + y_3 = a\left[\sin 800\ \pi t + \sin 802\ \pi t + \sin 804\ \pi t\right]$$
$$= a\left[2 \cdot \sin 802\ \pi t \cdot \cos 2\pi t + \sin 802\ \pi t\right]$$
$$= a\left(1 + 2\cos 2\pi t\right) \cdot \sin 802\ \pi t = A \sin 802\ \pi t$$

Where $A = a(1 + 2\cos 2\pi t)$ is the amplitude of the resultant wave.

A is maximum when $\cos 2\pi t = 1$ $\quad \therefore \; 2\pi t = 2\,K\pi \;\; [K = 0, 1, 2, ...]$

So, maximum sound will be heard when $t = 0, 1, 2, 3, ...$

So, the number of beats per sec. is 1

Again A will be minimum when $1 + 2\cos 2\pi t = 0$

$$\therefore \quad \cos 2\pi t = -\frac{1}{2} \qquad \therefore \quad 2\pi t = 2K\pi + \frac{2\pi}{3} \;\; [K = 0, 1, 2, ...]$$

So, the sound of lowest intensity will be heard when

$$t = K + \frac{1}{3} = \frac{1}{3}, \frac{4}{3}, \frac{7}{3}, ...\text{etc}$$

$$\therefore \quad \text{No. of beats per second, } n = \frac{1}{\frac{4}{3} - \frac{1}{3}} = 1$$

- **Ex. 3.21. A sonometer is provided with two stretched strings. Their tensions are in the ratio 8 : 1 ; lengths in the ratio 36 : 35 ; diameters in the ratio 4 : 1 and densities in the ratio 1 : 2. If the strings are sounded together how many beats per sec will be heard ?**

The frequency of the sound with higher pitch is 360 Hz.

○ **Solution :** The frequency of the fundamental of a stretched string is

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}} = \frac{1}{2lr}\sqrt{\frac{T}{\pi\rho}}$$

Here l = length of the string, T = tension, r = radius of the wire, ρ = density.

If n_1 and n_2 be the frequencies of the two wires of the sonometer then,

$$\frac{n_1}{n_2} = \frac{l_2}{l_1} \cdot \frac{r_2}{r_1}\sqrt{\frac{T_1}{T_2}} \cdot \sqrt{\frac{\rho_2}{\rho_1}} = \frac{35}{36} \times \frac{1}{4} \times \sqrt{\frac{8}{1}} \times \sqrt{\frac{2}{1}}$$

$$\therefore \quad \frac{n_1}{n_2} = \frac{35}{36} \qquad \text{Here } n_2 = 360$$

$$\therefore \quad n_1 = 360 \times \frac{35}{36} = 350 \text{ Hz}$$

$\therefore$ No. of beats per second or beat frequency = 360 – 350 = **10 Hz.**

● ● Short Answer Type Questions (with answers) ● ●

- **Question 1. What are the characteristics which distinguish one harmonic wave from another ?**

○ **Ans.** The distinguishing characteristics of a harmonic wave are :

(i) amplitude, (ii) frequency, (iii) wave velocity, (iv) phase.

- **Q. 2. If the mass providing tension to a string is immersed in water, how the frequency of sound emitted by the string changes.**

○ **Ans.** The frequency will decrease. **Reason :** The frequency $n \propto \sqrt{T}$ [T = tension]

Now, if the mass providing tension to the string is immersed in water, its apparent weight will decrease. Tension to the string will decrease. Hence its frequency of transverse vibration also decreases.

- **Q. 3. When an empty bucket gradually filled with water, the pitch of the sound produced gradually increases. Explain.**

○ **Ans.** The bucket may be treated as a pipe closed at one end. If the depth of the empty bucket be l then the frequency of the fundamental produced by the bucket is $n = \frac{V}{4l}$ [V = velocity of sound in air]

Now, as the bucket is gradually filled with water, the length of the air column also decreases. As a result the frequency of the fundamental increases. Hence the pitch of the emitted sound increases.

- **Q. 4. What is the characteristic form of a wave function that represents (i) a travelling wave and (ii) stationary wave ?**

○ **Ans.** (i) A travelling wave is characterised by wave functions of the type $y = f(vt + x)$ or $y = g(vt - x)$

(ii) A stationary wave is described by periodic function of the type $y = f(t)\, g(x)$

- **Q. 5. Two open pipes of equal lengths produce fundamental of different frequencies at the same temperature. How is it possible ?**

○ **Ans.** If the frequency of the fundamental emitted by an open pipe be n, then

$$n = \frac{V}{2(l + 0 \cdot 6d)} \quad [d = \text{diameter of the tube}]$$

Here 0·6 d is the total end correction. Evidently the frequency of the fundamental emitted by the two open pipes of equal length will be different if their diameters are different.

- **Q. 6. An open and a closed pipe produce fundamentals of equal frequencies. What is the ratio of their lengths ?**

○ **Ans.** Let l_c = length of the closed pipe and l_0 = length of the open pipe. Then the frequency of the fundamental emitted by closed pipe is $n_e = \frac{V}{4l_e}$ and the frequency of the fundamental emitted by the open pipe is $n_0 = \frac{V}{2l_0}$

Here $n_e = n_0$. So $\frac{V}{4l_c} = \frac{V}{2l_0}$ $\quad \therefore 4l_c = 2l_0 \quad \therefore \frac{l_c}{l_0} = \frac{1}{2}$

- **Q. 7. If the temperature of closed pipe or open pipe is raised how does the frequency of the emitted sound change ?**

○ **Ans.** The frequency of the sound produced by a closed pipe or an open pipe is directly proportional to the velocity of sound in air.

So, $n \propto V$; or, $\frac{n_t}{n_0} = \frac{V_t}{V_0} = \sqrt{\frac{T}{T_0}}$

where n_t = frequency at t°C and n_0 = frequency at 0°C.

$$\therefore \frac{n_t}{n_0} = \sqrt{\frac{273 + t}{273}} = \left(1 + \frac{t}{273}\right)^{1/2} = 1 + 0 \cdot 00183\, t \quad \therefore \frac{n_t - n_0}{n_0} = 0 \cdot 00183\, t = \mathbf{0 \cdot 183\, t\ \%}$$

- **Q. 8. A stretched string is producing transverse wave. Explain which harmonics will be suppressed when it is touched at 1/4 of its length ?**

○ **Ans.** The point where it is touched will be a node. So the harmonics whose one node falls on that point will be emitted and others will be suppressed. So, only 4th, 8th, 12th ..... harmonics will be emitted and all other harmonics will be suppressed.

● **Q. 9. How does the frequency of the fundamental emitted by a stretched string change in the following cases :**

(i) length is made double, (ii) tension raised to four times.

○ **Ans.** (i) The frequency of stretched string is $n \propto \frac{1}{l}$ [l = length of the string]

$\therefore$ $n\,l$ = constant. So, if the length of the string is doubled then, $n \times l = n_1 \times 2l \quad \therefore n_1 = \frac{n}{2}$

So, in the frequency will be halved.

(ii) Also the frequency of the string $n \propto \sqrt{T}$

$$\therefore \frac{n_1}{n} = \sqrt{\frac{T_1}{T}} = \sqrt{\frac{4T}{T}} = 2 \quad \therefore n_1 = 2n$$

So, in this case the frequency will be double.

● **Q. 10. What will happen to the frequency of the fundamental produced by an open pipe if its one mouth is suddenly closed ?**

○ **Ans.** The frequency of the fundamental will be half of its initial frequency.

Reason : In the case of an open pipe, the fundamental frequency is $n = \frac{V}{2l}$. If its one open end is suddenly closed, it will behave as a closed pipe. So frequency of the fundamental will be now $n' = \frac{V}{4l}$.

$$\therefore \quad n' = \frac{V}{2l} \cdot \frac{1}{2} \qquad \therefore \quad n' = \frac{n}{2}$$

●● Miscellaneous Examples ●●

● **Example 3.22. The wave represented by the equation $y = a\cos(Kx - \omega t)$ on superposition with another wave produces a stationary wave whose one node is formed at $x = 0$. Obtain the equation of the second wave.**

○ **Solution :** To produce stationary waves the superposing harmonic waves must be identical i.e. the frequency, velocity and amplitude of the waves should be equal. Also they must travel in the opposite directions along the same straight line. So, the second wave equation may be as

$$y_1 = a\sin(Kx + \omega t + \theta) \quad [\theta = \text{phase angle}]$$

So, the equation of the resultant stationary wave is $y' = y + y_1$

$$y' = a\left[\cos(Kx - \omega t) + \cos(Kx + \omega t + \theta)\right]$$

It is given that at $x = 0,\ y' = 0$

$$\therefore \quad a\left[\cos(-\omega t) + \cos(\omega t + \theta)\right] = 0$$

$$\therefore \quad \cos\omega t = -\cos(\omega t + \theta) = \cos(\omega t + \theta + \pi)$$

$$\therefore \quad \omega t = \omega t + \theta + \pi \qquad \therefore \quad \theta = -\pi$$

$\therefore$ The required equation, $y_1 = a\cos(Kx + \omega t - \pi) = -a\cos(Kx + \omega t)$

● **Ex. 3.23. A wire is kept on two bridges 25 cm apart and is stretched by 0·04 cm by the application of a tension. If the density of the material of the wire be 10 g/c.c. and Young's modulus 9×10^{11} dyne/cm^2, find the frequency of the fundamental tone of such a stretched string. [J.E.E. 1987]**

○ **Solution :** The frequency of the fundamental of a stretched string is

$$n=\frac{1}{2L}\sqrt{\frac{T}{m}} \quad [\text{ L = length of the string }]$$

$$\text{Here } Y=\frac{TL}{Al} \quad \therefore T=\frac{YAl}{L}$$

$$\therefore \quad n=\frac{1}{2L}\sqrt{\frac{YAl}{Lm}}=\frac{1}{2L}\sqrt{\frac{YAl}{LA\rho}}=\frac{1}{2L}\sqrt{\frac{Yl}{L\rho}} \quad [m=A\rho]$$

$$=\frac{1}{50}\sqrt{\frac{9\times10^{11}\times0\cdot04}{25\times10}}=\mathbf{240\ Hz}$$

∴ The required frequency = **240 Hz.**

● **Ex. 3.24. A uniform wire of length 12 m and mass 6 kg is hanging from a rigid support. A load of 2 kg is attached at the lower end. If a transverse wave pulse is produced at the lower end of the string, what will be its wavelength when it reaches the upper end of the string ? [I.I.T.]**

○ **Solution :** The velocity of transverse wave in a stretched string is

$V \alpha \sqrt{T}$ [T = tension of the string]

$$\therefore \quad \frac{V_1}{V_2}=\sqrt{\frac{T_1}{T_2}} \quad \text{or,} \quad \frac{n\lambda_1}{n\lambda_2}=\sqrt{\frac{T_1}{T_2}} \qquad \therefore \ \lambda_2=\lambda_1\sqrt{\frac{T_2}{T_1}}$$

$$\therefore \quad \lambda_2=0\cdot06\times\sqrt{\frac{8}{2}}=0\cdot12\ \text{m}$$

∴ The wavelength of the wave at the upper end = **0·12 m.**

● **Ex. 3.25. A rope of mass 0·1 kg and length 2·45 m is hanging from the roof of a room. What is the velocity of transverse wave at a distance of 0·5 m from the lower end ? How much time will the wave takes to travel the full length of the rope ?**

○ **Solution :** The tension at a point on the string at a distant h metre from the lower end is

$T = mh\times g$ [m = mass per unit length of the string]

∴ Velocity of transverse wave at that point

$$V=\sqrt{\frac{T}{m}}=\sqrt{\frac{mhg}{m}}=\sqrt{gh}$$

$$\therefore \quad V=\sqrt{9\cdot8\times0\cdot5} \quad [h=0\cdot5\ \text{m}]$$

$$=\mathbf{2\cdot21\ ms^{-1}}$$

Now, at the lowest point of the string tension T = 0 and so that velocity at the lowest point is also zero. If the length of the string be l then, the velocity of the wave at highest point will be $V=\sqrt{gl}$.

If the wave takes a time t sec to move from the lowest end to the highest end, then acceleration of the wave is

$$f=\frac{\sqrt{gl}-0}{t}=\frac{\sqrt{gl}}{t}$$

$$\therefore \quad l=\frac{1}{2}ft^2=\frac{1}{2}\frac{\sqrt{gl}}{t}.t^2 \quad \therefore \quad t=\frac{2l}{\sqrt{gl}}=2\sqrt{\frac{l}{g}} \quad \therefore \quad t=2\times\sqrt{\frac{2\cdot45}{9\cdot8}}=1\ \text{sec}$$

● **Ex. 3.26. One mouth of an open pipe is suddenly closed. It is found that the frequency of the third harmonic of the closed pipe is greater by 100 Hz than the frequency of the fundamental of the open pipe. Calculate the fundamental frequency of the open pipe.** **[I.I.T. 1996]**

○ **Solution :** If l be the length of the open pipe so its fundamental frequency is

$$n_1 = \frac{V}{2l} \quad [\text{ V = velocity of sound in air}]$$

With one end being closed, its fundamental frequency, $n_2 = \dfrac{V}{4l}$

and the frequency of the third harmonic of the closed pipe is $n_2' = 3n_2 = \dfrac{3V}{4l}$

Now, by the problem, $\dfrac{3V}{4l} = \dfrac{V}{2l} + 100$

$\therefore \quad \dfrac{3}{2}n_0 = n_0 + 100 \quad [n_0 = \text{fundamental frequency of the open pipe}]$

$\therefore \quad \mathbf{n_0 = 200\ Hz.}$

● **Ex. 3.27. At 10°C the frequency of the fundamental of an open pipe is 400 Hz. What will be its frequency at 40°C.**

○ **Solution :** The required relation $\dfrac{n_1}{n_2} = \dfrac{V_1}{V_2}$

$$\therefore \quad n_2 = n_1 \cdot \frac{V_2}{V_1} = n_1\sqrt{\frac{T_2}{T_1}} = 400 \times \sqrt{\frac{313}{283}}$$

$\therefore \quad \mathbf{n_2 = 420{\cdot}66\ Hz}$

● **Ex. 3.28. A wire of length 25 cm and mass 2·5 gm is under a fixed tension. The length of a closed pipe is 40 cm. When they are sounded together first overtone of the string and the fundamental of the closed pipe produce 8 beats per second. If the tension of the string is lowered beat frequency is found to decrease. If the velocity of sound in air be 320 m/s, calculate tension of the string.** **[J.E.E. 1993]**

○ **Solution :** The fundamental frequency of the closed pipe is $n = \dfrac{V}{4l} = \dfrac{32000}{4 \times 40} = 200\ \text{Hz}$

and frequency of the first overtone of the string, $n_1 = \dfrac{1}{l}\sqrt{\dfrac{T}{m}}$

$$\therefore \quad n_1 = \frac{1}{25} \times \sqrt{\frac{T}{0{\cdot}1}} \quad \left[\because m = \frac{2{\cdot}5}{25} = 0{\cdot}1\ \text{g/cm}\right]$$

By problem, $n_1 - n = 8$ or $n - n_1 = 8$

Now, if tension of the string is lowered, its frequency decreases. Since the number of beats per second also decreases, it means that $n_1 > n$

So, $n_1 - n = 8$

$$\therefore \quad \frac{1}{25}\sqrt{\frac{T}{0{\cdot}1}} - 200 = 8 \quad \therefore \quad \sqrt{\frac{T}{0{\cdot}1}} = 208 \times 50$$

$\therefore \quad T = (208 \times 50)^2 \times 0{\cdot}1 = 27{\cdot}04 \times 10^5$ dyne

$\therefore \quad$ Tension of the string = **27·04 N**

- **Ex. 3.29. 65 tuning fork are arranged in order of increasing frequencies. When any two consecutive tuning forks are sounded together 8 beats per second is obtained. First and the last tuning fork are in unison with the fundamental and the third overtone respectively of a stretched string. What is the frequency of the first overtone ?**

○ **Solution :** Let the frequencies of the tuning forks be $n_1, n_2, ... n_{65}$. Also given, the difference in frequencies of any two consecutive tuning forks is 8

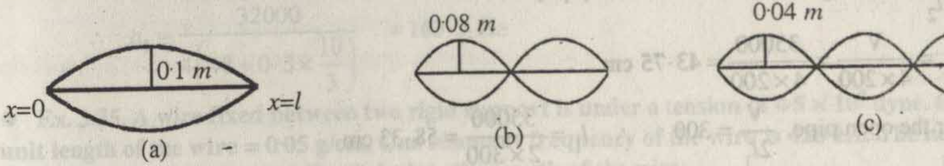
$\therefore \quad n_{65} = n_1 + (65 - 1) \times 8 = n_1 + 512$ . . . . (i)

Now, n_1 is equal to the fundamental frequency of a stretched string and n_{65} is equal to the third harmonic frequency of the same string. Since third harmonic frequency is exactly three times the fundamental frequency, we can write $n_{65} = 3n_1$ . . . . (ii)

Hence from equations (i) and (ii), $3n_1 - n_1 = 512 \;\; \therefore n_1 = 256$

$\therefore$ So, the frequency of the first tuning fork = **256 Hz.**

- **Ex. 3.30. Figures 3.18(a), (b) and (c) show different standing waves on a string of length 2·0m and linear mass density $4{\cdot}0 \times 10^{-2}$ kg m^{-1} under a tension of 100 N. The amplitudes of antinodes are indicated.**



(a) (b) (c)

Fig. 3·18

(i) Find the frequency of the three modes of vibration of the string.

(ii) Obtain the equation of transverse displacement y as a function of x and t for each mode.

○ **Solution :** Velocity of transverse wave in the string is

$$V = \sqrt{\frac{T}{m}} = \sqrt{\frac{100}{4 \times 10^{-2}}} = 50 \text{ m/s}$$

(i) The frequency of the string vibrating in p segments is $n_1 = \frac{p}{2l}\sqrt{\frac{T}{m}} = \frac{pV}{2l}$

Case (a), $p = 1$, so, $n_1 = \frac{50}{2 \times 2} = 12{\cdot}5$ Hz

Case (b), $p = 2$, so, $n_2 = \frac{V}{2l} = \frac{50}{2} = 25$ Hz

Case (c), $p = 3$, so, $n_3 = \frac{3V}{2l} = 3 \times 12{\cdot}5 = 37{\cdot}5$ Hz

(ii) The string is fixed at $x = 0, x = l$. It is given that at $t = 0$ the string is along x-axis. So at $t = 0$, $y = 0$. Hence displacement equation can be written as

$$y = 2A \cdot \sin\left(\frac{p\,\pi\,x}{l}\right) \cdot \cos\left(2\pi n_p t\right) \; [2A = \text{amplitude at antinode}]$$

Case (a), $2A = 0{\cdot}1$ m, $p = 1$, $n_1 = 12{\cdot}5$ Hz and $l = 2{\cdot}0$ m.

$$\therefore \quad y = 0{\cdot}1 \sin\left(\frac{\pi x}{2}\right) \cdot \cos(25\,\pi t)$$

Case (b), $2A = 0{\cdot}08$ m, $p = 2$, $n_2 = 25$ Hz and $l = 2{\cdot}0$ m

$$\therefore \quad y = 0{\cdot}08 \sin(\pi x) \cdot \cos(50\,\pi t)$$

Case (c), $2A = 0{\cdot}04\ m$, $p = 3$, $x_3 = 37{\cdot}5$ Hz and $l = 2.0$ m

$$\therefore \quad y = 0{\cdot}04 \sin\left(\frac{3\pi x}{2}\right)\cdot\cos(75\,\pi t)$$

- **Ex. 3.31. The frequency of the fundamental of an open pipe is 300 Hz. The frequency of the first overtone of the open pipe is equal to the frequency of the third overtone of a closed pipe. Calculate the length of the two pipes. Velocity of sound in air = 350 ms^{-1}.**

○ **Solution :** Let the frequency of the fundamental of the open pipe be n_1

Then the frequency of its first overtone $= 2n_1$

Again, if the frequency of the fundamental of the closed pipe be n_2 then the frequency of its third harmonic $= 3n_2$.

Now, by problem, $2n_1 = 3n_2 \quad \therefore \quad n_2 = \frac{2}{3}n_1 = \frac{2\times 300}{30} = 200$ Hz

$\therefore$ fundamental frequency of the closed pipe $n_2 = 200$ Hz

$$\therefore \quad \frac{V}{4l_2} = 200 \ [l_2 = \text{length of the closed pipe}]$$

$$\therefore \quad l_2 = \frac{V}{4\times 200} = \frac{35000}{4\times 200} = \mathbf{43{\cdot}75\ cm}$$

And for the open pipe, $\frac{V}{2l_1} = 300 \quad \therefore \quad l_1 = \frac{35000}{2\times 300} = 58{\cdot}33$ cm

$\therefore$ Length of the open pipe = **58·33 cm** and that of the closed pipe = **43·75 cm**

- **Ex. 3.32. In an experiment with a sonometer, the string is under tension due to a load attached at its free end. It is the found that the frequency of the fundamental of the string is equal to that of a tuning fork. Now the load is immersed in water and the tuning fork is found to produce 5 beats per second with the sonometer wire. If the density of the material of the load be 9 gm/c.c., what is the frequency of the tuning fork ?**

○ **Solution :** Let weight applied = W and m = mass per unit length of the wire.

$\therefore$ the frequency of the fork n = the frequency of the fundamental of the string $= \frac{1}{2l}\sqrt{\frac{W}{m}}$

When the load is immersed in water, its apparent load, $W_1 = W - \frac{W}{9}\times 1 = \frac{8}{9}W$

Now, the frequency of the fundamental, $n_1 = \frac{1}{2l}\sqrt{\frac{8W}{9m}}$

Now, $n - n_1 = 5$ and $\frac{n}{n_1} = \sqrt{\frac{9}{8}}$

$$\therefore \quad \frac{n}{n-5} = \sqrt{\frac{9}{8}} \quad \text{or,} \quad n = \mathbf{88 \cdot 36\ Hz}$$

$\therefore$ So, the frequency of the fork = **88·36 Hz.**

- **Ex. 3.33. The two ends of a wire is rigidly fixed. At 30°C the wire is taut with negligible tension. If the temperature is lowered to 20°C, calculate the velocity of transverse wave in the string. Given the coefficient of linear expansion of the wire, $\alpha = 18\times 10^{-6}/°C$; its Young's modulus $Y = 12\times 10^{11}$ dynes/cm^2, and density of the material of the wire = 6 g/c.c.** **[J.E.E. 1990]**

○ **Solution :** Velocity of transverse wave in the string, $V = \sqrt{\frac{T}{m}}$

Now. $T = YA\,\alpha\,t$ and $m = A\rho$

$$\therefore \quad V=\sqrt{\frac{YA\,\alpha\,t}{A\rho}}=\sqrt{\frac{Y\,\alpha\,t}{\rho}}=\sqrt{\frac{12\times10^{11}\times18\times10^{-6}\times10}{6}}=6\times10^{3}\ \text{cm/s}$$

$\therefore$ The velocity of transverse wave is **60 ms^{-1}.**

- **Ex. 3.34. The length of an open pipe is 48 cm and has definite diameter. The frequency if its fundamental is 320 Hz. The velocity of sound in air is 320 ms^{-1}. Calculate the diameter of the tube. If one of its two ends is closed, what will be the frequency of the fundamental now. [I.I.T., 1980]**

○ **Solution :** The fundamental frequency of the open pipe, $n=\dfrac{V}{2(l+0\cdot6D)}$

Here $n = 320$ Hz, $V = 32000$ cm s^{-1}, $l = 48$ cm and D = diameter

$$\therefore\ 320=\frac{32000}{2(48+0\cdot6D)} \qquad \therefore\ 48+0\cdot6D=50. \qquad \therefore\ D=3\cdot33\ \textbf{cm}$$

2nd Case : When one end is closed, the fundamental frequency is $n_1=\dfrac{V}{4(l+0\cdot3D)}$

$$\therefore\ n_1=\frac{32000}{4\left(48+0\cdot3\times\frac{10}{3}\right)}=\mathbf{163\cdot3\ Hz}$$

- **Ex. 3.35. A wire fixed between two rigid support is under a tension of 4·5 × 10^7 dyne. the mass per unit length of the wire = 0·05 g/cm. One resonant frequency of the wire is 420 Hz. The next resonant frequency of the wire is 490 Hz. Calculate the length of the wire. [I.I.T. 1971]**

○ **Solution :** Let the frequency of the p th harmonic = 420 Hz

So, the frequency of the $(p + 1)$ the harmonic = 490 Hz

$$\therefore\ \frac{p}{2l}\sqrt{\frac{T}{m}}=420 \quad\text{and}\quad \frac{\rho+1}{2l}\sqrt{\frac{T}{m}}=490$$

$$\therefore\ \frac{p+1}{p}=\frac{49}{42} \qquad \therefore\ p=6$$

$$\therefore\ \frac{6}{2l}\sqrt{\frac{4\cdot5\times10^{7}}{0\cdot05}}=420 \qquad \therefore\ l=\mathbf{214\cdot3\ cm}$$

- **Ex. 3.36. One end of a wire of radius r is soldered to one end of another a wire of radius $2r$. The combined wire is used as a sonometer wire under a tension T. The junction point is at the mid point between two bridges. When a stationary wave is formed, one node is formed at the junction, calculate the number of loops formed on the two wires. [I.I.T. 1976]**

○ **Solution :** Let the densities of the materials of the two wires be ρ_1 and ρ_2

$\therefore$ mass per unit length of the first wire, $m_1=\pi r^2\rho_1$

and that per unit length of the second wire, $m_2=4\pi r^2\rho_2$

$$\therefore\ \frac{m_1}{m_2}=\frac{\rho_1}{4\rho_2}$$

As the junction point lie at the mid point between the two bridges, the lengths of the two wires are equal. Let this length be l. Also the frequency of the two wires will be same and let this frequency be n.

Since the junction point is a node, integral number of loops are formed on both the wires. Let the number of loops on the two wires be x and y.

$$\therefore\quad n=\frac{x}{2l}\sqrt{\frac{\mathrm{T}}{m_1}}\quad\text{and}\quad n=\frac{y}{2l}\sqrt{\frac{\mathrm{T}}{m_2}}$$

$$\therefore\quad \frac{x}{y}=\sqrt{\frac{m_1}{m_2}}=\frac{1}{2}\sqrt{\frac{\rho_1}{\rho_2}}$$

In this case, $\rho_1=\rho_2$. So, $\frac{x}{y}=\frac{1}{2}$

- **Ex. 3.37. The midpoint of a horizontal metal rod of length 1 m is rigidly fixed. It is set into longitudinal vibration to produce logitudinal stationary wave and two nodes are formed on both sides of the mid point. The amplitude of the antinode is 2×10^{-6} m. Obtain the equation of motion of the point 2 cm away from the mid point. Also obtain the component waves which form the stationary wave. Young's modulus = 2×10^{11} N/m^2 ; density = 8000 kg/m^3 [I.I.T. 1994]**

○ **Solution :** The equation of the stationary wave $y=2a\cos\frac{2\pi x}{\lambda}\cdot\sin\frac{2\pi vt}{\lambda}$

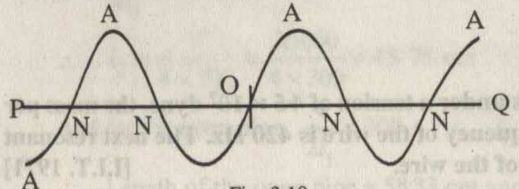


Fig. 3·19

where the symbols has usual meaning.

Let the equations of the two progressive waves be $y_1=a\sin\frac{2\pi}{\lambda}(vt-x)$ and $y_2=a\cdot\sin\frac{2\pi}{\lambda}(vt+x)$

Here amplitude of the antinodal point,

$2a=2\times10^{-6}$ m $\quad\therefore\quad a=10^{-6}$ m

Velocity of the longitudinal wave is $v=\sqrt{\frac{\mathrm{Y}}{\rho}}=\sqrt{\frac{2\times10^{11}}{8000}}=5\times10^{3}$ m/s

The midpoint is a node and the end points of the rod are two antinode.

The point O is the mid point of the rod PQ. 5 half waves will be produced in PQ.

$$\therefore\quad 5\frac{\lambda}{2}=1\qquad\therefore\quad \lambda=\frac{2}{5}=\mathbf{0\cdot4\ m}$$

So, the equation of the stationary wave is

$$y=2\times10^{-6}\cos\frac{2\pi x}{0\cdot4}\cdot\sin\frac{2\pi(5000)t}{0\cdot4}\,\mathrm{m}$$

$$=2\times10^{-6}\cos(5\pi x)\cdot\sin\left(25\pi\times10^{3}t\right)\,\mathrm{m}$$

The equation of motion of the two points located at 2 cm way from the midpoint i.e. the equation of motion of two points at distances of 52 or 48 cm from the left end of the rod are

$$y=2\times10^{-6}\cdot\cos(2\cdot6\pi)\cdot\sin\left(25\pi\times10^{3}t\right)\,\mathrm{m}$$

$$\text{or,}\quad y=2\times10^{-6}\cos(2\cdot4\pi)\cdot\sin\left(25\pi\times10^{3}t\right)\,\mathrm{m}$$

Hence the equation of the two progressive waves are

$$y_1=1\times10^{-6}\sin\frac{2\pi}{0\cdot4}\left(5\times10^{3}t-x\right)\,\mathrm{m}$$

$$\text{and}\quad y_2=1\times10^{-6}\sin\frac{2\pi}{0\cdot4}\left(5\times10^{3}t+x\right)\,\mathrm{m}$$

• E ◇ X ◇ E ◇ R ◇ C ◇ I ◇ S ◇ E •

(A) Short answer type questions :

1. State the principle of superposition.

2. What is a stationary wave ?

3. How stationary waves are formed ?

4. What is meant by beats ? When is it formed ?

5. How does the frequency of the fundamental produced by a stretched string change in the following cases :

(i) when tension in the string is made double,

(ii) diameter of the wire is made double,

(iii) length of the string is made double.

6. Give examples (i) where transverse stationary wave and (ii) where longitudinal stationary waves are formed.

7. An open pipe is producing its fundamental tone. Now one of its mouth is suddenly closed. What will be the change in frequency of the fundamental ?

8. The frequency of the fundamentals emitted by a closed pipe and a open pipe are equal. What is the ratio of the lengths of the two pipes ?

9. Will the frequency of sound emitted by an organ pipe (open or closed) change if the temperature is changed ? **[H.S. 1990]**

10. Why all harmonics are not present in a closed pipe ?

11. Why is the sound emitted by an open pipe is more musical than a closed pipe ?

12. Why beats are not observed in the case of light waves ?

13. What is the relation between the frequencies of the overtones of a stretched string with the frequency of its fundamental ?

14. How will the frequency of the fundamental emitted by an open change in the following cases :

(i) the temperature of air in the tube rises.

(ii) humidity of the air in the tube rises.

(iii) the tube is filled up with carbon-di-oxide gas instead of air.

(iv) the diameter of the tube is made double.

(v) one of its mouth is suddenly closed.

15. What is end correction in the case of a closed pipe and an open pipe ?

16. Will the frequencies of the fundamentals emitted by two open pipes of equal lengths be same, if the diameter of one tube is double the other ?

17. The diameter of an open pipe is double the diameter of another tube; but their lengths are equal. Will their fundamental frequencies be equal ? Explain.

18. A stretched string is struck at 1/3 of its length from one end. Which harmonics will be present ?

19. Which harmonics will be absent if a vibrating string is lightly touched at its mid point ?

20. What is the condition so that beats are audible ? **[H.S. 1998]**

21. The frequency of the fundamental of a stretched string is found to produce equal number of beats with two tuning forces. How is it possible ?

22. What will happen to the emitted sound if a stretched vibrating string is touched at 1/4 of its length from one end ?

23. Draw different modes of vibration of a stretched string.

24. What are beats ? When are they produced ? **[H.S. 2000]**

(B) Essay type questions :

1. (a) State and explain principle of superposition of waves.
   (b) Explain under what conditions stationary waves and beats are formed ?
3. What are stationary waves ? How are they formed ? Distinguish between stationary waves and progressive waves.
4. Explain mathematically how two progressive waves can produce stationary waves. What are nodes and antinodes. Obtain the position of these points on the stationary waves.
5. (a) State the laws of transverse vibration of a stretched string. **[H.S. 1999, 2000]**
   (b) What are overtone and harmonics ? Show on a stretched string fixed at the two ends, the first two modes of vibration. Locate the positions of nodes and antinodes on the string.
6. What are fundamental, harmonics and overtones ? Why all harmonics are overtones, but all overtones are not harmonics ? **[H.S. 2002]**
7. Show that for same lengths, the fundamental tone emitted by an open tube will be sharper than that emitted by a closed pipe. **[H.S. 2002]**
8. Show with diagrams that in the vibrations of an air column contained in a closed pipe only odd harmonics are present. **[H.S. 2000]**
9. Why each stringed instrument is provided with a hollow box ?
10. Draw the diagrams of the first three modes of transverse vibration of a stretched string clamped at both ends. Show the positions of nodes. **[H.S. 1980]**
11. Prove that the pitch of the fundamental tone emitted by an open pipe is one octave higher than that emitted by a closed pipe of the same length. **[H.S. 1998]**
12. What are beats ? How can you determine at unknown frequency with the help of beats ? **[H.S. 1988]**
13. Describe the superposition of a progressive waves and formation of stationary wave is an open pipe.
14. Determine the frequencies of the different modes of vibration of the air column in a pipe closed at one end. Show that only odd harmonics are present here. **[H.S. 1984]**
15. Show how the frequency of the vibrating air column inside the tube would change if temperature is raised. **[H.S. 1990]**

(C) Simple numerical problems :

1. The equation of a stationary wave is $y = 4\cos\frac{\pi x}{5}\sin 60\,\pi t$. Here x and y are in cm and t is in sec. Calculate the amplitude and velocity of the superposing waves. **[Ans.** 2 cm ; 300 cm/s]
2. A wire of length 40 cm and having area of cross section 2 mm^2 is stretched by 4 kg load applied at its free end. What will be the frequency of transverse vibration of the string ? Given, density of steel 7·7 g/c.c. and g = 980 cm/s^2. **[Ans.** 199·5 Hz]
3. The length of a wire is 80 cm and it weighs 40g and it is kept under a tension of 800 kg. What is the frequency of the transverse vibration of the string ? g = 10 m/s^2 **[Ans.** 250 Hz]
4. Two wires made of same metal and of equal lengths are put under tension by 288 gm and 628 gm loads respectively. If the ratio of the fundamentals of two wires be 5 : 3, calculate the ratio of their diameters. **[Ans.** 2 : 5]
5. A wire of length 3·6 m and having mass $1{\cdot}6 \times 10^{-3}$ kg is set into transverse vibration. Tension in the string is 19·6 N and it is vibrating by forming 6 loops. What is its frequency ? **[Ans.** 175 Hz]
6. Two progressive waves of equal amplitude and each having frequency 660 Hz and wave velocity 330 m/s produce stationary wave are superposition. Calculate : (i) distance between two consecutive antinodes and (ii) are node and next antinode. **[Ans.** (i) 25 cm (ii) 12·5 cm]
7. Calculate the percentage change in tension of a wire of fixed length so that frequency of transverse vibration of the string increases by 10%. **[Ans.** 21%]
8. Calculate the percentage change in frequency of fundamental produced by a stretched string if its tension is increased by 20% ? Length is held constant. **[Ans.** 9·54% increase]
9. The frequency of the fundamental emitted by a stretched string is 400 Hz. Calculate the percentage change in length of the string so that the frequency of the fundamental in reduced to 360 Hz. **[Ans.** 11·1% increase]

10. Two wires of equal length are made of same material and are stretched by equal tension. If the radii of the wires be in the ratio 3 : 4, calculate the ratio of the frequencies of the fundamentals produced by the two wires. **[Ans.** 4 : 3]

11. The frequency of the fundamental of the transverse vibration of a stretched string is 400 Hz. If the length is increased by 10%, what will be the changed fundamental frequency ? **[Ans.** 364 Hz]

12. Below are given mass (M), length (l) and tension (T) for two wires. Calculate the frequency of transverse vibration of the strings :

(a) $M = 3g$, $l = 30$ cm, $T = 36 \times 10^5$ dyne, (b) $M = 2{\cdot}5$ g ; $l = 25$ cm ; $T = 25 \times 10^5$ dyne. **[Ans.** (a) 100 Hz, (b) 100 Hz]

13. The velocity of transverse wave in a stretched string of length 60 cm is 150 ms^{-1}. Calculate the frequency of third overtone produced by the string. **[Ans.** 500 Hz]

14. The frequency of the fundamental of transverse vibration of a sonometer wire under tension is 250 Hz. Length of the wire is made double and tension is also increased so that its fundamental frequency remains unchanged. What is the ratio of the final and initial tension in the string ? **[Ans.** 4 : 1]

15. Stationary waves are formed by the superposition of the following two progressive waves : $y_1 = 5 \sin 2\pi(76t - 0 \cdot 12x)$ and $y_2 = 5 \sin 2\pi\,(76t - 0 \cdot 12x)$. Here distances are measured in cm units. Find the distance between two consecutive nodes. **[Ans.** 4·17 cm]

16. The length of a sonometer wire is 36 cm, diameter 1·0 mm and tension 31·4 N. If the frequency of the fundamental of transverse vibration of the string be 100 Hz, calculate the density of the material of the wire. **[Ans.** 7·716 g/c.c.]

17. The length of a sonometer wire is 1 m and tension 5 kg-wt. Tension is increased by 0·2 kg-wt. How its length be changed so that frequency of transverse vibration remain unchanged ? **[Ans.** 0·02 m decrease]

18. The ratio of the length of two wires made of same material is 2 : 3. If the diameters are equal, the frequency of fundamental of the shorter wire is one octave above the fundamental frequency of the larger wire. What is the ratio of tension of the two wires ? **[Ans.** 16 : 9] **[H.S. 1994]**

19. A sonometer wire emits a tone of frequency 150 Hz. Tension is increased in the ratio 9 : 16 and length is made double. What will be the frequency of the fundamental now ? **[Ans.** 100 Hz] **[H.S. 1995]**

20. In a string musical instrument, the distance between the two bridges is 80 cm and tension 40 N. It weighs 0·25 g/ meter.

(i) If a wave is produced at one end, after what time it will reach the other end ?

(ii) What is the frequency of the fundamental ?

(iii) What are the frequency of first two overtones ? **[Ans.** (i) 2×10^{-3} sec, (ii) 250 Hz, (iii) 500 Hz, 1000 Hz]

21. A sonometer wire of length 100 cm under a certain tension can produce resonance with a tuning fork. When the weight is increased by 100 gm, its length has to be shortend by 2 cm to produce resonance again. Calculate the initial load. **[Ans.** 2·475 kg]

22. When two tuning forks are sounded together 4 beats per second is produced. These tuning forks are in resonance with 128 cm and 130 cm respectively of the same wire kept under a fixed tension. Calculate the frequency of the two forks. **[Ans.** 256 Hz and 260 Hz] **[H.S. 2000]**

23. The length of the vertical tube filled with water is 200 cm. A vibrating tuning fork of frequency 256 Hz is hold near the open end of the tube. Water is slowly drained out from the botttom of the tube. Neglecting end correction, obtain the position of water level in the tube during the first and second resonance. Given, the velocity of sound in air = 320 m/s. **[Ans.** 168·75 cm, 106·75 cm] **[H.S. 2000]**

24. Two tuning forks A and B when sounded together can produce 5 beats per second. 40 cm length of a sonometer wire under a certain tension is in resonance with the tuning fork A and the second tuning fork B is in resonance with the length of 40·5 cm of the same sonometer wire kept under the same tension. Calculate the frequency of the two forks. **[Ans.** 405 Hz, 400 Hz] **[H.S. 1993]**

25. 24 tuning forks are arranged in order of increasing frequency. Any two consecutive tuning forks when sounded together produce 4 beats per second and the last fork is one octave above the first fork. Calculate the frequency of the first and the last tuning fork. **[Ans.** 92 ; 184 Hz**] [J.E.E. 1997]**

26. A transversely vibrating sonometer wire emits second harmonic which is resonance with the third harmonic produced by a second wire having length equal to half the length of the first wire. If the two wires have same mass per unit length, compare the tensions of the two wires. **[Ans.** 9 : 1]

27. The lengths, radii, densities and tensions of two wires A and B all are in the ratio 1 : 2. Which overtone of the wire B will be in resonance with the fundamental of the wire A ? **[Ans.** Third harmonic]

28. Two vibrating tuning forks can produce 5 beats per second. The frequency of one of the fork is 295 Hz. When some wax is attached to the prong of the other fork, three beats per second are produced now. Calculate the frequency of the second fork. **[Ans.** 300 Hz] **[H.S. 1990]**

29. The frequency of transverse vibration of a sonometer wire is.80 Hz. If the tension is increased in the ratio 1 : 9 and the length of wire is increased in the ratio 1 : 2. What will be the frequency of the sonometer now ? **[Ans.** 120 Hz]

30. Two vibrating tuning fork A and B can produce 5 beats per second. If both the forks A and B are in resonance with 36 cm and 37 cm respectively of a closed pipe. Calculate the frequency of the two forks. **[Ans.** 185 ; 180 Hz] **[H.S. 1988, 2002]**

31. What is the frequency of the fundamental of a closed pipe of length 16·3 cm ? Given velocity of sound in air = 334 ms^{-1}. Also calculate the frequency of the first two overtones. **[Ans.** 512 Hz, 1536 Hz, 2560 Hz]

32. Two open pipes of lengths 80 cm and 82 cm are producing fundamental tones. How many beats per second is formed ? Given the velocity of sound in air = 340 ms^{-1}. **[Ans.** 5·18 Hz]

33. A glass tube is partly filled with water. A vibrating tuning fork is placed on the open end of the tube. The lengths of the column for the first and second resonance are 33 cm and 100·5 cm. Calculate the end correction. **[Ans.** 0·75 cm]

34. In resonance air column experiment if a tuning fork of frequency 256 Hz is used the lengths of the air column for the first and second reaonance are 33·4 cm and 101·7 cm respectively. Calculate (i) velocity of sound and (ii) end correction. **[Ans.** 0·75 cm, 350 ms^{-1}]

35. When a tuning fork of frequency 256 Hz is used, the length of the resonant air column is 32·4 cm. But when another tuning fork of frequency 480 Hz is used, the length of the resonant column becomes 16·9 cm. Calculate (i) end correction and (ii) velocity of sound. **[Ans.** (i) 0·8 cm (ii) 340 ms^{-1}]

36. A tuning fork having a frequency of 340 Hz is vibrating just above a cylindrical tube of length 120 cm. Water is slowly poured into it. What is the minimum height of water required for resonance ? Velocity of sound = 332 ms^{-1}. **[Ans.** 46·76 cm] **[J.E.E., 1985]**

37. When two tuning forks A and B are sounded together 8 beats per sec. are produced. A is in resonance with a closed pipe of length 37·5 m and B produces resonance with 38.5 cm pipe closed at are end. Calculate the frequencies of the two forks. **[Ans.** 308 Hz, 300 Hz] **[I.I.T.]**

38. The first overtone emitted a closed pipe is in union with the third overtone of a closed pipe. Calculate the ratio of the lengths of the two pipes. **[Ans.** 3 : 8]

39. The frequency of the fundamental of an organ pipe at 40°C is found to be 250 Hz. If the temperature is lowered to 20°C, what will be the frequency now ? **[Ans.** 242 Hz]

40. At 15°C two organ pipes can produce 4 beats per second. If the temperature is raised to 40°C, how many beats will be produced in two seconds ? The velocity of sound in air at 0°C is 332 ms^{-1}. Neglect the expansion of the pipes. **[Ans.** 4·17 Hz]

41. The difference in frequencies of the fundamentals produced by an open pipe and a closed pipe is 25 Hz. The length of the open pipe is 166 cm. Calculate the length of the closed pipe. Given, the velocity of sound in air 332 ms^{-1}. **[Ans.** 110·7 cm]

42. The frequency of the fundamental produced by an open organ pipe of length 50 cm is equal to the frequency of the fundamental produced by a sonometer wire of length 75 cm. The mass of 1 cm length of the sonometer wire is 1 gm. Calculate its tension. The velocity of sound in air = 350 ms^{-1}. **[Ans** 25·0 × 10^8 dyne]

43. It is desired to tune the sonometer wire with a tuning fork of frequency 512 Hz. When vibrated together they produce 10 beats per second.

(a) by what percentage should the length of the string be altered to achieve tuning, the tension remaining unchanged ?

(b) by what percentage should be the tension in the string be altered to achieve tuning, the length of the string remaining unlatered ? **[Ans.** (a) 1·95%, (b) 4·02% or, 3·8%]

44. Calculate the percentage change in the frequency of the fundamental note emitted by a closed pipe of length 1·0 m if the temperature of the air is changed from 15°C to 30°C. Velocity of sound at 0°C = 330 ms^{-1}. **[Ans.** 2·63%]

45. The lower portion of a vertical glass tube is immersed in water. A vibrating tuning fork is placed on the open end of the tube. The first resonance occurs when the length of the air column is 25 cm and the second resonance occurs when the length of the air column is 77·2 cm. Obtain the value of end correction. **[Ans.** 1·1 cm]

46. A wire of length 50 cm is kept under tension by a load attached to its free end. Density of the material of the wire is 9 gm/c.c. When the wire is set into transverse vibration it is in reasonance with a tuning force of frequency of 256 Hz. When the load is fully immersed in water, how the length of the wire be decreased so that it is again in resonance with the tuning fork ? **[Ans.** 2·86 cm]

47. A brass rod of length 3 m is clamped at the middle. When the rod is set into longitudinal vibration it producesd a tone of 600 Hz. If the density of brass be 8·3 g/c.c calculate (i) velocity of sound in the rod, (ii) Young's modulus of the material of the rod. **[Ans.** (i) 3600 ms^{-1} (ii) $10{\cdot}76 \times 10^{11}$ dynes/cm^2]

48. An air column at 51°C produces 4 beats per second with a turning fork. When the temperature is lowered to 16°C, 1 beat per second is produced. Calculate the frequency of the fork. **[Ans.** 50 Hz] **[I.I.T.]**

49. The equation of two progressive waves are $y_1 = 2 \cdot 0 \sin \frac{2\pi}{3}(340t - x)$ metre and $y_2 = 2 \cdot 0 \sin \frac{2\pi}{3+y}(340t - x)$ metre. On superposition they produce 10 beats per second. Calculate the value of y. **[Ans.** 0·3 m]

50. A sonometer wire carries a metal block at the other end. When the block is in air resonant length with a tuning fork is found to be 90 cm. When the block is wholly immersed in water, the resonant length with the same tuning fork decreases to 80 cm. Find the relative density of the metal block. **[Ans.** 4·76]

(B) Harder numerical problems :

1. A wire is vibrating according to the following equation : $y = 5 \cos \frac{\pi x}{3} \cdot \sin 40\,\pi t$

Here x and y are in cm and t is in sec.

(i) Calculate amplitude and velocity of the two waves which produce the above vibration on superposition.

(ii) Calculate the distance between the two nearest points which are at rest.

(iii) Calculate the particle velocity at a distance x = 15 cm and time t = 9/8 sec. **[J.E.E. 1993]**

[Ans. (i) $y_1 = \frac{5}{2} \sin 40\,\pi\left(t - \frac{x}{120}\right)$; $y_2 = \frac{5}{2} \sin 40\,\pi\left(t + \frac{x}{120}\right)$ $a_1 = a_2 = 2{\cdot}5$ cm ; v = 120 m/s

(ii) 3 cm ; (iii) particle velocity = 0]

2. The equation of a progressive wave moving along a string is $y = 40 \cos \frac{\pi}{2}(0 \cdot 005x - 8 \cdot 0t)$. Here x and y are in cm and t in sec. Write the equation of the wave which on superposition with the given wave can produce stationary wave. What is the distance between the two consecutive nodes ? **[Ans.** $y_1 = 40 \cos \frac{\pi}{2}(0 \cdot 005x + 8 \cdot 0t)$; 4 cm]

3. The length of a steel wire in 1·5 m. Due to a tension applied to it, strain of 1% is produced. Calculate the frequency of the fundamental. Given, density of steel = $7{\cdot}7 \times 10^3$ kg/m^3 ; Young's modulus Y = $2{\cdot}2 \times 10^{11}$ N/m^2 **[Ans.** 178 Hz]

[**Hint :** Strain = 1/100 ; Stress = $T/\pi r^2$; $n = \frac{1}{2l}\sqrt{\frac{T}{\pi r^2}}$]

4. A wire of length 20 cm is rigidly fixed between two supports under negligible tension at 30°C. If the temperature of the wire is lowered to 10°C, calculate the velocity of transverse wave in the string. Given, Young's modulus of the string = 13×10^{10} N/m^2. Coefficient of linear expansion = 17×10^{-6} /°C and its density = 9000 kg/m^3. Calculate also the fundamental frequency. **[Ans.** 70 ms^{-1} ; 175 Hz]

5. A wire of length 100 cm and density 9 gm/c.c. is just taut between two fixed supports. Due to tension applied to it, its length increases by 0·05 cm. Calculate the lowest frequency of vibration of the wire. Given young's modulus of the wire = 9×10^{11} dyne/cm^2 **[Ans.** 35·4 Hz] **[I.I.T.]**

6. The temperature of a wire fixed rigidly at its two ends is lowered by 10°C. Calculate the tension in the string from the following data : area of cross section = 0·01 cm^2 ; $\alpha = 16 \times 10^{-6}$ /°C, Y = 2×10^{11} N/m^2. If the frequency of transverse vibration of the wire now be double the initial frequency of vibration, what was the initial tension ? **[Ans.** 32 N ; 10·67 N] **[J.E.E. 1980]**

7. One end of an aluminium wire of length 60 cm and area of cross-section 0·01 cm^2 in fixed on a wall. At its other end one end of a steel wire is attached. The steel wire being of length 86·6 cm passes over a pulley and a load of 10 kg is suspended from its free end. It is set into transverse vibration by external source. Find the lowest frequency of excitation for which stationary wave is observed such that the junction of the wire is a node. Density of aluminium = 2·6 g/c.c. and of steel = 7·8 g/c.c. **[Ans.** 323·6 Hz] **[I.I.T. 1976]**

8. A under water suimmer sends a sound signal to the surface. It produces 5 beats per second when compared with the fundamental note of a pipe 20 cm long closed at one end. What is the wave length of sound in water. ? [Given, velocities of sound in air and water are 360 m/s and 1500 m/s respectively.] **[Ans.** 3·3 m or 3·37 m] **[J.E.E. 1996]**

9. The diameter of a steel wire of length 34·6 cm is 0·5 mm. The frequency of the fundamental produced by the string under a tension of 5 kg is in resonance with a closed organ pipe at 15°C. Keeping everything unchanged the temperature of the organ pipe is lowered by 2°C. Now the wire and the pipe are set into vibration, what change is observed ? Density of steel = 7·8 g/c.c. **[Ans.** beats will be produced ; beat frequency = 1]

10. A wire of length 25 cm and mass 2·5 g is stretched by a fixed weight. The first overtone of the wire produces 8 beats per second with the fundamental of a closed pipe of length 40 cm. If the tension of the wire is decreased, number of beats is found to decrease. If the velocity of sound in air be 320 m/s, calculate the tension in the wire. **[Ans.** 27·04 N] **[J.E.E. 1993]**

11. The linear density of a wire is 0·05 g/cm. It is stretched between two rigid supports. Tension in the string $4{\cdot}5 \times 10^7$ dyne. The wire is in resonance with a frequency of 450 Hz. The next frequency with which it also produces resonance is 490 Hz. Calculate the length of the wire. **[Ans.** 214·3 cm] **[I.I.T.]**

[**Hints :** Assume pth harmonic = 420 Hz and $(p + 1)$ th harmonic = 490 Hz]

12. The initial temperature of air in an organ pipe is 10°C. At what temperature the frequency of the fundamental produced by the pipe will increase by 4%. **[Ans.** 32·25°C]

13. The length of a wire between two bridges is 25 cm and it is stretched by 0·04. The density of the material of the wire is 10 g/c.c. and its Young's modulus 9×10^{11} dyne/cm^2. Calculate the frequencies of the fundamental under this tension. **[Ans.** 240 Hz] **[J.E.E. 1987]**

14. The tension of a sonometer wire is 64 N. Its fundamental frequency is equal to that of a tuning fork. The distance between the two bridges of the sonometer is 10 cm and mass 1 gm. Now, the vibrating tuning fork is displaced with constant speed from the wire. A person standing near the wire hears one beat per second. If the velocity of sound in air be 300 m/s, calculate the density of the tuning fork. **[Ans.** 0·0752 m/s] **[I.I.T. 1985]**

15. The length of an organ pipe is 30 cm. How the length of the pipe be changed so that the frequency of the fundamental emitted by the pipe will not change inspite of the temperature rise from 7°C to 27°C of the pipe. **[Ans.** 1 cm decrease]

16. A closed pipe of length 1·5 m is full of a gas. Its fundamental frequency is in resonance with a tuning fork. The fundamental frequency of an air filled open pipe of same length as the closed pipe is also in resonance with the tuning fork. Calculate the velocity of sound in the gas of the closed pipe at 0°C. During experiment air temperature is 30°C and velocity of sound in air at this temperature is 360 m/s. **[Ans.** 683·4 m/s]

[**Hints :** Fundamental frequency of the closed pipe is $n_c = \frac{V}{4l} = \frac{V}{6}$ [temp. = 30°C] frequency of the fundamental of the open pipe is $n_0 = \frac{V}{2l} = \frac{360}{2l}$ =120 Hz [l = 1·5 m]

∴ $V/6 = 120$ ∴ $V = 720$ m/s ∴ $V_0 = 683{\cdot}4$ m/s]

17. A metal wire of length 1 mm is held on two knife edges separated by a distance of 50 cm. The tension in the wire is 100 N. The wire vebrating with its fundamental frequency and a vibrating tuning fork together produce 5 beats per second. The tension in the wire is then reduced to 81 N. When the two are excited, beats are heard at the same rate. Calculate (a) frequency of the fork and (b) density of the material of the wire.

[**Ans.** (a) 95 Hz, (b) $12{\cdot}74 \times 10^3$ kg/m^3] **[I.I.T. 1980]**

18. An aluminium wire of length 90 cm is clamped at the middle. It is set into longitudinal vibration so that it can produce fundamental frequency only. Given, density of aluminium = 2·6 g/c.c. and its Young's modulus = $7{\cdot}8 \times 10^{10}$ N/m^2, claculate (i) velocity of sound in aluminium, (ii) wavelength of sound in aluminium, (iii) frequency of sound produced, (iv) wavelength of sound produced in air. Velocity of sound in air is 340 m/s.

[**Ans.** (i) 5480 m/s (ii) 180 am, (iii) 3050 Hz ; (iv) 11·1 cm]

19. A block is suspended at the lower end of a sonometer wire. Its frequency of vibration is 500 Hz. When the block is immersed in water, the frequency of vibrtation of the wire becomes 460 Hz. But if the block is immersed in a liquid, the frequency of vibration becomes 480 Hz. Calculate the sp. gr. of the block and the liquid.

[**Ans.** 6·51 ; 0·51]

[**Hints :** $n \propto \sqrt{T}$ or, $T = Kn^2$; K = a constant, $W_1 = C\,(500)^2$, $W_2 = C\,(460)^2$, $W_3 = C\,(480)^2$, sp. gr. of the block,

$S_1 = \frac{W}{W_1 - W_2} = 6{\cdot}51$; sp. gr. of the liquid $S_2 = \frac{W_1 - W_3}{W_1 - W_2} = 0{\cdot}51$]

20. First overtone of an open pipe and first overtone of a closed pipe on superposition produce 2·2 number of beats per second. If the fundamental frequency of the closed pipe be 110 Hz. Calculate length of the pipes.

[**Ans.** l_0 = 75 cm ; l_c = 99·3 cm or 100·7 cm]

21. A sonometer has three wires of equal lengths. Ratio of their specific masses is 2 : 8 : 18 ; ratio of their tensions 12 : 12 : 27. Calculate the ratio of the frequencies of transverse vibration of the wires.

[**Ans.** 2 : 1 : 1] **[J.E.E. 1983]**

22. The length of a sonometer wire is 100 cm. How it be divided into three segments by bridges so that the fundamental frequencies of three segments inthe ratio 1 : 2 : 3.

[**Ans.** l_1 = 27·27 cm ; l_2 = 54·54 cm ; l_3 = 18·18 cm] **[Roorkee 1984]**

23. A wire of length 0·98 m is stretched by a length of 0·02 m. Its fundamental and the fundamental of an open pipe of length 0·5 m produce 8 beats per second. Number of beats decreases with the decrease of tension in the wire. Calculate Young's modulus of the wire. Density of the material of the wire = 10^4 kg/m^3 and velocity of sound in air = 292 ms^{-1}.

[**Ans.** $1{\cdot}764 \times 10^{11}$ N/m^2] **[Roorkee 1996]**

24. If the tension of the sonometer wire is increased by 44%, keeping its length unchanged, frequency of the fundamental increases by 6 Hz. What will be the change in fundamental frequency if length of the wire is increased by 20%, keeping tension unchanged ?

[**Ans.** 5 Hz] **[Roorkee 1998]**

25. Two identical sonometer wires have a fundamental frequency of 500 vibrations per second, when kept under the same tension. What fractional increase in the tension of the first wire would cause an occurance of 5 beats per sec., when both wire vibrate together ?

[**Ans.** 0·02] **[Roorkee 1995]**

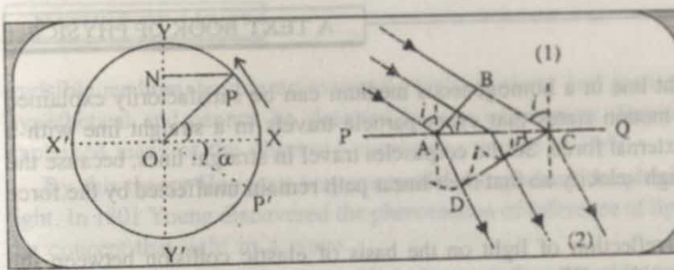
26. An open pipe filled with air has fundamental frequency of 500 Hz. The first harmonic of another closed organ pipe and filled with carbon di-oxide has same frequency as that of the first harmonic of the open organ pipe. Calculate the length of each organ pipe. Velocity of sound in air and in gas are 300 ms^{-1} and 264 ms^{-1} respectively.

[**Ans.** 30 cm ; 13·2 cm] **[I.I.T.]**

PHYSICAL OPTICS

• TOPICS DISCUSSED •

1. NATURE OF LIGHT 259
2. INTERFERENCE OF LIGHT 272



CHAPTER 1

NATURE OF LIGHT

● **TOPICS :** ○ Introduction; ○ Newton's Corpuscular theory of light; ○ Wave theory of light; ○ The electromagnetic theory of light; ○ The quantum theory of light; ○ Dual nature of light; ○ Origin of wave theory; ○ Wave motion; ○ Illumination at a point; ○ Wave front and Ray; ○ Huygen's Principle of wave propagation; ○ Reflection of light according to wave theory; ○ Refraction of light on the basis of wave theory; ○ Short answer type questions (with answers); ○ Exercise.

1.1. Introduction

In geometrical optics we have seen how mirrors and lenses form images of objects illuminated by light. We have discussed different types of aberrations associated with image formed and their remedy. We have also considered in detail the working principles, actions and uses of different optical instruments (e.g. camera, microscopes, telescopes and binoculars). In all these discussion we have assumed that **in a homogeneous medium light travels in a straight line.** But the questions like-what is light ? What is its nature and properties ? How does it actually propagate ? Geometrical optics does not deal with all these questions. Answers to all these questions and different experiments performed in vapour of these answers are discussed in wave optics also called physical optics.

Light, indeed, a form of energy. Radiant energy from the luminous source, on entering our eyes gives us the sensation of sight and enables us to see the object. Different theories regarding nature and propagation of light have been put forward by different scientists at different times. The theories are :

(i) The corpusular theory of light.

(ii) The wave theory of light.

(iii) The electromagnetic theory of light.

(iv) The quantum theory of light.

These theories will be discussed briefly.

Wave optics deals with the production, emission and propagation of light, its nature and the study of the wave related phenomena like interference, diffraction and polarisation.

1.2. Newton's Corpuscular theory of light

The originator of the **corpusculor theory of light** was Sir Isaac Newton. According to this theory, a luminous body continuously emits tiny, elastic particles called corpuscles in all directions. These particles has no weight and they spread out in all directions from the source. Their impact on the retina produces sensation of vision.

These corpuscles travel in straight lines when not acted upon by external forces. The fact that light seems to travel in straight lines and cast shadow behind the obstacles is the strongest evidence of particle nature of light.

By this corpuscular theory we can explain rectilinear propagation of light and the phenomena of reflection and refraction of light can also be explained.

- The fact that light travels in straight line in a homogeneous medium can be satisfactorily explained by this theory. Newton's first law of motion states that every particle travels in a straight line with a constant velocity unless disturbed by external force. So the corpuscles travel in straight lines, because the corpuscles are weighless and possess high velocity so that their linear path remain unaffected by the force of gravity.

- Newton could explain the laws of reflection of light on the basis of elastic collision between the particles of light with the surface it is incident. When light particles come and strike a reflecting surface, they bounce off the surface just as a ball bounces off a wall when thrown against it.

- The phenomenon of refraction was explained by stating that these corpuscles are attracted by the material of the medium and are allowed to enter into the second medium.

Defects of the corpuscular theory :

- (i) Although Newton explained the reflection and refraction of light on the basis of elastic collision and attraction between particles of light and the medium, he faced some difficulty in respect of simultaneous reflection and refraction. We know that when a beam of light is incident on the surface of a medium, the beam is partly reflected and partly refracted at the same time. On the basis of the corpuscular theory, this should mean that the medium partly bounces off the light particles and partly allowed them to enter into it. Why should there be such preferential treatment ? The argument put forward by Newton in solving the problem was not very convincing.

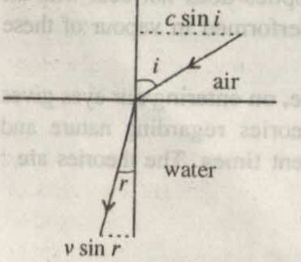


Fig. 1.1

- (ii) In Newton's corpuscular theory of refraction, the particles of light incident from rarer to denser medium experiences a force of attraction normal to the surface. So, the normal component of velocity increases while the component parallel the surface remains constant.

$$\therefore \ \mathrm{C} \,.\, \sin i = \upsilon \,.\, \sin r$$

$$\therefore \ \frac{\mathrm{C}}{\upsilon} = \frac{\sin r}{\sin i} \quad \text{or,} \ \frac{\upsilon}{\mathrm{C}} = \frac{\sin i}{\sin r} = {}_{a}\mu_{\omega}$$

$$\text{As } {}_{a}\mu_{\omega} > 1, \qquad \upsilon > \mathrm{C}$$

It means that the velocity of light in the medium should be greater than the velocity of light in air or vacuum.

But the experimental results of Foucault and Michelson show that the velocity of light in a rarer medium is higher than that in denser medium.

Besides, certain other observed phenomena like interference, colour of thin films, diffraction of light, polarisation etc. could not be explained in the basis of corpuscular theory of light. So, the corpuscular theory was discarded as an incomplete and inaccurate theory.

1.3. Wave theory of light

A Dutch physicist Christian Huygens put forward the wave theory of light. Later, Young, Fresnel and others established it on firm footing. According to this theory, light is a form of energy which advances forward as a **wave motion** from the luminous source and consists of simple harmonic disturbances, like sound waves. When a disturbance is created at any point in an optical medium, the disturbance spreads out in all directions in the form of waves just as waves are created on the surface of water when a stone is thrown into a pond and the waves spread over the entire surface of the pond.

Now, just a sound needs a material medium for its propagation, light was also given a 'medium' through which it travels. This medium is called 'ether'. Since light can travel through vacuum, therefore 'ether' was supposed to fill all space including vacuum. This ether is supposed to be massless, colourless, invisible medium, having an extremely high elasticity and practically no density. The 'ether' concept is hypothetical and cannot be detected by any experiment. Inspite of this, the wave theory very well explained many of the observed phenomena related to light.

By this theory Huygens very successfully explained the phenomenon of reflection and refraction of light. In 1801 Young discovered the phenomenon of inference of light. This phenomenon firmly established the concept that light in a wave.

Fresnel explained diffraction of light by wave theory and he gave a suitable explanation for rectilinear propagation of light. Also to explain **polarisation of light** wave, he assumed that light is a transverse wave.

Thus Fresnel's wave theory of light was established on solid ground.

➡ Defects of wave theory :

Although the wave theory of light achieved tremendous success in explaining different optical phenomena, but the concept of all pervading ether medium required for the propagation of light wave is not tangible. Because—the velocity of wave propagation is given by $v = \sqrt{\frac{E}{\rho}}$ where E is the elastic modulus and ρ is the density of the medium. Now, light has enormous velocity. So ether must have very high modulus of elasticity and low density. Obviously, these two properties are contradictory to each other. So, the existence of ether was discorded.

Also, wave theory could not explain photoelectric effect, compton scattering of X-rays.

1.4. The electromagnetic theory of light

Huygens and Fresnel assumed that light is a mechanical wave like sound wave and mechanical energy is transformed through light waves. In 1873 James Clerk Maxwell through mathematical calculation showed that light was not a mechanical wave, but an electromagnetic wave.

According to him, light propagates in the form of electromagnetic waves. In 1860, he enunciated his theory which came to known as **electromagnetic theory of light.** The theory says that light consists of electric and magnetic fields mutually perpendicular as well as transverse to the direction of propagation of light. The alternately expanding and collapsing magnetic and electric fields propagate in free space with velocity of light. Hertz actually demonstrated experimentally that electromagnetic waves propagate with velocity equal to that of light. According to this theory, the velocity of light in any medium is given by $V = \frac{C}{\sqrt{\mu K}}$ where C = ratio of esu and emu units of electric charge μ = permeability and K = dielectric constant of the medium. For air or vacuum $\mu = 1$ and $K = 1$, so $V = C$. In the laboratory C is found to be $2{\cdot}93 \times 10^8\ \text{ms}^{-1}$, while the velocity of light in vacuum was found to be $2{\cdot}99 \times 10^8\ \text{ms}^{-1}$. The closeness of the two values removed any doubt regarding the electromagnetic theory of light.

Also electromagnetic wave does not require any material medium for its propagation. During later years the theory received strong support when two important magneto-optical effects. viz. **Faraday effect** and **Zeeman effect** was discovered. These effects confirms the relation between light and magnetism. Thus electromagnetic theory of light was taken to be the correct representation of nature of light.

1.5. The quantum theory of light

During the later part of nineteenth century when corpuscular theory of light has already been discorded and classical wave theory has taken full control over the question of nature of light, Max Planck proposed a radically new theory of radiation which is very similar to corpuscular theory. The new theory was named **'quantum theory'** by Planck. The new theory was proposed after the discovery of several phenomena is connection with radiation, in general. The discoveries are : (i) Energy distribution in black body rediation (ii) Photoelectric effect (iii) Compton effect (iv) Raman effect and (v) line spectra produced by elementary substances.

All these phenomena cannot be explained by the wave theory or electromagnetic theory. In order to explain, the distribution of energy in the spectrum of a black body, Planck proposed **'quantum theory of radiation'.** According to this theory, radiation of energy from a black body is not a continuam process but takes place in **Packets of energy** called energy **quanta.** So according to Planck, radiation consists of large number of quantum of energy. Energy of each quantum is proportional to frequency ν of radiation. So, energy of each quantum is $E = h\nu$, h is called Planck's constant. Einstein extended the theory and applied it in the case of light. He called light quantum as **photon** whose energy is also $E = h\nu$. The quantum theory or photon theory satisfactorily explained photoelectric effect, Campton effect, Raman effect etc. but failed to explain interference diffraction and polarisation. There can be explained by wave theory of light.

✪ 1.6. Dual nature of light ✪

To explain adequately different optical phenomena it is found that to explain some of the phenomena, light is considered as waves and to explain other effects we have to consider photon nature of radiation. Light thus appears to have **dual nature**—wave and particle. The dualism has given birth to new branch of physics called quantum physics.

Later de Broglie has shown that the concept of dualism is not unique for light only but also this property is shown by subatomic particles like electron, proton and neutron etc. In fact the particle and wave ideas are complementary conceptions.

✪ 1.7. Origin of wave theory ✪

In 1678 Christian Huygens proposed the wave theory of light. According to this theory, light can propagate as waves in an elastic medium called ether, which pervades all matter and space. As the velocity light is extremely large, the medium must be highly elastic and should have very small density. The introduction of such a subtle and imponderable medium was necessary because no wave motion can take place without a medium. When these waves carrying energy are incident on the eye, sensation of vision is produced.

A wave is a disturbance, periodic both in space and time and is characterised by the phenomenon of undulation i.e. up and down motion spreaidng out of the source in all directions through the medium. The energy of disturbance is transferred from one part of the medium to another part and the particles of the medium only oscillate about the mean position of equilibrium. Any part of the medium does not move, but the profile of the waves move continuously.

Huygens assumed that the vibrations of the ether medium are similar to those produced by solids and liquids. He also assumes that the waves are longitudinal elastic waves in which the vibration of the particles is parallel to the direction of propagation of the wave.

By this wave concepts, Huygens could satisfactorily explain reflection, refraction and double refraction noticed in some crystals like calcite or quartz. However, the phenomenon of polarisation discovered by him could not be explained. Also rectilinear propagation of light could not be explained by wave theory. The difficulties were overcome, when Fresnel and Young suggested that light waves are transverse and not longitudinal as suggested by Huygens. In a transverse wave the vibration of the particles of ether medium take place in a direction perpendicular to the direction of propagation of the wave. Fresnel could also explain successfully the rectilinear propagation of the light wave by combining the effect of all secondary waves starting from different points of a primary wave front.

○ 1.8. Wave motion ○

Before the explanation of various optical phenomena is given on the basis of wave theory of light, it is necessary to understand what a wave motion is. A wave motion is produced in a medium due to periodic motion or vibration of its particles. As there is some sort of elastic connection between the adjacent particles of the medium, the vibration of single particle is not confined to itself, but is transferred from layer to layer. The instantaneous form of disturbance as a whole is called a wave *e.g.* the wave on the surface of water.

The simplest form of wave motion is one in which the particles of the medium perform simple harmonic motion (SHM). To under such a motion, we consider a particle P moving with uniform velocity v along a circle of radius r. The circle along which P moves is called circle of reference. As the generating particle P moves from X to Y, the foot of the particle N moves from O to Y. As the point P moves over the circle of reference the point N executes to and from motion along the diameter YY′. The motion of the foot of the perpendicular (N) is periodic and simple harmonic.

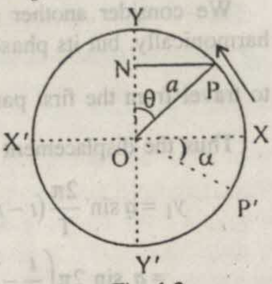


Fig. 1.2

Let the revolving particle move from X to P in time t describing an angle θ at the centre of the circle. So $\angle XOP = \theta = \omega t$ when ω is the uniform angular velocity. $\omega = \frac{2\pi}{T} = 2\pi n$, T is the time period. The displacement of the foot of the perpendicular is ON = y, where

$$y = a \sin \theta = a \sin \omega t \quad \ldots\ldots \text{(1.1)}$$

This is the equation of motion of N starting from O. The maximum displacement of N on either side of mean position O is called the **amplitude.**

Next, the velocity of the particle executing S.H.M. is $v = \frac{dy}{dt} = a\omega \cos \omega t$

$$\therefore \text{ Acceleration } f = \frac{dv}{dt} = -a\omega^2 \sin \omega t = -\omega^2 y$$

Thus, the acceleration of the particle executing SHM is proportional to the displacement from the mean position, O. This satisfies one of the conditions necessary for SHM. The nagative sign of acceleration indicates that the acceleration is always directed towards the mean position of equilibrium.

Instead of measuring the starting time from the instant $\theta = 0$ as above (i.e. when P is at X or N is at O) we can measure the time from the instant when P is at P′, say. Then the displacement along Y′OY at time t is

$$y = a \sin (\omega t - \alpha) \quad \ldots\ldots \text{(1.2)}$$

On the other hand, if the point P′ be above X, then the displacement is given by

$$y = a \sin (\omega t + \alpha) \quad \ldots\ldots \text{(1.3)}$$

The phase of a vibrating particle at any instant is the quantity which determines the state of motion i.e. the direction of motion as well as the displacement of the particle at that instant.

The **phase** is measured by the angle described by the radius OP [Fig. 1.2] on the circle of reference, since the particle left the mean position. So in equation (1.1) phase angle is (ωt) ; in equation (1.2) the phase angle is $(\omega t-\alpha)$ and in equation (1.3) it is $(\omega t+\alpha)$.

- **Epoch :** It is the initial phase i.e. the phase angle at $t = 0$. So, from equation (1.2), epoch is $-\alpha$ and from equation (1.3) epoch is $+\alpha$.

➤ *1.8.1. Equation of simple harmonic wave :*

So far, we have considered only the equation of motion of a single particle executing S.H.M. Now, we consider the equation of a wave motion i.e. the equation of vibration of all the particles in the medium as the wave travels through it with a velocity v.

Let, the equation of vibration of the single particle in the medium be

$$y=a\ \sin\ \omega t=a\ \sin\frac{2\pi}{\mathrm{T}}t.\ \ [\mathrm{T} = \text{time period}]$$

We consider another particle at a distance x from the first particle. It will also vibrate simple harmonically, but its phase of vibration will be different. Let t_1 be the time interval taken by the wave to travel from the first particle to the second particle. Then $t_1=\frac{x}{v}$.

Thus the displacement of the second particle will be given by the equation

$$y_1=a\sin\frac{2\pi}{\mathrm{T}}(t-t_1)=a\ \sin\frac{2\pi}{\mathrm{T}}\left(t=\frac{x}{v}\right)=a\ \sin\ 2\pi\left(\frac{t}{\mathrm{T}}-\frac{x}{\mathrm{T}v}\right)$$

$$=a\ \sin\ 2\pi\left(\frac{t}{\mathrm{T}}-\frac{x}{\lambda}\right)\qquad\left[\lambda=\frac{v}{n}=v\mathrm{T}\right]$$

$$=a\ \sin\frac{2\pi}{\lambda}(vt-x)\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1.4)$$

The equation of motion of any particle in the medium can be obtained by giving the particular value of x in equation (1.4). So, this equation represents the general equation of a wave motion.

✪ 1.9. Illumination at a point ✪

Illumination or intensity of illumination at a point is measured by the quantity of light energy passing per sec normally through a unit area surrounding that point.

During propagation of light waves, ether particles in the medium execute simple harmonic motions. The equation of vibration of any particle of the medium is

$$y=a\ \sin\ \frac{2\pi}{\lambda}(vt-x)\quad [v = \text{velocity of light wave}]$$

So, the velocity of any oscillating particle is $v_\rho=\frac{dy}{dt}=a\cdot\frac{2\pi}{\lambda}v\ \cos\ \frac{2\pi}{\lambda}(vt-x)$

and its acceleration, $f=\frac{d^2y}{dt^2}=-\frac{4\pi^2\upsilon^2}{\lambda^2}a\sin\ \frac{2\pi}{\lambda}(vt-x)$

$$\therefore\ f=\frac{4\pi^2\upsilon^2}{\lambda^2}x\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(\mathrm{i})$$

If m be the mass of the particle, its kinetic energy will be

$$\frac{1}{2}mv_{\rho}^{2}=\frac{1}{2}m\cdot\frac{4\pi^{2}\upsilon^{2}a^{2}}{\lambda^{2}}\cdot\cos^{2}\frac{2\pi}{\lambda}(vt-x)$$

The mean kinetic energy during a complete time period is

$$\frac{1}{T}\int_{0}^{T}\frac{1}{2}mv_{\rho}^{2}\,dt=\frac{m\pi^{2}\upsilon^{2}a^{2}}{\lambda^{2}T}\int_{0}^{T}2\cos^{2}\frac{2\pi}{\lambda}(vt-x)\,dt$$

$$=\frac{m\pi^{2}\upsilon^{2}a^{2}}{\lambda^{2}}\quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(ii)}$$

Again, potential energy of the particle is

$$=-\int_{0}^{y}mf\,dy=\frac{4\pi^{2}m\upsilon^{2}}{\lambda^{2}}\int_{0}^{y}y\,dy=\frac{2\pi^{2}m\upsilon^{2}y^{2}}{\lambda^{2}}=\frac{2\pi^{2}m\upsilon^{2}a^{2}}{\lambda^{2}}\cdot\sin^{2}\frac{2\pi}{\lambda}(vt-x)$$

So, the mean potential energy during a complete time period is

$$\frac{1}{T}\int_{0}^{T}\frac{2m\pi^{2}\upsilon^{2}a^{2}}{\lambda^{2}}\sin^{2}\frac{2\pi}{\lambda}(vt-x)\,dt=\frac{m\pi^{2}\upsilon^{2}a^{2}}{\lambda^{2}}$$

$$\therefore\ \text{Total energy} = \text{K.E.} + \text{P.E.} = \frac{2m\pi^{2}\upsilon^{2}a^{2}}{\lambda^{2}}$$

$$\therefore\ \text{Energy per unit volume or energy density } E_{0}=\frac{E}{V}=\frac{2\rho_{0}\pi^{2}\upsilon^{2}a^{2}}{\lambda^{2}}\quad[\rho_{0}=\text{density of the medium}]$$

$\therefore$ Intensity of illumination I = Energy density × velocity

$$\therefore\ I=E_{0}\nu=\frac{2\rho_{0}\pi^{2}\upsilon^{3}}{\lambda^{2}}\cdot a^{2}\qquad\therefore\ I\ \alpha\ a^{2}$$

So, intensity of illumination is proportional to the square of the amplitude.

✪ 1.10. Wave front and Ray ✪

According to wave theory of light, a source of light sends out disturbances in all directions. In a homogeneous medium, the disturbance travels with equal velocity in all directions from the source. So, the disturbance reaches all those points of the medium in phase, which are located at the same distance from the source and all such particles must vibrate in phase with each other. Evidently at any instant, the wave motion lies upon **the surface of a sphere** whose centre is the generating point and the radius is equal to the product of the velocity and time. The surface of the sphere is called **wave front**.

❖ **Definition : A wave front is defined as the envelope or trace drawn through all the points on a wave which are exactly in the same phase of vibration.**

Thus, a surface drawn along the crests or along the troughs of a water wave is a wave front.

Depending upon the shape of the source of light, wave front can be of the following three types :

● (i) **Spherical wave front :** If light is given by point source of light, and light travels with equal velocity in all directions, then spherical wave front is produced [Fig. 1.3(a)].

- (ii) **Cylindrical wave front :** When the source of light is linear in shape (e.g. a slit) a cylindrical wave front is produced. It is because, all the points which are equidistant from the linear source, lie on the surface of a cylinder [Fig. 1.3(b)].

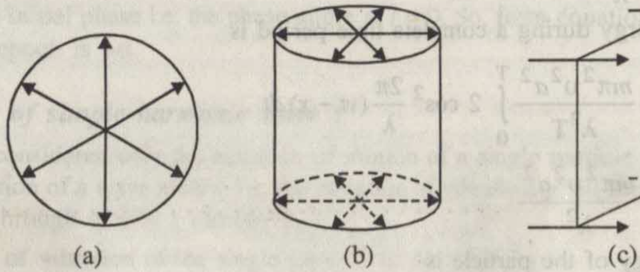


Fig. 1.3

- (iii) **Plane wave front :** A small part of a spherical or a cylindrical wave front originating from a distant source, will appear plane and hence it is called a plane wave front [Fig. 1.3(c)].

- **Ray of light :** A normal drawn to the wave front at any point gives the direction of propagation of the wave and is known as a ray. A ray indicates the direction along which the energy travels.

Fig. 1.4(a), (b) and (c) represent the wave fronts of a parallel beam, a convergent beam and diverging beam respectively. The wave front representing a parallel beam is a plane wave front, while the wave front representing a converging or a diverging beam of light is spherical in shape.

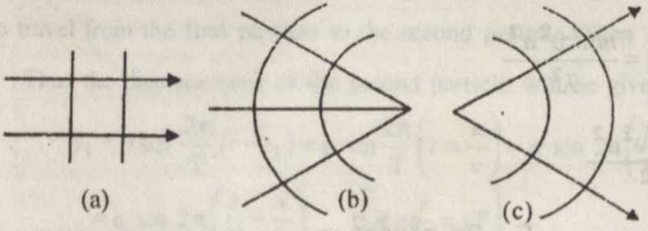


Fig. 1.4

1.11. Huygens' principle of wave propagation

According to wave theory, in an isotropic transparent medium light travels with definite velocity in all directions. Huygens' principle is a geometrical construction which is used to determine the new position of the wave front at a later time from its given position at any instant. So this principle gives us a geometrical picture of this mode of propagation. To explain the porpagation of light wave, Huygens followed the following principle for the construction of new wave front.

(i) Each point on the given wave front acts as a source of secondary wavelets sending out disturbance in all directions like the original source of light.

(ii) These wavelets are spherical in shape and move forward in a homogeneous medium with the same velocity with which the original wave travels.

(iii) The new position of the wave front at any instant called the secondary wave front is the envelope of the secondary wavelets at that instant.

These three assumptions are known as Huygens' principle.

Let us now apply this principle to find the subsequent position of the wave front after a time interval t.

We consider a point source of light. AB is a section of a spherical wave front at any time t. To find the new position of wave front at time $t + dt$, we consider a number of points a, b, c, d, ... on the primary wave front [Fig. 1.5]. These points acts as the sources of secondary wavelets. In time dt light travels a distance $v.\Delta t$ where v is the velocity of light.

Now, *a, b, c, d,* ..... as centres we draw spheres each of radius $v\Delta t$.

These spheres are called secondary wavelets originating from different points on the primary wave front AB. The envelope of these secondary wavelets, i.e. the spherical surface whose trace A_1B_1 touches all these spheres, is the new wave front at the end of the period envisaged. In this way the wave front will move forward parallel to itself.

It may be pointed out that the effective part of the secondary wavelets is the portion which lies on the forward secondary wavelets. Huygens assumed that the portion of the secondary wavelets which lies on the backward secondary wave front, does not exist at all. The reason is : if a particle was already under a forward velocity, it cannot go back simultaneously under the influence of the same. Hence a wave front, advancing in the forward direction cannot produce a backward wave front.

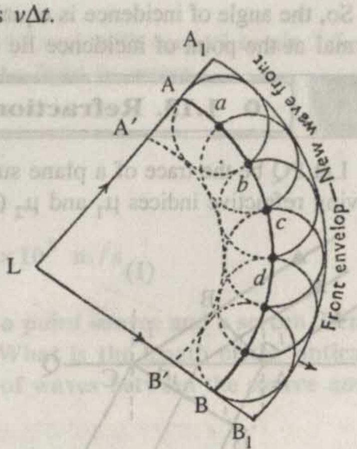


Fig. 1.5

1.12. Reflection of light according to wave theory

In the previous article we have seen how to draw a wave front at time $t + \Delta t$ from the wave front corresponding to the time t given out by a point source and how light energy propagates in all directions from point source of light. In this article we shall deduce the laws of reflection of light from Huygens construction. In fig. 1.6 MM′ is the section of the section of the reflecting surface in the plane of the paper. ABC is the section of the incident plane wave front perpendicular to the plane of the paper. PCE and QA are perpendicular to the wave front ABC and hence they are incident rays. The angle i between the incident ray and normal is the angle of incidence.

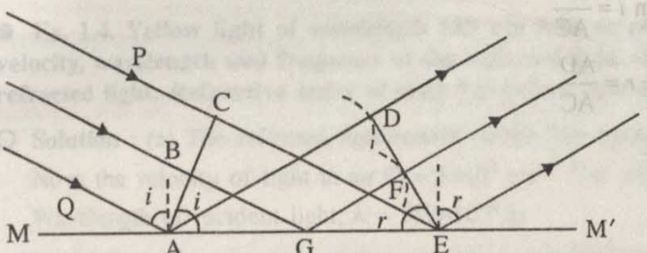


Fig. 1.6

At time $t = 0$, the point A of the wave front is incident on MM′, then the point B of the wave front falls on the surface MM′ and at later instant the point C of the wave front falls on MM′. Now point C is incident at E after a time $t = \dfrac{\text{CE}}{v}$ where v is the velocity of the wave in air. Now, according to Huygens principle, all the points A to E will be the sources of secondary wavelets one after another. Now we have to determine the position of the reflected wave front after the time t. The secondary wavelet formed at A at time $t = 0$ will travel a distance AD = CE after the time t. With the point A as centre draw an arc of a circle of radius AD = AE. This arc denotes the position of a reflected wave front. The point B of the incident wave front touch G and the secondary wavelet formed at G advances by GF = BG during the time t. Now with G as centre draw an arc of a circle of radius GF. Thus we can draw different arcs with the point A to E as centres. The tangent plane to all these arcs denotes the position of the reflected wave front ED after the time t. The straight lines drawn perpendicular to this wavefront ED denotes the reflected rays.

Now, in fig. the triangles ACE and AED are congruent. Because CE = AD and AE is common. Hence $\angle\text{CAE} = \angle\text{DEA}$ or $\angle i = \angle r$

So, the angle of incidence is equal to angle of reflection. Also, the reflected wave, incident wave and normal at the point of incidence lie in the same plane (here in the plane of the paper).

1.13. Refraction of light on the basis of wave theory

Let PQ be the trace of a plane surface perpendicular to the plane of the paper, separating two media having refractive indices μ_1 and μ_2 ($\mu_2 > \mu_1$) [Fig. 1.7]. Assume that v_1 and v_2 be the velocities of light in the two media respectively. Let AB is the trace of the plane wave front perpendicular to the plane of the paper travelling in the first medium (r.i. μ_1) and incident on PQ. We required to obtain the position of refracted wave front after a time t. We assume $v_1 > v_2$. As the wave front AB sweeps over the surface PQ, each point of the surface becomes the centre of secondary wavelets which travels in the second medium.

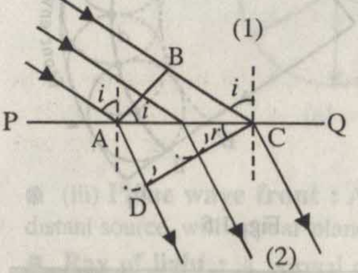


Fig. 1.7

At $t = 0$, the point A of the wave front AB touches the surface PQ and after a time t, the end point B of the wave front touch the surface PQ at the point C. So $BC = \upsilon_1 t$. During this time wavelet produced at A advances a distance $AD = \upsilon_2 t$ in the second medium (r.i. μ_2). With A as centre and radius AD an arc of a circle is drawn (shown by dotted curve). This circle denotes the spherical wavelet formed at A t sec later. The tangent plane CD from the point C denotes the position of the refracted wave front.

Now, in triangle ABC, $\sin \angle BAC = \sin i = \dfrac{BC}{AC}$

and in triangle ACD, $\sin \angle ACD = \sin r = \dfrac{AD}{AC}$

$$\therefore \quad \frac{\sin i}{\sin r} = \frac{BC}{AD} = \frac{v_1 t}{v_2 t} = \frac{v_1}{v_2} = \mu_2 = \frac{\mu_2}{\mu_1}$$

$$\therefore \quad \mu_1 \sin i = \mu_2 \sin r$$

This is Snell's law. Again, incident ray, refracted ray and the normal at the point of incident lie in the plane of the paper. The law of refraction is applicable to any type of wave.

The absolute refractive index of a medium is its refractive index with respect to vacuum. It is given by $\mu = \dfrac{C}{v}$ [C = velocity of light in vacuum, v = velocity of light in the medium]

Note : When a ray of light is refracted from one medium to another, its velocity changes. But frequency remain unchanged. As $v = n\lambda$, so velocity v changes because wavelength of the light λ changes.

- **Example 1.1. The velocity of light in vacuum $C = 3 \times 10^8$ ms^{-1} and in water $2{\cdot}25 \times 10^8$ ms^{-1}. What is the ratio of wavelengths of light in the two media ?**

○ **Solution :** $C = n_1\lambda_1, \quad v = n_2\lambda_2$

$$\therefore \quad \frac{C}{v} = \frac{n_1\lambda_1}{n_2\lambda_2} = \frac{\lambda_1}{\lambda_2} \qquad [\because n_1 = n_2]$$

$$\therefore \quad \frac{\lambda_1}{\lambda_2} = \frac{3}{2{\cdot}25} = \frac{4}{3}. \quad \lambda_1 = \lambda_2 = 4 : 3$$

● **Ex. 1.2. Refractive index of glass with respect to water is 1·14 and with respect to air 1·5. If velocity of light in air be 3×10⁸ ms⁻¹, calculate the velocity of the light in water.**

○ **Solution :** $_{\omega}\mu_g = 1\cdot 4 = \dfrac{\text{Velocity of light in water}}{\text{Velocity of light in glass}} = \dfrac{v_\omega}{v_g}$

$$\therefore\ v_\omega = 1\cdot 14 \times v_g$$

But, $v_g = \dfrac{C}{\mu_g}$ $\quad\therefore\ v_\omega == 1\cdot 14 \times \dfrac{C}{\mu_g} = \dfrac{1\cdot 14 \times 3 \times 10^8}{1\cdot 5} = \mathbf{2\cdot 28 \times 10^8\ m/s}$

● **Ex. 1.3. A glass plate 3 mm thick of r.i. 1·5 is placed between a point source and a screen 3 cm away. The source emits light of wavelength 6000Å in vacuum. What is the length of the optical path between the source and the screen ? Calculate the number of waves between the source and screen.**

○ **Solution :** The air path between source and screen = 3 – 0·3 = 2·7 cm.

Equivalent air path for thickness of glass plate $= \mu t = 1\cdot 5 \times 0\cdot 3 = 0\cdot 45$ cm

∴ Total optical path = 2·7 + 0·45 = **3·15 cm**

And number of waves between the source and screen is

$$n = \frac{\text{Optical path}}{\text{Wave length}} = -\frac{3\cdot 15}{6000 \times 10^{-8}} = \mathbf{5\cdot 25 \times 10^4}$$

● **Ex. 1.4. Yellow light of wavelength 589 nm falls on plane glass plate from air. (a) Calculate velocity, wavelength and frequency of the reflected light, (b) also calculate these quantities for the refracted light. Refractive index of glass for yellow light is 1·5.**

○ **Solution :** (a) The reflected light returns to the first medium (air).

Now, the velocity of light in air C = 3×10^8 ms^{-1}. The velocity of reflected light C = 3×10^8 ms^{-1}

Wavelength of incident light, $\lambda = 589 \times 10^{-9}$ m

∴ Frequency of the incident light, $n = \dfrac{C}{\lambda} = \dfrac{3\times 10^8}{589 \times 10^{-9}} = 5\cdot 09 \times 10^{14}$ Hz

The frequency and wavelength of the reflected light will be same as those of incident light.

∴ Wavelength of the reflected light = $\mathbf{5\cdot 89 \times 10^{-7} m}$

and its frequency = $\mathbf{5\cdot 09 \times 10^{14}}$ **Hz.**

(b) The velocity of light in glass, $v_g = \dfrac{C}{\mu_g} \dfrac{3\times 310^8}{1\cdot 5} = \mathbf{2\times 10^8}$ **ms**$^{-1}$

The frequency does not change i.e. $n' = n = \mathbf{5\cdot 09 \times 10^{14}\ Hz}$

The wavelength of the refracted light, $\lambda_g = \dfrac{v_g}{n} = \dfrac{2\times 10^8}{5\cdot 9 \times 10^{14}} = 3929 \times 10^{-10}$m

$$\therefore\ \lambda_g = 3429\ \text{A}^\circ$$

● **Ex. 1.5. A parallel beam of light having width 5 cm is incident on a glass plate from air. The angle of incidence is 30°. What is the thickness of the beam after refraction in glass ? μ_g = 1·5**

○ **Solution :** The situation is shown in fig. 1.8. AB = 5 cm and angle of incidence is 30°.

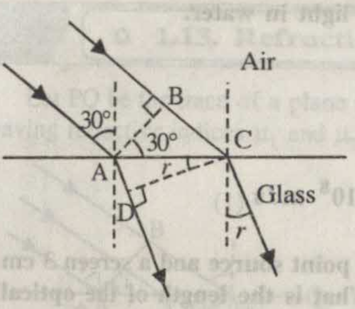


Fig. 1.8

$$\therefore \; AC = \frac{AB}{\cos 30^\circ} = \frac{5}{\sqrt{3}/2} = \frac{10}{\sqrt{3}} \text{ cm}$$

The thickness of the refracted beam = CD.

If r be the angle of refraction, $\sin 30^\circ = 1{\cdot}5 \sin r$

$$\therefore \; \sin r = \frac{1}{3} \text{ and } \cos r = \sqrt{1 - \frac{1}{9}} = \frac{\sqrt{8}}{3}$$

From triangle ACD, $DC = AC \cos r = \frac{10}{\sqrt{3}} \times \frac{\sqrt{8}}{3} = 1 = \mathbf{5{\cdot}44 \text{ cm}}$

●● Short Answer Type Questions (with answers) ●●

● **Question 1. What is a wave front ?**

○ **Ans.** Wave front is a surface obtained by joining particles of the medium which are in the same state of vibration.

● **Q. 2. What is a ray of light ?**

○ **Ans.** An arrow drawn normal to the wave front and pointing in the direction of propagation of the disturbance is called a ray of light.

● **Q. 3. What is the shape of the wave front originating from (i) a point source and (ii) a line source ?**

○ **Ans.** (i) The wave front originating from a point source is spherical in shape. It is because, locus of all such points which are equidistant from the point source, is a sphere.

(ii) The wave front originating from a lime source is cylindrical in shape. It is because, all the points which are equidistant from the line source lie on the surface of cylinder.

● **Q. 4. What is the shape of the wave front of a beam of parallel rays ?**

○ **Ans.** The wave front of a parallel beam of light is plane in shape.

● **Q. 5. Why is the contribution of a wavelets on back of the secondary wave front zero ?**

○ **Ans.** The contribution of a wavelet in any direction making angle θ with the normal to the wavelet is proportional to $\frac{1}{2}(1 + \cos \theta)$

Now, for wavelet lying on the back of the secondary wave front θ = 180°.

So, the factor $\frac{1}{2}(1 + \cos \theta) = \frac{1}{2}(1 + \cos 180^\circ) = 0.$

● **Q. 6. In what respect is Huygen's wave theory of light similar to Maxwell's electromagnetic theory ? In what respect are the two theories different ?**

○ **Ans. Similarities :** According to both the theories, light is a wave motion.

Dissimilarities : According to electromagnetic theory, light does not require any medium for its propagation. But according to wave theory, a medium is necessary for the propagation of light waves. For this Huygens assumed the existence of all pervading ether medium.

EXERCISE

A. Short answer type questions :

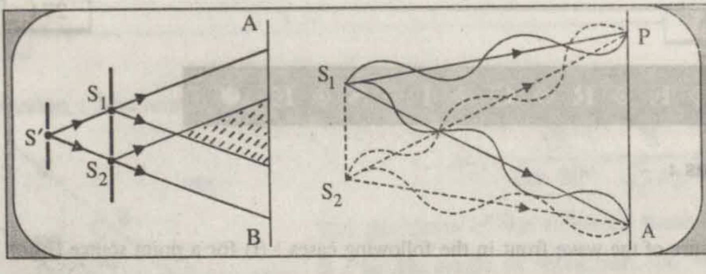
1. What is wave theory of light ?
2. What is a wave front ? What the nature of the wave front in the following cases : (i) for a point source (ii)for a line source ?
3. What is the shape of the wave front diverging from a point source ?
4. What is Huygens' principle ?
5. What is corpuscular theory of light ?
6. What you know about ether medium ?
7. What is electromagnetic theory of light ?
8. What is quantum theory of radiation ?
9. What photoelectric effect, common effect cannot be explained by wave theory of light ?
10. What is a photon ? How its energy is measured ?

B. Essay type questions :

1. Discuss different theories of light.
2. State Huygens' principle and show how it can be used to explain the law of refletion of light.
3. State explain Huygens' principle. Deduce the laws of refraction from it.

C. Numerical problems :

1. The wave length of a monochromatic ray in vacuum is 5896A°. Calculate the velocity and wavelength of the ray in glass. Velocity of light in vacuum = 3×10^8 ms^{-1} and $\mu_g = 1{\cdot}5$. **[Ans. $2{\cdot}10^8$ ms^{-1} ; 3930 A°]**



CHAPTER 2

INTERFERENCE OF LIGHT

● **TOPICS :** ○ Wave; ○ Principle Superposition of waves; ○ Interference of light; ○ Young's experiment or interference of light waves; ○ Analytical explanation of interference; ○ Conditons for sustained interference; ○ Interference of light and principle of conservaion of energy; ○ Coherent sources and experimental method of their production; ○ Explanation and theory of Young's experiment; ○ Numerical examples; ○ Short answer type questions (with answers); ○ Exercise.

2.1. Wave

When a disturbance is created on the otherwise calm water of a pond, waves are created and energy of disturbance is transferred to other points by the waves. Assume that a stone is dropped at the point 'S' on the water in the pond, waves starts moving from the point. It is found that during propagation of wave, water goes up and then goes down from the initial level of water. In fig. 2.1. AB represents the level of water when it not disturbed. The portions of water at P, R etc. goes up from AB. These are called **crests** whereas the portions of water at Q, S etc. go down from AB. These are called **troughs**. The distance between two crests or between two troughs is called **wavelength, λ**. In the fig. PR = λ or QS = λ. The distance of crest or trough from the initial horizontal water level is called **amplitude** of the wave and it is denoted by '*a*'.

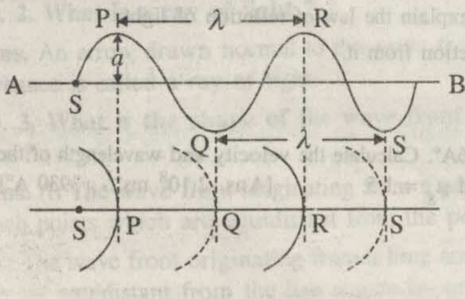


Fig. 2.1

During propagation of the wave, particles of water execute simple harmonic motion in the vertical direction. This type of vibration is called **transverse vibration** and corresponding waves are called transverse waves. The states of vibration of the vibrating particle is called phase of the wave.

Equation of motion :

The equation of motion of the particle in S.H.M. is given by

$$y = a \sin \omega t \quad [\ a = \text{amplitude of motion}\ ]$$

Next, to obtain the equation of wave motion, we have consider the motion of all the particles. Assume, during propagation of the wave, the equation of vibration of any particle is $y = a \sin \omega t$. Another particle at a distance x from the first particle also executes simple harmonic motion. So, both the particles are vibrating identically, but there is a phase difference between the particles.

Now, phase difference is 2π when the path difference is λ.

So for path difference x, the phase difference is $\theta = \dfrac{2\pi}{\lambda} x$

So, the instantaneous phase of the second particle $= \left(\omega t - \dfrac{2\pi}{\lambda} x \right)$

∴ The equation of motion of the second particle is

$$y = a\sin\left(\omega t - \frac{2\pi}{\lambda}x\right) = a\sin\left(\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x\right) = a\sin\frac{2\pi}{\lambda}(vt - x)$$

It is the general equation of wave motion.

2.2. Principle Superposition of waves

When two or move waves trains pass through a region of a medium, the distribution of light intensity or energy in the region can be determined by the principle of superposition of the waves. According to the principle—

When two or more waves simultaneously pass through a point of a medium, the resultant displacement at any instant at the point is the algebraic sum of the displacements produced by the individual waves provided the displacements are small.

Let the displacement of a point of the medium due to the action of one wave at any instant y_1 and that due to the other wave at the same instant by y_2. Then the resultant displacement y of the point due to simultaneous action of the two waves is

$$y = y_1 \pm y_2$$

when the displacements are in the same direction positive sign is taken, but the negative sign is considered when the displacements are opposite.

The **phenomenon of interference of light waves** was explained by Thomas Young in 1802 by applying the principle of superposition.

In the region of superposition the waves act independent of the other waves. Also after the superposition, at the region of cross over, the wave trains emerge as if they have not interfered at all. Each wave train retains its individual characteristics. Each wave train behave as if others are absent.

2.3. Interference of light

The phenomenon of interference of light is due to the superposition of two wave trains within the region of cross over, let us first consider the waves produced on the surface of water. In fig. 2.2 A and B are two sources which produce waves of equal amplitude and constant phase difference.

Waves spread out on the surface of water which are circular in shape. Crests have been shown by full circles and troughs by dotted line circles. At any instant, the particles will be under the action of displacements due to both the waves. From the principle superposition, we know that the resultant displacement at a point on the surface of water is obtained by algebraically summing up the displacements produced by the two waves separately at that point. The points shown by circles in the diagram will have minimum displacement because the crest of one wave falls on the trough of the other and the resultant displacement is zero. The waves are said to have **interfered destructively** with each other at these points. Intensity of wave there is also zero. On the other hand, the points shown by crosses in the diagram will have maximum displacement because, either the crest of one will combine

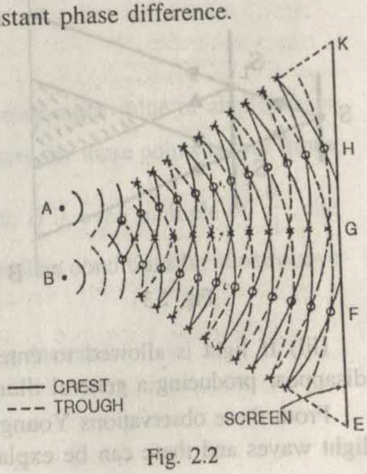


Fig. 2.2

with the crest of the other or the trough of one will combine with the trough of the other. In such a case, the amplitude of the displacement is twice the amplitude of either of the waves and the intensity is quadrupled. The waves, at those points are said to **interfere constructively with each other.**

It should be remembered that there is no **loss of energy due to interference.** The energy is only transferred from the points of minimum displacement to the points of maximum displacements.

The same thing happens in the case of light. Instead of surface of water, if we suppose that A and B are sources of light continuously sending out **identical** waves in the surrounding optical medium and EGK is a screen, then we find very bright points at E, G, K etc. and dark points at F, H etc. It means that the screen will be intersected by alternate bright and dark points, known as **interference fringes.**

2.4. Young's experiment on interference of light waves

It has been stated that when two or more waves are incident at a point, the resultant amplitude or intensity at the point can be determined by the principle of superposition of waves. When two sound waves pass through a point simultaneously under special condition silence may be produced at the point. In the case of light, similarly two light waves may produce darkness at a point under a certain condition (the conditions will be discussed later)

The experiment on interference conclusively confirms the wave nature of light.

Young's experiment :

Thomas Young, an English scientist conclusively established for the first time, the wave nature of light by his classical experiment described below.

A free pencil of light rays is admitted through a narrow slit S′. The light coming out of the first slit in then divided into two parts by passing it through two other slits S_1 and S_2. These slits are placed parallel to the first and they are very close and are equidistant from S′. Light from the slits S_1 and S_2 falls on the screen AB. So the screen is simultaneously illuminated by light coming from both the sources S_1 and S_2.

In the shaded region [Fig. 2.3], the light from the two sources S_1 and S_2 overlap. By using monochromatic light Young observed a pattern of alternate dark and bright bands in the overlapping region. At the centre of the screen, the intensity of light is maximum and it is called central maximum.

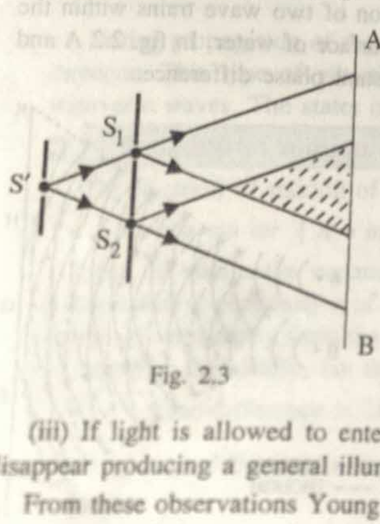


Fig. 2.3

Young observed some characteristics of the dark and bright bands :

(i) The interference pattern disappears, if one of the two slits is closed. It shows that interference pattern is due to superposition of waves from the two slits.

(ii) As the separation between the two slits is gradually diminished the spacing between the bands increases and as the separation is increased the bands becomes narrow and ultimately disappears.

(iii) If light is allowed to enter directly through S_1 and S_2 and the slit S' is removed the bands disappear producing a general illumination on the screen.

From these observations Young concluded that the above phenomena is due to **interference** of two ight waves and these can be explained from the principle of superposition of two waves.

2.5. Analytical explanation of Interference

Let S_1 and S_2 be two coherent monochromatic point sources of light. They emit light of same wavelength and have constant phase relationship. The amplitude and wavelength of waves produced by both the sources are 'a' and 'λ'. The wave trains produced by two sources superpose at the point P [Fig. 2.4]. The resultant displacement of the point P will be the algebraic sum of the displacements produced by the individual waves. Let the displacement at P at time t produced by the two source S_1 and S_2 be given by

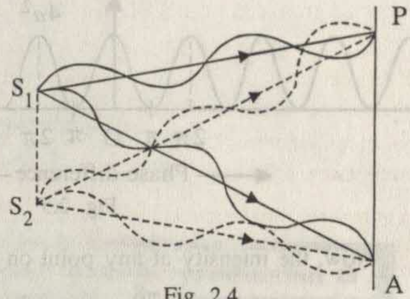


Fig. 2.4

$$y_1 = a\sin\frac{2\pi}{\lambda}(vt - x)$$

$$y_2 = a\sin\frac{2\pi}{\lambda}[vt - (x+\delta)]$$

Here δ = path difference between the two waves. The resultant displacement due to the superposition of two waves is $y = y_1 + y_2 = a\sin\frac{2\pi}{\lambda}(vt - x) + a\sin\frac{2\pi}{\lambda}[vt - (x+\delta)]$

$$= 2a\cos\frac{\pi\delta}{\lambda}\cdot\sin\frac{2\pi}{\lambda}\left[vt - \left(x + \frac{\delta}{2}\right)\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots (2.2)$$

Therefore, the resultant vibration is also sinusoidal in nature having the same period, but of different amplitude. The amplitude is given by $2a\cos\frac{\pi\delta}{\lambda}$. Evently the amplitude varies with δ.

(i) Condition for destructive interference :

When $\delta = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, (2n+1)\frac{\lambda}{2}, 2a\cos\frac{\pi\delta}{\lambda} = 0$. The resultant amplitude will be zero for all these values of δ. So, the intensity is zero at those points for which path difference is odd multiple of $\frac{\lambda}{2}$. These points are **dark points.** At these points, the waves interfere destructively with each other. Hence the condition for **destructive interference** is $\delta = (2n+1)\frac{\lambda}{2},\ n = 0, 1, 2, \ldots\ldots$

(ii) Condition for construction interference :

When $\delta = 0, \lambda, 2\lambda, \ldots\ldots, n\lambda, 2a\cos\frac{\pi\delta}{\lambda} = \pm 2a$ (maximum). So, the maximum value of amplitude = double the amplitude of the individual wave. Hence the maximum intensity for these points is four times the intensity due to individual wave. At all these points where the path δ is even multiple of $\frac{\lambda}{2}$, the waves will interfere constructively. These points are called **bright points.** The conditions for constructive interference is $\delta = n\lambda,\ x = 0, 1, 2, 3, \ldots\ldots$

- **A. Discussion :** Total intensity at the point P depends on following factors :

(i) The path difference of the point P from the sources S_1 and S_2

$$\delta = S_2P - S_1P$$

(ii) Phase difference between the two waves produced by the two sources. This phase difference between the two waves will remain constant when the sources are coherent.

(iii) Due to superposition or interference of two waves, the resultant intensity or amplitude varies at different points. The resultant amplitude of the wave is

$$A = 2a \cos\frac{\pi\delta}{\lambda}$$

So, the magnitude of A varies from 0 to $2a$ and intensity varies from 0 to 1 = $4a^2$ as intensity is proportional to square of amplitude.

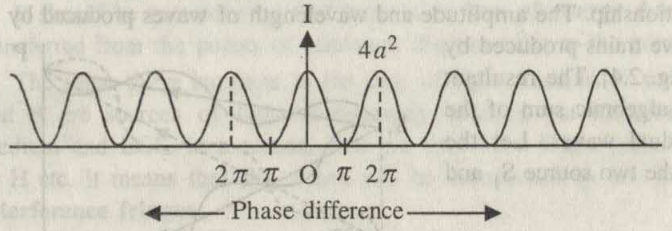


Fig. 2.5

Now, the intensity at any point on the screen is given by I α A^2

$$\therefore\ I = K\,4a^2 \cos^2\frac{\pi\delta}{\lambda} = I_0 \cos^2\varphi \left[I_0 = 4a^2 K \text{ and } \varphi = \frac{\pi\delta}{\lambda} \right] \ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

So, at the region of interference intensity obeys cosine square law. In fig. 2.5 we show the variation intensity with phase angle.

2.6. Conditions for sustained interference

The interference pattern in which the positions of maximum and minimum intensity of light remain fixed all along the screen, is called sustained or permanent interference pattern.

For producing sustained interference, following conditions must be satisfied :

(i) The two sources must be coherent. It means that the two sources vibrate either in the same phase or have a constant phase difference between them.

(ii) The sources should emit light of same wavelength and same amplitude.

(iii) The sources should be narrow and close to each other to observe distinct fringes.

(iv) At minima, the path difference between the waves should be odd multiple of $\frac{\lambda}{2}$ and at maxima, even multiple of $\frac{\lambda}{2}$.

(v) The two light waves must be in the same state of polarisation.

2.7. Interference of light and principle of conservation of energy

When two light waves interfere, darkness is produced at some points. It appears that energy is destroyed at the points of destructive interference and created at the points of constructive interference. So, the law of conservation of energy appears to be violated, because energy can not be destroyed and it can not be created. But this is not correct. The energy is merely redistributed in the space. The energy is only transferred from the region of destructive interference to the region of constructive interference.

Let amplitude of a single wave be a and it gives out energy I (α a^2) at a point. Similarly the second wave given the same energy I (α a^2). Then the contribution to the total energy at a point due to two waves is 2I (α $2a^2$). Now, let these two waves interfere constructively at a point. Then the intensity I_{max} at the point is I_{max} α $(2a)^2$ or, I_{max} α $4a^2$.

Similarly if the two interfere destructively at a point, then intensity I_{min} at that point is

$I_{min} \alpha (a - a) \quad \therefore I_{min} \alpha 0$

In a region containing one maximum and one minimum, energy $\alpha (4a^2 + 0)$

$\therefore$ Energy $\alpha\ 4a^2$

$\therefore$ So, average energy is $\frac{4a^2}{2} = 2a^2$

It is equal to the total energy contribution by the two waves individually.

Thus due to interference, there is merely a redistribution of energy.

Hence the principle of conservation of energy is not violated.

◎ 2.8. Coherent sources and experimental method of their production ◎

It has already been stated that to obtain sustained or permanent interference pattern the sources (i) must emit light wave continuously and (ii) must be coherent i.e. they must have constant phase difference.

If two independent sources are taken, their phase difference changes continuously and so they cannot produce interference pattern. Hence to obtain coherent sources we require to derive them from the same source. There are two practical methods of obtaining two coherent sources from a single source.

(a) Method involving **division of amplitude :** In this method we require an extended source. In this case the amplitude of the wave is divided into two by reflection, refraction of both to get two coherent sources. So, here interference takes place between a ray and its reflected or refracted component or between two reflected or refracted components of the same ray. Some of the examples of this class are (i) the interference effect in this films and (ii) Newton's ring (iii) Michelson's interferometer.

(b) **Methods involving division of wave front :** In this method we require a point source or a line source, from which light waves spread out as spherical or cylindrical waves. Any two points on such a spherical or cylindrical wave front are taken as the interfering sources, which are evidently coherent. Names of some of the instruments where this method of division of wave front is employed are : (i)Fresnel's bi-prism, (ii) Lloyd's mirror and (ii) Diffraction grating.

◎ 2.9. Explanation and theory of Young's experiment ◎

➲ **Explanation :** We have already discussed Young's double-slit experiment on interference of light waves. In this experiment two coherent sources are obtained **by the method of division of wave front.** Coherent sources are a pair of sources which are very close and they produce waves of equal wavelength and amplitude. Also phase difference remains constant.

Young in his experiment first showed that when two rays of light are superposed, alternate bright and dark bands are produced. This process is called interference of light. Bright band is produced when the waves meet in same phase and dark band is produced when they meet in opposite phase.

In this experiment a narrow slit is illuminated by a mono-chromatic beam of light. If we consider a section of the slit in the plane of the paper, it will practically be a point at L [Fig. 2.6]. From this point spherical waves originate and then fall on the two narrow adjacent slits, whose sections in the plane of the paper are points A and B. Evidently A and B lie on the same wave front. According to

Huygen's principle, A and B will act as secondary sources and emit fresh spherical wavelets. In the space beyond A and B the waves superpose. Two sets of wavefront starting from A and B are shown in fig. Crests are shown by thick curves and troughs by dotted curves. At the positions when two troughs or two crests intersect, the waves meet in same phase. These points are connected by thick straight lines. The points where these straight lines must the screen SS_1, brightness will be maximum. At these points **constructive interference** takes place. On the other hands, the broken straight lines are the loci of the points of intersection where crests and troughs from A and B superpose and cancel each other. Hence the points when the broken lines meet SS_1 corresponds to minimum intensity arising from **destructive interference** of two wave trains for A and B. Parallel interference fringes or bands which are alternately bright and dark are, therefore, formed on the screen.

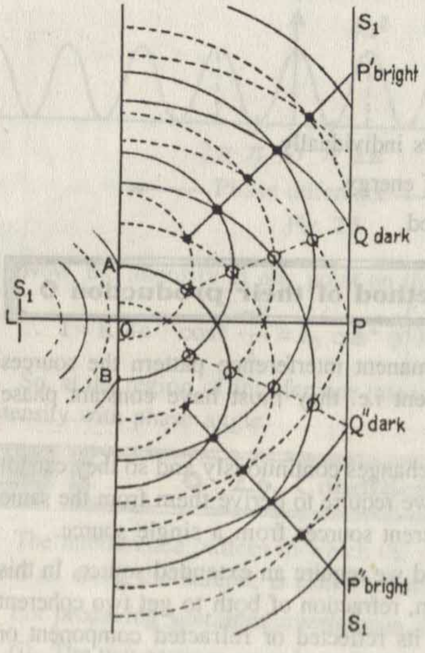


Fig. 2.6

- **Note :** It is to be noted that fig. 2.4 and 2.6 represent the same state of things in respect of interference of two waves.

- **Theory :** To determine whether at any point P on the screen S_1S_2, there will be formation of a dark or a bright fringe, we consider the fig. 2.7. Let AB = $2d$ and O is the midpoint of AB. The distance of the screen from O is OP_0 = D. Now, P_0 is equidistant from A and B. So the waves from A and B will meet in same phase at P_0. So light intensity will be maximum at P_0. It is called **central maximum point.**

Next we consider a point P on the screen at a distance x from P_0. So, $P_0P = x$. AP and BP are joined. As these distances are unequal, there will be difference in phase between the waves reaching P from the sources A and B. So, whether the intensity at P will be maximum or minimum that depends on the path difference.

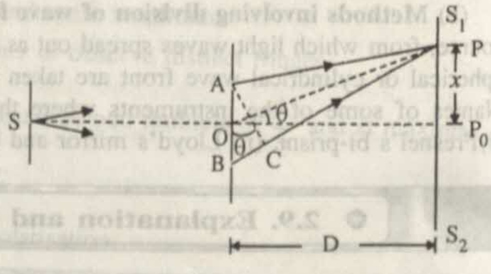


Fig. 2.7

The path difference is $\delta = BP - AP = BC$

[Here AC is normal on BP]

$\therefore\ \delta = AB \sin\theta \quad [\theta = \angle BAC]$

Since the point P is close to P_0. $\theta = \angle POP_0$

$$\therefore\ \delta = 2d.\sin\angle POP_0 = 2d\cdot\frac{x}{PO} = 2d\frac{x}{D}\ \text{(approx.)}$$

Now, from the theory of interference, the amplitude of the resultant wave is $A = 2a\cos\frac{\pi\delta}{\lambda}$ where $\delta = \frac{2xd}{D}$ and λ = wavelength.

- (i) **Bright fringe :** Condition so that the waves meet at P in same phase and P will be a bright point is

$$\cos\frac{\pi\delta}{\lambda} = \pm 1 \qquad \therefore\ \frac{\pi\delta}{\lambda} = n\pi \quad [n = 0, 1, 2, 3, \ldots]$$

$$\therefore\ \delta = \frac{2d.x}{D} = n\lambda$$

$$\therefore\ x = \frac{n\,D\,\lambda}{2d} \qquad \text{(2.3)}$$

Putting $n = 1, 2, 3, \ldots..$ we can determine the positions of first, second, third .... bright fringes on both sides of P_0 (central maximum)

If the distances of n-th and $(n + 1)$-th bright bands be x_n and x_{n+1} from P_0 then

$$x_{n+1} - x_n = (n+1)\frac{\lambda D}{2d} - \frac{n\lambda D}{2d} = \frac{D\lambda}{2d}$$

$$\therefore\ \beta = \frac{D\lambda}{2D} \qquad \text{(2.4)}$$

So, the distance between two consecutive bright fringes is $\beta = \dfrac{D\lambda}{2d}$

● Table : Positions of bright fringes.

Fringe	n	Path difference	Distance from the centre, x
Central	0	0	0
First	1	λ	$\frac{D\lambda}{2d}$
Second	2	2λ	$\frac{2D\lambda}{2d}$
Third	3	3λ	$\frac{3D\lambda}{2d}$
...	...	...	...
...	...	...	...
n-th	n	$n\lambda$	$\frac{nD\lambda}{2d}$

● (ii) **Dark fringes :** If two waves meet in opposite phase at P, it will be the position of dark fringe. The condition for the point P to be dark point is

$$\cos\frac{\pi\delta}{\lambda} = 0 \qquad \therefore\ \frac{\pi\delta}{\lambda} = (2n+1)\frac{\lambda}{2}\ [n = 0, 1, 2, \ldots..]$$

$$\therefore\ \delta = \frac{2d\cdot x}{D} = (2n+1)\frac{\lambda}{2} \qquad \therefore\ x = \frac{D}{2d}(2n+1)\frac{\lambda}{2}$$

Putting $n = 0, 1, 2, \ldots..$ we can determine the positions of 1st, 2nd, 3rd ..... dark fringes a both sides of the point P_0

It can be shown in the same way as before that the distance between two consecutive dark fringes is $\beta = \dfrac{D\lambda}{2d}$.

- **Table : Position of dark bands.**

Fringe	n	Path difference	Distance from the centre, x
First	1	$\frac{1}{2}\lambda$	$\frac{1}{2}\frac{D\lambda}{2d}$
Second	2	$\frac{3}{2}\lambda$	$\frac{3}{2}\frac{D\lambda}{2d}$
Third	3	$\frac{5}{2}\lambda$	$\frac{5}{2}\frac{D\lambda}{2d}$
...	...	...	...
...	...	...	...
n-th	m	$\left(m-\frac{1}{2}\right)\lambda$	$\frac{2m-1}{2}\cdot\frac{D\lambda}{2d}$

Thus the distance between, consecutive bright and dark fringes is same. This distance is called **fringe width.**

$\therefore$ Fringe width, $\beta = \frac{D\lambda}{2d}$

Hence, the width of the fringe is (i) directly proportional to wavelength, $\beta \propto \lambda$, (ii) directly proportional to distance of the screen from the sources $\beta \propto D$ and (iii) inversely proportional to distance between the source, $\beta \propto \frac{1}{2d}$.

Also width of the fringe does not depend on the order of the fringe. If the light be monochromatic, the distance from the fringe to the next fringe is always equal.

Since the fringes are obtained at any position of the screen beyond AB, they are **non-localised fringes.**

●● Numerical Examples ●●

- **Example 2.1. The width of the two slits in Young's experiment is in the ratio 4 : 9. Find the ratio of intensity of the maximum to the intensity at the minimum in the interference pattern.**

○ **Solution :** The intensity of light due to a slit is directly proportional to the width of the slit. So,

$$\frac{I_1}{I_2} = \frac{W_1}{W_2} \qquad \text{Here } \frac{W_1}{W_2} = \frac{4}{9}$$

$$\therefore \frac{I_1}{I_2} = \frac{4}{9} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

If a_1 and a_2 are amplitudes of the waves from the two slits, then

$$\frac{I_1}{I_2} = \frac{a_1^2}{a_2^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(ii)}$$

From (i) and (ii), $\frac{a_1^2}{a_2^2} = \frac{4}{9}$

$\therefore \ \frac{a_1}{a_2} = \frac{2}{3} \quad \therefore \ a_2 = 1\cdot5\, a_1$

Now, $\frac{I_{max}}{I_{min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2}$

$$= \frac{(a_1 + 1\cdot5 a_1)^2}{(a_1 + 1\cdot5 a_2)^2} = 25$$

$\therefore$ $I_{max} : I_{min}$ = **25 : 1.**

● **Ex. 2.2. Green light of wavelength 5100A° from a narrow slit is incident on a double slit. If over all separation of 10 fringes on a screen 10 cm away is 2 cm, find the slit separation.**

○ **Solution :** The required relation : $\beta = \frac{\lambda D}{2d}$

Here $\lambda = 5100 \times 10^{-8}$ cm, D = 200 cm, $10\beta = 2$ cm or $\beta = 0\cdot2$ cm, $2d$ = ?

$$\therefore \ 2d = \frac{\lambda D}{\beta} = \frac{5100 \times 10^{-8} \times 200}{0\cdot2} = \mathbf{0\cdot051\,cm}$$

● **Ex. 2.3. In a Young's double slit experiment, the fringe width obtained is 0·6 cm, when the wavelength of light is 4800A°. If the distance between the screen and the slit is reduced to half, what should be the wavelength of light used to obtain fringes 0·0045 m wide ?**

○ **Solution :** In the first case, $\lambda = 4800$ A° $= 48 \times 10^{-8}$ m, $\beta = 0\cdot6 \times 10^{-2}$ m

Now, $\beta = \frac{D}{2d} \cdot \lambda \qquad \therefore \ \frac{2d}{D} = \frac{\lambda}{\beta} = \frac{4800 \times 10^{-10}}{0\cdot6 \times 10^{-2}} = 8 \times 10^{-5}$

When the distance between the screen and the slits is reduced to half of its initial value :

Here $\beta' = 0\cdot0045$ m, $D' = \frac{D}{2}$ and $2d' = 2d$

If λ' be the wavelength of light, the, $\beta' = \frac{D'\lambda'}{2d'}$

$$\therefore \ \lambda' = \beta' \frac{2d'}{D'} = 0\cdot0045 \times \frac{2d}{D/2} = 0\cdot009 \times \frac{2d}{D}$$

$$\therefore \ \lambda' = 0\cdot009 \times 8 \times 10^{-5} = \mathbf{7\cdot2 \times 10^{-7}\ m} = 7200A°$$

● **Ex. 2.4. A double slit is illuminated by light of wavelength 6000A°. Calculate (a) the angular position of 10th maximum in radians and (b) separation of two adjacent minima.**

○ **Solution :** Wavelength, $\lambda = 6\times10^{-7}$ m. $2d = 10^{-3}$ m and $\mathrm{D} = 1\,\mathrm{m}$

(a) The angular position of n-th maximum, $\theta_n = \dfrac{x_n}{\mathrm{D}} = \dfrac{n\mathrm{D}\lambda}{2d}\times\dfrac{1}{\mathrm{D}} = \dfrac{n\lambda}{2d}$

So, the angular position of 10th maximum, $\theta_{10} = \dfrac{10\times6\times10^{-7}}{10^{-3}} = 0\cdot006\,\mathrm{rad}$

(b) Separation between the two adjacent minima i.e. fringe width

$$\beta = \frac{\mathrm{D}\lambda}{2d} = \frac{10\times6\times10^{-7}}{10^{-3}} = 0\cdot6\times10^{-3}\ \mathrm{mm} = \mathbf{0\cdot6\ mm}$$

● **Ex. 2.5. In Young's experiment, the separation between the two slits is 0·02 cm. The interference fringes due to a light of wavelength 6000A° is formed at a distance of 80 cm on the screen. Calculate the distance of the fifth bright fringe.**

○ **Solution :** Width of each fringe $\beta = \dfrac{\mathrm{D}\lambda}{2d}$

$\therefore$ Width of n number of fringes $n\beta = \dfrac{n\mathrm{D}\lambda}{2d}$

$\therefore\ 5\beta = \dfrac{5\times80\times6\times10^{-5}}{0\cdot02} = 1\cdot2\,\mathrm{cm}$

$\mathrm{D} = 80\,\mathrm{cm}$

$\lambda = 6\times10^{-5}\,\mathrm{cm}$

$2d = 0\cdot02\,\mathrm{cm}$

$n = 5$

$\therefore$ The distance of fifth bright fringe = **1·2 cm**

● **Ex. 2.6. In a Young's double slit experiment, the fringes are formed at a distance of 1m from double slit of separation 0·12 mm. Calculate the distance of 3rd dark band from the centre of the screen. Given, wavelength of light = 6000A°.**

○ **Solution :** For $(n + 1)$ the dark fringe, $x_{n+1} = \left(n+\dfrac{1}{2}\right)\dfrac{\mathrm{D}\lambda}{2d}$

For the third dark fringe, $n = 2$

Also, $2d = 0\cdot012\,\mathrm{cm}$, $\mathrm{D} = 100\,\mathrm{cm}$, $\lambda = 6\times10^{-5}\,\mathrm{cm}$

$$\therefore\ x_3 = \left(2+\frac{1}{2}\right)\times\frac{100\times6\times10^{-5}}{0\cdot012}$$

$$= \mathbf{1\cdot25\ cm}$$

● **Ex. 2.7. Two slits 0·0125 cm apart are illuminated by light of wavelength 4500A°. The screen is one metre away from the slit. Find the separation between the second bright fringes on both sides of central maximum.**

○ **Solution :** The distance of nth bright fringe from central maximum is $x_n = n\cdot\dfrac{\mathrm{D}\lambda}{2d}$

So, the distance of the second bright fringe from central maximum is $x_2 = 2\cdot\dfrac{\mathrm{D}\lambda}{2d}$

$$= 2\times\frac{1\times45\times10^{-8}}{0\cdot125\times10^{-3}}\,m = 7\cdot2\times10^{-3}\ \mathrm{m}$$

Hence, separation between the second bright fringes formed on both sides of the central maximum is $2x_2 = 2\times 7\cdot 2\times 10^{-3} = 14\cdot 4\times 10^{-3}$ m

Alternative method : Separation between the second bright fringes on both sides of the central maximum = width of four dark fringes = 4β.

● **Ex. 2.8. In Young's experiment, the width of the fringe obtained with light of wavelength 6000A° is 2·0 mm. What will be the fringe width, if the entire apparatus is immersed in a liquid of refraction index 1·33 ?**

○ **Solution : In air :** the width of the fringe, $\beta = \frac{D\lambda}{2d}$

$$\therefore \frac{D}{2d} = \frac{\beta}{\lambda} = \frac{2\cdot 0\times 10^{-3}}{6\times 10^{-7}} \quad [\beta = 2\times 10^{-3} \text{ m}; \lambda = 6\times 10^{-7} \text{ m}]$$

$$= \frac{1}{3}\times 10^{4}$$

In a liquid : Let λ′ be the wavelength of the light in the liquid

$$\therefore \lambda' = \frac{\lambda}{\mu} = \frac{6\times 10^{-7}}{1\cdot 33} = 4\cdot 51\times 10^{-7} \text{ m}$$

∴ The present fringe width, $\beta' = \frac{D\lambda'}{2d}$

$$= \frac{\beta}{\lambda}\cdot\lambda' = \frac{1}{3}\times 10^{4}\times 4\cdot 5\times 10^{-7}$$

$$\therefore \beta' = 1\cdot 5\times 10^{-3} \text{ m} = \mathbf{1\cdot 5\,mm}$$

●● Short Answer Type Questions (with answers) ●●

● **Question 1. What the conditions for two light waves to be coherent ?**

○ **Ans.** (i) The light waves should have same wavelength and same amplitude.

(ii) The two light waves should either be in phase should have constant phase difference.

● **Q. 2. What happens to the interference pattern if the phase difference between the two sources continuously varies ?**

○ **Ans.** If the phase difference between the two sources continuously varies, the position of minima and maxima will also vary. So, the interference pattern will not be sustained one and it will not be visible.

● **Q. 3. What is the ratio of the slit widths, when the amplitudes of the light waves from them have a ratio $\sqrt{2} : 1$?**

○ **Ans.** Here $\frac{a_1}{a_2} = \sqrt{2}$. Now, the ratio of the slit widths is

$$\frac{W_1}{W_2} = \frac{a_1^2}{a_2^2} = \left(\frac{\sqrt{2}}{1}\right) = 2$$

$$\therefore W_1 : W_2 = 2 : 1$$

● **Q. 4. If the two slits in Young's double slit experiment have width ratio 4 : 1, deduce the ratio of intensity at a maxima and minima in the interference pattern.**

○ **Ans.** $\frac{W_1}{W_2} = \frac{a_1^2}{a_2^2} = \frac{4}{1}$ or, $\frac{a_1}{a_2} = \frac{2}{1}$

$$\therefore a_1 = 2a_2$$

If I_1 and I_2 are intensities of light at maxima and minima respectively, then

$$\frac{I_1}{I_2} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \frac{(2a_2 + a_2)^2}{(2a_2 - a_2)^2} = \frac{9}{1}$$

$$\therefore I_1 : I_2 = 9 : 1$$

● **Q. 5. In Young's double slit experiment, the separation of the slits is doubled and the distance between the slits and the screen is halved. How will it affect the fringe width ?**

○ **Ans.** In the interference pattern, the fringe width is given by $\beta = \frac{D\lambda}{2d}$

When $2d$ is doubled and D is halved, β becomes one-fourth. So the fringe width reduces to one-fourth of its previous value.

● **Q. 6. Two light waves of amplitudes a_1 and a_2 interfere with each other. Find the ratio of the intensities of a maxima to that of a minima.**

○ **Ans.** When two light waves of amplitudes a_1 and a_2 differing in phase by φ interfere, the intensity of the resultant light is given by

$$I = a_1^2 + a_2^2 + 2a_1a_2 \cos\varphi$$

The intensity will be maximum when $\varphi = 0$. $I_{max} = (a_1 + a_2)^2$

On the other hand, the intensity of light will be minimum when $\varphi = \pi$

$$\therefore I_{min} = (a_1 - a_2)^2$$

$$\therefore \frac{I_{max}}{I_{min}} = \frac{(a_2 + a_2)^2}{(a_1 - a_2)^2}$$

● E ◈ X ◈ E ◈ R ◈ C ◈ I ◈ S ◈ E ●

A. Short answer type questions :

1. What is meant by principle of superposition of waves ?
2. What is interference of light ?
3. Define coherent source of light ?
4. State the essential conditions for the two sources to be coherent.
5. What happens to the interference pattern ? When one of the slits is closed ?
6. The ratio of the width of the slits in Young's double slit experiment is 4 : 1, find the ratio of intensity at the maxima and minima in the interference pattern.
7. What happens to fringe width, when the separation between the sources is increased ?
8. Can white light produce interference ? What is its nature ?
9. Write the necessary conditions for producing sustained interference fringes.
10. Why do we not see the phenomenon of interference in the case of waves emitted by two candles.
11. In Young's double slit experiment the following operations are made state briefly what change will be observed in the fringe system ?
    (i) The slit is illuminated with white light.
    (ii) The slit width is made half.
12. Mention the conditions to get bright fringe and dark fringe during interference of two waves.
13. Why two independent sources cannot produce interference ?

B. Essay type questions :

1. What are coherent sources of light ? State the necessary conditions for sustained interference pattern ? Derive an expression for fringe width using Young's double slit method for interference of light.
2. Draw the labelled diagram for the experimental determination of the wavelength of light by Young's interference experiment. Derive the formula used.
3. Deduce the conditions for maxima and minima in Young's double slit experiment. Find the expression for fringe width.

C. Numerical problems :

1. Two coherent sources whose intensity ratio is 81 : 1 produce interference fringes calculate the ratio of intensity of maxima and minima in the fringe system. **[Ans.** 25 : 16] **[CBSE 1996]**
2. In a Young's double slit experiment, the distance between the slits and the screen is 1 m. The separation between the slits is 5 mm and the fringe width is found to be 0·1 mm. Calculate the wavelength of light used. **[Ans.** 5000A°]
3. In a Young's double slit experiment, interference fringes are produced on a screed placed at 1·5 m from the screen. Separation between the slits is 0·3 mm and illuminated by light of wavelength 6400A°. Find the fringe width. **[Ans.** 3·2 mm]
4. In a Young's double slit experiment, the fringe width is 6 mm with a monochromatic light of wavelength 4000A°. Now the distance between the screen and the slit is reduced to half. Calculate the wavelength of light required to obtain fringes of 5 mm width. **[Ans.** 6666·7 A°]
5. The separation between the two slits in Young's double slit experiment on interference is 0·1 mm, while the distance between the plane of the slits and the screen is 50 cm. Calculate the distance between the first maximum and central maximum on the screen if monochromatic light of wavelength 5000A° is used. **[Ans.** 0·25 cm]

6. In a Young's experiment, the separation of four bright fringes is 2·5 mm, when light of wavelength 6250 A° is used. The distance between the slit and the screen is 80 cm. Calculate the separation of the two slits. **[Ans.** 0·08 cm]

7. In a Young's experiment, the separation between the slits is 0·28 mm and the screen is 1·4 m away. The distance between the central bright fringe and the fourth bright fringe is 1·2 cm. Find the frequency of light used. Velocity of light is 3×10^8 m/s. **[Ans.** 5×10^{14} Hz]

8. In a Young's experiment, the slits are 0·2 mm apart. The interference fringes for light of wavelength 6000A° are formed on a screen 1·5 m away from the slits. Calculate (i) the angular position of the third maxima (ii) the angular position of fifth minima and (iii) finge width. **[Ans.** (i) 0·009 rad (ii) 0·0135 rad (iii) 4·5 mm]

9. Laser light of wavelength 660 nm incident on a pair of slits produce an interference pattern in which bright fringes are separated by 7·8 mm. A second light produces an interference pattern in which the fringes are separated by 6·5 mm. Calculate the wavelength of second light. **[Ans.** 550 nm]

10. Find the ratio of intensitives of two points P and Q on a screen in Young's double slit experiment when waves from sources S_1 and S_2 has phase difference of (i) 0° (ii) $\pi/2$ respectively. **[Ans.** (i) 2I (ii) $\sqrt{2}$ I]

11. In Young's double slit experiment, two slits are separated by 3 mm and illuminated by light of wavelength 480 nm. The screen is at 2m from the plane of the slits. Calculate the separation between the 8th bright fringe and 3rd dark fringe observed with respect to the central bright fringe. **[Ans.** $1{\cdot}76\times10^{-3}$m]

ELECTROSTATICS

• TOPICS DISCUSSED •

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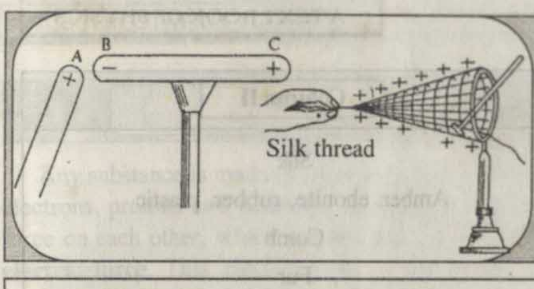


CHAPTER 1

ELECTROSTATIC CHARGE AND ITS PROPERTY

● TOPICS : ○ Introduction; ○ Frictional electricity and two types of charges; ○ Two kinds of charges; ○ Electric charge; ○ Explanation of frictional electricity; ○ Quantisation of charge; ○ Conservation of charge; ○ Conductors and insulators; ○ Electrostatic Induction; ○ Induction precedes attraction; ○ Charges reside only on the surface of a conductor; ○ Action of highly charged points; ○ Lightning Arrester; ○ Short answer type questions (with answers); ○ Exercise.

1.1. Introduction

Electrostatics is an important branch of physics which deals with electric charges at rest. Here we come across a new physical quantity called electric charge like mass, length and time.

1.2. Frictional electricity and two types of charges

Electricity can be produced by simply rubbing two bodies. It is called **frictional electricity.** In 600 B.C. Greek Philosopher Thales casually noticed that amber*, when rubbed with wool, acquires the property of attracting objects like a small piece of paper, dry leaves etc. There are many other substances like amber, which can also acquire the property of attracting light objects on being rubbed by suitable substances. For example, a glass rod on being rubbed with silk, an ebonite rod on being rubbed with fur, a plastic comb on being run through dry hair etc. acquire a power to attract light bodies such as small pieces of paper. The amber, glass rod, ebonite rod or plastic comb are said to be **electrified or charged with electricity**. Also wool (in the case of amber) silk (in case of glass rod) and fur (in case of ebonite rod) all are found to be electrically charged.

Frictional electricity is the electricity developed on substances, when they are rubbed with each other.

The electric charge cannot move from one part of the object to the other part. It remains static at some part of the object. For this reason frictional electricity is also known as **static electricity.** The branch of physics which deal with static electricity is called **electrostatics.**

Frictional electricity, under suitable condition may accumulate to a dangerous extent. When a truck containing petrol moves speedily the oil is jerked violently and electric charges are developed due to rubbing. This accumulated charges may cause sparking. As petrol is highly inflammable, it may cause violent explosion. To prevent such accident, charges are not allowed to accumulate. A metallic chain is allowed to dangle on the road from the body of the track. Then frictional charges as soon as they are produced leak away to the earth through the chain. So they cannot accumulate and cause of accident is averted.

It has been observed from experiment that when substances in column I are rubbed with substances listed in column II, they acquire positive charges while substances listed in column II acquire negative charges. It is also found that any two charged objects in the same column repel each other. On the other hand, two charged objects from different columns attract each other.

---

* Amber is a yellow resineous substance. It is hardened sap of a tree-similar for a pine tree.

Column I	Column II
Glass	Silk
Wool, Flannel,	Amber, ebonite, rubber, plastic
Dry hair	Comb
Ebonite	Fur

1.3. Two kinds of charges

We can easily show that electric charges are of two types. For this we take a **glass rod** and suspend it with a thread after rubbing it with **silk.** Now another glass rod rubbed with silk is brought near the first rod, the two rods are found to repel each other. Similarly, two ebonite rods each rubbed with fur are found to repel each other.

But when an ebonite rod rubbed with fur is brought near a glass rod rubbed with silk, the two rods are found to attract each other. From these observations we conclude :

(i) The electric charge developed on glass rod (rubbed with silk) is different for the charge developed on ebonite rod (rubbed with fur).

(ii) The electric charges of same kind (charges developed on two glass rods or on two ebonite rods) repel each other, while those of different kinds (charges developed on a glass rod and an ebonite rod) attract each other.

We may, therefore, conclude that **like charges repel each other and unlike charges attract each other.**

The electricity developed in glass due to rubbing with silk is treated as **positive** while that developed in ebonite due to rubbing with fur is **negative.** It is to be noted that positive and negative are simple names and they have no further significance except that they denote opposite kind of electricity. Similarly, where an ebonite rod is rubbed with fur, ebonite rod becomes negatively charged and fur positively charged.

It should be borne in mind that the nature of charge developed on a body depends on the body and the rubber. The table below shows that the same substance may develop both positive and negative electricity when rubbed with suitable substance. If an object in the first column is rubbed against the object given in second column the object in the first column will acquire positive charge, while that in the second column will acquire negative charge.

• The name of the object, which acquires •

Positive Charge	Negative Charge
Glass rod	Silk cloth
Fur	Ebonite rod
Woollen cloth	Plastic object
Woollen cloth	Amber
Silk	Amber
Metal	Resin

✪ 1.4. Electric charge ✪

Any substance is made of small particles called atoms, which are made of three elementary particles—electrons, protons and neutrons. In addition to gravitational force, electrons and protons exert an extra force on each other, which is quite large as compared to gravitational force. This extra force is actually **electric force**. This electric force is due to electric charge. So protons and electrons possess extra property called electric charge.

Electric charge is a scalar quantity. Its SI unit is **column** (C) A proton carries a positive charge $+e$ and electron a negative charge $-e$, where $e = 1{\cdot}6 \times 10^{-19}$ coulomb.

Normally an atom contains equal number of protons and electrons, and as such it is electrically neutral. Hence any object which consists of atoms is electrically neutral. If the number of electrons in an atom is different from the number of protons then the atom is said to be charged and that the object is also charged.

✪ 1.5. Explanation of frictional electricity ✪

The origin of frictional electricity and charging of a body by friction can be explained on the basis of exchange of electrons between the rubbing body and the rubbed body.

When two different objects (such as a glass rod and silk) are rubbed together they get electrified. It happens due to transfer of electrons from one object to the other. What happens is : when glass rod is rubbed with silk, some of the electrons are torn off from the atoms or molecules in the glass due to mechanical friction. This is because electrons are held less lightly in glass than in silk. these 'torn off' electrons are transferred to the silk. As the glass rod loses electrons it becomes positively charged. On the other hand, silk acquires an equal amount of negative charge on gaining electrons.

So, frictional electricity is due to actual transfer of electrons form one object to another. In general, **a negatively charged body possesses excess of electron while a positively charged body has a deficit of electrons.** All the phenomena in electrostatic can be explained from the concept of transfer of electrons.

✪ 1.6. Quantisation of charge ✪

Previously it was throughout that the charge on a body can be increased continuously. It was in the year 1897 that J. J. Thomson discovered electron. In 1911 Millikan successfully showed that charges on tiny oil drops are exact multiples of elementary charges. The magnitude of charge on a proton or an electron ($e = 1{\cdot}6 \times 10^{-19}$C) is called elementary charge. Since the protons and electrons are the only charged particles constituting the matter, the charge on an object must be integral multiple of $\pm e$. It means that the charge possessed by an object cannot be a fraction of $\pm e$. Mathematically, the charge on any object must always be equal to $q = \pm ne$, when n is an integer. Also, the charge on an object can be increased or decreased in steps of e. **It is known as quantization of charge or discrete nature of charge.**

Thus, the statement 'quantisation of charge' means that all observable charges are integral multiple of elementary charge e where $e = 1{\cdot}6 \times 10^{-19}$C.

For many practical purposes or for large scale phenomena, we ignore quantisation of charge and regard the charge as something continuous. It is because step size e of electric charge is very small. Due to this, number of elementary charges involved becomes extremely large, even when electric charge of a body changes by a very small amount. For example, let a charge of 1μC (10^{-6}C) is transferred from one body to another during charging by friction. As elementary charge is equal; to $e = 1{\cdot}6 \times 10^{-19}$C, the number of elementary charge involved is

$$n = \frac{q}{e} = \frac{10^{-6}\,\mathrm{C}}{1{\cdot}6 \times 10^{-19}\,\mathrm{C}} = 6{\cdot}25 \times 10^{12}$$

Evidently, n is very large quantity. For this reason, quantisation of charge is not observed.

1.7. Conservation of charge

Electric charge or quantity of electricity is conserved in the same manner as linear momentum, energy, angular momentum etc. The law of conservation of charge states that, **for an isolated system, the net charge always remains constant** i.e. the algebraic sum of positive and negative charges in an isolated system remains constant.

The law, like the law of conservation of momentum or of energy, is one of the fundamental law of physics. There is no exception to this law and it is universal.

In a physical process, the charge may get transferred from one part of the system another, but net charge will always remain constant. In other words, **charge cannot be created nor destroyed.**

We consider the following examples to illustrate the law of conservation of electric charge.

(i) When a glass rod is rubbed with silk, positive charge appears on glass rod, while an equal amount of negative charge appears on the silk. In this way, the net charge on the glass-silk system remains zero before and after rubbing.

(ii) Next we consider nuclear fission of uranium ${}_{92}\mathrm{U}^{235}$ by a neutron (${}_0\mathrm{n}^1$). Here barium (${}_{50}\mathrm{Ba}^{141}$), Krypton (${}_{36}\mathrm{Kr}^{92}$) and three neutrons are produced along with liberation of energy. This nuclear reaction is represented as

$${}_0n^1 + {}_{92}\mathrm{U}^{235} \rightarrow {}_{56}\mathrm{Ba}^{141} + {}_{36}\mathrm{K}r^{92} + 3\,{}_0n_1 + \text{energy}$$

Total charge before fission = + $92e$; total charge after fission = $56e + 36e = 92e$

So, the total charge is conserved.

1.8. Conductors and insulators

Materials are broadly classified into two class—conductors and insulators.

Conductors : Substances through which electricity can easily pass are called conductors. Metals are in general good conductor of electricity.

Insulators : Substances through which electricity can not easily pass are called insulators. Glass, ebonite, rubber etc. are insulators.

In metals free electrons are current carriers and there is large number of free electrons in metals. It is found that the elements, in which the valence shell is less than half filled are found to be good conductors. For example, in metals such as copper, aluminium, silver etc., the valence shell contains three or less electrons. Since the atom has a tendency to have filled valence shell, these valance electrons move through the metal in random manner. There electrons take part in conductor process.

On the other hand, in insulators there is no free electrons. They are all bound to the nucleus. So they cannot conduct electricity.

Insulators are also called **dielectric.** This valence electrons in the atoms of a dielectric on tightly bound to their nuclei.

1.9. Electrostatic Induction

We know that an uncharged body contains equal amount of positive and negative charges. The charges are distributed evenly over the entire volume of the bodies. If a charged body is held near an uncharged body, its nearer end developes opposite charge and further end similar charge. In fig. 1.1 A is a positively charged rod. An uncharged body BC provided with an insulated handle is placed near A. Due to influence of A, charges on BC will get separated. Negative charges will be attracted towards A and positive charges will be repelled. So end B will develop negative charge and end C will develop positive charge. **This phenomenon of temporary electrification of a body by the influence of a neighbouring charge is called electrostatic induction.** The charge of A is called inducing charge and the charges on both the ends of BC are called induced charge. The charge induced at end B of the body BC is called **bound charge** and the charge induced at end C is called **free induced charge**. They are so named because if the body is momentarily touched with a finger, the charge at C immediately goes to the earth, but the charge at B is held bound by the opposing charge of the inducing body.

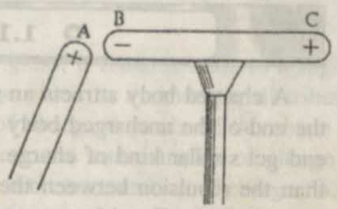


Fig. 1.1

➤ *1.9.1. Explanation from electronic theory* :

The phenomenon of electrostatic induction can be explained by electronic theory. A conductor contains a large number of free electrons. These electrons are not attached to any particular atom. In fig. 1.1 the positive charge of glass rod A attracts the free electrons of the body BC to the end B, which, therefore, gets surplus of electrons and becomes negatively charged. As surplus electrons came to the end B, there occurs a deficiency of electrons at the other end C, which therefore, becomes positively charged.

➤ *1.9.2. Equal amount of opposite charges are produced due to induction* :

We can easily show that due to electrostatic induction equal amount of opposite charges (positive and negative) are developed in the body which is placed near a charged body. Consider fig. 1.2 two metal spheres A and B supported on insulated handle are placed side by side touching each other. A positively charged rod C is brought near A. Charges will be induced on A and B. Now the sphere B is moved away and then the rod C is removed from the vicinity. Both the spheres A and B are found to be charged. The sphere A is negatively and the sphere B positively. Then the spheres are touched with each other. On examination it will be found that neither of the spheres has any charge. The negative charge of the sphere A has completely neutralised the positive charge on the sphere B.

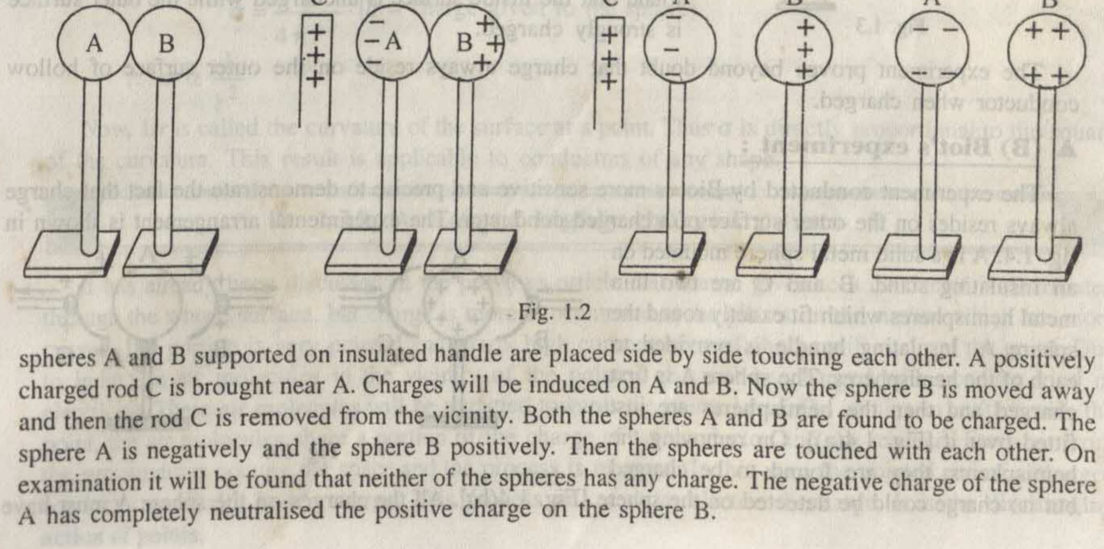


Fig. 1.2

The following facts has to be noted in connection as the electrostatic induction :

(i) Both kinds of electricity—positive and negative—are developed simultaneously by induction.

(ii) Induced positive and induced negative charges are equal in amount.

(iii) The induced charges are temporary. They neutralise each other as soon as the inducing body is removed.

✪ 1.10. Induction precedes attraction ✪

A charged body attracts an uncharged body. The reason of attraction is induction. Due to induction, the end of the uncharged body nearest to the charged body gets opposite kind of charge and the further end get similar kind of charge. The attraction between the nearer opposite charges is evidently greater than the repulsion between the two farther similar charges. As a result the inducing body attracts the induced body. For this reason, it is said that induction precedes attraction.

✪ 1.11. Charges reside only on the surface of a conductor ✪

The electric charge given to a conductor distributes over the outer surface of the conductor only. No charge is found to exist on the inside of a solid conductor or on the inner surface of a hollow conductor. The following experiments demonstrate this phenomenon.

▲ (A) Faraday's butterfly-net experiment :

In this experiment a hollow conducts is a conical shaped butterfly net. The net could be turned inside out by means of an insulating silk thread attached to the apex of the net. It is mounted on a brass ring supported on an insulating stand [Fig. 1.3]. The net is charged strongly. On testing it will found that only the outer surface of the net is strongly charged and there is no charge on the inner surface.

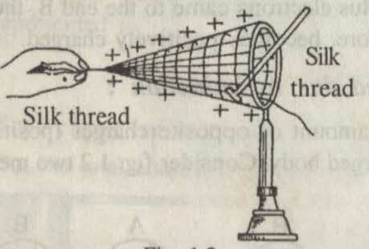


Fig. 1.3

The net is then pulled inside out, so that the outer surface now becomes the inner surface of the net. Once again it will be found that the inside surface is uncharged while the outer surface is strongly charged.

The experiment proves beyond doubt that charge always reside on the outer surface of hollow conductor when charged.

▲ (B) Biot's experiment :

The experiment conducted by Biot is more sensitive and precise to demonstrate the fact that charge always resides on the outer surface of a charged conductor. The experimental arrangement is shown in fig. 1.4. A is a solid metal sphere mounted on an insulating stand. B and C are two thin metal hemispheres which fit exactly round the sphere A. Insulating handle is provided to each of the hemispheres. The sphere A is first charged and then the hemispheres are just fitted over it [Fig. 1.4(a)]. On removing the hemispheres, they are found to be charged but no charge could be detected on the sphere [Fig. 1.4(b)]. All the charges on the sphere A must have

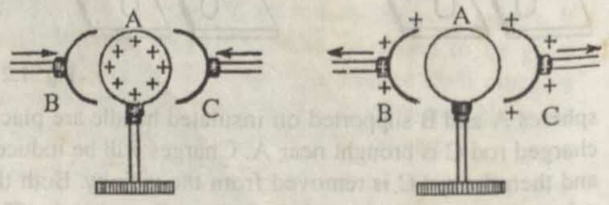


Fig. 1.4

passed to the hemispheres. The reason is : when the hemispheres fully conver the sphere A and come in contact with A, they form a single conductor whose outer surface is the outer surface of the hemispheres. The charges, therefore, leave the inner sphere A and spread over the outer surface of the hemispheres.

➤ *1.11.1. Distribution of charge on the surface of a charged conductor and surface density of charge:*

When charge is given to a non-conductors like amber, ebonite or glass, it remains static in a region of the body. The charge cannot move over the surface. But a charge given to a conductor distributes itself over the surface of the conductor. The nature of distribution of charge depends on the shape of the conductor. If the surface has different curvature at different points, the concentration of charge is also different at different points. Higher the curvature, the greater is the concentration of charge. With a metallic sphere, whether solid or hollow, the charge spreads uniformly over the surface. In other cases, the distribution is not uniform the concentration is more at points and less where the surface is more nearly straight.

Fig. 1.5 shows qualitatively how charge is distributed over the surfaces of conductors of different shapes. The distance of the dotted lines from any point on the surface of a conductor represents the concentration of charge at that point. The first conductor being spherical, the distribution of charge over the surface is uniform but in other two cases the situation is different [Fig. 1.5].

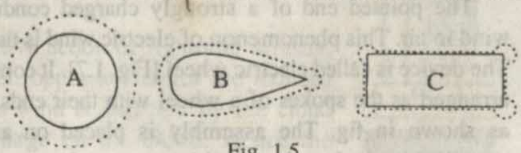


Fig. 1.5

● **Surface density of charge :** The degree of concentration of charge over a surface of a charged conductor (solid or hollow) is expressed by the concept of surface density of charge. It is defined as the quantity of charge per unit area of the surface of a conductor.

$$\text{Surface density, } \sigma = \frac{q}{s} = \frac{\text{charge}}{\text{surface area}}$$

In the case of spherical conductor of radius r, its uniform surface density of charge is

$$\sigma = \frac{q}{4\pi r^2} \quad [q = \text{charge given to the sphere}]$$

$$\therefore \ \sigma \propto \frac{1}{r^2}$$

Now, $1/r$ is called the curvature of the surface at a point. Thus σ is directly proportional to the square of the curvature. This result is applicable to conductors of any shape.

❂ 1.12. Action of highly charged points ❂

It has already been discussed in the previous article that charge given to a conductor is distributed through the whose surface, but charge is more concentrated at the places on the surface which are more curved. If a portion is very pointed and sharp, high concentration of charge will occur at the point. Due to induction air molecules in the vicinity of the pointed end will be charged with opposite kind of electricity. These air molecules will be attracted towards the pointed end. On coming in contact with the point, the air molecules share a portion of the charge of the conductor and are repelled. Fresh air from the surroundings occupy the space and the process is repeated. In this way, the pointed end of a charged conductor gradually losses charge. This process is known as electric discharge. It is the **discharging action of points.**

So, a conductor which is required to retain its charge for a long time should be rounded so that there is no sharp point.

➤ *1.12.1. Electric wind* :

Suppose a conductor mounted on an insulating stand has a sharp point. It is strongly charged by an electric machine. Due to the discharging action of the charged point the similarly charged air molecules are repelled and they move away with considerable speed. It appears that as if the charges are pumped into the conductor by the machine and are sprayed by the conductor through the pointed end in the vicinity. This phenomenon is called **spraying action of points.**

The flow of charged air molecules away from the pointed end of the conductor is called electric wind. The formation of electric wind is demonstrated by keeping a candle with burning flame near the point. When the conductor is strongly charged the candle flame is found to be deflected by the strong draught.

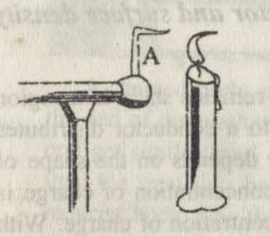


Fig. 1.6

➤ *1.12.2. Electric wheel* :

The pointed end of a strongly charged conductor forms electric wind in air. This phenomenon of electric wind is used to rotate a wheel. The device is called electric wheel [Fig. 1.7]. It consists of several wires arranged as the spokes of a wheel with their ends bent at right angles as shown in fig. The assembly is placed on an insulated pointed metallic stand when the stand is connected to an electric machine, the wheel stands rotating.

● **Explanation :** When the wheel is charged strongly, the pointed ends of the wire causes electric discharge. Similarly charged air molecules are repelled. The charged air molecules move away. As a reaction, the air molecules exerts equal and opposite repulsive force on the wheel. The resulting reaction on the wires causes the wheel to rotate in the opposite direction.

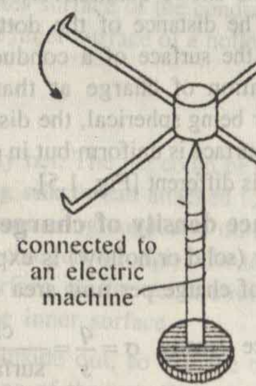


Fig. 1.7

➤ *1.12.3. Action of points as collector of charge* :

Sharp points of a highly charged conductor facilitate the discharging of the conductor. They can also act as collector of charge when the conductor itself remains uncharged. The action of points as collector of charges can be shown by the following experiment.

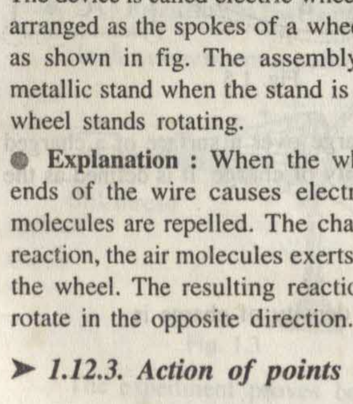


Fig. 1.8

A metallic needle is placed on the disc of an uncharged gold leaf electroscope as the fig. 1.8. The pointed end is projected out as in fig. As a result the electroscope is converted into a conductor with a pointed end. Now, a charged (say, positively) rod is brought near to the point of the needle. Immediately the leaves diverge. The divergence remain unchanged even when the rod removed. On testing it is found to be positively charged, while the rod becomes free of charge. It is usual to describe the process by saying that charge has been 'taken off' the rod by the pointed end of the needle. Actual explanation is as follows :

The positively charged rod induced negative charge on the pointed end of the needle and positive charge on the leaves. Now, the pointed end begins to discharge and the air particle will be charged negatively. The positively charge rod will attract negatively charged air particles on it. So the rod will gradually be neutralised. The needle due to discharge will also be neutralised. So the ultimate result is : electroscope gains the positive charge and the rod loses it.

✪ 1.13. Lightning Arrester ✪

Lightning arrester is a device which protects tall building or structure from damage due to lightning. It is a thick copper strip fixed to the outside wall of the building. Its lower end is buried into moist earth. Its other end is projected above the highest point of the building and ends in several sharp spikes [Fig. 1.9].

At the time of thunderstorm, cloud becomes heavily charged. Large electric sparks, known as **lightnings** occur between two charged clouds or between a cloud and the earth. Lightning between charged cloud and the earth is a cause of danger to us. In this case lightning occurs between charge on the cloud and the induced opposite charge on the highest structure. Thus tall trees and high-rise buildings are targets of direct lightning strike.

Discharging action of points is the basis of protective action of lightning arrester. When a strongly charged cloud comes over the arrester, opposite charges are induced on the pointed spikes, an electric wind is set up. This wind is attracted by the cloud and rises to the cloud and partly neutralises the charge on the cloud. Hence the possibility of lightning strike on the building decreases considerably. Even after this, lightning flash may pass. Lightning strikes the spikes. The spikes being earthed, provide the path of least resistance and the electricity easily flows to the earth. Thus the building is protect and remain safe from the lightning strike.

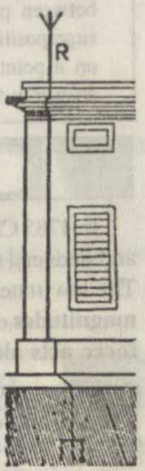
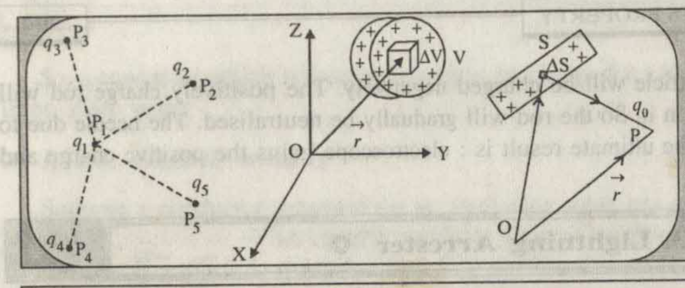


Fig. 1.9

● E ◈ X ◈ E ◈ R ◈ C ◈ I ◈ S ◈ E ●

A. Short answer type questions :

1. What happens when a glass rod is rubbed with silk cloth ?
2. When an object is said to be a conductor and when an insulator ?
3. Repulsion is the surer test of electrification. Explain.
4. Induction precedes attraction. Explain.
5. Petrol tankers are provided with metal chains which touch the gound—why ?
6. What we mean be 'free charge' and 'bound charge' in connection with electrostatic induction ?
7. What we mean be discharging action of points ?
8. What is the basis of action of lightning arrestor ?



CHAPTER 2

COULOMB'S LAW

● **TOPICS :** ○ Introduction; ○ Relative permeability; ○ Coulomb's law in vector form; ○ Force between point charges in terms of position vectors; ○ Force between many charges : the principle of superposition; ○ Continuous charge distribution; ○ Force due to continuous charge distribution; ○ Force on a point charge placed on the axis of a uniformly charged ring; ○ Short answer type questions (with answers); ○ Exercise.

❂ 2.1. Introduction ❂

In 1785 Charles Augustin de Coulomb first measured electrical force between fixed charges quantitatively and deduced the law governing them. It is a law which governs the force between two fixed point charges. The law states that **the force between two point charges is directly proportional to the product of the magnitudes of the charges and inversely proportional to the square of the distance between them. This force acts along the line joining the charges.**

Let q_1 and q_2 be two point charges at a distance r apart as in fig. 2.1. Then by Coulomb's law, the force F between the charges is

$$F \propto q_1 q_2 \text{ and } F \propto \frac{1}{r^2}$$

Combining these two, $F \propto \frac{q_1 q_2}{r^2}$

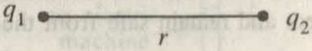


Fig. 2.1

$$\therefore \; F = k\frac{q_1 q_2}{r^2} \quad \ldots\ldots (2.1)$$

where k is a constant of proportionality whose value depends on the nature of the medium in which the two charges are located and also on the system of units adopted to measure q_1, q_2 and r.

In SI system and if the charges be in air (or vacuum) $k = \frac{1}{4\pi\,\varepsilon_0}$ where ε_0 is called absolute permittivity of free space or air.

So, the equation (2.1) is written as $F = \frac{1}{4\pi\,\varepsilon_0}\frac{q_1 q_2}{r^2}$ (for air or vacuum) ……(2.2)

Now, $\varepsilon_0 = 8{\cdot}854 \times 10^{-12}\,C^2\,N^{-1}m^{-2}$ and $\frac{1}{4\pi\,\varepsilon_0} = 9 \times 10^9\,Nm^2C^{-2}$

$$\therefore \; F = 9 \times 10^9 \frac{q_1 q_2}{r^2} \text{ (N) (for air)} \quad \ldots\ldots (2.3)$$

On the other hand, in C.G.S. system if the charges be in air, $k = 1$ in equation (2.1). Hence

$$F = \frac{q_1 q_2}{r^2} \quad \ldots\ldots (2.4)$$

Unit of charge and its definition : (i) In SI system unit of charge is 1 Coulomb (C). In equation (2.3), let $q_1 = q_2 = q$, $r = 1\text{m}$ and $F = 9\times10^9$ N. Then

$$9\times10^9 = 9\times10^9 \times \frac{q\times q}{1} \qquad \therefore\ q = \pm1\text{C}$$

Therefore, **if two identical point charges (having equal magnitude and same nature) placed at a separation of 1 m in air repel each other with a force of 9×10⁹ N, then each charge is 1 Coulomb.**

(ii) In C.G.S. system, unit of charge is Statcoulomb (Stat C)

In equation (2.4), let $q_1 = q_2 = q$, $r = 1$ cm, $F = 1$ dyne. Then

$$1 = 1\times\frac{q\times q}{1} \qquad \therefore\ q = \pm1 \text{ stat C}$$

Therefore, if two identical point charges placed at a separtion of 1 cm in air repel each other with a force of 1 dyne, then each charge is I Statcoulomb.

Relation : It can be shown that **I C = 3×10⁹ stat C**

- **Note :** In electromagnetic c.g.s. system, the unit of charge is known as electromagnetic unit (emu) of charge.

I C = 1/10 emu charge.

2.2. Relative permeability (dielectric constant)

From experiment it is found that if the two charges are situated in some other medium (other than air), the force between the charges is greatly affected. For example, when two charges are placed at the same distance in water, the force between the two charges reduces to 1/80 of the force between the same two charges placed in air. Reason is : the value of absolute permittivity of water is about 80 times the absolute permittivity of air.

The force between two charges q_1 and q_2 situated at a distance r in a medium is

$$F = \frac{1}{4\pi\,\varepsilon}\,\frac{q_1q_2}{r^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.5)$$

where ε = **absolute permittivity** of the medium.

For vacuum or air $\varepsilon = \varepsilon_0$. For any other medium the ratio $\frac{\varepsilon}{\varepsilon_0}$ is called **relative permittivity** or **dielectric constant** or **specific inductive capacity of the medium.** The ratio is denoted by ε_r or K.

$$\therefore\ K = \frac{\varepsilon}{\varepsilon_0} \quad \therefore\ \varepsilon = K\,\varepsilon_0$$

$$\therefore\ F = \frac{1}{4\pi\,\varepsilon_0}\,\frac{1}{K}\,\frac{q_1q_2}{r^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.6)$$

The equation (2.2) determines the force between two charges q_1 and q_2 placed in air at a separation r. And the equation (2.6) gives the force between the same two charges palced in other medium at the same separation r. We write

$$F_{\text{air}} = \frac{1}{4\pi\,\varepsilon_0}\,\frac{q_1q_2}{r^2} \text{ and } F_{\text{med}} = \frac{1}{4\pi\,\varepsilon_0}\cdot\frac{1}{K}\,\frac{q_1q_2}{r^2}\,F_{air} \quad \therefore\ K = \frac{F_{\text{air}}}{F_{\text{med}}}$$

Hence, **dielectric constant of a medium is defined as the ratio of the force between two charges placed at a distance in air to the force between the same two charges placed at the same separtion in that medium.**

The following table gives the values of some insulators at 20°C

Material	Dielectric constant	Material	Dielectric constant
Vacuum	1·00	amber	2·800
air (1 atm)	1·00059	Porcelain	7·00
Water	80·000	Paraffined paper	2·1–2·300
mica	3–6·00	Turpentine	2·230
Alcohol	25·00	Germanium	16·00

2.3. Coulomb's law in vector form

We consider two point charges q_1 and q_2 kept separated by a distance r.

(i) First we consider the case when $q_1q_2 > 0$. Here either q_1 and q_2 both positive or both negative. So, the charges repel each other as shown in fig. 2.2.

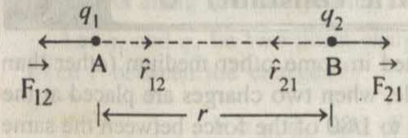


Fig. 2.2

Let $\vec{F}_{12}$ be the force exerted on q_1 due to the charge q_2 and $\vec{F}_{21}$ be the force exerted on q_2 due to the charge q_1.

Now, we define unit vector $\hat{r}_{12}$ pointing from q_1 to q_2 and $\hat{r}_{21}$, a unit vector pointing from q_2 to q_1.

$$\therefore \ \vec{F}_{12} = \frac{1}{4\pi\,\varepsilon_0}\frac{q_1q_2}{r^2}\,\hat{r}_{21} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2.7)$$

$$\text{and } \vec{F}_{21} = \frac{1}{4\pi\,\varepsilon_0}\frac{q_1q_2}{r^2}\,\hat{r}_{12} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2.8)$$

As $\hat{r}_{21} = -\hat{r}_{12}$ it follows $\vec{F}_{12} = -\vec{F}_{21}$

$$\text{and } |\vec{F}_{12}| = |\vec{F}_{21}| = \frac{1}{4\pi\,\varepsilon_0}\frac{q_1q_2}{r^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2.9)$$

(ii) Next we consider the case $q_1q_2 < 0$ i.e. the case when the two charges are unlike. Now, the force is attractive as shown in fig. 2.3. Proceeding as above, it can be obtained that

$$\vec{F}_{12} = \frac{1}{4\pi\,\varepsilon_0}\frac{q_1q_2}{r^2}\,\hat{r}_{12}$$

$$\text{and } \vec{F}_{21} = \frac{1}{4\pi\,\varepsilon_0}\frac{q_1q_2}{r^2}\,\hat{r}_{21}$$

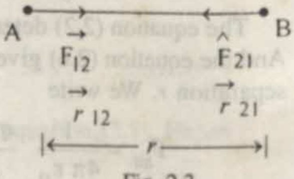


Fig. 2.3

● **Note :** From Coulomb's law in vector form, it follows that the electrostatic force between two charges is a **central force** i.e. it acts along the line joining the two charges.

2.4. Force between point charges in terms of position vectors

To introduce the concept of position vectors in the expression for the force between two point charges, we consider three dimensional rectangular Cartesian co-ordinate system OXYZ [Fig. 2.4]. Two point charges q_1 and q_2 are situated in space at the points A and B whose position vectors are $\overrightarrow{\text{OA}} = \hat{r_1}$ and $\overrightarrow{\text{OB}} = \hat{r_2}$. The distance of separation between the charges is r and $\hat{r}_{21}$ is a unit vector from q_2 to q_1

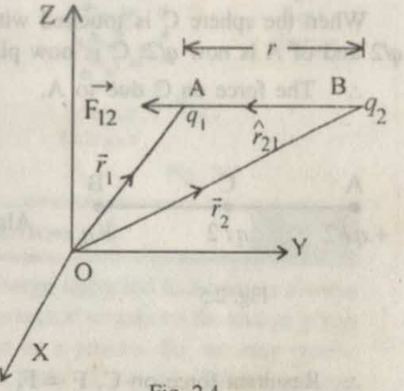


Fig. 2.4

Then the force on q_1 due to q_2 is $\overrightarrow{\text{F}}_{12} = \dfrac{1}{4\pi\,\varepsilon_0}\dfrac{q_1 q_2}{r^2}\hat{r}_{21}$

It can also be written as,

$$\overrightarrow{\text{F}}_{12} = \frac{1}{4\pi\,\varepsilon_0}\frac{q_1 q_2}{r^3}\left(r\ \hat{r}_{21}\right) = \frac{1}{4\pi\,\varepsilon_0}\frac{q_1 q_2}{r^3}\hat{r}_{21}$$

Now, by triangle law of addition of vectors, we write

$$\overrightarrow{\text{OB}} + \overrightarrow{\text{BA}} = \overrightarrow{\text{OA}} \qquad \text{or, } \overrightarrow{\text{BA}} = \overrightarrow{\text{OA}} - \overrightarrow{\text{OB}} \qquad \therefore\ \hat{r}_{21} = \hat{r}_1 - \hat{r}_2$$

$$\overrightarrow{\text{F}}_{12} = \frac{1}{4\pi\,\varepsilon_0}\frac{q_1 q_2}{r^3}\left(\vec{r}_1 - \vec{r}_2\right) = \frac{1}{4\pi\,\varepsilon_0}\frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^3}\left(\vec{r}_1 - \vec{r}_2\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (2.10)$$

Similarly, the force q_2 due to q_1 is $\overrightarrow{\text{F}}_{21} = \dfrac{1}{4\pi\,\varepsilon_0}\dfrac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^3}\left(\vec{r}_2 - \vec{r}_1\right)$ . . . . (2.11)

The two equations (2.10) and (2.11) gives the most general vector expressions of Coulomb's law.

- **Example 2.1. Compare the electrostatic force and gravitational force between proton and electron in a hydrogen atom. Given, electronic charge $e = 1\cdot6\times10^{-14}$C, mass of electron, $m_e = 9\cdot1\times10^{-31}$ kg, mass of proton, $m_p = 1\cdot7\times10^{-27}$ kg and $\text{G} = 6\cdot67\times10^{-11}$ Nm²Kg⁻²**

○ **Solution :** The electrostatic force between proton and electron is

$$\text{F}_e = 9\times10^9\,\frac{e^2}{r^2} = \frac{9\times10^9\times(1\cdot6)^2\times10^{-38}}{r^2}$$

$e = 1\cdot6\times10^{-19}$
$m_e = 9\cdot1\times10^{-31}$ Kg
$m_p = 1\cdot7\times10^{-27}$ Kg

Gravitational force between proton and electron is

$$\text{F}_g = \text{G}\,\frac{m_e m_p}{r^2} = 6\cdot67\times10^{-11}\times\frac{9\cdot1\times10^{-31}\times1\cdot7\times10^{-27}}{r^2}$$

$$\therefore\ \frac{\text{F}_e}{\text{F}_g} = \frac{9\times10^9}{6\cdot67\times10^{-11}}\times\frac{(1\cdot6)^2\times10^{-38}}{9\cdot1\times1\cdot710^{-58}} = 2\cdot26\times10^{39}$$

- **Ex. 2.2. When two identical metal spheres A and B are given equal amount of positive charge, they repel each other with a force of 2×10^{-5} N. Another identical sphere C is now touched with the sphere A and then placed at the midpoint of AB. Find the resultant force on the sphere C.**

○ **Solution :** Let initially spheres A and B each be given a charge q and their separation is r. So in the first case $\frac{1}{4\pi\,\varepsilon_0}\cdot\frac{q^2}{r^2}=2\times10^{-5}$ .... (i)

When the sphere C is touched with A, the charge will be equally divided. So, the charge of C is $q/2$ and of A is now $q/2$. C is now placed at C. The situation is shown in fig. 2.5.

$\therefore$ The force on C due to A,

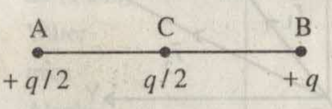


Fig. 2.5

$$F_1=\frac{1}{4\pi\,\varepsilon_0}\,\frac{(q/2)^2}{(r/2)^2}=\frac{1}{4\pi\,\varepsilon_0}\,\frac{q^2}{r^2}\text{ along }\overrightarrow{AC}$$

Also, force on C due to B,

$$F_2=\frac{1}{4\pi\,\varepsilon_0}\,\frac{q^2/2}{(r/2)^2}=\frac{1}{4\pi\,\varepsilon_0}\,\frac{2q^2}{r^2}\text{ along }\overrightarrow{BC}$$

$\therefore$ Resultant force on C, $F=F_2-F_1=\frac{1}{4\pi\,\varepsilon_0}\times\frac{q^2}{r^2}=2\times10^{-5}\text{N along }\overrightarrow{BC}$

✪ 2.5. Force between many charges : the principle of superposition ✪

The equation (2.7) of Art. 2.3 gives the force between two point charges. Here it is assumed that there is no charge in the neighbourhood. Now if we consider a number of charges in a region of space, then how to calculate resultant force on a particular charge ?

Let $q_1, q_2, q_3, \ldots\ldots$ are the charges located $P_1, P_2, P_3, \ldots\ldots$ [Fig. 2.6]. Let we like to calculate total force on q_1 situated at P_1.

In this case, we apply the **principle of superposition**. This principle gives a method of finding the force on a charge when a group of charges are interacting. The principle states that **when a number of charges are interacting, the total force on a given charge is the vector sum of the individual forces exerted by all other charges on the given charge.**

Evidently, it is assumed that the force between any two charges is not affected by the presence of other charges. The total force on q_1 at P_1 due to all other charges is $\vec{F}_1=\vec{F}_{12}+\vec{F}_{13}+\vec{F}_{14}+\ldots\ldots$

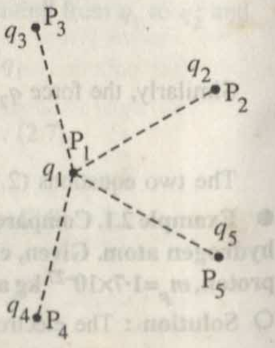


Fig. 2.6

$$=\frac{1}{4\pi\,\varepsilon_0}\left[\frac{q_1q_2}{r_{21}^2}\,\hat{r}_{21}+\frac{q_1q_3}{r_{31}^2}\,\hat{r}_{41}+\frac{q_1q_4}{r_{41}^2}\,\hat{r}_{41}+\ldots\ldots\right]$$

The same procedure can be adopted for finding the force on any other charge due to the remaining other charges. So, the total force on the charge q_2 located at P_2 due to all other charges is

$$\vec{F}_2=\vec{F}_{21}+\vec{F}_{23}+\vec{F}_{24}+\ldots\ldots.$$

$$=\frac{1}{4\pi\,\varepsilon_0}\left[\frac{q_1q_2}{r_{12}^2}\,\hat{r}_{12}+\frac{q_1q_3}{r_{32}^2}\,\hat{r}_{32}+\frac{q_1q_4}{r_{42}^2}\,\hat{r}_{42}+\ldots\ldots\right]$$

We can express the total force on a charge in short form. Let N charges $q_1, q_2, \ldots\ldots\ldots q_n$ ($n = 1, 2, 3, \ldots\ldots\ldots n$) are distributed in space. We calculate the total force on q_0 due to these n charges [Fig. 2.7]. The total force on the test charge q_0 is

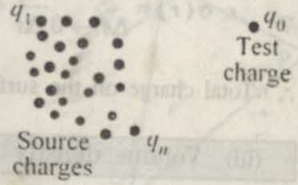


Fig. 2.7

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots\ldots = \sum_{1}^{N} \vec{F}_n$$

$$= \frac{1}{4\pi\,\varepsilon_0} \sum_{n=1}^{n} \frac{q_0 q_n}{r_n^2}\,\hat{r}_n \ldots\ldots\ldots\ldots (2.12)$$

2.6. Continuous charge distribution

It has already been stated that electric charge is quantised so that charge imparted to a body is always an integral multiple of elementary charge of an electron or a proton. In practical situations the charge given to a body is usually very large compared to the charge of an electron or a proton. So, we may ignore the concept of quantisation of charge and assume that on a charged body of reasonable size, charge distribution on the surface of the body is continuous. On a uniform body charge distribution is assumed to be uniform.

We consider three types of charge distribution on a charged body. These are

(i) Linear charge distribution (one dimensional charge distribution)
(ii) Surface charge distribution (two dimensional)
(iii) Volume charge distribution (three dimensional)

(i) Linear charge distribution :

When the charge is distributed along a line (curved or straight), the charge distribution is called linear.

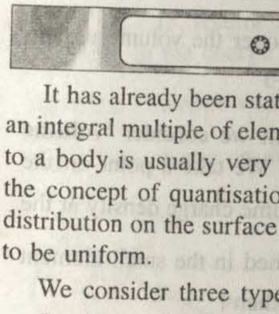


Fig. 2.8

The fig. 2.8 shows the uniform distribution of charge q over a conducting wire of length 'l'. If the charge is uniformly distributed then linear charge density is $\lambda = \frac{q}{l}\,\text{Cm}^{-1}$

If the distribution of charge is not uniform, we consider 'linear density at a point'. Δl is a small element of line l. Its position vector is $\vec{r}$ and carries a charge Δq. Then the linear charge density at the point under consideration is $\lambda(\vec{r}) = \underset{\Delta l \to 0}{Lt} \frac{\Delta q}{\Delta l} = \frac{dq}{dl}$ $\quad \therefore\ dq = \lambda(\vec{r})\,dl$

$\therefore$ Total charge on the line l is $q = \int_l \lambda(\vec{r})\,dl$

(ii) Surface density of charge :

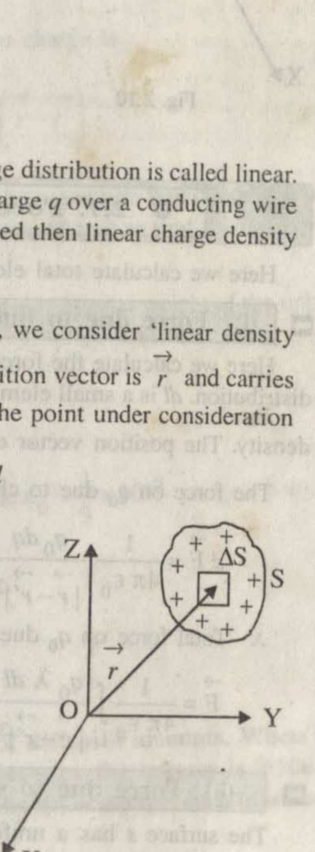


Fig. 2.9

Surface density of charge is charge per unit area and is denoted by σ. So its SI unit Cm^{-2}. If the charge q is uniformly distributed over the surface s, surface density of charge is $\sigma = \frac{q}{s}$.

If the charge is not uniformly distributed we consider 'surface density at a point' we take a point onthe surface having position vector $\vec{r}$. The surface density of charge at the point is $\sigma(\vec{r})$.

If Δq be the charge contained in the small element Δs around the point under consideration, then

$$\sigma(\vec{r}) = \underset{\Delta s \to 0}{Lt} \frac{\Delta q}{\Delta s} = \frac{dq}{ds} \qquad \therefore\ dq = \sigma(\vec{r})\,ds$$

$\therefore$ Total charge on the surface s, $q = \int_l \sigma(\vec{r})\,ds$

(iii) Volume density of charge :

We define volume charge density as the charge per unit volume. It is denoted by ρ and its SI unit Cm^{-3}.

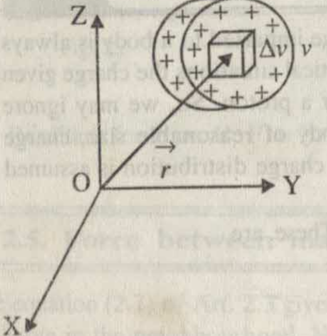


Fig. 2.10

Let a charge q is distributed uniformly over the volume v, then the volume density of charge is $\rho = \frac{q}{v}\text{cm}^{-3}$

If the charge distribution is not uniform we consider **'volume charge density at a point'** of the surface. We take a point on the volume having position vector $\vec{r}$. The volume charge density at the point is $\rho(\vec{r})$. If Δq be the charge contained in the small element Δs around the point under consideration, then

$$\rho(\vec{r}) = \underset{\Delta v \to 0}{Lt} \frac{\Delta q}{\Delta v} = \frac{dq}{dv} \qquad \therefore\ dq = \rho(\vec{r})\,dv$$

$\therefore$ Total charge on the surface s, $q = \int_v \rho(\vec{r})\,dv$

2.7. Force due to continuous charge distribution

Here we calculate total electrostatic force on a charge q_0 at a point r due to charge distribution

(i) Force due to linear charge distribution :

Here we calculate the force on q_0 due to a line charge L (straight or curved) having uniform charge distribution. dl is a small element of the line charge L. Charge on dl is $dq = \lambda\, dl$ where λ is linear charge density. The position vector of the point where q_0 is located is $\vec{r}$ and that of element dl is $\vec{r}'$.

The force on q_0 due to charge dq on the line element dl is

$$d\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_0\,dq}{|\vec{r}-\vec{r}'|^3}\left(\vec{r}-\vec{r}'\right)$$

$\therefore$ Total force on q_0 due to whole line charge distribution is

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\int_L \frac{q_0\,\lambda\,dl}{|\vec{r}-\vec{r}'|^3}\left(\vec{r}-\vec{r}'\right) \quad \ldots\ldots\ldots\ (2.13)$$

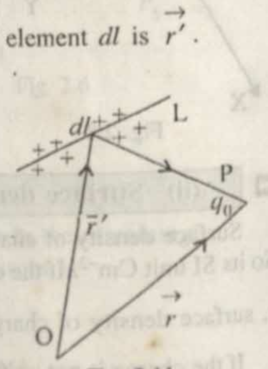


Fig. 2.11

(ii) Force due to surface distribution of charge :

The surface s has a uniform surface charge distribution σ. The charges on elementary area ds is $dq = \sigma.\, ds$. Proceeding as be force we write the force on q_0 due to the dq on ds as

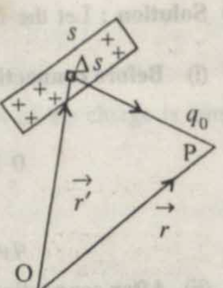


Fig. 2.12

$$d\vec{F} = \frac{1}{4\pi\,\varepsilon_0}\,\frac{q_0\,dq}{|\vec{r}-\vec{r}'|^3}\cdot\left(\vec{r}-\vec{r}'\right)$$

$$\therefore \text{ Total force } d\vec{F} = \frac{1}{4\pi\,\varepsilon_0}\,q_0\int\frac{\sigma\,ds}{|\vec{r}-\vec{r}'|^3}\left(\vec{r}-\vec{r}'\right) \;\ldots\ldots\; (2.14)$$

(iii) Force due to volume distribution of charge :

In fig. 2.13, volume v has uniform charge distribution. We require to calculate the force in the charge q_0 due to the charged volume v having volume charge density ρ. Position vector of the point where q_0 is located is $\vec{r}'$, and that of elementary volume element is $\vec{r}$. Now charge on dv, $dq = \rho.\ dv$.

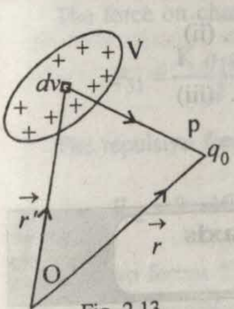


Fig. 2.13

So, the force on charge q_0 due to the charge dq on volume element is

$$d\vec{F} = \frac{1}{4\pi\,\varepsilon_0}\,\frac{q_0\,dq}{|\vec{r}-\vec{r}'|^3}\cdot\left(\vec{r}-\vec{r}'\right)$$

$\therefore$ Total force on q_0 due to whose volume charge is

$$d\vec{F} = \frac{1}{4\pi\,\varepsilon_0}\,q_0\int\frac{\rho\,dv}{|\vec{r}-\vec{r}'|^3}\left(\vec{r}-\vec{r}'\right)$$

● **Example 2.3. A pith ball of mass 9×11^{-5} kg carries of charge 5 μc. Another pith ball of equal mass is placed 2 cm above the first pith ball. What charge be given to the second ball so that the balls will remain in equilibrium ?**

○ **Solution :** The two balls will remain in equilibrium when electrostatic repulsion = weight of the upper ball.

$$\frac{1}{4\pi\,\varepsilon_0}\,\frac{q_1\,q_2}{r^2} = mg \qquad q_2 = \frac{mg\,r^2}{q_1}\left(4\pi\,\varepsilon_0\right)$$

Here $mg = 9\times10^{-5}\times9\cdot8$ N, $r = 2\times10^{-2}m$, $q_1 = 5\times10^{-6}$ C and $4\pi\,\varepsilon_0 = \frac{1}{9}\times10^{-9}$

$$q_2 = \frac{9\times10^{-5}\times9\cdot8\times\left(2\times10^{-2}\right)^2}{5\times10^{-6}\times9}\times10^{-9}$$

$$= \frac{9\cdot8\times4}{5}\times10^{-12} = 7\cdot84\times10^{-12}\ \text{C}$$

● **Ex. 2.4. Two small similar spheres are given opposite charges of different amounts. When the separation between the spheres is 0·5 m, the force of attraction between the spheres is 0·108 N. The spheres are connected momentarily by a conducting wire. Now the spheres repel each other with force of 0·036 N. Calculate initial charge of both the spheres.**

○ **Solution :** Let the initial charges of the spheres be q_1 and q_2.

(i) **Before connection :** By Coulomb's law, $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$

$$0\cdot108 = 9\times10^9 \times \frac{q_1q_2}{(0\cdot5)^2} \quad [F = 0\cdot108 \text{ N}, r = 0\cdot5 \text{ m}]$$

$$\therefore \; q_1q_2 = 3\cdot0\times10^{-12} \text{ (Coulomb)}^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (i)$$

(ii) **After connection :** When the spheres are connected by a wire, charge will flow from one sphere to another until their potentials are equal. Since the spheres are identical, each sphere will acquire equal amount of charge.

Now, charge on each sphere $= \frac{q_1 - q_2}{2}$ So, the spheres will repel each other.

$$0\cdot036 = 9\times10^9 \times \frac{(q_1-q_2)^2}{4\times(0\cdot5)^2} \;\therefore\; (q_1-q_2) = 2\times10^{-6} \text{ Coulomb} \ldots\ldots (ii)$$

Adding (i) and (ii), $q_1 + q_2 = 4 \times 10^{-6}$ Coulomb ……………………….(iii)

Hence from (ii) and (iii), $q_1 = 3\cdot0 \times 10^{-6}$ C (+ ve)

$q_2 = 3\cdot0 \times 10^{-6}$ C (– ve)

2.8. Force on a point charge placed on the axis of a uniformly charged ring

Let O be the centre and r the radius of the thin ring. It is charged uniformly with $+\lambda$ coulomb charge per unit length. [Fig. 2.14]. At P on the axis of the ring at a distance x from the centre a charge $-Q$ is placed. We have to calculate the axial force on the charge due to the charged ring.

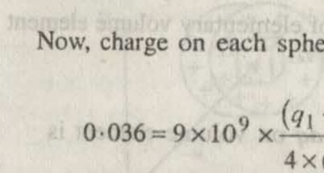


Fig. 2.14

We consider an elementary length dl at A of the ring. Charge on dl is $dq = \lambda dl$.

The force of attraction between the charge dq and $-Q$ is

$$F = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q\,dq}{a^2} \text{ along } \overrightarrow{PA}$$

Its two rectangular moments are : $F_x = F\cos\theta$, $F_y = F\sin\theta$

If we consider other elementary portions of the ring, due to symmetry all F_y components will cancel and all F_x components will be added. Hence the total force on the charge $-Q$ along the axis of the ring is

$$F = \sum F_x = \frac{1}{4\pi\varepsilon_0}\sum\frac{Q\,dq}{a^2}\cdot\cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{Q\lambda}{a^2}\cdot\frac{x}{a}\cdot\sum dl$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q\lambda x}{\left(r^2+x^2\right)^{3/2}}\cdot 2\pi r = \frac{1}{4\pi\varepsilon_0}\frac{2\pi Q\lambda x r}{\left(r^2+x^2\right)^{3/2}}$$

If $r >> x$, $r^2 + x^2 = r^2$ $\;\therefore\; F = \frac{1}{2\varepsilon_0}\frac{Q\lambda}{r^2}\cdot x$

∴ Acceleration of the charge, $f = \frac{1}{2\,\varepsilon_0}\frac{Q\lambda}{mr^2}\cdot x$ $\therefore\ f \propto x$

Here m is the mass

∴ Acceleration is proportional to distance from the centre. So, the motion of the charge is simple harmonic

∴ Time period, $T = 2\pi\sqrt{\frac{2\,\varepsilon_0 mr^2}{Q\lambda}} = 2\pi r\sqrt{\frac{2\,\varepsilon_0 m}{Q\lambda}}$

●● Numerical Examples ●●

● **Ex. 2.5. Three charges, 3μc, 4μc and 5μc are placed at the corners of an equilateral triangle of side 0·1 m each. What is the magnitude and direction of the resultant force on the charge of 5μc.**

○ **Solution :** The charges $q_1 = 3\mu c$, $q_2 = 4\mu c$ and $q_3 = 5\mu c$ are placed at the angular points A, B and C respectively of the equilateral triangle ABC [Fig. 2.15].

The force on charge q_3 due to charge q_1 along AC (repulsive)

$$F_{31} = \frac{K\,q_1 q_3}{r^2} = 9\times10^9\times\frac{3\times10^{-6}\times5\times10^{-6}}{(0\cdot1)^2} = \mathbf{13\cdot5\ N}$$

The repulsive force on charge q_3 due to charge q_2 along BC

$$F_{32} = 9\times10^9\cdot\frac{q_3 q_2}{r^2} = \frac{9\times10^9\times4\times10^{-6}\times5\times10^{-6}}{(0\cdot1)^2} = \mathbf{18\,N}$$

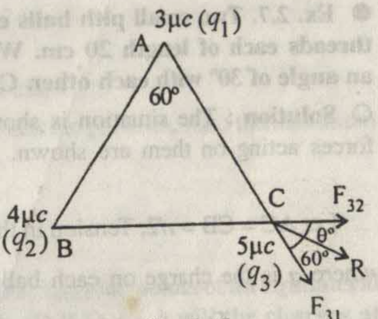


Fig. 2.15

The two forces F_{31} and F_{32} are inclined at 60° each other.

∴ Resultant force on the charge at C is

$$F = \sqrt{F_{31}^2 + F_{32}^2 + 2\,F_{31}\cdot F_{32}\cdot\cos 60^\circ}$$

$$= \sqrt{(13\cdot5)^2 + (18)^2 + 2\times13\cdot5\times18\cdot0\times\frac{1}{2}} = 27\cdot37\text{ N along }\overrightarrow{CR}$$

$$\tan\theta = \frac{F_{31}\sin 60^\circ}{F_{31} + F_{32}\cos 60^\circ} = \frac{13\cdot5\times\sqrt{\frac{3}{2}}}{13\cdot5 + 18\times\frac{1}{2}} = \mathbf{0\cdot5195} \qquad \therefore\ \theta = 27^\circ 33'$$

● **Ex. 2.6. Two negative point charges each of magnitude 8×10⁻⁸C and another positive charge Q are placed on the same straight line. At what position and for what value of Q, the system will remain in equilibrium ? What is the nature of the equilibrium ?**

○ **Solution :** Each of the two negative charges q_1 ($= -8\times10^{-8}$ C) are placed at the two points A and B on the line AB. [Fig. 2.16]. + Q charge is placed at C on AB. Let AQ = r_1 and BQ = r_2.

(i) **Equilibrium of Q :** For the equilibrium, the charge Q has to be placed at a point in between A and B. Evidently the force on Q due to charge at A be equal and opposite to the force on Q due to the charge at B. So, $|F_{CA}| = |F_{CB}|$

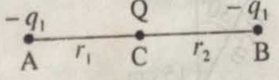


Fig. 2.16

$$\therefore\ \frac{1}{4\pi\,\varepsilon_0}\frac{Qq_1}{r_1^2} = \frac{1}{4\pi\,\varepsilon_0}\cdot\frac{Qq_1}{r_2^2} \qquad \therefore\ r_1 = r_2 = r\ \text{(say)}$$

So, the charge Q will be at equal distance from the charges at A and B.

(ii) **Equilibrium of the charge q_1 at A and B :** For the equilibrium of the system, each of the charge $-q_1$ situated at A and B must be in equilibrium. So, $|F_{AC}|=|F_{AB}|$ or, $|F_{BC}|=|F_{BA}|$

$$\therefore \quad \frac{Qq_1}{r^2}=\frac{q_1^2}{(2r)^2} \Rightarrow Q=\frac{q_1}{4}$$

Hence for the equilibrium of q_1 charges A and B is

$$Q_1=\frac{q_1}{4}=\frac{8\times10^{-8}}{4}=2\times10^{-5}\text{ Coulomb}$$

Nature equilibrium : When the charge Q is displaced towards A, F_{CA} will increase and F_{CB} will decrease. So Q will gradually move towards A. So the equilibrium of the system is lost. Hence the equilibrium is unstable equilibrium.

But if Q is displaced at right angle to AB, the resultant of F_{CA} and F_{CB} will act an Q and it returns to its initial position. So, in this case the equilibrium is stable.

● **Ex. 2.7. Two small pith balls each of mass 0·5 g are suspended from the same point by two silk threads each of length 20 cm. When they are given equal and similar charges the threads make an angle of 30° with each other. Calculate the charge on each pith ball.**

○ **Solution :** The situation is shown in fig. 2.17. At equilibrium, the balls are at A and B. Different forces acting on them are shown.

Let AC = CB = $r/2$, Tension in the string is T. The repulsive force between the balls is $F=\frac{1}{4\pi\epsilon_0}\frac{q^2}{r^2}$ where q is the charge on each ball.

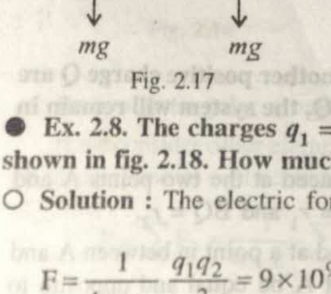


Fig. 2.17

$\therefore$ T cos 15° = mg and T sin 15° = F

$\therefore$ tan 15° = F/mg $\qquad \therefore$ F = mg tan 15°

$$\therefore \quad 9\times10^9\times\frac{q^2}{r^2}=5\times10^{-4}\times9\cdot8\times0\cdot2679$$

$$\therefore \quad q=3\cdot82\times10^{-7}\,r\text{ Coulomb}$$

Now, $\frac{r}{2}=\text{OB}\sin15^\circ \qquad \therefore r=2\times0\cdot2\times0\cdot2588=0\cdot1035$

$$\therefore \quad q=3\cdot82\times10^{-7}\times0\cdot1035 \;=\mathbf{3\cdot954\times10^{-8}\ C.}$$

● **Ex. 2.8. The charges $q_1 = -1{\cdot}0$ μc, $q_2 = +3{\cdot}0$ μc and $q_3 = -2{\cdot}0$ μc are placed at three points as shown in fig. 2.18. How much force will be acting on q_1 ?**

○ **Solution :** The electric force on q_1 due to q_2 is

$$F=\frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}=9\times10^9\times\frac{3\times10^{-12}}{(0\cdot15)^2}=1\cdot2\text{ N along }\overrightarrow{OX}\text{ [Fig. 2.19]}$$

Also, the force on q_1 due to q_3 is

$$F_2=9\times10^9\times\frac{2\times10^{-12}}{(0\cdot1)^2}=1\cdot8\text{ N along }\overrightarrow{OB}\text{ [Fig. 2.19]}$$

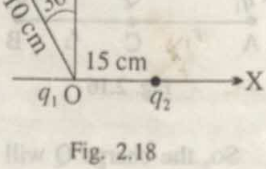


Fig. 2.18

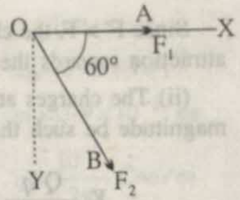


Fig. 2.19

$\therefore$ The resultant force on q_1, $F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos\theta}$

$\therefore$ $F = \sqrt{(1\cdot2)^2 + (1\cdot8)^2 + 2\times1\cdot2\times1\cdot8\times\cos 60^\circ}$ $[\theta = 60^\circ]$

$= \mathbf{2\cdot6\ N}$

If this force makes angle α with x-axis, then

$$\tan\alpha = \frac{F_2\ \sin\theta}{F_1 + F_2\ \cos\theta} = \frac{1\cdot8\times\sin 60^\circ}{1\cdot2+1\cdot8\ \cos 60^\circ} = \frac{1\cdot56}{2\cdot1} = 0\cdot74 \qquad \therefore\ \alpha = 36\cdot5^\circ$$

● **Ex. 2.9. Two particles are at a separation of 10 cm in air. How a charge of 20 stat C be divided between the two points so that the force between them will be maximum. Calculate this maximum force.** **[I. I. T.]**

○ **Solution :** Assume that the first particle is given a charge q and the second particle $(20 - q)$ stat C.

Then the repulsive force between the charges is $F = \dfrac{q(20-q)}{(10)^2}$ dyne

So, F will be maximum when $q\ (20 - q)$ be maximum.

Let $x = q\ (20 - q) = 20q - q^2 = 100 - (q - 10)^2$

Evidently x will be maximum when $q - 10 = 0$ $\qquad \therefore\ q = 10$ stat C.

The force will be maximum when the charge is equally divided between the particles. Also the maximum force is $F = \dfrac{10\times10}{(10)^2} = 1$ dyne

Note : The problem is solved in c.g.s. system.

● **Ex. 2.10. Three charges each of magnitude q are placed at the three angular points of an equilateral triangle. A fourth charge Q is placed at the centre of the triangle. (i) if Q = – q will the charges at the angular points will move towards the centre or away from the centre ? (ii) for what value of Q, the charges will remain at rest.** **[I. I. T.]**

○ **Solution :** Consider the fig. 2.20. The force on the charge q at A due to the charge q at B is

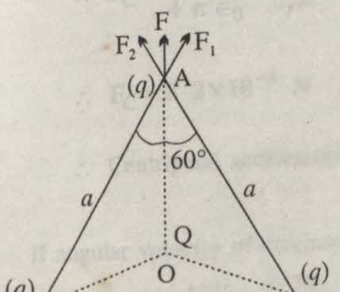


Fig. 2.20

$$F_1 = K\frac{q^2}{a^2} \text{ alone } \overrightarrow{BA} \quad \left[K = \frac{1}{4\pi\in_0},\ a = \text{ length of a side of triangle}\right]$$

Similarly, the force on the charge q at A due to the charge q at C is

$$F_1 = K\frac{q^2}{a^2} \text{ alone } \overrightarrow{CA}$$

The resultant force,

$$F^2 = F_1^2 + F_2^2 + 2F_1F_2\ \cos\theta = 2F_1\ (1+\cos 60^\circ) \quad \left[\ \therefore\ |\overrightarrow{F_1}| = |\overrightarrow{F_2}|\right]$$

$$F = \sqrt{3}\ K\frac{q^2}{a^2} \text{ alone } \overrightarrow{OA}$$

(i) If a charge Q = – q is placed at O, force of attraction of this charge ont he charge q at A is

$$F' = K\frac{q^2}{(OA)^2} = K\cdot\frac{q^2}{(a/\sqrt{3})^2} = \frac{3Kq^2}{a^2} \text{ along OA } [OA = OB = OC = a/\sqrt{3}]$$

Since $F' > F$, the charge q at A will move towards O. The charges at B and C will feel same force of attraction towards the centre at O.

(ii) The charges at the angular points will remain in equilibrium if the charge Q is negative and its magnitude be such that the resultant of the forces of attraction on Q be equal to F.

$$\therefore\ K\frac{Qq}{(OA)^2} = \sqrt{3}\ K\frac{q^2}{a^2} \quad \text{or,}\ \frac{3Qq}{a^2} = \sqrt{3}\ \frac{q^2}{a^2} \quad \therefore\ Q = \frac{q}{\sqrt{3}}.$$

$\therefore$ The required charge $Q = \dfrac{q}{\sqrt{3}}$

● **Ex. 2.11. Four charges Q, q, Q and q are placed at the four angular points of a square ABCD. If the total force on the charge at C be zero, obtain the relation between Q and q.**

○ **Solution :** The situation is shown in fig. 2.21.

The resultant force on the charge Q at C will be zero if $\overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3} = 0$ . . . . (i)

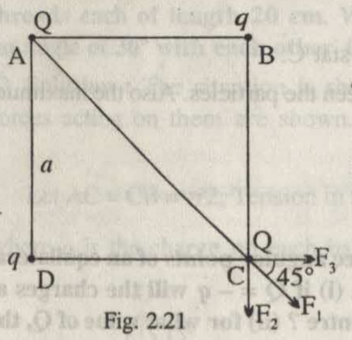


Fig. 2.21

Where F_1, F_2 and F_3 are the forces on the charge at C due to the charges at A, B and D respectively.

From the fig. it is evident that resultant force on Q at C will be zero if the charges at B and D are negative.

Now, $\overrightarrow{F_1} = F_1 \cos 45^\circ\ \hat{i} - F_1 \sin 45^\circ\ \hat{j}$;

$\overrightarrow{F_2} = -F_2\ \hat{j}$ and $= \overrightarrow{F_3} = F_3\ \hat{i}$

$\therefore\ F_1 \cos 45^\circ\ \hat{i} - F_1 \sin 45^\circ\ \hat{j} - \overrightarrow{F_2}\ \hat{j} + F_3\ \hat{i} = 0$

or, $(F_1 \cos 45^\circ + F_3)\ \hat{i} + (-F_1 \sin 45^\circ - F_2)\ \hat{j} = 0$

$\therefore\ F_1 \cos 45^\circ + F_3 = 0$ or, $F_1 \cos 45^\circ = -F_3$

and $F_1 \sin 45^\circ + F_2 = 0$ or, $F_1 \sin 45^\circ = -F_2$

Now, $F_1 = K\dfrac{Q^2}{2a^2}$ and $F_2 = K\dfrac{Qq}{a^2}$ $\left[K = \dfrac{1}{4\pi\varepsilon_0}\right]$

$$\therefore\ K\frac{Q^2}{2a^2} \times \frac{1}{\sqrt{2}} = \frac{KQq}{a^2} \qquad \therefore\ Q = -2\sqrt{2}\cdot q$$

It is the required relation.

● **Ex. 2.12. A thin ring of radius 1 m is charged uniformly with a charge of 1×10^{-5} Coulomb. A particle of mass 0·9 gm carrying a charge of -10^{-6} Coulomb is placed on the axis at a distance of 1 cm from its centre. Show that the negatively charged particle executes simple harmonic motion and calculate its time period.** **[I. I. T. 1982]**

○ **Solution :** For the first part vide article **2·8**

Second Part : Time period, $T = 2\pi a\sqrt{\dfrac{2\,\epsilon_0\,m}{Qq}}$

$$\therefore\ T = 2\pi\times 1\times\sqrt{\frac{1\times 8\cdot 85\times 10^{-12}\times 0\cdot 9\times 10^{-3}\times 2\pi}{10^{-6}\times 10^{-5}}}$$

$$= 2\times 3\cdot 14\sqrt{2\times\pi\times 18\times 8\cdot 85\times 10^{-5}} = \mathbf{0\cdot 63\ sec.}$$

$a = 1\ m$

$\epsilon_0 = 8\cdot 85\times 10^{-12}$

$m = 0\cdot 9\times 10^{-3}\ kg$

$q = \dfrac{10^{-5}}{2\pi}\ Cou/m$

$Q = 10^{-6}\ Cou$

● **Ex. 2.13. The force of repulsion between two point charges kept at a certain separation in air is 9 dyne. But when they are placed in a dielectric medium under the same separation, the force is 4 dyne. Calculate dielectric constant of the medium.**

○ **Solution :** In the first case, $F_1 = K\,\dfrac{q_1 q_2}{r^2}\quad\left[K = \dfrac{1}{4\,\pi\,\epsilon_0}\right]$

and in the second case, $F_2 = \dfrac{K}{\epsilon_r}\,\dfrac{q_1 q_2}{r^2}$

$$\therefore\ \frac{F_1}{F_2} = \epsilon_r \qquad \epsilon_r = \frac{9}{4} = 2\cdot 25$$

∴ Dielectric constant of the medium = **2·25.**

● **Ex. 2.14. In a hydrogen atom electron revolves round the proton in a circular orbit of radius 0·53A°. Calculate centripetal acceleration and angular velocity of the electron.** [Charge of an electron $= 1\cdot 6\times 10^{-19}$ C and its mass $= 9\cdot 1\times 10^{-31}$ kg].

○ **Solution :** The force of attraction between electron and proton provides the necessary centripetal force F_C.

$$\therefore\ F_C = \frac{1}{4\,\pi\,\epsilon_0}\,\frac{q_1 q_2}{r^2} = 9\times 10^9\times\frac{\left(1\cdot 6\times 10^{-19}\right)^2}{\left(0\cdot 53\times 10^{-10}\right)^2}$$

$$\therefore\ \mathbf{F_C = 8\cdot 2\times 10^{-8}\ N}$$

$$\therefore\ \text{Centripetal acceleration, } f_c = \frac{F_C}{m} = \frac{8\cdot 2\times 10^{-8}}{9\cdot 1\times 10^{-31}} = \mathbf{9\cdot 01\times 10^{22}\ ms^{-2}}$$

If angular velocity of electron be ω, then $f_c = \omega^2 r$

$$\therefore\ W = \sqrt{\frac{f_c}{r}} = \sqrt{\frac{9\cdot 01\times 10^{22}}{0\cdot 53\times 10^{-10}}} = \mathbf{4\cdot 12\times 10^{16}\ rad/s}$$

● **Ex. 2.15. A particle of mass *m* and carrying charge $-q_1$ is moving around a charge $+q_2$ along a circular orbit of radius *r*. Prove that the period of revolution of the charge $-q_1$ about q_2 is**

$$T = \sqrt{\frac{16\,\pi^3\,\epsilon_0\,mr^3}{q_1 q_2}}$$

○ **Solution :** Let the speed of the charge $-q_1$ around the fixed positive charge $+q_2$ be v. The necessary centripetal force is provided by the electrostatic force of attraction between them.

$$\therefore \frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{r^2} \qquad \therefore \upsilon = \left(\frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{mr}\right)^{1/2}$$

If T be the time period of revolution, $T = \frac{2\pi r}{v}$

$$\therefore T = 2\pi r \times \sqrt{\frac{4\pi\varepsilon_0 mr}{q_1 q_2}} = \sqrt{\frac{16\pi^3 \varepsilon_0 mr^3}{q_1 q_2}}$$

● **Ex. 2.16. It is required to hold four equal charges $+q$ in equilibrium at the corner of a square. Find the point charge that will do this, if placed at the centre of the square.**

○ **Solution :** The situation is shown in fig. 2.22. The charge Q is placed at the centre of the square to keep all charges at the corners of the square at equilibrium. Evidently, the charge Q must be negative.

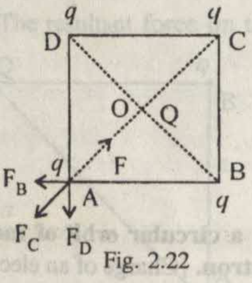


Fig. 2.22

If the net force on a charge at any angular point (say A) is zero, then by symmetry it follows that the net force experienced by charges at other angular points will also be zero.

Now the charge at A will experience forces F_B, F_C and F_D due to charges at B, C and D respectively. F is the force on charge at A due to change at the centre O. As the charge at A is in equilibrium.

$$F = F_C + F_B \cos 45^\circ + F_D \cos 45^\circ$$

$$\frac{1}{4\pi\varepsilon_0} \cdot \frac{Qq}{(CA)^2} = \frac{1}{4\pi\varepsilon_0}\left[\frac{q^2}{(CA)^2} + \frac{q^2}{\sqrt{2}\,a^2} + \frac{q^2}{\sqrt{2}\,a^2}\right]$$

$$= \frac{Q}{\left(a/\sqrt{2}\right)^2} = q\left[\frac{1}{\left(\sqrt{2}\,a\right)^2} + \frac{2}{\sqrt{2}\,a^2}\right]$$

$$\therefore Q = \frac{a^2}{2} q\left[\frac{1}{2a^2} + \frac{\sqrt{2}}{a^2}\right] = \frac{a^2 q}{2} \times \frac{1+2\sqrt{2}}{2a^2} = \frac{\left(1+2\sqrt{2}\right)q}{4}$$

$\therefore$ The required charge $= -\frac{\left(1+2\sqrt{2}\right)q}{4}$

● ● Short Answer Type Questions (with answers) ● ●

● **Question 1. Define dielectric constant of a medium from Coulamb's law of force between two point charges.**

○ **Ans.** The force between two point charges placed in air is $F_a = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$

If these charges be in the medium having dielectric constant K, then the force is·

$$F_m = \frac{1}{4\,\pi\,\epsilon_0}\cdot\frac{1}{K}\cdot\frac{q_1 q_2}{r^2} = \frac{1}{K}\;F_a$$

$$\therefore \text{ Dielectric constant } K = \frac{F_a}{F_m} = \frac{\text{force between two charges in air}}{\text{force between the charges in the medium}}$$

● **Q. 2. Name the unit of charges in c.g.s. and the SI system. Obtain relation between them.**

○ **Ans.** Unit of charges in c.g.s. system is Statcoulomb and in SI it is Coulomb.

If two similar charges placed at a separation of 1 cm in air repel each other with a force of 1 dyne, then each charge is 1 Statcoulomb.

Similarly, if two similar charges placed at a separation of 1 m in air repel each other with a force of 9×10^9 N then each charge is 1 C.

Relation between stat C and C :

$$\frac{F_1\ (\text{Newton})}{F_2\ (\text{dyne})} = \frac{(1\,C)^2}{1\ m^2}\times\frac{1\ cm^2}{(1\ \text{Stat}\ C)^2}$$

$$\text{or,}\quad \frac{9\times10^9}{10^{-5}} = \frac{(1C)^2\times10^{-4}\ m}{(1\,\text{Stat}\ C)^2\times 1 m^2}\qquad \therefore\ \mathbf{1\ C = 3\times10^9\ Stat\ C}$$

● **Q. 3. q_1 and q_2 are two point charges kept at a separation *d*. There is no such point in the field where electrostatic force is zero. What is your inference ?**

○ **Ans.** From this observation, we conclude that the charges are equal in magnitude, but of opposite sign ; *i.e.* $q_1 = -q_2$

● **Q. 4. A metal rod of length 10 cm is given a charge 8×10^{-8} C. Calculate linear charge density of the rod.**

○ **Ans.** The amount of charge per unit length of a charged conductor is called linear charge density (λ).

$$\therefore\ \lambda = \frac{\textbf{total charge}}{\textbf{length of the conductor}} = \frac{q}{L} = \frac{8\times10^{-8}}{0\cdot1} = 8\times10^{-7}\ \textbf{Cou.m}^{-1}$$

● **Q. 5. A charge + Q is placed on the line joining the charges + q_1 and + q_2. Will the charge + Q can remain in equilibrium. If it remains, what the nature of equilibrium ?**

○ **Ans. First Part :** If the force of repulsion of + q_1 on + Q be equal and opposite to the force of repulsion of + q_2 on + Q, then the charge + Q will remain in equilibrium.

2nd Part : This equilibrium is stable. Because, if the charge + Q is slightly displaced along the line joining the charges + q_1 and + q_2, the repulsive force on + Q will increase. So it will return to the equilibrium position. So, the equilibrium of + Q is stable.

● **Q. 6. A charge of – Q is placed on the line joining the charges +q_1 and +q_2, can the charge – Q remain in equilibrium ? If yes, what is the nature of equilibrium.**

○ **Ans.** If the position of – Q be such that the forces of repulsion of it due to the charges + q_1 and + q_2 are equal and opposite, then the charge – Q, will remain in equilibrium.

2nd part : The equilibrium is unstable. Because if the charge – Q_1 is slightly displaced along any direction along the line joining the charges, the force of attraction on it will increase and it will not return to the point of equilibrium. Hence the equilibrium is unstable.

● E ◇ X ◇ E ◇ R ◇ C ◇ I ◇ S ◇ E ●

◢ A. Short answer type questions :

1. Define 1 esu charge from Coulomb's law.
2. What is SI unit of charge ? Define this unit from Coulomb's law.
3. Obtain the relation between the units of charge in c.g.s. and in SI system.
4. What we mean by permittivity and dielectric constant of a medium ?
5. Mention the difference between gravitational force and electrostatic force.
6. What we mean by (i) linear charge density (ii) surface charge density and (iii) volume charge density ?
7. Electrostatic force between two point charges is a central force. Why ?
8. Define dielectric constant of a medium.
9. Dielectric constant of a medium is 80. What is its permittivity ?
10. State the principle of superposition of forces in electrostatics.
11. Write Coulomb's law in vector from.
12. The force of attraction between two point charges palced at a separation d in a medium is F. What distance apart should these be kept in the same medium so that force between them becomes F/3 ? **[Ans.** $\sqrt{3}\,d$**]**

◢ B. Essay type questions :

1. State and explain Coulomb's law. Define unit of charge both in c.g.s. in SI system. Obtain a relation between these two units. **[W. B. H. S. 1996]**
2. A uniform thin circular ring is uniformly charged. Radius of the ring is 'a'. A point charge $-q$ is placed on its axis at a distance r ($r >> a$) from the centre. Show that the motion of the charge is simple harmonic. Find its time-period.
3. Distinguish between linear, surface and volume charge density. Obtain expression for the force on a charge q due to continuous charge distribution over a volume.
4. State the principle of superposition of charge. Hence express the force on a charge q due to discrete distribution of n charges in terms of their position vectors.

◢ C. Simple numerical problems :

1. Two negative point charges each of magnitude 2 esu. and another charge q are placed on a straight line. If the system is in equilibrium find the position, magnitude and nature of the charge q.
[Ans. (i) q be placed at the midpoint, (ii) $q = \pm$ 0·5 esu**] [W. B. H. S. 2002]**
2. Two identical charges are placed at a separation of 3 cm in air. If a force of 4·5 kg-wt acts between the charges, find the magnitude of each charge. **[Ans.** $\pm 2{\cdot}1 \times 10^{-6}$ C**]**
3. A path ball of mass 0·05 g is given a charge of $\frac{10}{3}\times10^{-8}$C and is placed at a height of 10 cm from a charge + Q. For what value of Q the path ball will remain in equilibirum ? **[Ans.** $\frac{49}{3}\times10^{-9}$C **]**
4. Calculate the electrostatic force between a proton and an electron. Compare this force with the gravitational force between the particles. Given, mass of electron, $m = 9{\cdot}1\times10^{-31}$ kg and its charge $1{\cdot}6\times10^{-19}$ C. Distance between proton and electron = 0·53A°. **[Ans.** $8{\cdot}2\times10^{-8}$ N ; $9{\cdot}2\times10^{31}$**]**
5. Two point charges q_1 and q_2 are at a separation of 3 m in air. Sum of the magnitudes of the two charges is 20 μc. If the two charges repel each other with a force of 0·075 N, calculate each charge.**[Ans.** 15 μc, 5 μc**]**

6. Two identical small balls are given charges $q_1 = 10^{-3}$ C and $q_2 = -0{\cdot}33 \times 10^{-3}$ C respectively. They are put in contact momentarily and then kept at a separation of 20 cm. Calculate the force between the balls.
[**Ans.** $2{\cdot}5\times10^4$ N (repulsion)]

7. Two small and similar spheres are given charges + 12 μc and – 2μc. Calculate (i) electric force between the spheres when the separation is 0·1 m. (ii) the spheres are put in contact momentarily and then kept at a separation of 0·1 m. What is the force between the spheres ?
[**Ans.** (i) 21·6 N (attractive), (ii) 22·5 N repulsive]

8. The force of attraction between two charges in air in 40 N. When the charges are kept in a medium at equal separation, the force of attraction reduces of 5 N. Calculate dielectric constant of the medium. [**Ans.** 8]

9. Two unit negative charges and another charge $+q$ are placed on a straight line. Calculate the position and magnitude of q so that the system will remain in equilibrium. Is the equilibrium is stable or unstable ?
[**Ans.** q = 0·25 unit (at equal distance from each charge) it is unstable equilibrium] **[J.E.E. 1985]**

10. The force of repulsion between two small charged sphere is 2×10^{-3} N. The charge of one sphere is double the other. When the separation between the spheres is increased by 10 cm the force of repulsion becomes 5×10^{-4} N. Calculate (i) the charge on each sphere and (ii) initial distance of separation between the spheres.
[**Ans.** $66{\cdot}66\times10^{-9}$ C ; $33{\cdot}33\times10^{-9}$ C ; 0·1 m]

11. (a) Two small insulated metal spheres A and B are at a separation of 50 cm. If charge of each sphere be $6{\cdot}5\times10^{-7}$C, what is the force of repulsion between the spheres ?

(b) What will be the force of repulsion if (i) charge of each sphere is made double and distance is made half, (ii) the two spheres are kept immersed in water. (Dielectric constant of water K = 80)

[**Ans.** (a) $1{\cdot}521\times10^{-2}$N, (b) (i) 0·2434 N, (ii) $\dfrac{1{\cdot}521\times10^{-2}}{80} = 1{\cdot}9\times10^{-4}$ N]

12. The mass of a pith ball is 0·1 gm. When it is given a charge of –20 stat C, if floats in air due to the force of attraction due to a charged particle placed 2 cm above the pith ball. What is the charge on the particle ?
[**Ans.** 19·6 stat C]

13. Three charges 3, 4 and 5 stat C are placed at three angular points of an equilateral triangle of side 0·1 m. Calculate the total force on the largest charge.
[**Ans.** $6{\cdot}73\times10^{-6}$N]

14. Two identical balls A and B each of mass 0·2 gm and charged with equal amount of positive charges are suspended by two strings of length 50 cm from a point. Due to repulsion, the separation between the balls is 50 cm. Calculate the charge on each ball. $g = 9{\cdot}8$ m/s^2
[**Ans.** $1{\cdot}8\times10^{-7}$C]

15. Two similar balls each of mass m are suspended by two silk threads from a common point. Each is given a charge q. If the deflection of the string from the vertical line be θ, show that $x = \sqrt{\dfrac{q^2 l}{2\pi \epsilon_0 \, mg}}$ [x = separation between the balls after deflection]

16. Total charge of two small spheres both charged positively is $5{\cdot}0\times10^{-5}$C. When the separation between the two is 2·0m, the force of repulsion is F=1·0N, calculate the charge on each sphere. [**Ans.** $3´85\times10^{-5}$C ; $1{\cdot}15\times10^{-5}$C]

17. Two positive charges each of magnitude Q=√2 μc are placed on two opposite angular points of a square. On the other two angular points two identical charges each of magnitude q are placed. Calculate q so that the force on the charge Q is zero.
[**Ans.** – 0·5 μc]

18. Two identical small spheres being charged oppositely. When placed at a separation of 50 cm attract each other with a force of 0·108N. The spheres are connected momentarily by a wire. Now the force of repulsion between the spheres is $3{\cdot}60\times10^{-2}$N. Calculate the initial charge on each sphere. [**Ans.** ±1·0 μc, ∓3·0 μc]

19. Three charges are placed on the horizontal surface in air. One charge q_1=8 esu is at (0, 0), second charge q_2 =12·5 esu at (0, 5) and the third charge q_3 = – 24 esu is located at (8, 0). Calculate the total force on q_1.
[**Ans.** F = 5×10^{-5} N ; θ = $\tan^{-1}$ (4/3) with X-axis]

20. Three charges 1·0, 2·0 and – 3·0 esu are located at the angular points A, B and C of the equilateral triangle ABC of side 10 cm. Calculate total force on a unit charge placed D, the midpoint of AB.

[**Ans.** $\frac{\sqrt{2}}{25}$ dyne at 45° with DA]

21. An unchanged small sphere is put in contact with a charged sphere identical with the first sphere. Now, the spheres are placed at a separation of 4 cm. The force repulsion between the spheres is 9×10^{-5}N. Calculate the initial charge of the charged sphere. [**Ans.** 8×10^{-9} C]

22. Each of two particles each of mass m kg. is given a charge q units. The particles are suspended by two strings each of length l. Due to repulsion both the strings make angle of 45° with the horizontal. Prove that $q = l\sqrt{8\pi \in_0 mg}$.

23. Four particles having charges $+\frac{5}{3}, +\frac{10}{3}, +\frac{5}{3}$ and $-\frac{5}{3}$ nC are placed at the four corners of a square of side 10 cm. Another charge $Q = \frac{2}{3}$ nC is placed at the point of intersection O of the two diagonals. Calculate the force on Q. [**Ans.** 6×10^{-6}N force O to D]

24. The charge Q is divided into q and Q–q and are placed at a certain distance. What would be the relation between q and Q so that the force of repulsion is maximum ? [**Ans.** q = Q/2]

25. Two small spheres each of mass m and carrying a charge q are suspended by two strings each of length l from a point. At equilibrium both the strings make angle θ with the vertical. Show that

$$q^2 = 4\pi \in_0 \left(4mg\,l^2 \cdot \sin^2\theta \cdot \tan\theta\right)$$

26. Two small and similar metal spheres each of mass 3 gm are given equal amount of charge of same nature. When they are suspended by two strings each of length 13 cm, due to mutual repulsion they remain separated by a distance of 10 cm. Calculate charge on each sphere and also tension on the strings.

[**Ans.** 350 esu ; 3·25 g-wt] [**W.B.H.S. 1995**]

27. Four point charges 2μc, –5μc, 2μc and –5μc are located at the corners of a square ABCD of side 10 cm. What is the force on 1μc placed at the centre of the square ? [**Ans.** Zero]

◢ D. Harder numerical problems :

1. Four charges each of $\frac{4}{3} \times 10^{-9}$C are placed at the four corners of a square of side 10 cm. What charge be placed at the centre of the square so that the system will remain in equilibrium ? [**Ans.** $-1{\cdot}29 \times 10^{-6}$C]

2. Two point charges + 4q and +q are at a distance 'a' apart. Where should a third charge Q be placed between them so that the system is in equilibrium ? What will be the magnitude and sign of the charge Q ?

[**Ans.** At a distance $\frac{2}{3}$ a from the charge + 4q ; $Q = -\frac{4}{9}q$]

3. Three charges each $q = 2{\cdot}0 \times 10^{-6}$ C are located at the three angular points of a right angled triangle of sides 3 cm, 4 cm and 5 cm. Calculate the force on the charge located at the angular point adjacent to the right angle.

[**Ans.** 45·9N, $\theta = \tan^{-1} 16/9$ with side 4 cm]

4. Two spheres charged with identical charges are suspended from a common point by two strings of equal length. At equilibrium the strings make 30° with each other. Now the spheres are immersed in a liquid of density 0·8 g / c.c., but the angle between the two strings remains unchanged. Calculate dielectric constant of the liquid. Density of the spheres = 1·6 g/c.c. [**Ans.** K = 2]

5. Two particles each of mass 5g and carrying $1{\cdot}0 \times 10^{-7}$ C charge are kept on a horizontal table. At equilibrium they are at a separation of 10 cm. If coefficient of friction for both the particles be equal, calculate its value.

[**Ans.** μ = 0·18]

6. A charge of + 1×10^{-5} C is uniformly distributed on a thin wire ring of diameter 1 m. A particle of mass 0·9 gm and carrying – 1×10^{-6} C is placed on the axis at a distance of 1 cm from the centre. Prove that the negatively charged particle execute simple harmonic motion. Calculate its time period. [**Ans.** T = 0·63 sec] [**I. I. T.**]

7. A small sphere of mass m and having charge q is rotated in a vertical circle by a string of length l. An identical sphere carrying equal amount of similar charge is placed at the centre of the circle. What should be the minimum velocity of the first sphere at the lowest point of its path so that it will be able to describe the circle ?

[**Ans.** $v_1 = \sqrt{5\,gl - \frac{q^2}{lm}}$]

8. A shower of protons from outer space deposits equal charges + q on the earth and the moon and the electrostatic repulsion between them exactly counter balances the gravitational attraction. How large is q ?

[M = mass of earth, m = mass of the moon] [**Ans.** $q = \sqrt{(4\pi \in_0)\,GMm}$] [**J. E. E. 1998**]

9. Three small similar spheres each of mass 0·1 g are suspended from a point by three strings each of length 20 cm. What charge be given to each sphere so that each string makes angle of 30° with the vertical ? Assume that charges are equal. [**Ans.** $33{\cdot}0 \times 10^{-9}$ C] [**J. E. E. 1984**]

10. Four positive point charges each of magnitude Q are placed at the four corners of a square of side 'a'. A particle of mass m and carrying unit positive charge is placed at a height h from the centre of the square. For what value of Q, the particle will remain in equilibrium ?

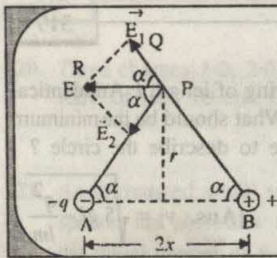
[**Ans.** $\frac{\pi \in_0 mg}{h}\left[h^2 + \frac{a^2}{2}\right]^{3/2}$]

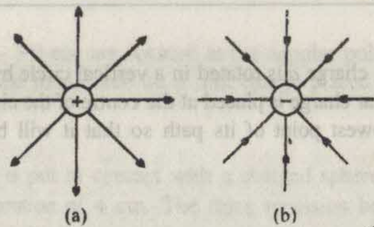
11. Three charges 10^{-4} C, -10^{-4} C and 10^{-4} C are located at the three angular points A, B and C of an equilateral triangle ABC. Calculate the magnitude and direction of the force on the charge at C.

[**Ans.** 5·625N parallel to AB]

12. Two positive point charges each of magnitude Q are placed at a separation of $2a$. Another charge of magnitude q is placed at the midpoint between the fixed charged. Show that (i) if q is positive and is slightly displaced along the line, joining the charges, it executes simple harmonic motion (ii) if q is negative and is slightly displaced perpendicular to the lime joining the charges, it also executes simple harmonic motion. Calculate the ratio of the frequency in the two cases.

[**Ans.** $\frac{n_1}{n_2} = \sqrt{2}$]





(a) (b)

CHAPTER 3
ELECTRIC FIELD INTENSITY

● **TOPICS :** ○ Electric field; ○ Intensity of electric field $\vec{E}$; ○ Calculation of electric field in different cases; ○ Electric field due to distribution of charges; ○ Electric field due to continuous charge distribution; ○ Intensity of electric field due to an electric dipole; ○ Field intensity due to a uniform charge distribution; ○ Effect of electric field on a point charge; ○ Torque on an electric dipole in an electric field; ○ Work done to deflecta dipole in an electric field; ○ Electric field near a charged conductor—Coulomb's theorem; ○ Electric lines of force; ○ Maps of electric field in different cases; ○ To calculate of electric intensity in terms of lines of force; ○ Coulomb's theorem from lines of force concept; ○ Short answers type questions (with anwers); ○ Exercise.

○ 3.1. Electric field ○

We know that when a body is taken near another body, they exert gravitational force on each other. This force is effective even when they are not in contact. Also if a magnet or a magnetic substance is brought into the force field due to a magnet, a magnetic force acts on them. So, these are the examples of force which **'acts at a distance'**. Similarly, the force of attraction or repulsion between two charged particles is effective even when they are not in contact. Thus when two charges are placed a distance apart they exert force on each other. So electric force is also **a force which acts at a distance.**

In order to explain, the force between charged particles we introduce the concept of **electric field**, similar to gravitational field and magnetic field which are introduced to explain the force between masses and between magnets respectively.

So, it is assumed that a charge called **source charge** creates an electric field in the space around it. To feel the existence of the electric field, another charge called **test charge** is to be brought into the field. Hence an electric field is said to exist in a region if a test charge experiences a force when brought into the region.

Thus, **electric field due to a charge is the space around the charge in which any other charge is acted upon by an electrostatic force.**

The electric field of a charge is the space property by virtue of which the charge modifies the space around itself. As a result any other charges brought in the space around the charge will experience an electric force. It is to be noted that the source charge does not experience any force due to the electric field produced by it.

○ 3.2. Intensity of electric field, $\vec{E}$ ○

An electric field exist around a charged body. Theoretically, this field is extended upto infinity. But actually its effect is observed upto a certain distance.

Field intensity :

The electric field intensity at a point in the electric field (due to source charge) is the force on a unit positive charge (test charge) placed at the point. The electric field intensity is also called strength of the electric field. The direction of the electric field is same as the direction of the force experienced by the positive charge in the field.

Let a test charge q_0 placed at a point in the field feels a force F then the electric field at the point is

$$\vec{E} = \frac{\vec{F}}{q_0} \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (3.1)$$

$$\therefore \ \vec{F} = \vec{E}\, q_0 \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (3.1a)$$

Evidently $\vec{E}$ vector is directed along $\vec{F}$ vector. It is being assumed that the test charge q_0 is very small so that it does not disturb the existing electric field. Hence equation (3.1) can be written as

$$\vec{E} = \underset{q \to 0}{Lt} \frac{\vec{F}}{q_0} \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (3.2)$$

The idea of considering q_0 be to vanishingly small is that on placing the test charge at the point of observation, the source charge is not disturbed.

The unit of electric field in SI is NC^{-1} and in c.g.s. system it is **dyne per esu.**

- **Relation :** 1 dyne/esu $= \dfrac{10^{-5}\text{ N}}{\frac{1}{3}\times 10^{-9}} = \mathbf{3\times10^{4}}\ \text{NC}^{-1}$

Positive charge moves in the direction of the applied field.

Negative charge moves in the direction opposite to the applied field.

Its dimension, $[E] = \dfrac{[F]}{[q]} = \dfrac{MLT^{-2}}{[AT]} = [MLT^{-3}A^{-1}]$

Importance of the concept of electric field.

The concept of electric field helps us to understand the mechanism by which two charges placed at a distance exert force on each other.

Through electric field a charge exerts a force on another charge situated at a distance. Also, as an alternative to Coulomb's law in electrostatics, the concept of electric field gives us a method to find the force experienced by a charge due to another in terms of electric field produced by it.

- **Example 3.1. An oil drop carries a charge of 12*e*. It is the equilibrium in an electric field of intensity 2·55×10⁴ V / m. If the density of oil be 1·26 g/c.c. Calculate the radius of the oil drop. Given *e* = 1·6×10⁻¹⁹ C and *g* = 9·8 m/s².**

○ **Solution :** The oil drop is at rest in the electric field. So its weight is balanced by the electric force on it.

$$\therefore \ mg = Eq$$

$$\text{or, } \frac{4}{3}\pi r^3 \rho g = Eq \qquad \text{or, } r^3 = \frac{3Eq}{4\pi\rho g}$$

$$\therefore \ r^3 = \frac{3\times 2\cdot55\times10^4 \times 19\cdot2\times10^{-19}}{4\pi\times1\cdot26\times10^3\times9\cdot8}$$

$$\therefore \ r = 9\cdot80\times10^{-7}\text{ m} = 9\cdot8\times10^{-4}\text{ mm}$$

$$q = 12\times1\cdot6\times10^{-19}$$

$$= 19\cdot2\times10^{-19}\text{C}$$

$$E = 2\cdot55\times10^4\ V/m$$

$$\rho = 1\cdot26\times10^3\ kg/m^3$$

$$g = 9\cdot8\ m/s^2$$

- **Ex. 3.2. A particle of mass 10^{-3} kg and a charge 5 μC is thrown at a speed 20 ms^{-1} against a uniform electric field of strength 2×10^5 NC^{-1}. Calculate the distance at which it comes to rest momentarily.**

○ **Solution :** Here $m = 10^{-3}$ kg, $q = 5 \times 10^{-6}$C, $u = 20$ ms^{-1} and $E = 2 \times 10^5$ N/C

From the principle of conservation of energy.

Work done against the electric force = initial kinetic energy

$\therefore$ $Eq.\, x = 1/2\, mu^2$ [x = required distance]

$$\therefore x = \frac{mu^2}{2\,Eq} = \frac{10^{-3} \times (20)^2}{2 \times 2 \times 10^5 \times 5 \times 10^{-6}} = 0 \cdot 2 \text{ m}$$

$\therefore$ The particle will stop after travelling **0·2 m** in the field.

❂ 3.3. Calculation of electric field in different cases ❂

➤ *3.3.1. Electric field due to point charge :*

The point P is situated at a distance r from a point charge + Q [Fig. 3.1]. We require to calculate the field at P.

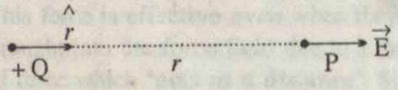


Fig. 3.1

Test charge q_0 is placed at P. The force on q_0 due to the charge Q according to Coulomb's law is

$$F = \frac{1}{4\pi\,\varepsilon_0}\,\frac{Qq_0}{r^2}$$

So, the electric field at P is $E = \dfrac{F}{q_0} = \dfrac{1}{4\pi\,\varepsilon_0}\,\dfrac{Q}{r^2}$

$$\therefore\ \vec{E} = \frac{1}{4\pi\,\varepsilon_0}\,\frac{Q}{r^2} \cdot \hat{r} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3.4)$$

- **Alternative Method :**

➤ *3.3.2. Electric field due to a point charge :*

The charge + Q is located at A whose position vector is $\vec{r_1}$ [Fig. 3.2]. We require to calculate electric field at B with position vector $\vec{r_0}$. The test charge q_0 placed at B will experience a force of

$$\vec{F} = \frac{1}{4\pi\,\varepsilon_0}\,\frac{Qq_0}{r^2}\,\hat{r}$$

$\hat{r}$ is the unit vector along $\vec{AB}$. Now, $r = |\vec{r}| = |\vec{r_0} - \vec{r_1}|$

So, the field intencity at B is $\vec{E} = \underset{q_0 \to 0}{Lt}\ \dfrac{F}{q_0}$

$$\therefore\ E = \underset{q_0 \to 0}{Lt}\left(\frac{1}{q_0}\,\frac{1}{4\pi\,\varepsilon_0}\,\frac{Qq_0}{r^2}\,\hat{r}\right)$$

$$\therefore\ \vec{E} = \frac{1}{4\pi\,\varepsilon_0}\,\frac{Q}{|\vec{r_0} - \vec{r_1}|^3}\left(\vec{r_0} - \vec{r_1}\right) \ldots\ldots\ldots\ldots\ldots\ldots (3.5)$$

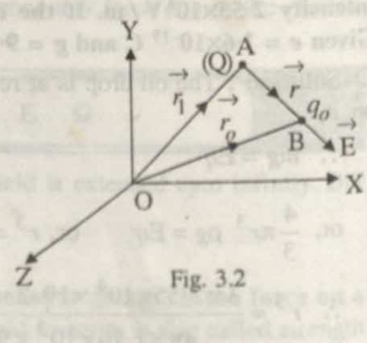


Fig. 3.2

Evidently equation (3.5) gives **electric field in terms of the position vectors of the source charge and the observation point.**

3.4. Electric field due to distribution of charges

Electric field intensity at a point due to discrete distribution of charges can be found by applying the **principle of superposition** of electric fields.

According to the superposition principle of electric field, **the electric field at any point due to a group of charges is equal to the vector sum of the electric fields produced by each charge individually, when all other charges are assumed to be absent.**

In fig. 3.3, we show n discrete point charges $q_1, q_2, q_3, \ldots\ldots, q_n$ distributed in space. We require to calculate electric field at P due to all these point charges.

For this we put a vanishingly small positive test charge q_0 at P. Let the forces exerted on q_0 at P due to the charges $q_1, q_2, q_3, \ldots\ldots, q_n$ be $\vec{F}_1, \vec{F}_2, \vec{F}_3 \ldots\ldots, \vec{F}_n$ respectively.

If $\vec{F}_i$ is the force due to the i th charge q_i on the test charge q_0

then $\vec{F}_i = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_i\, q_0}{r_i^2}\hat{r}_i$

r_1 is the distance of the test charge q_0 from the i-th charge q_i

and $\hat{r}_i$ is unit vector from charge q_i to q_0.

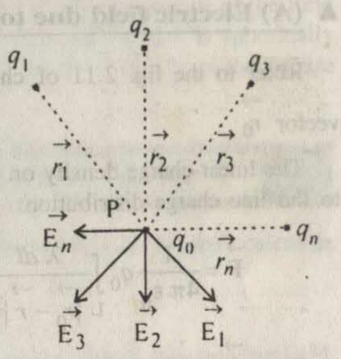


Fig. 3.3

So, the electric field intensity at P due to q_i is

$$\vec{E}_i = \underset{q_0 \to 0}{Lt}\frac{\vec{F}_i}{q_0} = \underset{q_0 \to 0}{Lt}\left(\frac{1}{q_0}\cdot\frac{1}{4\pi\varepsilon_0}\frac{q_i\, q_0}{r_i^2}\hat{r}_i\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_i}{r_i^2}\hat{r}_i \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.6)$$

The field is directed along the direction of the force on charge q_0 by q_i.

Similarly, the electric fields $\vec{E}_1, \vec{E}_2 \ldots\ldots, \vec{E}_n$ at P due to the charges $q_1, q_2, \ldots\ldots, q_n$ acts along the direction of the forces on the test charge due to them [Fig. 3.3]. Then the total electric field at P due to all the charges is

$$\vec{E}_1 = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \ldots\ldots + \vec{E}_n = \sum_{i-1}^{n}\vec{E}_i = \sum_{i=1}^{n}\frac{1}{4\pi\varepsilon_0}\frac{q_i}{r_i^2}\hat{r}_i \text{ [by equation (3.6)]}$$

$$\therefore\ \vec{E} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i^2}\hat{r}_i \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.7)$$

Following equation (3.5), electric field at P in terms of the position vectors of source charges and the observation point is

$$\vec{E} = \frac{1}{4\pi\,\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{|\vec{r_0}-\vec{r_i}|^3}\left(\vec{r_0}-\vec{r_i}\right) \quad \cdots\cdots\cdots\cdots (3.8)$$

✪ 3.5. Electric field due to continuous charge distribution ✪

Here we calculate electric field at a point due to a continuous distribution of charge in the following cases :

▲ (A) Electric field due to a line charge distribution :

Refer to the fig. 2.11 of chapter 2. Let $\vec{E}$ be the electric field at the point P having position vector $\vec{r_0}$.

The linear charge density on the line is λ. Now, the force on small test charge q_0 at the point P due to the line charge distribution

$$\vec{F} = \frac{1}{4\pi\,\varepsilon_0}q_0\int_L \frac{\lambda\, dl}{|\vec{r_0}-\vec{r}\,|^3}\left(\vec{r_0}-\vec{r}\right)$$

Here $\vec{r}$ is the position vector of the element dl of the line having charge λdl

So, the electric field at the location $\vec{r_0}$ due to line charge distribution is

$$\vec{E} = \underset{q_0\to 0}{Lt}\frac{\vec{F}}{q_0} = \frac{1}{4\pi\,\varepsilon_0}\int_L \frac{\lambda\, dl}{|\vec{r_0}-\vec{r}\,|^3}\left(\vec{r_0}-\vec{r}\right) \quad \cdots\cdots\cdots\cdots (3.9)$$

▲ (B) Electric field due to surface distribution of charge :

Let $\vec{E}$ be the electric field at the point P having position vector $\vec{r_0}$ due to surface distribution of charge over the surface S having surface density of charge σ. [vide fig. 2.12 of ch. 2] Now, the force on the vanishingly small test charge q_0 having position vector $\vec{r_0}$ due to surface charge distribution is

$$\vec{F} = \frac{1}{4\pi\,\varepsilon_0}q_0\int_S \frac{\sigma\, ds}{|\vec{r_0}-\vec{r}\,|^3}\left(\vec{r_0}-\vec{r}\right)$$

∴ So, the electric field E at P is $\vec{E} = \underset{q_0\to 0}{Lt}\frac{\vec{F}}{q_0}$

$$\therefore\ E = \frac{1}{4\pi\,\varepsilon_0}\int \frac{\sigma\, ds}{|\vec{r_0}-\vec{r}\,|^3}\left(\vec{r_0}-\vec{r}\right) \quad \cdots\cdots\cdots\cdots (3.10)$$

▲ (C) Electric field due to a volume charge distribution :

Consider the fig. 2.13 of chapter 2. $\vec{r_0}$ is the position vector of P where we want to calculate the field due to volume charge distribution having volume density ρ. The charge of elementray volume dv is ρdv Its position vector is $\vec{r}$. The electric field $\vec{E}$ is

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \int_v \frac{\rho\, dv}{|\vec{r_0} - \vec{r}|^3} \left(\vec{r_0} - \vec{r}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.11)$$

- **A discussion :** If the charge Q is positive, electric field is directed radially outward from the charge. Also electric field is spherically symmetric. It means that for a given value of r the magnitude of the field intensity is equal in all directions. Hence a point charge creates an electric field which is spherically symmetric around it and intensity is directed radially outward. Intensity varies inversely as the square of the distance.

✪ 3.6. Intensity of electric field due to an electric dipole ✪

Two equal and opposite charges kept at a small separation form an electric dipole. We shall calculate field intensity due to an electric dipole in three different cases :

▲ (A) Field intensity at a point on the axis of the dipole :

AB is an electric dipole. P is a point on the axis of the dipole where we require to calculate the field. AB = $2x$ and the point P is at a distance r from the middle of the dipole. The electric field at P due to the charge $+q$ at B is

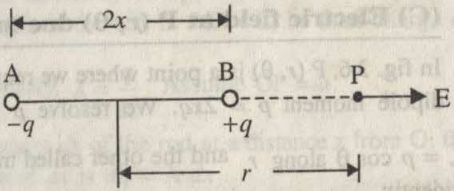


Fig. 3.4

$$E_1 = \frac{1}{4\pi\varepsilon_0} \frac{q}{(BP)^2} \text{ along } \vec{BP}$$

$$\therefore \ E_1 = \frac{1}{4\pi\varepsilon_0} \frac{q}{(r-x)^2} \text{ along } \vec{BP}$$

Also, electric field at P due to $-q$ at A, $E_2 = \dfrac{1}{4\pi\varepsilon_0} \dfrac{q}{(r+x)^2}$ along $\vec{PA}$

$$\therefore \text{ Resultant field at P, } E = E_1 - E_2 = \frac{1}{4\pi\varepsilon_0} q \left[\frac{1}{(r-x)^2} - \frac{1}{(r+x)^2}\right] \text{ along } \vec{BP}$$

$$\therefore \ E = \frac{q}{4\pi\varepsilon_0} \frac{4rx}{\left(r^2 - x^2\right)^2} = \frac{1}{4\pi\varepsilon_0} \frac{2pr}{\left(r^2-x^2\right)^2} \quad [p = 2xq = \text{electric dipole moment}]$$

If $r >> x$, $r^2 - x^2 = r^2$

$$\therefore \ E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2p}{r^3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.12)$$

▲ (B) Electric field at a point on the perpendicular bisector of the electric dipole :

We require to calculate electric field intensity at P on the perpendicular bisector of the electric dipole AB [Fig. 3.5]. The electric field at P due to the charge + q at B is

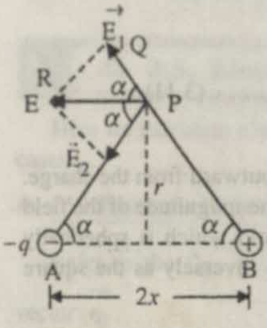


Fig. 3.5

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{BP^2} \text{ along } \overrightarrow{PQ}$$

and electric field at P due to $-q$ charge at A is

$$E_2 = \frac{1}{4\pi\varepsilon_0 .}\frac{q}{(AP)^2} \text{ along } \overrightarrow{PA}$$

The resultant field at P is

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = E^2 = E_1^2 + E_2 + 2E_1\ E_2\ \cos 2\alpha$$

$$\therefore\ E^2 = 2E_1^2 + 2E_1^2 \cos 2\alpha \left[\because |\vec{E}_1| = |\vec{E}_2|\right]$$

$$\therefore\ E = 2E_1 \cos\alpha = \frac{2}{4\pi\varepsilon_0}\frac{q}{r^2+x^2}\cos\alpha = \frac{2}{4\pi\varepsilon_0}\frac{qx}{r^2+x^2}\left[\because \cos\alpha = \frac{x}{\sqrt{r^2+x^2}}\right]$$

$$\therefore\ E = \frac{1}{4\pi\varepsilon_0}\frac{p}{\left(r^2+x^2\right)^{3/2}} = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}\ [\text{where } r >> x] \quad \ldots\ldots\ldots\ldots\ldots\ (3.13)$$

▲ (C) Electric field at P (r, θ) due to the dipole :

In fig. 3.6, P (r, θ) is a point where we require to calculate electric field due to the electric dipole AB of dipole moment $p = 2xq$. We resolve p into two components : one called radial component is $p_r = p\cos\theta$ along $\vec{r}$ and the other called transverse component is $p_\theta = p\sin\theta$ perpendicular to $\vec{r}$. Evidently.

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}_r}{r^3} = \frac{2\vec{p}\cos\theta}{\left(4\pi\varepsilon_0\right)r^3}$$

$$\text{and } \vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}_\theta}{r^3} = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\cos\theta}{r^3}$$

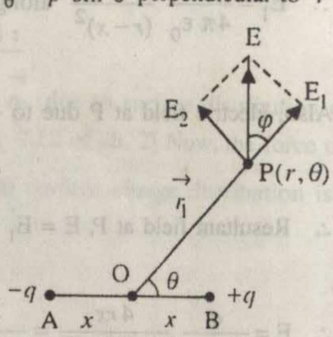


Fig. 3.6

The magnitude of the resultant intensity is

$$E = \sqrt{E_1^2 + E_2^2} = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}\sqrt{1+3\cos^2\theta} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (3.14)$$

Its angle of inclination with radius vector OP is $\phi = \frac{E_2}{E_1} = \frac{1}{2}\tan\theta$

3.7. Field intensity due to a uniform charge distribution

Here we consider three cases :

(A) Field intensity at a point on the axis of the uniformly charged rod.

The rod AB of length L is given a charge q [Fig. 3.7].

So its linear charge density, $\lambda = q/\mathrm{L}$. To calculate the field intensity at P on the axis.

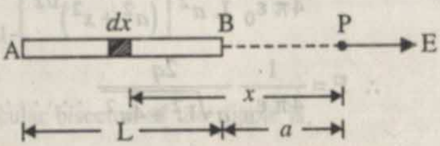


Fig. 3.7

Now, we consider a small element dx of the rod at a distance x from P. The charge on dx, $dq = \lambda . dx$. So, the electric field at P due dx is

$$d\vec{\mathrm{E}}_1 = \frac{1}{4\pi\,\varepsilon_0}\,\frac{dq}{x^2}\ \hat{x} \text{ along } \overrightarrow{\mathrm{BP}}$$

Now, the electric field at P due all other element acts in the same direction. So, the total field at P is

$$\mathrm{E} = \int_a^{a+\mathrm{L}} \frac{\lambda}{4\pi\,\varepsilon_0}\,\frac{dx}{x^2} = \frac{\lambda}{4\pi\,\varepsilon_0}\left[\frac{1}{a} - \frac{1}{a+\mathrm{L}}\right] = \frac{1}{4\pi\,\varepsilon_0}\,\frac{q}{a(a+\mathrm{L})}$$

If $a >> \mathrm{L}$, $\mathrm{E} = \dfrac{1}{4\pi\,\varepsilon_0}\,\dfrac{q}{a^2}$ . . . . (3.15)

So, under this condition, the charged rod behaves as a point charge.

(B) Find intensity at a point on the perpendicular bisector of a uniformly charged rod.

P is a point on the perpendicular bisector (ON) of the uniformly charged rod AB. We require to calculate field at P due to the charged rod. [Fig. 3.8]. Length of the rod is AB = L, its total charge q.

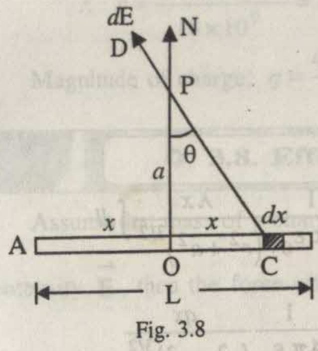


Fig. 3.8

So, the linear charge density, $\lambda = \dfrac{q}{\mathrm{L}}$. Assume OP = a.

Consider a small element dx of the rod at a distance x from O, the midpoint of AB, charge of dx is $dq = \lambda\ dx$

So, the field at P due to dq is

$$d\,\mathrm{E} = \frac{1}{4\pi\,\varepsilon_0}\,\frac{dq}{(\mathrm{CP})^2} \text{ along } \overrightarrow{\mathrm{PD}}$$

Due to symmetry the field at P will be directed along OP if the charge q is positive.

Now, the component of dE along OP is $d\mathrm{E}_y = d\mathrm{E} . \cos\theta$

$$\therefore\ d\,\mathrm{E}_y = \frac{1}{4\pi\,\varepsilon_0}\,\frac{dq}{(\mathrm{CP})^2}\cdot\cos\theta$$

$$= \frac{1}{4\pi\,\varepsilon_0}\,\frac{\lambda\,dx}{a^2+x^2}\cdot\frac{a}{\sqrt{a^2+x^2}} = \frac{1}{4\pi\,\varepsilon_0}\,\frac{aq\,dx}{\mathrm{L}\left(a^2+x^2\right)^{3/2}}$$

$$\therefore\ \text{Total electric field intensity, } \mathrm{E} = \int d\,\mathrm{E}_y = \frac{aq}{(4\pi\,\varepsilon_0)\,\mathrm{L}}\int_{-\mathrm{L}/2}^{\mathrm{L}/2}\frac{dx}{\left(a^2+x^2\right)^{3/2}}$$

We put $x = a \tan\theta$, $dx = a \sec^2\theta \, d\theta$

$$\therefore \int \frac{dx}{\left(a^2+x^2\right)^{3/2}} = \int \frac{a \cdot \sec^2\theta \cdot d\theta}{a^2 \sec^3\theta} = \frac{1}{a^2}\int \cos\theta \, d\theta = \frac{\sin\theta}{a^2} = \frac{1}{a^2}\frac{x}{\left(a^2+x^2\right)^{1/2}}$$

$$\therefore E = \frac{1}{4\pi\varepsilon_0}\frac{aq}{L a^2}\left[\frac{x}{\left(a^2+x^2\right)^{1/2}}\right]_{-L/2}^{L/2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{aq}{L a^2} \cdot \frac{2L}{\sqrt{L^2+4a^2}}$$

$$\therefore E = \frac{1}{4\pi\varepsilon_0}\frac{2q}{a\sqrt{L^2+4a^2}} \quad \ldots\ldots (3.16)$$

▲ (C) Field intensity at a point on the axis of a uniformly charged ring.

A thin wire ring of radius r with centre at O is given a charge q. So, its uniform linear charge density is $\lambda = \frac{q}{2\pi r}$. We require to calculate the electric field at P at a distance x from the centre of the ring on the axis.

Consider a small element dl of the ring. It carries a charge $dq = \lambda \, dl$.

The field at P due to dq of dl is $dE = \frac{1}{4\pi\varepsilon_0} \cdot \frac{dq}{a^2}$ along PA [Fig. 3.9]

Due to symmetry the resultant field at P acts along the axis of the ring. Now, the component of dE along the axis is $dE_x = dE \cos\theta$

$$\therefore dE_x = \frac{1}{4\pi\varepsilon_0}\frac{\lambda \, dl}{a^2} \cdot \cos\theta$$

$\therefore$ Total field intensity at P due to the whole ring is

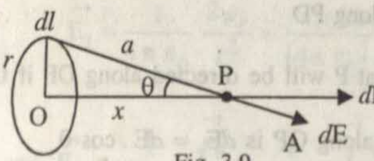


Fig. 3.9

$$E = \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{a^2}\int dl \cdot \frac{x}{a} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda x}{\left(r^2+a^2\right)^{3/2}}\int dl$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\lambda x (2\pi r)}{\left(r^2+x^2\right)^{3/2}} = \frac{1}{4\pi\varepsilon_0}\frac{qx}{\left(r^2+x^2\right)^{3/2}}$$

$$\therefore E = \frac{1}{4\pi\varepsilon_0}\frac{qx}{\left(r^2+x^2\right)^{3/2}} \quad \ldots\ldots (3.17)$$

- **Ex. 3.3. Two charges 1μc and –1μc placed at a separation of 2·0 cm form an electric dipole. Calculate the electric field in the following cases.**

(i) at a point on the axis distant 50 cm from the centre of the dipole.

(ii) at a point on the perpendicular bisector of the dipole 50 cm from the centre of the dipole.

○ **Solution :** (i) Electric dipole moment of the dipole, $p = 2xq$

$$\therefore\ P = 2\cdot 0\times 10^{-2}\times 10^{-6} = 2\times 10^{-8}\ \text{Cou.m}$$

The field at a point on the axis, $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{2p}{r^3}$

$$\therefore\ E = 9\times 10^9\times\frac{2\times 2\times 10^8}{(0\cdot 5)^3}\ [r = 0\cdot 5\ \text{m}]$$

$= \mathbf{2880\ NC^{-1}}$ along the axis.

(ii) The electric field intensity at a point on the perpendicular bisector of the dipole is

$$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3} = \frac{1}{2}E_1 = 1440\ \text{NC}^{-1}\quad \text{(Parallel to the dipole)}$$

● **Ex. 3.4. An electric intensity at a point 20 cm from the centre on the axis of an electric dipole is 10^5 NC^{-1}. Calculate (i) magnitude of each charge (ii) dipole moment if length of the dipole be 10 mm.**

$E = 10^5\ \text{NC}^{-1}$
$r = 0\cdot 2$ m
$2x = 0\cdot 01$ m

○ **Solution :** Electric field, $E = 9\times 10^9\ \dfrac{2p}{r^3}$

$$\therefore\ \text{Dipole moment,}\ p = \frac{E\,r^3}{2\times 9\times 10^9}$$

$$\therefore\ p = \frac{10^5\times(0\cdot 2)^3}{18\times 10^9} = \mathbf{4\cdot 4\times 10^{-8}\ \text{Cou.m.}}$$

Magnitude of charge, $q = \dfrac{4\cdot 4\times 10^{-8}}{0\cdot 01} = 4\cdot 4\ \mu\text{C}$

✪ 3.8. Effect of electric field on a point charge ✪

Assume that mass of a charged particle is m and its charge is q. If it is placed in an electric field of intensity $\vec{E}$, then the force on the particle is $\vec{F} = \vec{E}q$

$\therefore$ Acceleration of the particle in the field is $\vec{a} = \dfrac{\vec{F}}{m} = \left(\dfrac{q}{m}\right)\vec{E}$

$\dfrac{q}{m}$ is called specific charge of the particle.

✪ 3.9. Torque on an electric dipole in an electric field ✪

We consider an electric dipole consisting of two charges $+q$ and $-q$ separated by a distance $2x$. Its dipole moment, $p = 2xq$. Magnetic moment is a vector quantity and is directed from negative to positive charge.

Assume that the electric dipole is inclined at angle θ with the direction of the applied electric field $\vec{E}$ [Fig. 3.10]. Two equal and opposite forces $\vec{F}$ and $-\vec{F}$ acts on the dipole. Here F = qE. Evidently total force on the dipole is zero. But at the deflected position, a torque acts on the dipole. The magnitude of the torque is $\tau = qE(2x \cdot \sin\theta) = (2xq)E\sin\theta = pE\sin\theta$

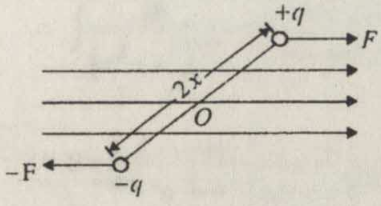


Fig. 3.10

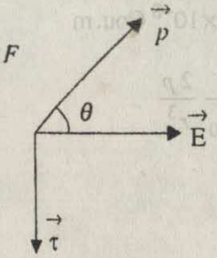


Fig. 3.11

$$\therefore \vec{\tau} = \vec{p} \times \vec{E} \quad \ldots\ldots\ldots\ldots (3.14)$$

The torque is at right angle to the plane containing $\vec{p}$ and $\vec{E}$ as shown in fig. 3.11.

3.10. Work done to deflect a dipole in an electric field

An electric dipole placed freely in an electric field will remain parallel to the electric field. To deflect the dipole from its equilibrium position some work has to be done. That work done is stored in the dipole in the form of potential energy.

As the dipole rotates in the field, at any instant it makes angle α with the direction of the field [Fig. 3.12]

∴ Torque on the dipole at the moment, $\tau = pE\sin\alpha$. Let it be deflected by an infinitesimally small angle $d\alpha$, so that torque on the dipole remain nearly constant. Work done for this small deflection is $dw = \tau\, d\alpha = pE\sin\alpha \cdot d\alpha$

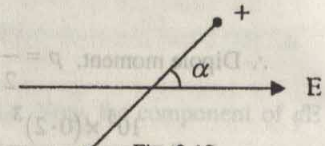


Fig. 3.12

Total work done for producing a deflection θ is $W = \int_0^\theta pE\sin\alpha\, d\alpha$

$$\therefore \quad W = pE(1-\cos\theta)$$

∴ Potential energy of the dipole at this position is, U = $pE(1-\cos\theta)$

If the angle of deflection α varies from 90° to θ then change of potential energy is

$$U(\theta) - U(90^\circ) = \int_{90^\circ}^{\theta^0} pE \cdot \sin\alpha\, d\alpha = pE[-\cos\alpha]_{90^\circ}^{\theta} = -pE\cos\theta = -\vec{p}\cdot\vec{E}$$

If U(90°) be taken to be zero $U(\theta) = -\vec{p}\cdot\vec{E} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3.15)$

● **Ex. 3.5. An electric dipole is formed by two charges +1μc and −1μc kept separated by a distance of 2·0 cm. How much torque is required to deflect the dipole by 30° in a uniform electric field of intensity 2·5×10⁴ NC⁻¹.**

○ **Solution :** Torque required is $\tau = pE\sin\theta$

$$\therefore \tau = 2\times10^{-8} \times 2\cdot5\times10^{-4} \times \sin 30^\circ$$

$$= \mathbf{2\cdot5\times10^{-4}\ N-m}$$

$p = 2xq = 0\cdot02\times10^{-6} = 2\times10^{-8}\ \text{C}-\text{m}$

$E = 2\cdot5\times10^{4}\ \text{NC}^{-1}$

$\theta = 30^\circ$

- **Ex. 3.6. How much work is required to be done to deflect an electric dipole of dipole moment $2{\cdot}0\times10^{-8}$ C-m through 60° in an electric field $1{\cdot}0\times10^{5}$ NC^{-1}.**

○ **Solution :** Work done, $W = pE\,(1-\cos\theta)$

$$\therefore\ W = 2{\cdot}0\times10^{-8}\times10^{5}\,(1-\cos 60^\circ)$$

$$= 10^{-3}\ \textbf{joule}$$

$p = 2{\cdot}0\times10^{-8}\ \text{C}-\text{m}$

$E = 1{\cdot}0\times10^{5}\ \text{NC}^{-1}$

$\theta = 60^\circ$

3.11. Electric field near a charged conductor—Coulomb's theorem

A hollow metal sphere of radius r is situated in a medium of dielectric constant K and it is given a charge + Q. We require to calculate electric field intensity at a point P near the conductor [Fig. 3.13]

By definition, electric force on a unit positive charge placed at P, gives the measurement of electric field at the point.

To calculate the electric field at any point due to charged sphere it will be assumed that its whole charge is at its centre. Now, the distance of the point P from its centre is equal to its radius, because the point P is very close to the surface of the sphere. So the force on a unit positive charge or electric intensity at the point P is

$$E = \frac{1}{4\pi\epsilon_0}\,\frac{1}{K}\cdot\frac{Q}{r^2}\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (3.16)$$

If σ be the measure of surface density of charge, $Q = 4\pi r^2\sigma$

$$\therefore\ E = \frac{1}{4\pi\epsilon_0}\,\frac{1}{K}\cdot\frac{4\pi r^2\sigma}{r^2} = \frac{1}{\epsilon_0 K}\cdot\sigma\ \ldots\ldots\ (3.16a)$$

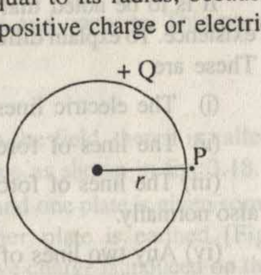


Fig. 3.13

The equation (3.16a) does not contain radius of the sphere. It means that it will be applicable to any type of charged conductor. So, in general, the electric field at any point near a charged conductor placed in a medium of dielectric constant K is $E = \frac{1}{\epsilon_0 K}\times$ surface density of charge

This is known as **Coulomb's theorem.**

- **Ex. 3.7. A sphere of radius 10 cm is given a charge 2 μC. If the sphere is situated in air, what will be electric field at a point near the surface of the sphere ? $\epsilon_0 = 8{\cdot}854\times10^{-12}\,C^2N^{-1}m^{-2}$**

○ **Solution :** $E = \frac{1}{\epsilon_0 K}\sigma$

Here $\epsilon_0 = 8{\cdot}854\times10^{-12}$, $K = 1$, $\sigma = \frac{Q}{4\pi r^2} = \frac{2\times10^{-6}}{4\pi(0{\cdot}1)^2} = \frac{1}{2\pi}10^{-4}\ C/m^2$

$$\therefore\ E = \frac{10^{12}}{8{\cdot}854}\times\frac{10^{-4}}{2\pi} = \mathbf{1{\cdot}8\times10^{6}\ NC^{-1}}$$

3.12. Electric lines of force

The nautre of the electric field can be ascertained if magnitude and direction of the electric field can be known at different points of the electric field. The magnitude of the field intensity and its direction can be determined from the force and its direction on a vanishingly small charge q_0 placed in the field. The concept of lines of force introduced by Faraday provides a convenient way of studying the nature of electric field.

Just as in the case of magnetic field, we can also visualize the electric field in terms of lines of force. If a free positive charge be placed in an electric field, it will experience a definite force in a definite direction. So, a free positive charge will move along a definite path in an electric field. This path is defined as **electric line of force**.

Hence an electric line of force is a path along which a free positive charge moves and a tangent drawn at any point on the path gives the direction of the resultant electric field at the point [Fig. 3.14]. Generally, the direction of electric intensity varies from point to point in the field. For this reason, the lines of force are usually curved. In fig. 3.14, the direction of the electric field E_A at A is different from the direction of the field E_B at B.

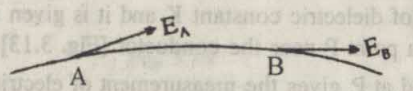


Fig. 3.14

It is to be noted that the concept of lines of force is imaginary. The lines of force do not have real existence. To explain different experimental observations, the lines of force are attributed some characteristics. These are :

(i) The electric lines of force always start from positive charges and end on the negative charges.

(ii) The lines of force always tend to contract along the length.

(iii) The lines of force start normally from the surface of a charged conductor and meet the surface also normally.

(iv) Any two lines of force never intersect. Because if they intersect, at the point of intersection two tangents can be drawn on two lines. It implies that the resultant field at a point is in two directions which is impossible.

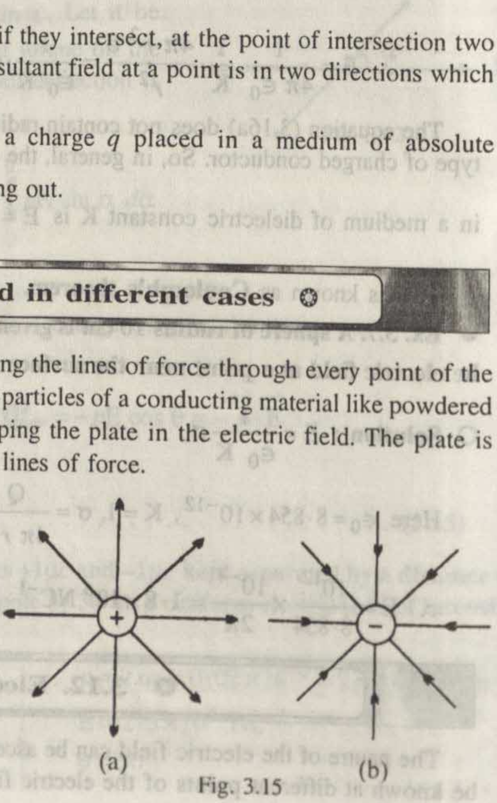
For the sake of calculation it is taken that from a charge q placed in a medium of absolute permittivity ε_0, $\dfrac{q}{\varepsilon_0}$ number of lines of force are coming out.

3.13. Maps of electric field in different cases

An electric field in a region can be mapped by drawing the lines of force through every point of the field. This is done experimentally by sprinkling very light particles of a conducting material like powdered gypsum salt on a non-conducting plate and then by keeping the plate in the electric field. The plate is gently tapped when the particles will arrange along the lines of force.

We discuss below the maps of electric field in the following cases.

- **(i) A point charge :** If we consider an isolated point charge, the lines of force is straight and radial. The lines of force due to an isolated positive and a negative charge are shown in fig. 3.15(a) and (b). For the positive charge the lines of force will be directed radially outwards and for negative charge, the lines of force will be directed radially inwards.



(a) (b)

Fig. 3.15

● **(ii) An electric dipole or two unlike equal charges :** The electric lines of force due to an electric dipole is shown in fig. 3.16. The lines of force start from the positive charge, curve around and converge to the negative charge. The magnitude and direction of electric intensity are different at different point around the dipole.

● **(iii) Two like equal charges :** In fig. 3.17, we see the lines of force due to two like equal positive charges. All the lines of force start from the charges. The lines of force are supposed to go to infinity. No line of force pass through the point marked x in between the charges. The resultant intensity at this point is zero i.e. the intensity of the electric fields due to the two charges are equal and opposite. This point is called **neutral point**.

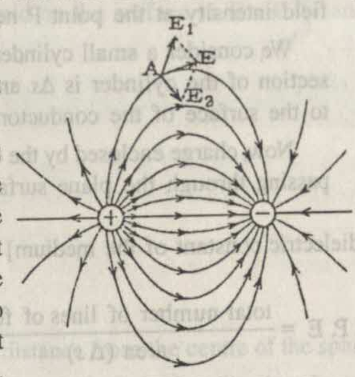


Fig. 3.16

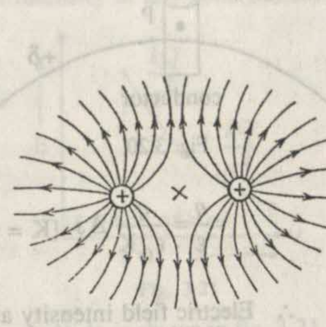


Fig. 3.17

➤ *3.13.1. Uniform electric field* :

If the magnitude and direction of the field intensity be same at all points in the field, then it is called a uniform electric field. Such a field is represented by equidistant parallel lines as shown in fig. 3.18.

Example : Two identical parallel metal plates are plated at small separation and one plate is given some positive charges and the other plate is earthed [Fig. 3.19]. Equal amount of negative charge is induced on the other plate. The lines of force are shown in the fig. So, in the space between the plates uniform electric field is produced.

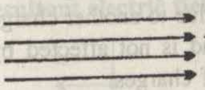


Fig. 3.18

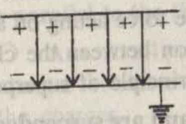


Fig. 3.19

✪ 3.14. To calculate of electric intensity in terms of lines of force ✪

Assume that a charge +Q is placed in a medium of absolute permittivity $\in$. A sphere of radius r is imagined with the charge +Q at the centre. Lines of force from the charge emerge out radially outwards and pass perpendicular to the surface of the sphere.

$\therefore$ Total number of lines of force coming out of the charge q is

$$\phi = \frac{q}{\in} = \frac{q}{\in_0 K} \quad \text{[K = dielectric constant of the medium]}$$

$\therefore$ Lines of force passing normally through unit area is $= \dfrac{1}{4\pi r^2}\,\dfrac{q}{\in_0 K} = \dfrac{1}{4\pi \in_0}\,\dfrac{q}{K r^2}$

Comparing with the equation (3.16) we see that it is the electric intensity at any point on the surface of the sphere. Hence **electric intensity at a point in the electric field is the number of lines of force passing normally through a unit area around the point**.

✪ 3.15. Coulomb's theorem from lines of force concept ✪

We have already seen that Coulomb's theorem gives the electric field intensity at a point near a charged conductor.

Assume that a charged conductor be placed in a medium of absolute permittivity $\in$. Surface density of the charge on the conductor is σ. We required to caculate electric field intensity at the point P near the charged conductor.

We consider a small cylinder enclosing the point P. Area of cross section of the cylinder is Δs and its curved surface is perpendicular to the surface of the conductor.

Now, charge enclosed by the cylinder, $q = 6\cdot\Delta s$. Hence lines of force passing through the plane surface of the cylinder

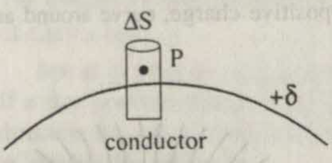


Fig. 3.20

$$=\frac{q}{\varepsilon}=\frac{q}{\varepsilon_0 K}\,\Delta s \quad [\text{K = dielectric constant of the medium}]$$

$$\therefore \text{ Electric field intensity at P, } E = \frac{\text{total number of lines of force}}{\text{area }(\Delta s)} \qquad \therefore\ E=\frac{\sigma}{\varepsilon_0 K}$$

This is Coulomb's theorem.

• • Short Answer Type Questions (with answers) • •

● **Question 1. A charge q_1 exerts a force on q_2. Now, another q_3 is brought near these charges. Will the force exerted on q_2 by q_1 will increase, or decrease or will remain unchanged ?**

○ **Ans.** The force exerted on q_2 by q_1 will remain same before and after the charge q_3 is brought near to them. The reason is : If the medium remains same, the force acting on a charge due to another charge depends on the magnitude of the charges and separation between the charges and is not affected by the presence of other charges. This follows from the principle of superposition of charges.

● **Q. 2. Each of two small balls in given a charge + q and are suspended from a stand by two strings of length l. If the whose arrangement is taken in an artificial satellite orbiting a planet, what will be the tension and angle between the strings ?**

○ **Ans.** The angle between the strings will be 180° and tension on each string is $T=\frac{1}{4\pi\varepsilon_0}\,\frac{q^2}{(2l)^2}$, is q is in Coulomb and l is in metre.

Explanation : In an orbiting artificial satellite everything inside the satellite remains in weightless condition. In this case as the weight of both the balls is zero, due to repulsion between the balls, the strings will be horizontal. So the angle between the strings in 180°.

Also, tension in a string will be equal to force of repulsion between to two balls. It can be determined by Coulomb's law.

● **Q. 3. Two particles each carrying a charge $2\cdot0 \times 10^{-8}$C are connected by an insulating string of length 1 m and are placed on a horizontal table. What is the tension in the string ?**

○ **Ans.** Tension in the string is the force of repulsion between the charge particles. So, the tension is

$$T=\frac{1}{4\pi\varepsilon_0}\,\frac{q_1 q_2}{r^2}=9\times10^9\times\frac{4\times10^{-16}}{(1)^2}=2\cdot6\times10^{-6}\,\text{N}$$

● **Q. 4. A hollow conducting sphere is given a charg + Q. Show how the intensity at a point varies from inside to outside the sphere. Show by graph.**

○ **Ans.** There is no charge and no lines of force inside the charged hollow sphere. So, there is no electric field inside the sphere. The electric field on the surface is $E = k\dfrac{Q}{r^2}$ and intensity at any point outside the sphere is $E = \dfrac{kQ}{x^2}$ $\left[k = \dfrac{1}{4\pi\varepsilon_0}\right]$

$\therefore\ E = 0$ when $x < r$

$E = \dfrac{kQ}{r^2}$ when $x = r$

$E = \dfrac{kQ}{x^2}$ when $x > r$

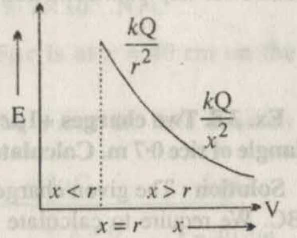


Fig. 3·21

The variation of electric field with distance from the centre of the sphere outside is shown in fig. 3.21.

● **Q. 5. Dielectric constant of a medium is 2. Calculate its electric permittivity.**

○ **Ans.** Electric permittivity, $\varepsilon = \varepsilon_0\ K$

$\therefore\ \varepsilon = 8\cdot85\times10^{-12}\times2 = 17\cdot70\times10^{-12}\ C^2N^{-1}m^{-2}$

● **Q. 6. What is the electric field at centre of a charged ring ?**

○ **Ans.** Electric field intensity at the centre of a charged ring is zero.

● **Q. 7. The charges + q and – q are placed at a separation d. At which points the direction of the resultant electric field will be parallel to the line joining the charges ?**

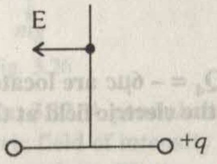


Fig. 3.22

○ **Ans.** At all points on the perpendicular bisector of the line joining the charges the electric field will be parallel to the axis of the dipole formed by the charges.

● **Q. 8. A charge Q is divided into two parts and kept at a certain separation. How the division be made so that the force between the charges be maximum ?**

○ **Ans.** Let the charges be q and $(Q - q)$ and placed at a separation r.

Now, the force between the charges, $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q(Q-q)}{r^2} = kq(Q-q)$ where $k = \dfrac{1}{4\pi\varepsilon_0}\dfrac{1}{r^2}$ is a constant.

So, the force will be maximum when $\dfrac{dF}{dq} = 0,$

Now, $\dfrac{dF}{dq} = K\dfrac{d}{dq}[Qq - q^2] = K(Q - 2q)$

$$\therefore\ Q - 2q = 0 \qquad \therefore\ q = \frac{Q}{2}$$

So, when the charges are divided equally, the force between them will be maximum.

● **Q. 9. Is electrostatic field conservative or non-conservative ?**

○ **Ans.** Electric field is conservative.

- **Reason :** A field is conservative when the work done is independent of the path followed and depends only on the initial and final position. Also over a closed path work done is zero. Now, in an electric field, work done to bring a charge from one point to another depends on initial and final position. Work done is same for any path followed. So, the electric field is conservative.

●●Numerical Examples ●●

- **Ex. 3.8. Two charges +1μc and –1μc are placed at the angular points of the base of an equilateral triangle of sice 0·7 m. Calculate the electric field at the apex of the triangle. [J. E. E. 2001]**

○ **Solution :** The given charges $q_1 = +1\mu c$ and $q_2 = -1\mu c$ are placed at the points B and C of the triangle ABC. We require to calculate the electric field at the point A.

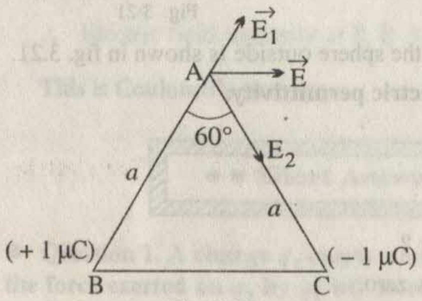


Fig. 3.23

The electric field at A due to the charge q_1 at B is $E_1 = k\dfrac{q_1}{a^2}$ along $\overrightarrow{BA}$ and electric field at A due to the charge q_2 at C is

$$E_2 = k\frac{q_1}{a^2} \text{ along } \overrightarrow{AC}$$

∴ Resultant field at A is,

$$E = \sqrt{E_1^2 + E_2^2 + 2E_1\,E_2 \cos 120^\circ} \quad [\text{Here } E_1 = E_2]$$

$$\therefore E = E_1$$

$$\therefore E = E_1 = \frac{kq_1}{a^2} \quad \therefore E = 9\times10^9\,\frac{10^{-6}}{(0\cdot7)^2} = 18\cdot37\times10^3\,\text{NC}^{-1}$$

- **Ex. 3.9. The four point charges $Q_1 = +2\mu c$, $Q_2 = +4\mu c$, $Q_3 = +6\mu c$ and $Q_4 = -6\mu c$ are located at the angular point of a square. The length of each diagonal is 2 m. Calculate the electric field at the point of intersection of the diagonals.**

○ **Solution :** The given charges are placed at the angular points A, B, C and D of the square ABCD. The diagonals intersect at O, where we want to calculate the field.

Electric field at O due to the charge Q_1 at A is

$$E_1 = 9\times10^9\,\frac{Q_1}{(OA)^2} = 9\times10^9\times\frac{2\times10^{-6}}{(1)^2} = 18\times10^3\,\text{NC}^{-1} \text{ along } \overrightarrow{OC}$$

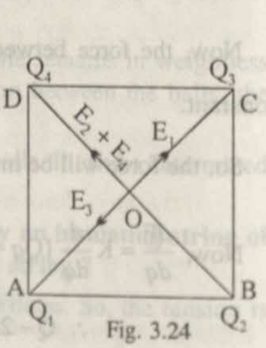


Fig. 3.24

Electric field at O due to the charge Q_2 at B is

$$E_2 = 9\times10^9\,\frac{Q_2}{(OB)^2} = 9\times10^9\times\frac{4\times10^{-6}}{(1)^2} = 36\times10^3\,\text{NC}^{-1} \text{ along } \overrightarrow{OD}$$

Electric field at O due to the charge Q_3 at C is

$$E_3 = 9\times10^9\,\frac{Q_3}{(1)^2} = 9\times10^9\times\frac{6\times10^{-6}}{(1)^2} = 54\times10^3\,\text{NC}^{-1} \text{ along } \overrightarrow{OA}$$

Electric field at O due to the charge Q_4 at O is

$$E_4 = 9\times10^9\,\frac{6\times10^{-6}}{(1)^2} = 54\times10^3 \text{ along } \overrightarrow{OD}$$

∴ Total field intensity along OA, $E' = E_3 - E_1 = (54-18)\times10^3 = 3\cdot6\times10^4\ \text{NC}^{-1}$

and total field intensity along OD, $E'' = E_2 + E_4 = 9\cdot0\times10^4\ \text{NC}^{-1}$

∴ Resultant intensity at O, $E=\sqrt{(E')^2+(E'')^2}=10^4\sqrt{(3\cdot6)^2+9^2}=\mathbf{9\cdot7\times10^4\ N/C}$

● **Ex. 3.10. A charge + 3μc is placed at *x* = 0 and another charge – 5μc is at *x* = 40 cm on the *x*-axis. At which point, the field intensity will be zero ?**

○ **Solution :** The situation is shown in fig. 3.25. Let at the point P on the *x*-axis the field intensity is zero. OP = *x* cm.

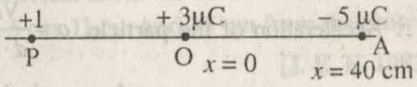


Fig. 3.25

$$\therefore\ k\cdot\frac{3}{(OP)^2}=k\frac{5}{(AP)^2}\quad\text{or,}\ \frac{3}{x^2}=\frac{5}{(40+x)^2}$$

$$\therefore\ \frac{40+x}{x}=\sqrt{\frac{5}{3}}=1\cdot29\quad\therefore\ 40+x=1\cdot29x\quad\therefore\ x=137\cdot91\text{cm}$$

So, electric field will be zero on the *x*-axis at a distance of 137·91 cm from the origin.

● **Ex. 3.11. An oil drop of mass 10^{-18} kg is given a charge of 2*e* (*e* = $1\cdot6\times10^{-19}$ C). What electric field is required to be applied to keep the oil drop in equilibrium ?**

○ **Solution :** Let the required electric field in the upward direction is E. The condition for the oil drop to remain floating in space is $Eq = mg$

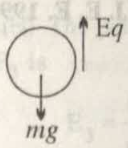


Fig. 3.26

Here $q=2\times1\cdot6\times10^{-19}=3\cdot2\times10^{-19}\text{C},\ m=10^{-18}\,kg$

and $g=9\cdot8\,m/s^2$

$$\therefore\ E=\frac{mg}{q}=\frac{10^{-18}\times9\cdot8}{3\cdot2\times10^{-19}}=20\cdot42\ \text{NC}^{-1}$$

● **Ex. 3.12. A pendulum bob of mass 80 mg and carrying 2×10^{-8}C is in equilibrium in a horizontal electric field of intensity 2×10^4 Vm^{-1}. What is the tension in the string. Calculate also the angle the string makes with the vertical.**

○ **Solution :** Consider the fig. 3.27. The forces acting on the pendulum bob are shown in fig. Considering equilibrium of the bob $T\cos\theta = mg$, $T\sin\theta = qE$

$$\therefore\ T=\sqrt{(mg)^2+(qE)^2}$$

Here $mg=80\times10^{-6}\times9\cdot8=7\cdot84\times10^{-4}\ \text{N}$

$$qE=2\times10^{-8}\times2\times10^4=4\times10^{-4}\ \text{N}$$

∴ Tension is the string, $T=10^{-4}\sqrt{(7\cdot84)^2+(4)^2}=\mathbf{8\cdot8\times10^{-4}\ N}$

Also, $\tan\theta=\frac{qE}{mg}=\frac{4}{7\cdot84}=0\cdot51\qquad\therefore\ \theta=\mathbf{27^\circ3'}$

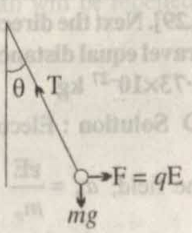


Fig. 3.27

● **Ex. 3.13. A particle carrying a charge $1\cdot6\times10^{-19}$C enters midway between the plates of a parallel plate capacitor. The initial velocity of the particle is parallel to the plates. A potential difference of the 300V is applied to the capacitor plates. If the length of each plate is 10 cm and separation 2 cm, calculate the high test initial velocity for which the particle will not be able to come out of the plates, the mass of the particle is 12×10^{-24} kg.** **[J. E. E. 1996]**

○ **Solution :** The plates P and Q is horizontal at a separation of 2 cm. Let v m/s be the maximum velocity of the particle with which it can be projected so that it just cannot go out of the plate [Fig. 3.28]. Deflection of the charged particle is $y = 0{\cdot}01$ m. The time for which the particle remains between the plates is $t = \frac{l}{v}$ [$l = 0{\cdot}1$ cm]

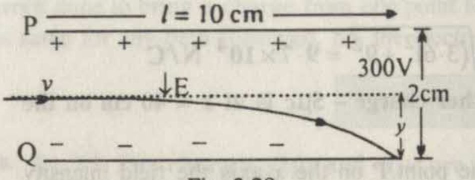


Fig. 3.28

Now, downward force on the particle is

$$F = Eq = \frac{Vq}{d} \quad [V = 300V,\ d = 0{\cdot}02 \text{ cm}]$$

$\therefore$ Acceleration of the particle, $a = \frac{Vq}{d \cdot m}$

$\therefore$ Vertical deflection, $y = \frac{1}{2}at^2 = \frac{1}{2}\frac{Vq}{d \cdot m} \times \frac{l^2}{v^2}$

$$\therefore \quad v^2 = \frac{1}{2}\frac{Vql^2}{d \cdot m \cdot y} = \frac{1}{2} \times \frac{300 \times 1{\cdot}6 \times 10^{-19} \times (0{\cdot}1)^2}{0{\cdot}02 \times 12 \times 10^{-24} \times 0{\cdot}01} = 10^8 \quad \therefore \quad v = 10^4 \text{ ms}^{-1}$$

So, the required velocity = $\mathbf{10^4}$ **ms^{-1}**

● **Ex. 3.14. A water drop carrying a charge of one electron is floating in equilibrium in an electric field of earth. If the intensity of the electric field be 3V.cm^{-1}, calculate radius of the water drop. Given, $e = 4{\cdot}805 \times 10^{-10}$ esu and $g = 980$ cm/s^2.** **[J. E. E. 1995]**

○ **Solution :** The water drop is in equilibrium in the electric field

$\therefore$ Weight of the water drop = electric force.

$$\therefore \ mg = eE \qquad \therefore \ \frac{4}{3}\pi r^3 \times 1 \times g = eE$$

$$\therefore \ r^3 = \frac{3eE}{4\pi g} = \frac{3 \times 4{\cdot}805 \times 10^{-10}}{100 \times 4\pi \times 980}$$

$$\therefore \ r = \mathbf{1{\cdot}05 \times 10^{-5}} \textbf{ cm}$$

$e = 4{\cdot}805 \times 10^{-10}$ esu

$E = \frac{1}{100}$ esu/cm

$g = 980$ cm/s^2

● **Ex. 3.15. An electron travels a distance of 1·5 cm through an electric field of intensity 2×10^4 NC^{-1} [Fig. 3.29]. Next the direction of the electric field is reversed with its magnitude remaining unchanged. A proton travel equal distance of 1·5 cm. Calculate the time taken by of electron and proton. Mass of proton, m_p = 1·73×10^{-27} kg.**

○ **Solution :** Electrons move opposite to the electric field [Fig. 3.29(i)]. Acceleration of the electron in the field, $a_e = \frac{eE}{m_e}$ [m_e = mass of electron]. If it travels a distance h in time t_1, $t_1 = \sqrt{\frac{2h}{a_e}} = \sqrt{\frac{2hm_e}{eE}}$

$$\therefore \ t_1 = \sqrt{\frac{2 \times 1{\cdot}5 \times 10^{-2} \times 9{\cdot}1 \times 10^{-31}}{1{\cdot}6 \times 10^{-19} \times 2 \times 10^4}} = \mathbf{2{\cdot}9 \times 10^{-9}} \textbf{ sec}$$

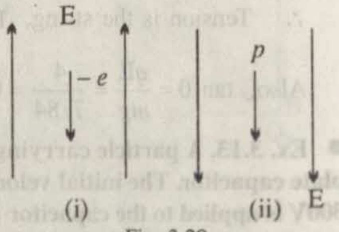


Fig. 3.29

But proton mass in the direction of the electric field [Fig. 3.29 (ii)]

$\therefore$ Acceleration of proton, $a_p = \dfrac{eE}{m_p}$

If the proton travels a distance h in time t_2 then, $t_2 = \sqrt{\dfrac{2hm_p}{eE}}$

$$t_1 = \sqrt{\frac{2\times1\cdot5\times10^{-2}\times1\cdot673\times10^{-27}}{1\cdot6\times10^{-19}\times2\times10^{4}}} = 1\cdot3\times10^{-7}\ \text{sec}$$

● **Ex. 3.16. A charge of 20 esu is kept at the centre of a spherical shell. Its inner radius is 10 cm and outer radius 12 cm. Calculate the electric field at distance 5 cm, 11 cm and 15 cm from the centre.** **[J. E. E. 1982]**

○ **Solution :** The situation is shown in fig. 3.30. The charge Q = + 20 esu is placed at the centre O of the spherical shell. We require to calculate electric fields at the points P_1, P_2 and P_3 where OP_1 = 5 cm, OP_2 = 11 cm and OP_3 = 15 cm.

(i) The electric field at P_1 is $E_1 = \dfrac{Q}{r_1^2} = \dfrac{20}{(5)^2} = 0\cdot8 = 0\cdot8\ \text{dyne/esu}$

(ii) The point P_2 (OP_2 = 11 cm) is situated within the metal. Lines for force cannot pass through the metal. So electric field at P_2 is $E_2 = 0$.

(iii) The point P_3 is situated outside the field. So, the field intensity at P_3 is

$$E_3 = \frac{Q}{r_3^2} = \frac{20}{(15)^2}$$

$$= 0\cdot088\ \text{dyne/esu}$$

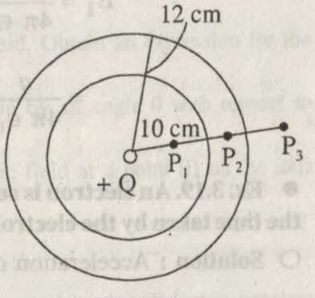


Fig. 3.30

● **Ex. 3.17. The plates of a parallel plate capacitor is horizontal. A small metal ball is suspended by a light string between the plates. If the ball is given a charge + *q* and the upper plate is positively charged. Calculate the time period of oscillation of the metal ball which is assumed to behave as a simple pendulum.** **[J. E. E. 1988]**

○ **Solution :** The upper plate and the metal ball both are positively charged. The ball will be repelled downwards. So its effective weight will increase.

$\therefore$ Effective weight, $mg' = mg + qE$

Here E = intensity of the electric field, m = mass of the bob.

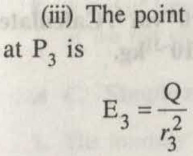


Fig. 3.31

$\therefore$ Effective acceleration due to gravity, $g' = g + \dfrac{qE}{m}$

$$\therefore \text{Time period, } T = 2\pi\sqrt{\frac{l}{g'}} = 2\pi\sqrt{\frac{l}{g+\dfrac{qE}{m}}} = 2\pi\sqrt{\frac{ml}{mg+qE}}$$

● **Ex. 3.18. Infinite number of positive charges each of magnitude *q* is placed on the *x*-axis at the points *x* = 1, *x* = 2, *x* = 4, *x* = 8, ... what will be intensity of the electric field at *x* = 0. Also calculate the electric field if alternate charges are of opposite signs.** **[J. E. E. 1997]**

○ **Solution : 1st case :** The arrangement of charges are shown in fig. 3.32. All the charges (each + q) are placed on the x-axis at the point $x = 1, 2, 4, 8$ . . . . . So, the resultant intensity at $x = 0$ is

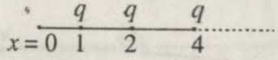


Fig. 3.32

$$E = \frac{1}{4\pi \in_0}\left[\frac{q}{1^2}+\frac{q}{2^2}+\frac{q}{4^2}+\frac{q}{8^2}+\ldots\ldots\right]$$

$$= \frac{q}{4\pi \in_0}\left[\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{4^2}+\frac{1}{8^2}+\ldots\ldots\right] = \frac{q}{4\pi \in_0}\left[1+\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\ldots\ldots\right]$$

$$= \frac{q}{4\pi \in_0}\,\frac{1}{1-\frac{1}{4}} = \frac{1}{4\pi \in_0}\,\frac{4q}{3} \text{ SI units}$$

2nd case : Eelctric intensity at $x = 0$

$$E_1 = \frac{1}{4\pi \in_0}\left[\frac{q}{1^2}-\frac{q}{2^2}+\frac{q}{4^2}-\frac{q}{8^2}+\ldots\ldots\right]$$

$$= \frac{q}{4\pi \in_0}\cdot\frac{1}{1-\frac{1}{4}} = \frac{1}{4\pi \in_0}\,\frac{4q}{5} \text{ SI units}$$

● **Ex. 3.19. An electron is set in motion from rest in an electric field of intensity $1{\cdot}5\times10^4$ vm^{-1}. Calculate the time taken by the electron to travel a distance of 10 cm. mass of electron, $m = 9{\cdot}1\times10^{-31}$kg.**

○ **Solution :** Acceleration of the electron in the electric field is

$$a = \frac{aE}{m} = \frac{1{\cdot}6\times10^{-19}\times1{\cdot}5\times10^{4}}{9{\cdot}1\times10^{-31}} = 2{\cdot}637\times10^{5}\ \text{m/s}^2$$

∴ The required time to travel a distance of 10 cm is $t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2\times0{\cdot}1}{2{\cdot}637\times10^{15}}} = \mathbf{8{\cdot}71\times10^{-9}}$ **sec**

• E ◇ X ◇ E ◇ R ◇ C ◇ I ◇ S ◇ E •

◢ A. Objective type questions :

1. What we mean by electric field and electric field intensity ?
2. Give the definitions of electric field intensity both in c.g.s. and in SI system. How are they related ?
3. How is the direction of electric intensity is determined ?
4. Why there is no lines of force inside a charged hollow conductor ? What is its significance in the case of electric field ?
5. A hollow conducting sphere of radius r is given a charge q.
   What is the electric field at a point whose distance x from the centre is given by (i) $x < r$ (ii) $x > r$.
   Represent this variation of electric intensity with distance graphically.
6. What are electric lines of force ? Why are they usually curved ?
7. Map the electric field due to an electric dipole.
8. What we mean by uniform electric field ? How is it represented ?

9. Map the electric field due to a charge q when (i) $q < 0$ (ii) $q > 0$.

10. How much electric flux is obtained from a charge of a coulomb placed in a medium of absolute permittivity $\in$.

11. Show that the two units of electric fields NC^{-1} and Vm^{-1} are equivalent.

◢ B. Essay type questions :

1. (a) State and explain Coulomb's law. Hence define electric field intensity. What is SI unit of electric intensity ? How is it defined ?

(b) Show that electric field is conservative.

2. Obtain an expression for electric field due to an isolated point charge.

3. (a) What is an electric dipole ? Define dipole moment. Give its unit.

(b) Draw electric lines of force in the following cases :

(i) isolated positive charge, (ii) isolated negative charge, (iii) electric dipole.

4. Obtain an expression for the electric field at a point an axis of an electric dipole.

5. Obtain an expression for the electric field at a point on the perpendicular bisector of an electric dipole. What is the direction of the field ?

6. A magnetic dipole is kept at an angle θ with respect to a uniform electric field. Obtain an expression for the torque acting on it. Hence define dipole moment.

7. Obtain an expression for the work done required to deflect an electric dipole by an angle θ with respect to a uniform electric field.

8. A rod of length L is uniformly charged. Obtain an expression for the electric field at a point (i) on the axis of the rod and (ii) at a point on the perpendicular bisector of the rod.

◢ C. Simple numerical problems :

1. The intensity of the electric field is $E = 10^4\ NC^{-1}$. A block of dielectric material having dielectric constant $K = 8$ is placed in the field. What is electric field inside the material ? **[Ans.** $1{\cdot}25\times10^3\ NC^{-1}$]

2. If a metal block is placed in the field in sum No. 1, what will be the electric field inside the metal ? [Zero; as $K = \infty$ for a metal]

3. What will be the force acting on a charge 10^{-6} C in an electric field of intensity $2 \times 10^5\ NC^{-1}$. **[Ans.** 0·2 N]

4. Two small conductors carry charges 10^{-8} C and 2×10^{-8} C respectively. If the separation between the conductors be 30 cm, calculate the electric field at the midpoint joining the conductors. **[Ans.** 4000 NC^{-1}]

5. Two point charges + 10 μc and 4 μc are placed at a separation of 12 cm. At which point electric field intensity will be zero ? **[Ans.** At a distance of 4 cm from the first charge (towards the second charge)]

6. Three charges $+4\times10^{-8}C, -\dfrac{2}{3}\times10^{-8}C$ and $+\dfrac{2}{3}\times10^{-8}C$ are placed at the angular points A, B and C of the equilateral triangle ABC of side 10 cm. Calculate the electric field at the midpoint of BC.

[Ans. $6{\cdot}78 \times 10^4$ N/C, 0 = 45°]

7. Calculate the electric field intensities at a distance of 40 cm and 80 cm from a point charge +50μ*c*.

[Ans. $5{\cdot}6 \times 10^5\ NC^{-1}$; $1{\cdot}4 \times 10^5\ NC^{-1}$]

8. The electric field at a distance of 0·5 m from a point charge is 2·7 NC^{-1}. What is the magnitude of the point charge ? **[Ans.** $7{\cdot}5 \times 10^{-11}$ C]

9. The measurement of three sides of a triangle ABC is AB = 5 cm, BC = 4 cm and CA = 3 cm. If the charges $1{\cdot}2\times10^{-12}$ C and $-\ 1{\cdot}6\times10^{-12}$ C are placed at two points A and B respectively, calculate the magnitude and direction of electric field at C. **[Ans.** 15 NC^{-1}, $\theta = \tan^{-1}(1{\cdot}33)$ with CB]

10. Two equal charges $1{\cdot}41 \times 10^{-7}$C are placed on two opposite angular points of square of side 30 cm. Calculate the electric field at the other two angular points. [**Ans.** 2×10^4 NC^{-1}]

11. Two point charges $q_1 = 2{\cdot}0 \times 10^{-8}$C and $q_2 = 1{\cdot}6 \times 10^{-7}$C are placed at a separation of 5·0 cm. Calculate the electric field at the point P located 3 cm from q_1 and 4 cm from q_2. [**Ans.** $9{\cdot}2 \times 10^5$ NC^{-1}]

12. An oil drop of mass 10^{-6} g carries a charge of $q = +20e$. It is made to float in space by applying an electric field. What is the intensity of the electric field ? [$g = 10$ m/s^2] [**Ans.** $3{\cdot}125 \times 10$ NC^{-1}]

13. A charged oil drop of mass 2·0 mg is floating in space in an electric field of intensity 1000 NC^{-1}. The electric field is directed downward. What is the charge on the oil drop ? [$g = 10$ ms^{-2}] [**Ans** – 0·02 μc]

14. Two charges $q_1 = +25 \times 10^{-9}$ C and $q_2 = -25 \times 10^{-9}$C form a dipole with the separation between the charges being 6 m. Calculate the electric field at a distance of 4 m from the centre of the dipole in the following case: (i) When the point is situated on the axis of the dipole and (ii) When the point is situated on the perpendicular bisector of the dipole. [**Ans.** (i) 55·1 NC^{-1} (ii) 10·8 NC^{-1}]

15. An electric dipole is situated in air. The electric field on the axis at a distance of 10 cm from the centre of the dipole $3{\cdot}75 \times 10^5$ NC^{-1} and at a distance of 20 cm from the centrre of 3×10^4 NC^{-1}. What is the length of the dipole ? [**Ans.** 10 cm]

16. A metal sphere of radius 0·5 cm is kept immersed in a liquid of specific gravity 0·8. When the sphere is given a charge of 10^{-8}C and an electric field of intensity 36×10^{-5} Vm^{-1} is applied, the sphere floats in the liquid. Calculate the density of the material of the sphere. [**Ans.** 7800 kg/m^3]

17. A sphere of radius 10 cm is given a charge of $\frac{1}{3}\times 10^{-6}$C. If dielectric constant of the medium be 4, calculate electric field at a point near the surface of the sphere. [**Ans.** $7{\cdot}5 \times 10^4$ NC^{-1}]

18. Two horizontal metal plates are kept at a separation of 2 cm and a potential difference of 600 V is applied to them. A charged oil drop remains at rest between the plates. Radius of the oil drop is 0·1 mm and density of oil 800 kg/m^3. Calculate the charge on the oil drop. [**Ans.** $1{\cdot}095 \times 10^{-12}$C]

19. The intensity of electric field between the horizontal plates of a parallel plate capacitor is 3700 Vm^{-1}. An electron is projected parallel to the plates with a velocity of $3{\cdot}6\times10^7$ m/s. If the length of each plate be 20cm calculate the deflection of the electron when it is in the field of the capacitor. [**Ans.** 0·01 m]

◢ D. Harder numerical problems :

1. Two equal negative charges $-q$ are fixed at points $(0, a)$ and $(0, -a)$ on the y-axis. A positive charge Q is released from rest at a point $(2a, 0)$ on the x-axis. Describe the motion of the charge Q.

[**Ans.** $F = \frac{2qQ}{4\pi\varepsilon_0}\frac{x}{\left(a^2+x^2\right)^{3/2}}$, the motion is not simple harmonic]

2. A simple pendulum consists of a small sphere of mass m suspended by a thread of length l. The sphere carries a positive charge q. The pendulum is placed in a uniform electric field of strength E directed vertically upwards. With what period will the pendulum oscillate if the electrostatic force acting on the sphere is less than the gravitational force ? Assume the oscillation to be small.

$$\left[\textbf{Ans :}\ T = 2\pi\sqrt{\frac{l}{\left(g-\frac{qE}{m}\right)}}\right]$$

3. Eight charges are placed at the eight corners of a cube of side 'a'. Calculate the force experienced by each charge q.

$$\left[\textbf{Ans :}\ \frac{q^2}{4\pi\varepsilon_0 a^2}\left(\frac{1}{3}+\sqrt{\frac{3}{2}}+\sqrt{3}\right)\right]$$

4. A metal ring of radius R is given a charge q. Calculate the electric field due to the charge (i) at the centre of the ring and (ii) at a point on the axis at a distance R from the centre. [**Ans.** (i) Zero, (ii) $\frac{q^2}{4\pi\varepsilon_0}\frac{\sqrt{2}q}{4R^2}$]

5. AB and CD are the two perpendicular diameters of a circle of radius 20π cm. Four charges + 10, + 10, + 10 and – 10 esu are placed at the points A, B, C and D respectively. Calculate intensity of the electric field at the centre O of the circle. [**Ans.** 0·2 dyne/esu along $\overrightarrow{CD}$]

6. Two charges $+q$ and $+q'$ are at a separation of d. Now a dielectric slab of thickness t and having dielectric constant K is introduced between the charges. Show that the force of attraction between the charges becomes

$$F = \frac{1}{4\pi\varepsilon_0}\frac{qq'}{\left[(d-t)+\sqrt{K}\cdot t\right]^2}$$

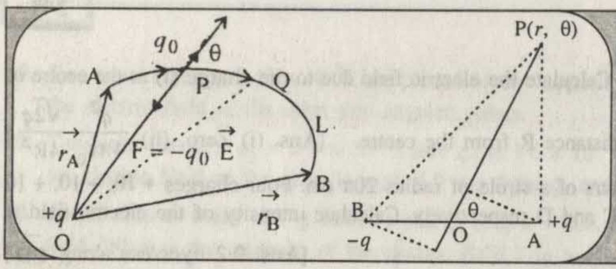
7. A straight conductor of length $2l$ is uniformly charged with amount of charge. What is the electric intensity on the perpendicular besector of the conductor at a distance 'd' from the centre ? $\left[\textbf{Ans :}\ \frac{1}{4\pi\varepsilon_0}\frac{1}{d\sqrt{l^2+d^2}}\right]$

8. A thin ring made of copper wire and having radius 'a' is given a charge q. What is the electric field on the axis at a distance x from the centre of the ring ? $\left[\textbf{Ans :}\ E = \frac{1}{4\pi\varepsilon_0}\frac{qx}{\left(a^2+x^2\right)^{3/2}}\right]$

9. A uniform rod of length L is given a charge Q. If the charge is uniformly distributed, calculate the electric field at a point on its axis at a distance 'a' from one end. $\left[\textbf{Ans :}\ \frac{1}{4\pi\varepsilon_0}\frac{Q}{a\,(L+a)}\right]$

10. A positively charged oil drop remains stationary in the electric field between the horizontal parallel plates separated by a distance of 1 cm. If the charge on the drop be $3{\cdot}2\times10^{-19}$C and the mass of the droplet be 10^{-13} kg, what is the potential difference between the plates ? Now, if the polarity of the plates be reversed what is the instantaneous acceleration of the droplet ? [**Ans.** $3{\cdot}4 \times 10^{-9}$ C, 19·6 m/s^2]

11. An electron is liberated from the lower of two large plates separated by a distance of 50 mm. The upper plate has a potential of 250 volts relative to the lower. How long does the electron take to reach it ? Mass of electron $= 9{\cdot}1 \times 10^{-31}$ kg ; charge $= 1{\cdot}6 \times 10^{-19}$C. [**Ans.** $1{\cdot}06 \times 10^{-8}$ sec]



CHAPTER 4

ELECTRIC POTENTIAL

● **TOPICS :** ○ Electric potential; ○ The line integral, work; ○ Conservative nature of electrostatic field; ○ Line integral of an electric field over a closed path is zero; ○ Physical significance of line integral of electric field; ○ Concept of electrostatic potential difference and potential; ○ Electric field as a gradient of potential; ○ Potential due to some charge distributions; ○ Electric potential at any point due to an electric dipole; ○ Electric potential energy; ○ Work done in deflecting an electrice dipole in a uniform electric field; ○ Unit of electrostatic energy : Electron volt (*e*V); ○ Equipotential surface; ○ Drawing of equipotential surface in different cases; ○ Short answer type questions (with answers); ○ Exercise.

4.1. Electric Potential

In the previous chapter we learned that electric field at a point in the electric field can be described in terms of the quantity $\vec{E}$. The vector $\vec{E}$ is called the electric field intensity at a point in the field. This method is similar to the concept of force in solving problems on mechanics. In the present chapter we introduce the concept of **electric potential** to describe electric field around a charge. It is a scalar quantity and electric field can be described easily by the term potential. The concept of potential is based on the energy consideration. We shall see that there is a close relation between potential and intensity.

The nature of electric potential is best understood if we compare it with gravitational potential energy. To raise a body from the earth's surface we require to perform some work which remain stored in the body itself in the form of potential energy. Now, if it is allowed to fall freely, it falls with increasing velocity and kinetic energy. Here stored potential energy inside the body is converted to equal amount of kinetic energy. If free to move a body tends to assume the position of minimum potential energy i.e. it tends to move from higher position to lower position. We say that in the earth's gravitational field, depending on height two places have different gravitational potential. Each and everybody has a natural tendency to move downwards.

Similar to gravitational potential we think of electrostatic potential in electric field due to a charge. A positive charge creates an electric field around it. If another positive charge is brought into that field, the second charge tends to move away from the first charge. It means that a force always acts on a charge placed in the electric field. This force on a positive charge is directed in the direction of the field intensity.

In the case of negative charge, the force is opposite. Evidently, if the charge is displaced opposite to the force, work is done against the force. The potential energy of the charge increases. On the other hand, if it is allowed to move freely, it moves in the direction of the force due to the electric field and its potential energy gradually decreases. It is said that **electric potential** exists at every point around the first charge.

To measure the potential at a point in the electric field we consider a unit positive charge as a **test charge.**

A positively charged body always tends to move from higher potential to lower potential similar to the tendency of a body to move from higher potential to lower potential in the gravitational field. Hence, electric potential of a charged body is its electrostatic condition which determines whether it will give charge to another body or not.

The concept of electric potential can be best understood in terms of a very important properly of the electric field, namely, that it is **conservative**.

⭘ 4.2. The line integral, work ⭘

In mechanics, line integral plays a very fundamental role in problems involving work. The concept of line integral is equally important in electrostatics. From fig. 4.1, the work done on a body depends on the component of force acting in the direction of motion. In differential form, the elementary work for a small displacement *dl* is $dW = F \cos \theta \cdot dl$

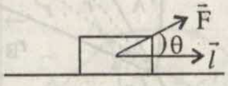


Fig. 4.1

For finite displacement, work done is

$$W = \int_A^B F \cos \theta \cdot dl = \int_A^B \vec{F} \cdot \vec{dl} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4.1)$$

Where we integrate from the beginning to the end of the path from A to B. The equation (4.1) is called the line integral of the force field. The integral may be regarded as the average value of F between A and B times the length of the interval from A to B.

If the region over which we integrate is 'an area in a plane' or 'surface in space', then instead of line integral, we consider surface integral. If the region over which we integrate is a 'volume in space', then we talk of volume integral. In all the three types of integral, the region of integration is always divided into a large number of small parts (called elements). The contribution of each part is taken into account and finally all the contributions are added up. By increasing the number of terms indefinitely, the sum tends to definite value. This definite value is called integral.

⭘ 4.3. Conservative nature of electrostatic field ⭘

A field or force is said to be conservative, if the work done by the field or force between two points is independent of the path followed between the two points. Further, work done by the conservative force or field over a closed path is always zero.

Any force which obeys inverse square law is conservative in nature. So, gravitational and electrostatic force are conservative forces. Then work done by electrostatic force between two points is same for all the paths between the two points and over a closed path work done will be zero.

We assume a system of electric charges (an assembly of discrete point charges or continuous charge distributions) which produces an electric field $\vec{E}$ in the space surrounding the charge system. Now we consider two points in space and connect them by a curve L. Then the line integral of the electric field from A to B along the curve is $\int_A^B \vec{E} \cdot \vec{dl}$.

We now ask the question : Will the value of the line integral depend upon the path followed to go from A to B. We show in the following that it will not ; the line integral will only depend on the starting and end points A and B and not on L.

This is the basic property of electrostatic field. **So for an electrostatic field, the line integral $\int_A^B \vec{E}\cdot\vec{dl}$ is same for all paths, depending only on the end points A and B.**

This property of electric field lead to the concept of electric potential. Electric field can also be represented in terms of potential. Before doing so, we shall first prove that the line integral of electric field between two points is independent of path followed between the two points.

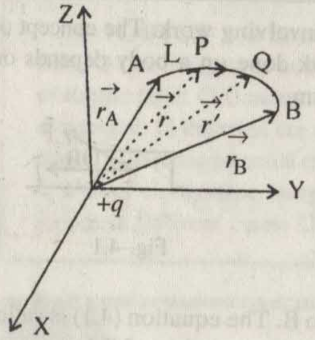


Fig. 4.2

For the sake of convenience, let a point charge + q be located at the origin of three dimensional rectangular co-ordinate system [Fig. 4.2]. We consider any two points A and B and connect them by a curve L. Let $\vec{r}_A$ and $\vec{r}_B$ be the position vectors of A and B respectively.

We shall try to evaluate the line integral $\int_A^B \vec{E}\cdot\vec{dl}$

The electric field $\vec{E}$ due to a point charge q at a point $\vec{r}$ is given by $\vec{E} = \frac{1}{4\pi\,\varepsilon_0}\frac{q}{r^2}\hat{r} = \frac{q}{4\pi\,\varepsilon_0}\frac{\vec{r}}{r^3}$

Thus the line integral to be evaluated is

$$\underset{\text{Path L}}{\int_A^B} \vec{E}\cdot\vec{dl} = \frac{q}{4\pi\,\varepsilon_0}\underset{\text{Path L}}{\int_A^B}\frac{\vec{r}\cdot\vec{dl}}{r^3}$$

In order to evaluate the integral we divide the path L into a large (infinite) number of small (infinitesimal) segments such as $\vec{PQ} = \vec{dl}$. Let the positions vectors of P and Q are $\vec{r}$ and $\vec{r}'$ respectively.

Applying the triangle law of vector addition, $\vec{r}' = \vec{r} + \vec{dl}$

$$\therefore\ (r')^2 = \left(\vec{r}+\vec{dl}\right)\cdot\left(\vec{r}+\vec{dl}\right) = \vec{r}\cdot\vec{r} + \vec{r}\cdot\vec{dl} + \vec{dl}\cdot\vec{r} + \vec{dl}\cdot\vec{dl}$$

$$\therefore\ (r')^2 = r^2 + 2\,\vec{r}\cdot\vec{dl} + dl^2 \quad \left[\because\ \vec{r}\cdot\vec{dl} = \vec{dl}\cdot\vec{r}\right]$$

$$= r^2 + 2\,\vec{r}\cdot\vec{dl} \quad [dl^2 \text{ is neglected as it is very small}]$$

$$\therefore\ \vec{r}\cdot\vec{dl} = \frac{1}{2}\left(r'^2 - r^2\right) = \frac{1}{2}(r'-r)\ (r'-r)$$

$$= \frac{1}{2}\cdot 2r\cdot(r'-r) \quad [\because r' \approx r]$$

$$= r\ (r'-r)$$

$$\therefore \text{ The line integral} = \frac{q}{4\pi \in_0}\int_A^B \frac{r'-r}{r^2} = \frac{q}{4\pi \in_0}\int_A^B \frac{r+dr-r}{r^2} = \frac{q}{4\pi \in_0}\int_A^B \frac{dr}{r^2}$$

$$= \frac{q}{4\pi \in_0}\left[-\frac{1}{r}\right]_{r_A}^{r_B} = \frac{q}{4\pi \in_0}\left(\frac{1}{r_A}-\frac{1}{r_B}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (4.3)$$

This result shows that the line integral is independent of the particular path L. It depends only on the initial and final position. The same result (4.3) holds if the charge is located at any general point rather than at the origin.

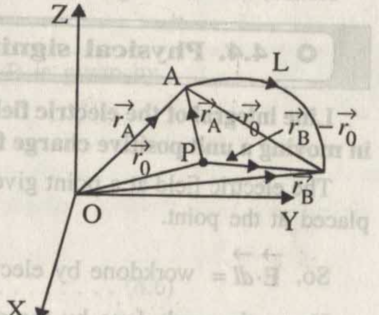


Fig. 4.3

Let the single point charge q is located at p having position vector $\vec{r}_0$. The position vectors of A and B with reference to p are given by $\left(\vec{r}_A - \vec{r}_0\right)$ and $\left(\vec{r}_B - \vec{r}_0\right)$ respectively.

$$\therefore \int_A^B \vec{E}\cdot\vec{dl} = \frac{q}{4\pi \in_0}\left[\frac{1}{|\vec{r}_A - \vec{r}_0|} - \frac{1}{|\vec{r}_B - \vec{r}_0|}\right] \ldots\ldots (4.4)$$

We can easily extend this result to an arbitrary electrostatic field due to any system of charges. It follows from the principle of superposition that the electric field at a point, due to any system of charges, is the vector sum of the fields due to each of the charges in the system.

- **A Note :** The above result does not hold if the electric field is not electrostatic, e.g. fields due to moving charges. Thus if the electric field varies with time or a function of velocity of charges, the line integral of such a field would not be independent of the path.

➤ *4.3.1. Line integral of an electric field over a closed path is zero :*

We consider a closed path (a loop) as shown in fig. 4.4. The loop starts from the point A and finally ends at A itself. We divide the total path into two parts L_1 (from A to B) and L_2 (from B back to A)

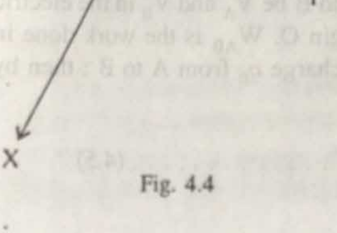


Fig. 4.4

Now, the line integral of the electric field between A and B along the path L_1 is given by

$$\int_A^B \vec{E}\cdot\vec{dl} = \frac{1}{4\pi \in_0} q\left(\frac{1}{r_A}-\frac{1}{r_B}\right)$$

(Path L_1)

Similarly, the line integral of electric field between the points B and A along the path L_2 is

$$\int_B^A \vec{E}\cdot\vec{dl} = \frac{1}{4\pi \in_0} q\left(\frac{1}{r_B}-\frac{1}{r_A}\right)$$

(Path L_2)

$$\therefore \quad \underset{\text{(Path } L_1)}{\int_A^B \vec{E}\cdot \vec{dl}} + \underset{\text{(Path } L_2)}{\int_B^A \vec{E}\cdot \vec{dl}} = \frac{1}{4\pi\epsilon_0} q\left(\frac{1}{r_A}-\frac{1}{r_B}\right)+\frac{1}{4\pi\epsilon_0} q\left(\frac{1}{r_B}-\frac{1}{r_A}\right)=0$$

$$\therefore \quad \underset{\text{along loop}}{\oint \vec{E}\cdot \vec{dl}} = \underset{\text{along } L_1}{\int_A^B \vec{E}\cdot \vec{dl}} + \underset{\text{along } L_2}{\int_B^A \vec{E}\cdot \vec{dl}} = 0$$

Hence, the line integral of an electric field along a closed path in an electric field is always zero.

4.4. Physical significance of line integral of the electric field

Line integral of the electric field between two points represents the work done by the electric field in moving a unit positive charge from one point to another in the field.

The electric field at a point gives the measurement of force experienced by a unit positive test charge placed at the point.

So, $\vec{E}\cdot\vec{dl} =$ workdone by electric field in moving a unit positive charge through $\vec{dl}$

Hence the work done by the electric field in moving a unit positive test charge from a point A to another point B is

$$W = \int_A^B \vec{E}\cdot\vec{dl}$$

Obviously, work done in moving a unit positive test charge against the electric field from A to B is

$$W = -\int_A^B \vec{E}\cdot\vec{dl}$$

4.5. Concept of electrostatic potential difference and potential

In fig. 4.5, a point charge $+q$ is situated at the origin O of co-ordinate system. The electric field due to $+q$ will be directed radially outwards from the charge. We consider two point A and B in the electric field due to the charge $+q$. L is the arbitrary path connecting A and B.

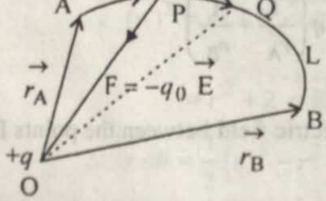


Fig. 4.5

The electrostatic potential difference between two points B and A is the work done to move a unit positive charge from A to B against the electrostatic force due to the electric field.

Let the potentials at the points A and B be V_A and V_B in the electric field due to the charge $+q$ at the origin O. W_{AB} is the work done in moving a vanishingly small positive charge q_0 from A to B ; then by definition,

$$V_B - V_A = \frac{W_{AB}}{q_0} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.5)$$

Assume that the test charge q_0 be at P when OP = r and $\vec{E}$ be the electric field at P due to the charge q at O.

So, the force on the test charge at P = $q_0 \vec{E}$

To keep the charge in equilibrium, an external force $\vec{F} = -q_0 \vec{E}$ has to be applied on the test charge so as to balance the force due to $+q$

So, the work done for infinitesimal displacement $\vec{PQ} = \vec{dl}$ is

$$dW = \vec{F}\cdot\vec{dl} = \left(-q_0\ \vec{E}\right)\cdot\vec{dl} = -q_0\left(\vec{E}\cdot\vec{dl}\right)$$

Hence, the work done in moving the test charge q_0 from A to B is given by

$$W_{AB} = \int_A^B dw = \int_A^B -q_0\ \vec{E}\cdot\vec{dl} = -q_0\int_A^B \vec{E}\cdot\vec{dl}$$

$$\therefore\ \frac{W_{AB}}{q_0} = -\int_A^B \vec{E}\cdot\vec{dl} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.6)$$

From the equations (4.5) and (4.6) : $V_B - V_A = -\int_A^B \vec{E}\cdot\vec{dl}$

Hence electrostatic potential difference between two points B and A in the electric field is equal to the negative of the line integral of the electric field between the points A and B.

Electrostatic Potential :

To define and express electrostatic potential at a point in the electric field, we assume that the point A is located at infinity where the electrostatic potential is taken to be zero. So, electrostatic potential difference between point B and at a point A at infinity is equal to the electrostatic potential at B. Hence potential at a point is defined as :

The electrostatic potential at a point in the electric field is defined as the work done to bring a unit positive charge from infinity upto the point against the electrostatic force due to the electric field.

$\therefore$ Electric potential, $V = \dfrac{W}{q_0}$

where W is the work done to bring a vanishingly small positive charge q_0 from infinity upto the point.

$$\therefore\ W = Vq_0 = -q_0\int_\infty^B \vec{E}\cdot\vec{dl} \qquad \therefore\ V = -\int_\infty^B \vec{E}\cdot\vec{dl} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.7)$$

Thus electrostatic potential at a point in an electric field is equal to the negative of the line integral of the electric field between infinity and the point.

- **Exercise 4.1. A charge of 4C is transferred from a point at potential –10V to another point whose potential is V volt. If 100 J of work is done in the process, find the value of V.**

○ **Solution :** The potential difference between two points is $V_B - V_A = \frac{W_{AB}}{q_0}$

$$\therefore\ V-(-10)=\frac{100}{4} \qquad \therefore\ V=15 \textbf{ volts.}$$

Unit of Potential difference :

The SI unit of potential difference is volt. The potential difference between two points is 1 volt if 1 joule of work is done in moving a charge of 1 coulomb from one point to another (without acceleration) against electrostatic force due to electric field. So,

$$1\text{ volt}=\frac{1\text{ Joule}}{1\text{ Coulomb}}=\mathbf{1\ JC^{-1}}$$

4.6. Electric field as a gradient of potential

A point charge $+q$ is placed at O. P and Q are two points at distances r and $r - dr$ respectively from the charge $+ q$. [Fig. 4.6] Potential at P is V and at Q it is V + dV. Let dw be the work done in moving a vanishingly small positive test charge q_0 from P to Q. Then,

$$V+dV-V=\frac{dw}{q_0} \qquad \therefore\ dV=\frac{dw}{q_0} \cdots\cdots\cdots\cdots \text{(i)}$$

Let $\vec{E}$ be the electric field at P due to the charge $+ q$ at O.

Then the test charge q_0 experiences a force of $q_0 \vec{E}$ and the external force required to move the test charge (without acceleration) against the electric field $\vec{E}$ is $\vec{F}=-q_0\vec{E}$

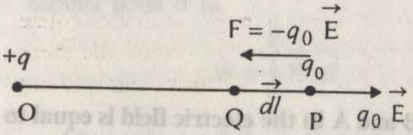


Fig. 4.6

So, the work done for a small displacement $\vec{PQ}=\vec{dl}$ is

$$dw=\left(-q_0\ \vec{E}\right)\cdot\vec{dl}=-q_0\ E\ dl\cos 180^\circ=q_0\ E\ dl$$

As the distance decreases in the direction of $\vec{dl}$, the distance dl is taken as $-dr$. The above equation is written as $dw = -q_0\ E\ dr$

$$\therefore\ \frac{dw}{q_0}=-E\ dr \cdots\cdots\cdots\cdots \text{(ii)}$$

Hence from (i) and (ii), $dV = -E dr$

$$\therefore\ E=-\frac{dV}{dr} \cdots\cdots\cdots\cdots \text{(4.8)}$$

Hence the electric field at a point is equal to the negative gradient of the electrostatic potential at the point. The negative sign indicates that $\vec{E}$ is always directed in the direction of decrease of electric potential.

Unit of potential gradient or electric field is Vm^{-1}.

4.7. Potential due to some charge distributions

Now, we calculate potential at a point due to different charge distributions.

(A) Potential at a point due to a point charge :

We calculate potential at P due to a charge + q. Here OP = r potential at P is given by

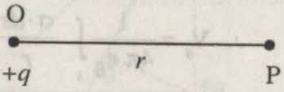


Fig. 4.7

$$V = -\int_{\infty}^{r} E\, dr$$

$$\therefore\ V = -\int_{\infty}^{r} \frac{1}{4\pi\,\varepsilon_0}\frac{q}{r^2}dr$$

$$\therefore\ \mathbf{V = \frac{1}{4\pi\,\varepsilon_0}\frac{q}{r}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.9)$$

The potential is spherically symmetric around the point i.e. it depends on r for a given charge q. So its value only depends on the distance of the point from the charge.

(B) Potential due to a system of charges :

Potential at a point due to a system of charges is obtained by applying superposition principle. Thus to find the electric potential at a point in the electric field due to two or more charges, we first calculate the potential due to each charge, assuming that all other charges are absent, and then simply add these individual contributions. Since electric potential is scalar, the addition here is the ordinary sum and not the vector sum.

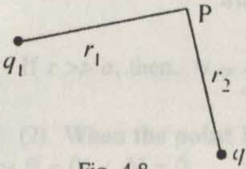


Fig. 4.8

● **(i) Two point charges :** Here we calculate the potential at P due to the two point charges q_1 and q_2 located at distance r_1 and r_2 from P. It is shown in fig. 4.8. The potential at P is simply given by

$$V = V_1 + V_2 = \frac{1}{4\pi\,\varepsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right) \ldots\ldots\ldots\ldots (4.10)$$

● **(ii) Many point charges :** Here we calculate the potential at P due to a group of point charges $q_1, q_2, \ldots\ldots q_n$ situated at distances $r_1, r_2, \ldots\ldots r_n$ from the point P. The potential at P due to a system of these N discrete point charges is give by

$$V = \sum_{n=1}^{N} V_n = \frac{1}{4\pi\,\varepsilon_0}\sum_{n=1}^{N}\frac{q_n}{r_n} \ldots\ldots\ldots\ldots (4.11)$$

where $n = 1, 2, 3, \ldots\ldots$, N ; V_n is the potential due to n-th charge q_n which is at a distance r_n from the point P.

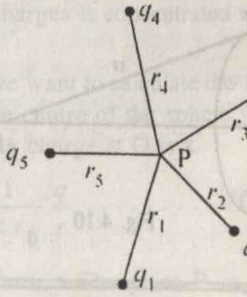


Fig. 4.9

● **(iii) Continuous charge distribution :** If the charge distribution be continuous, the summation in equation (4.11) is replaced by integration.

$$\therefore\ V = \frac{1}{4\pi\,\varepsilon_0}\int\frac{dq}{r} \ldots\ldots\ldots\ldots\ldots\ldots (4.12)$$

Where dq is differential element of charge distribution and r its distance from the point at which potential is to be calculated.

(a) If the charge is uniformly distributed along a line l, then $dq = \lambda\, dl$, here λ is the linear charge density and we have

$$V_l = \frac{1}{4\pi\,\varepsilon_0}\int \frac{\lambda\, dl}{r} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.13)$$

(b) If the charge is uniformly distributed over an area s, then $dq = \sigma\cdot ds$ when σ is the surface density of charge and we write equation (4.12) as surface integral.

$$\therefore\ V_s = \frac{1}{4\pi\,\varepsilon_0}\int_s \frac{\sigma\cdot ds}{r} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.14)$$

(c) If the charge is continuously distributed inside a volume V then $dq = \rho\cdot dv$, when ρ is the volume density of charge. Here the integral in (4.12) is volume integral.

$$V = \frac{1}{4\pi\,\varepsilon_0}\int_s \frac{\rho\, dv}{r} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.15)$$

- **Example 4.2. The electric field at a point due to a point charge is 20 NC^{-1} and electric potential at that point is 10 JC^{-1}. Calculate the distance of the point from the charge and magnitude of charge. [C. B. S. E. 1996]**

○ **Solution :** Electric intensity, $E = \frac{1}{4\pi\,\varepsilon_0}\frac{q}{r^2} = 20$

and Electric potential, $V = \frac{1}{4\pi\,\varepsilon_0}\frac{q}{r} = 10 \quad \ldots\ldots\ldots\ldots\ldots\ldots$ (i)

$$\therefore\ r = \frac{10}{20}\, m = \mathbf{50\ cm}$$

From the equation (i), $9\times10^9 \times \frac{q}{0\cdot5} = 10 \quad \therefore\ q = \mathbf{5\cdot56 \times 10^{-10} C}$

$\therefore$ Distance of the point is **50 cm** and charge, $q = \mathbf{5\cdot56 \times 10^{-10} C}$

- **Ex. 4.3. Calculate the potential at a point on the axis of a ring of charge of radius r. Charge on the ring is q. The distance of the point from the centre of the ring is x.**

○ **Solution :** Considering the fig. 4.10. The ring of radius r is given a charge q.

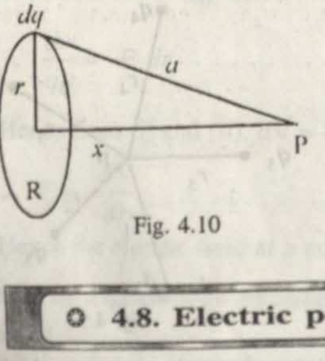


Fig. 4.10

So its linear charge density $\lambda = \frac{q}{2\pi r}$

Now, the potential at P due to an elementary charge $dq = \lambda\, dl$

$$\text{is } dv = \frac{1}{4\pi\,\varepsilon_0}\int \frac{\lambda\, dl}{a} = \frac{1}{4\pi\,\varepsilon_0}\frac{\lambda}{a}\times 2\pi r$$

$$= \frac{1}{4\pi\,\varepsilon_0}\frac{q}{r} = \frac{1}{4\pi\,\varepsilon_0}\frac{q}{\sqrt{r^2+x^2}}\ [q = 2\pi r\lambda]$$

○ 4.8. Electric potential at any point due to an electric dipole ○

An electric dipole is formed when two charges of equal magnitude and opposite sign (say, $+q$ and $-q$) are placed at a small separation.

In fig. 4.11, AB is an electric dipole with its centre at O. P is a any point of polar co-ordinates (r, θ) where we want to calculate electric potential due to the dipole. AB = $2a$.

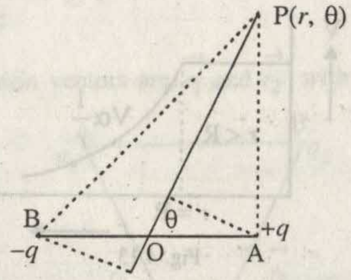


Fig. 4.11

The potential at P due to $+q$ is $V_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{PA}$

and potential at P due to $-q$ is $V_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{-q}{PA}$

$\therefore$ Total potential at P due to electric dipole is

$$V = V_1 + V_2 = \frac{1}{4\pi\varepsilon_0} \cdot q\left[\frac{1}{PA} - \frac{1}{PB}\right]$$

Now, PA = $r - a\cos\theta$ and PB = $r + a\cos\theta$

$$\therefore V = \frac{1}{4\pi\varepsilon_0} \cdot q\left[\frac{1}{r-a\cos\theta} - \frac{1}{r+a\cos\theta}\right] = \frac{1}{4\pi\varepsilon_0} \cdot q \cdot \frac{2a\cos\theta}{r^2 - a^2\cos^2\theta}$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{p\cos\theta}{r^2 - a^2\cos^2\theta} \quad [p = 2aq = \text{electric dipole moment of the dipole}]$$

- **Special cases :** (i) We can calculate the electric potential due to the dipole in the following two cases:

(i) **When the point P lies on the axis of the dipole.** In this case $\theta = 0^\circ$, $\cos\theta = 1$. Then

$$V = \frac{1}{4\pi\varepsilon_0} \frac{P}{r^2 - a^2}$$

If $r >> a$, then, $V = \frac{1}{4\pi\varepsilon_0} \frac{P}{r^2}$

(2) **When the point P is situated on the perpendicular bisector of the dipole**. In this case $\theta = 90^\circ$, $\cos\theta = 0$. $\therefore$ V = 0.

➤ *4.8.1. Potential due to a charged conductor (hollow or solid) :*

● (a) **At an external point :** Let a sphere of radius R is given a charge q. The charge resides on the outer surface. For calculation of potential, we shall assume that all the charges is concentrated at its centre.

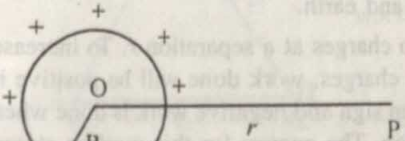


Fig. 4.12

Assume that the point P where we want to calculate the field be at a distance r $(r > R)$ from the centre of the sphere (Fig. 4.12). So the distance of P from the charge at O is r.

So, the potential at P is $V = \frac{1}{4\pi\varepsilon_0} \frac{q}{r}$ . . . . (i)

● (b) **At a point on the surface :** The point P is at a distance R from the charge q.

$\therefore$ The potential at the point P on the surface is $V = \frac{1}{4\pi\varepsilon_0} \frac{q}{R}$ . . . . (ii)

- (c) **At a point inside the conductor :** We know that the electric intensity E is related to electric potential V by $E=-\frac{dv}{dr}$

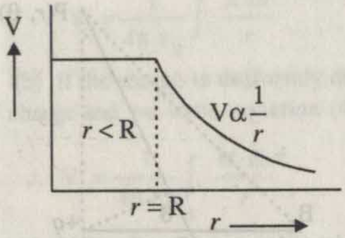


Fig. 4.13

But electric field at a point inside the conductor is zero.

$$\therefore \quad \frac{dv}{dr}=0 \text{ or, } V = \text{Constant}$$

So, potential at every point inside the conductor is same and is equal to potential at the surface.

$$\therefore \quad V=\frac{1}{4\pi\varepsilon_0}\frac{q}{R} \quad \ldots\ldots\ldots\ldots\ldots\ldots (iii)$$

The variation of potential with distance from the centre of the charged sphere is shown in fig. 4.13.

- **Ex. 4.4. Two point charges 1·5 μc and 2·5 μc are placed at a separation of 30 cm. Find electric potential (i) at the mid-point of the line joining the two charges and (ii) at a point 10 cm from the midpoint on the perpendicular besector of the line joining the two charges.**

○ **Solution :** The situation is shown in fig. 4.14.

$$q_1=1\cdot5\mu c,\ q_2=2\cdot5\mu c,\ r_1=r_2=\sqrt{100+225}=18\,\text{cm}$$

(i) The potential at the mid-point O is $V=\frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r}+\frac{q_2}{r}\right)$

$$\therefore \quad V=9\times10^9\times\frac{1}{0\cdot15}(1\cdot5+2\cdot5)\times10^{-6}$$

$$=9\times10^3\times\frac{4}{0\cdot15}=\mathbf{2\cdot4\times10^5 \ volt}$$

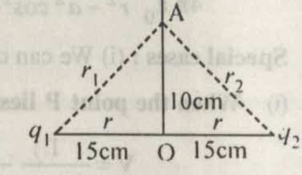


Fig. 4.14

(ii) Potential at A on the perpendicular bisector

$$V_1=9\times10^9\times\left(\frac{q_1}{r_1}+\frac{q_2}{r_2}\right)=9\times10^9\times\frac{1}{0\cdot18}\times4=\mathbf{2\cdot0\times10^5 \ volt}$$

✪ 4.9. Electric potential energy ✪

When we raise a body from the earth's surface, some work is done against the earth's gravity. This work is stored as **potential** energy in the system – earth and the body. If we now release the body, the stored potential energy of the system changes steadily into kinetic energy as the body falls. Ultimately the body comes to rest on earth. Its whole of kinetic energy (equal to stored potential energy at the raised position) is transformed into heat energy in the system – body and earth.

An identical situation exists in electrostatics. We consider two charges at a separation r. To increase the separation between the charges, work done will be positive if the charges are of opposition sign and negative work is done when the charges are of same sign. The energy for this work is stored in the system of two charges as **electric potential energy**: This energy, like all varieties of potential energy, can be transformed into other forms. For example, if q_1 and q_2 are of opposite sign and we release them, they will accelerate towards each other. So here potential energy is converted to kinetic energy of the accelerating charges.

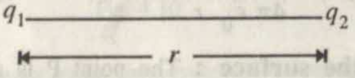


Fig. 4.15

❖ **Definition : We define electric potential energy of a system of point charges as the work required to assemble this system of charges by bringing them from infinite distances.**

Here we assume that the charges were at rest when they were infinitely separated.

➤ *4.9.1. Potential energy of a system of two point charges :*

We consider two point charges q_1 and q_2 at A and B whose position vectors are $\vec{r_1}$ and $\vec{r_2}$ with respect to the origin O of co-ordinate system.

To calculate the electric potential energy of the system of two charges. We assume that the charges be initially at infinite distance from each other. Now, the charge q_1 is brought to its original position A from infinity. For this no work is done. Reason is : no electrostatic force due to any other charge opposes the motion of q_1. Next the charge q_2 is moved to its original position B. Now, its motion is opposed by the electric field produced by the charge q_1 and work has to be done. The work required in moving charge q_2 from infinity to point B in the electric field of charge q_1 is given by

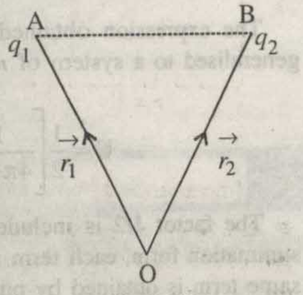


Fig. 4.16

$$W = (\text{Electric potential at B due to } q_1) \times q_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{AB} \times q_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{|\vec{r_1} - \vec{r_2}|}$$

$\therefore$ Mutual potential energy of the two charges is, $U = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1 q_2}{|\vec{r_1} - \vec{r_2}|}$

If the distance $\left|\vec{r_1} - \vec{r_2}\right| = r_{12}$ then, $U = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1 q_2}{r_{12}}$ . . . . (4.16)

The equation (4.16) gives the electric potential energy of a system of two charges.

➤ *4.9.2. Potential energy of a system of three point charges :*

Three point charges q_1, q_2 and q_3 are situated at the points A, B and C at distances r_{12}, r_{13} and r_{23} respectfully as shown in fig. 4.17. Let us calculate the electric potential energy of the system of three charges.

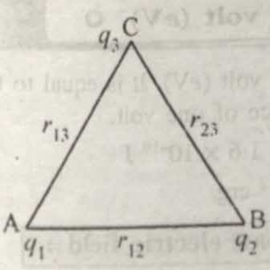


Fig. 4.17

(i) First of all, remove all the three charges to infinite distance from each other. And then move the charge q_1 to the position A. For this, no work is done as no electrostatics force opposes its motion.

(ii) Now, move the charge q_2 from infinity to the point B. In this case, work done is, W_{12} = (Potential at B due to the charge A) × q_2

$$\therefore\ W = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$$

(iii) Finally, the charge q_3 is moved from infinity to the point C. Work done is given by

W′ = (Potential at C due to the charges at A and B) × q_3

$$= \left(\frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{13}} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_{23}}\right) q_3$$

So, the total work done in order to bring all the three charges at their respective positions and hence the electric potential energy of the system of three point charges is given by

$$U = \frac{1}{4\pi\,\varepsilon_0}\left(\frac{q_1\,q_2}{r_{12}} + \frac{q_1\,q_3}{r_{13}} + \frac{q_2\,q_3}{r_{23}}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(4.17)}$$

➤ *4.9.3. Potential energy of a system of n point charges :*

The expression obtained for potential energy of a system of three point charges can easily be generalised to a system of *n* point charges. The potential energy for a system of *n* point charges is

$$U = \frac{1}{2}\left[\frac{1}{4\pi\,\varepsilon_0}\sum_{j=1}^{n}\sum_{i=1}^{n}\frac{q_i\,q_j}{r_{ij}}\right]$$

The factor 1/2 is included for the reason that when potential energy of the system of charges in summation form, each term is counted twice. For example, a term is obtained for $i = 1$ and $j = 2$. The same term is obtained by putting $i = 2$ and $j = 1$.

✪ 4.10. Work done in deflecting an electric dipole in a uniform electric field ✪

Suppose an electric dipole having dipole moment p is placed in an electric field of intensity E. Initially it is parallel to the field. Now, it is rotated by an angle α. The opposing torque acting on the dipole at this instant is $\tau = p\text{E} \sin\alpha$

Next the dipole further is rotated by an infinitesimally small angle $d\alpha$ against the torque acting on it, then work done is $dw = \tau \cdot d\alpha = p\text{E}\sin\alpha \cdot d\alpha$

∴ Total work done for total deflection Q is $\text{W} = \int_0^0 p\text{E}\sin\alpha \cdot d\alpha = p\text{E}\,(1 - \cos\theta)$

This work is stored in the dipole in the form of **potential energy**.

∴ Potential energy of the dipole, $\mathbf{U = \mathit{p}E\,(1 - \cos\theta)}$

✪ 4.11. Unit of electrostatic energy : Electron volt (*e*V) ✪

Electrostatic energy of a charged particle in an electric field is electron volt (*e*V). It is equal to the energy acquired by an electron when allowed through a potential difference of one volt.

∴ 1 *e*V = 1 electronic charge × 1 volt = $1{\cdot}6 \times 10^{-19}$ coulomb × volt = $1{\cdot}6 \times 10^{-19}$ J

∴ In S.I., 1 *e*V = $1{\cdot}6 \times 10^{-19}$ J and in c.g.s. system, 1 *e*V = $1{\cdot}6 \times 10^{-12}$ erg

➡ Kinetic energy of an electron (or a charged particle) in an electric field :

When a charged particle is released in an electric field, it experiences a force and moves with increasing velocity. So its kinetic energy gradually increases.

Let a charged particle having charge q coulomb passes through a potential difference of V volt. Work done on the particle is $\text{W} = \text{V}q$

∴ Increase of kinetic energy $\text{E}_\text{K} = \text{W} = \text{V}q$

$$\therefore \frac{1}{2}mv^2 = Vq \quad [v = \text{velocity acquired by the particle}]$$

$$\therefore v = \sqrt{\frac{2\,Vq}{m}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.18)$$

If the charged particle be electron, then

$$v = \sqrt{\frac{2\,Ve}{m}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.19)$$

e/m is called specific charge of electric

Its value is **$e/m = 1{\cdot}77 \times 10^{11}$ C/kg**

✪ 4.12. Equipotential surface ✪

❖ **Definition : Any two dimensional surface, which has same electrostatic potential at every point on the surface, is called an Equipotential surface.**

Such a surface may be a surface of a body or simply a surface in space. As for example, the surface of a charged conductor is an equipotential surface.

The equipotential surfaces can be drawn, through any region, in which there is electric field. An equipotential surface is obtained by joining all the points which are at the same potential in the electric field.

Properties of equipotential surface :

Equipotential surfaces are found to posses the following properties :

- (i) **Two equipotential surfaces never intersect :**

Any two equipotential surfaces cannot intersect. Because, if they intersect, at the point of intersection, there will be two values of electric potential. Evidently, it is not possible. So, any two equipotential surface cannot intersect.

- (ii) **No work is done in moving a test charge over an equipotential surface :**

If fig. 4.18, S is an equipotential surface. A and B are two close points on the surface. Assume that electric field $\vec{E}$ makes an angle θ with the equipotential surface. Now, work done to move a charge q from A to B along the surface is

$W = E\cos\theta\, q\,(AB) \ldots\ldots\ldots (i)$

Also work done is given by

$W = (V_A - V_B)\, q \ldots\ldots\ldots (ii)$

When V_A and V_B are the potentials at A and B respectively.

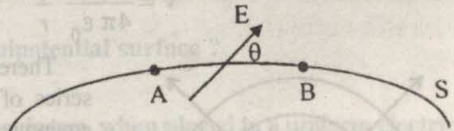


Fig. 4.18

$\therefore qE\cos\theta \cdot (AB) = (V_A - V_B)\, q$

But $V_A = V_B$, since the points A and B are on an equipotential surface

$\therefore\ qE\cos\theta\,(AB) = 0 \qquad \therefore \cos\theta = 0 \quad [\because q\,E(AB) \neq 0]$

$\therefore\ \theta = 90^\circ.$

If mean that electric field is perpendicular to the equipotential surface.

- (iii) **The equipotential surfaces can be used to distinguish a strong field and a weak field :**

The equipotential surfaces are close together in strong electric field and they are far apart in weak field. It can be explained in the following way.

We know that $E = -\frac{dV}{dr}$

$$\therefore\ dr = \frac{-dV}{E}$$

For equal change in value of dV i.e. dV = constant, we get $dr \propto \frac{1}{E}$

It means that the spacing between the equipotential surfaces will be smaller in the regions, where the electric field is stronger and vice versa.

- (iv) **The direction of the electric field is obtained from the idea of equipotential surface :**

We have the relation $E = -\frac{dV}{dr}$

The negative sign implies that the electric field is directed from stronger potential point to weaker potential point.

So, the direction of the electric field is from the equipotential surfaces which are close to each other to those which are more and more away from each other, provided all the surfaces are drawn for the same in value of dV.

➤ *4.12.1. Drawing of equipotential surface in different cases :*

We draw equipotential surfaces in the following cases :

- (i) **For an isolated point charge :** We consider an isolted point charge q. At a distance r from the charge, the potential is $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$

$$\therefore\ V \propto \frac{1}{r}$$

If means that the potential at all points equidistant from the charge is same.

So, a sphere of a radius r with centre at q is an equipotential surface having potential

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

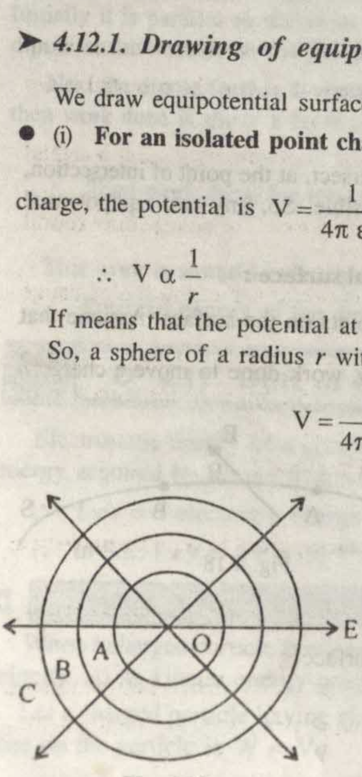


Fig. 4.19

Therefore, for a point charge, equipotential surfaces will be a series of concentric spheres with the charge at the centre. Three equipotential surfaces (A, B, C) for a point charge at O are shown [fig. 4.19]. The electric field is perpendicular to the equipotential surfaces.

- (ii) **For a uniform electric field :** In a uniform electric field, its intensity and direction is same at every point in the field. Such a field is represented by a set of equidistant parallel arrows. Now, for a uniform electric field $\frac{dV}{dr}$ = constant.

$$\therefore\ dV \propto dr$$

Here the equipotential surfaces (A′, B′, C′) are parallel to each other [Fig. 4.20]. They will be equidistant from each other when they differ by the same value of *d*V i.e. when potential of A′ – Potential of B′ = Potential of B′ – Potential of C′.

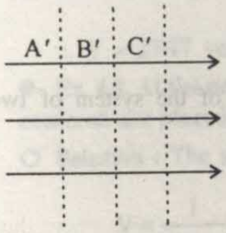
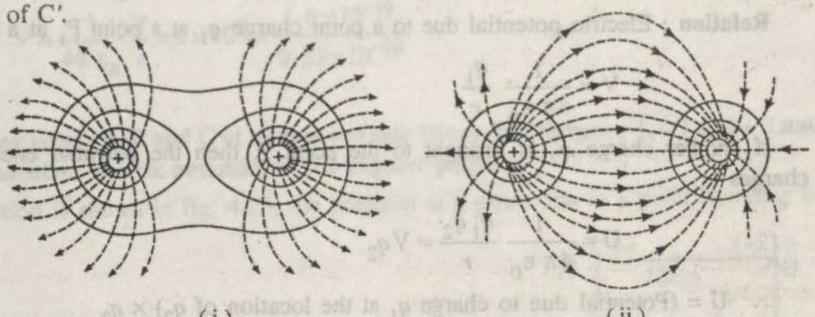


Fig. 4.20



(i)

(ii)

Fig. 4.21

● (iii) **For a system of two point charges :** Fig. 4.21(i) and (ii) represent the equipotential surfaces for two equal positive charges and for a pair of equal and opposite charges respectively [Continuous curved lines]. Dotted lines represent to the electric lines of force.

● ● Short Answer Type Questions (with answers) ● ●

● **Question 1. If the electric field E is zero at a point, the electric potential V most also be zero at that point. Is this statement's true ? Give examples.**

○ **Ans.** If the electric field E at a point is zero, the electric potential V is not necessarily zero at the point.

For example : **(i)** The electric field is zero inside a charged hollow conductor. But the potential is not zero in that region. It is a region of constant potential.

(ii) The electric field is zero exactly midway between two equal charges of same sign, but potential at this point is twice that due to a single charge.

● **Q. 2. A charge 5 μc is placed at a point. What is the work required to carry 1C of charge once round it in a circle of radius 5 cm ?**

○ **Ans.** Work done is zero. **Reason :** All the points on the circular path are at same potential. Change of potential being zero, work done is also zero.

● **Q. 3. What is the shape of the equipotential surfaces for a uniform electric field ?**

○ **Ans.** For a uniform electric field, equipotential surfaces are planes at right angle to the direction of electric field.

● **Q. 4. What is the direction of electric field w.r.t. an equipotential surface ?**

○ **Ans.** It is along the normal to the equipotential surface.

● **Q. 5. When is the potential energy of an electric dipole maximum, when placed in a uniform electric field ?**

○ **Ans.** The potential energy of an electric dipole is maximum, when it is aligned antiparallel to the electric field.

● **Q. 6. Distinguish between electric potential and potential energy and give the relation between them.**

○ **Ans.** The **electrostatic potential** at a point in an electric field in the work done to bring a unit positive charge from infinity upto the point against the force due to the electric field. So, $V = \frac{W}{q_0}$.

The electric potential energy of a system of point charges is the work done to bring the charges constituting the system to their respective location from infinity.

Relation : Electric potential due to a point charge q_1 at a point P_1 at a distance r is given by

$$V = \frac{1}{4\pi \varepsilon_0} \frac{q_1}{r}$$

If another charge q_2 is brought to the point P, then the potential energy of the system of two charges is

$$U = \frac{1}{4\pi \varepsilon_0} \frac{q_1 q_2}{r} = V q_2$$

∴ U = (Potential due to charge q_1 at the location of q_2) × q_2

• • Numerical Examples • •

- **Ex. 4.5. Two point charges + 12 μC and 8 μC are initially at a separation of 10 cm. Find the work done in bringing them 4 cm closer.**

○ **Solution :** Work done = Final potential energy, U_f – initial potential energy, U_i

$$\therefore W = U_f - U_i$$

Now, $U_i = \frac{1}{4\pi \epsilon_0} \frac{q_1 q_2}{r_1} = 9\times 10^9 \times \frac{12\times 8\times 10^{-12}}{0\cdot 1} \text{J} = 8\cdot 64 \text{ J}$

and $U_f = 9\times 10^9 \times \frac{q_1 q_2}{r_2} = 9\times 10^9 \times \frac{96\times 10^{-12}}{0\cdot 06} = 14\cdot 4 \text{ J}$

∴ Work done, W = 14·4 – 8·64 = **5·76 J**

- **Ex. 4.6. Two large metal plates A and B are kept horizontally at a separation of 2·5 cm [Fig.4.22]. The plate B are kept at a potential of 2500 V relative to the plate A. An electron is liberated from a point near the middle of the plate A. After what time, the electron may reach the plate ? Electronic charge = 1·6 × 10^{-19}C and mass of electron = 9·1 × 10^{-31} kg.**

○ **Solution :** The electric field between the plates will be taken to be uniform. The electric field between the plates is E = V/*d*.

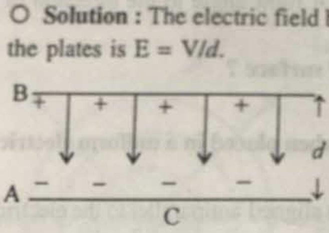


Fig. 4.22

Force on the electron, F = E*e*

$$\therefore \text{Acceleration of the electron, } a = \frac{F}{m} = \frac{Ee}{m} = \frac{Ve}{d\cdot m}$$

$$\therefore a = \frac{2500\times 1\cdot 6\times 10^{-19}}{2\cdot 5\times 10^{-2}\times 9\cdot 1\times 10^{-31}} = 1\cdot 76\times 10^{16} \text{ ms}^{-2}$$

Let the electron takes a time '*t*' to reach the plate B.

$$\therefore \alpha = \frac{1}{2} at^2 \qquad \therefore t = \sqrt{\frac{2d}{a}} \qquad \therefore t = \sqrt{\frac{2\times 2\cdot 5\times 10^{-2}}{1\cdot 76\times 10^{16}}} = 1\cdot 70\times 10^{-9} \text{ sec}$$

- **Ex. 4.7. What is the potential at a point distant 0·53 A° from a proton of charge $q = +1{\cdot}6 \times 10^{-19}$ C.**

○ **Solution :** Potential $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} = 9\times10^9 \times \frac{1{\cdot}6\times10^{-19}}{0{\cdot}53\times10^{-10}}$

$\therefore$ V = 27·17 **volts**

- **Ex. 4.8. At the angular points A, B and C of a square of side 10 cm the charges + 1, – 2 and + 3 nano coulomb are placed. Calculate electric potential at the angular point D.**

○ **Solution :** The situation is shown in fig. 4.23, the potential at a point due to a point charge q is

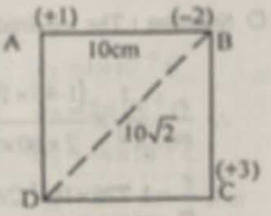


Fig. 4.23

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

$\therefore$ Total potential at D due to the charges at A, B and C of the square

ABCD is $V = 9\times10^9\left[\frac{1\times10^{-9}}{10\times10^{-2}} + \frac{-2\times10^{-9}}{10\sqrt{2}\times10^{-2}} + \frac{3\times10^{-9}}{10\times10^{-2}}\right]$

$$= 90\left[1-\sqrt{2}+3\right] = 232{\cdot}74 \text{ volt}$$

- **Ex. 4.9. A particle of mass 50 mg and carrying a charge 5×10^{-9}C is moving towards a static charge 10^{-8}C with an initial velocity of 0·5 m/s. Calculate the nearest distance of approach of the particle.**

○ **Solution :** Let the particle can approach upto a distance 'd' towards the static charge.

So, increase of potential energy = loss of kinetic energy

$$\therefore \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{d} = \frac{1}{2}mv^2$$

$$\therefore 9\times10^9 \times \frac{5\times10^{-17}}{d} = \frac{1}{2}\times5\times10^{-5}\times0{\cdot}25 \qquad \therefore d = 7{\cdot}2\,\text{cm}$$

$q_1 q_2 = 5\times10^{-17}\,\text{C}^2$
$m = 50\times10^{-6}$ kg
$v = 0{\cdot}5$ m/s

- **Ex. 4.10. A particle of mass 40 mg carries a charge 5×10^{-9}C. It is moving towards a charge 10^{-8}C. When the particle is at a distance of 10 cm, it velocity is 50 cm/s. Calculate its least distance of approach towards the stationary charge ?**

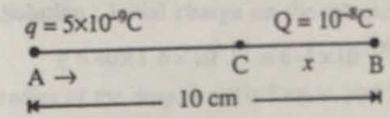


Fig. 4.24

○ **Solution :** Initially the particle is at A. As initial distance from the stationary charge Q = 10^{-8} C at B is 10 cm. So, AB = 10 cm. Let the particle can approach upto C when it stops momentarily. Let CB = x cm.

Now, from the principle of conservation of energy.

Sum of PE and KE at A = sum of PE and KE at B

$$\therefore \frac{1}{2}mv^2 + \frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq}{(AB)} = 0 + \frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq}{(CB)}$$

$$\therefore \frac{Qq}{4\pi\varepsilon_0}\left[\frac{1}{CB} - \frac{1}{AB}\right] = \frac{1}{2}mv^2$$

$Qq = 5\times10^{-17}\,\text{C}^2$
$m = 4\times10^{-5}$ g
$\text{CB} = x\times10^{-2}$ m
AB = 0·1 m
$v = 0{\cdot}5$ m/s

$$\therefore\ 9\times10^{9}\times5\times10^{-17}\left[\frac{100}{x}-\frac{100}{10}\right]=\frac{1}{2}\times4\times10^{-5}\times(0\cdot5)^{2}$$

$$\text{or, } 45\times10^{-6}\left(\frac{1}{x}-\frac{1}{10}\right)=0\cdot5\times10^{-5}\quad\therefore\ \frac{1}{x}-\frac{1}{10}=\frac{1}{9}$$

$$\therefore\ x=4\cdot74\ \text{cm}$$

● **Ex. 4.11. When an electron is allowed to pass through a potential difference of 60 KV from rest, it acquires a velocity of 1·46 × 10^{10} cm/s. Calculate specific charge of electron. [J. E. E. 1994]**

○ **Solution :** The required relation is $\frac{1}{2}mv^{2}=eV$

$$\therefore\ \frac{e}{m}=\frac{v^{2}}{2V}=\frac{\left(1\cdot46\times10^{8}\right)^{2}}{2\times60\times10^{3}}\quad [v=1\cdot46\times10^{8}\ \text{m/s},\ V=60\times10^{3}\ \text{volt}]$$

$$\therefore\ \frac{e}{m}=\mathbf{1\cdot776\times10^{11}\,C/kg}$$

● **Ex. 4.12. A positively charged oil drop is held stationary between two parallel horizontal plates. The spacing between the plates is 1 cm. the mass of the oil drop is 10^{-11}g and its charge 3·2 × 10^{-9}C. What is the potential difference between the plates ? If the polarity between the plates is reversed, calculate the acceleration of the oil drop ? [I. I. T.]**

○ **Solution :** The oil drop is floating between the plates. So, electric force on the oil drop = weight of the oil drop

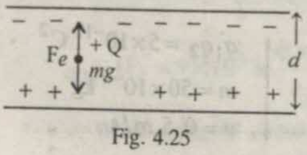


Fig. 4.25

$$\therefore\ EQ=mg$$

$$\text{or, } \frac{V}{d}Q=mg\qquad\therefore\ V=\frac{mg\,d}{Q}$$

$$\therefore\ V=\frac{10^{-14}\times9\cdot8\times10^{-2}}{3\cdot2\times10^{-19}}$$

$m=10^{-14}$ kg
$g=9\cdot8\,\text{m/s}^{2}$
$d=10^{-2}$ m
$Q=3\cdot2\times10^{-19}$ C

$$\therefore\ V=\mathbf{3\cdot062\times10^{3}}\ \textbf{volt}$$

Second Part : When the polarity of the plates is reversed, total force on the electron is F = mg+EQ

$$\therefore\ \text{Its acceleration, } a=\frac{F}{m}=g+\frac{EQ}{m}=g+\frac{VQ}{m\cdot d}$$

$$\therefore\ a=9\cdot8+\frac{3062\times3\cdot2\times10^{-19}}{10^{-14}\times10^{-2}}=\mathbf{19\cdot6\ m/s^{2}}$$

● **Ex. 4.13. A small sphere of mass 1 gm and carrying a charge 10^{-6}C passes from the point A to another point B. Potential at A is 6000V and that at B is zero. What should be the velocity of the sphere at A so that its velocity at B is 20 ms^{-1} ?**

○ **Solution :** Change of kinetic energy = work done

$$\therefore\ \frac{1}{2}m\left(v^{2}-u^{2}\right)=Vq$$

$v=20$ m/s
$u=$?
V = 6000 volt
$q=10^{-6}$C
$m=10^{-14}$ kg

$$\therefore\ v^2-u^2=\frac{2Vq}{m}=\frac{2\times 6000\times 10^{-6}}{10^{-3}}=12$$

$$\therefore\ 400-u^2=12 \qquad \therefore\ u^2=388 \qquad \therefore\ u=\mathbf{19{\cdot}7\ m/s}$$

∴ Required velocity = **19·7 m/s**

- **Ex. 4.14. Three charges 1 μc, 2 μc and – 4 μc are placed at the three angular points of an equilateral triangle of side 10 cm. Calculate electric potential energy of the system.**

○ **Solution :** Total potential energy, $U=\frac{1}{4\pi\,\varepsilon_0}\left[\frac{q_1\,q_2}{r_{12}}+\frac{q_2\,q_3}{r_{23}}+\frac{q_3\,q_1}{r_{13}}\right]$

$$\therefore\ U=9\times 10^9\times\frac{1}{r^{12}}(q_1q_2+q_2q_3+q_3q_1)$$

$$=\frac{9\times 10^9}{0{\cdot}1}\times 10^{-12}\ (1\times 2-2\times 4-1\times 4)=9\times 10^{-12}\ (2-8-4)$$

$$\therefore\ U=\mathbf{-\ 0{\cdot}9\ J}$$

$q_1=1\times 10^{-6}\ \text{C}$
$q_2=2\times 10^{-6}\ \text{C}$
$q_3=-4\times 10^{-6}\ \text{C}$
$r_{12}=10^{-1}\ \text{m}=0{\cdot}1\ \text{m}$

- **Ex. 4.15. Three charges *q*, 2*q* and 8*q* are placed along a straight line of length of 9 cm. Where the charges be placed so that potential energy of the system is minimum. Calculate the electric field intensity at the site of *q* due to the other two charges.** **[I. I. T.]**

○ **Solution :** Assume that the charge 2*q* be placed between the other two charges. Let the distance between *q* and 2*q* is *x* metre. The distance between 2*q* and 8*q* is (0·09 – *x*) m.

The potential energy of the whole system

$$U=9\times 10^9\left[\frac{q\times 2q}{x}+\frac{2q\times 8q}{0{\cdot}09-x}+\frac{q\times 8q}{0{\cdot}09}\right]=18q^2\times 10^9\left[\frac{1}{x}+\frac{8}{0{\cdot}09-x}+\frac{4}{0{\cdot}09}\right]$$

For minimum value of $U=\frac{dU}{dx}=0$

$$\therefore\ -\frac{1}{x^2}+\frac{8}{(0{\cdot}09-x)^2}=0 \qquad \therefore\ x=0{\cdot}0215\,\text{m}=\mathbf{2{\cdot}15\,cm}$$

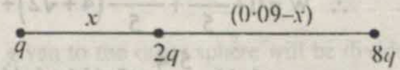


Fig. 4.26

- **Ex. 4.16. A spherical oil drop of radius 10^{-4} cm has on it a charge of 40 electrons. Calculate energy required to place an additional electron on the drop. Charge on an electron, $e=1{\cdot}6\times 10^{-19}$C.**

○ **Solution :** Initial charge on the spherical drop, *q* = 40*e*

$$q=40\times 1{\cdot}6\times 10^{-19}=6{\cdot}4\times 10^{-18}\text{C}$$

radius of the drop, $r=10^{-4}$ cm $=10^{-6}$ m

Potential on the surface of the drop, $V=\frac{1}{4\pi\,\varepsilon_0}\times\frac{q}{r}=9\times 10^9\times\frac{6{\cdot}4\times 10^{-18}}{10^{-6}}$

$$\therefore\ V=5{\cdot}76\times 10^{-2}\ \text{volt}$$

Hence, the energy required to place an additional electron on the drop is

$$U=V\cdot e=5{\cdot}76\times 10^{-2}\times 1{\cdot}6\times 10^{-19}=\mathbf{9{\cdot}22\times 10^{-21}\,J}$$

- **Ex. 4.17. Four point charges 2 μc, 4 μc, – 6 μc and 8 μc are placed on the four angular points of a square of side 5 cm. Calculate potential energy of the system.**

○ **Solution :** First we assume that all the charges are at infinity. Now, the charge $q_1 = 2\ \mu c$ is brought to A. For this work done $W_1 = 0$, as there is no electric field.

Next the charge $q_2 = 4\ \mu c$ is brought from infinity upto to pooint B.

Work done is W_2 = (Potential at B due to the charge at A) × q_2

$$= 9\times10^9 \times \frac{q_1 q_2}{\cdot05} = 9\times10^9 \times \frac{8\times10^{-12}}{5\times10^{-2}} = \frac{72}{5}\times10^{-1} = \frac{7\cdot2}{5}\ \text{J}$$

Next the charge $q_3 = -6\ \mu c$ is brought from infinity upto C.

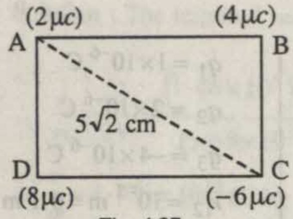


Fig. 4.27

Work done, W_3 = (potential at C) × $q_3 = 9\times10^9 \left(\frac{q_1}{0\cdot05\sqrt{2}} + \frac{q_2}{0\cdot05}\right)\times q_3$

$$= 9\times10^9 \times \frac{1}{5}\left(\frac{2}{\sqrt{2}}+4\right)\times(-6)\times10^{-12}\times10^2$$

$$= -\frac{54}{5}\times10^{-1}\times\left(\sqrt{2}+4\right) = -\frac{5\cdot4}{5}\left(4+\sqrt{2}\right)$$

Finally the charge $q_4 = 8\ \mu c$ is brought from infinity upto D

∴ Work done, W_4 = (Potential at D) × $q_4 = 9\times10^9\left(\frac{2\times10^{-6}}{0\cdot05} + \frac{4\times10^{-6}}{0\cdot05\sqrt{2}} \frac{6\times10^{-6}}{0\cdot05}\right)\times q_4$

$$= \frac{9}{5}\times10^5\left(2+\frac{4}{\sqrt{2}}-6\right)\times8\times10^{-6} = \frac{7\cdot2}{5}\left(-4+2\sqrt{2}\right)$$

Total work done, $W = W_1 + W_2 + W_3 + W_4$

$$\therefore\ W = 0 + \frac{7\cdot2}{5} + \frac{-5\cdot4}{5}\left(4+\sqrt{2}\right) + \frac{7\cdot2}{5}\left(-4+2\sqrt{2}\right)$$

$$= 1\cdot44 - \frac{5\cdot4}{5}\times5\cdot414 + 1\cdot44\,(-4+2\cdot828) = \mathbf{-6\cdot10\ J}$$

● **Ex. 4.18. An infinite number of charges, each equal to q are placed along the X-axis at $x = 1$, $x = 2$, $x = 4$, $x = 8$, ... and so on. Calculate the potential and the electric field at the point $x = 0$ due to this set of charges. What will be the potential and electric field if in the above set of charges, the alternate charges have opposite signs.**

○ **Solution :** The situation is shown in fig. 4.28.

(i) The electric potential at $x = 0$ due the set of charges each equal to $+q$ is

$$V = \frac{1}{4\pi\,\varepsilon_0}\left[\frac{q}{1}+\frac{q}{2}+\frac{q}{4}+\frac{q}{8}+\ldots\ldots\right]$$

$$= \frac{1}{4\pi\,\varepsilon_0}\cdot q\left[\frac{1}{1}+\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\ldots\ldots\right]$$

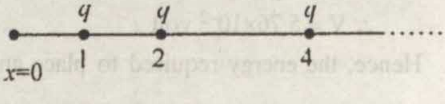


Fig. 4.28

$$= \frac{q}{4\pi\,\varepsilon_0}\left(\frac{1}{1-\frac{1}{2}}\right) = \frac{1}{2}\cdot\frac{q}{4\pi\,\varepsilon_0} = \frac{1}{4\pi\,\varepsilon_0}\,\frac{q}{2}$$

Since all the charges are of same sign, the electric intensities at $x = 0$ due to all the charges are in the same direction.

Electric intensity at $x = 0$ due to all the charges is given by

$$E=\frac{1}{4\pi\,\varepsilon_0}\left[\frac{q}{1^2}+\frac{q}{2^2}+\frac{q}{4^2}+\frac{q}{8^2}+\ldots..\right]=\frac{q}{4\pi\,\varepsilon_0}\left[\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{2^4}+\frac{1}{2^6}+\ldots..\right]$$

$$=\frac{q}{4\pi\,\varepsilon_0}\left(\frac{1}{1-\frac{1}{4}}\right)=\frac{1}{4\pi\,\varepsilon_0}\left(\frac{4q}{3}\right)$$

(ii) Let us now consider a case when the charges are of opposite sign. In this case, the potential at $x = 0$ is given by

$$V'=\frac{1}{4\pi\,\varepsilon_0}\left[\frac{q}{1}-\frac{q}{2}-\frac{q}{2^2}-\frac{q}{2^3}+\ldots..\right]=\frac{q}{4\pi\,\varepsilon_0}\left(\frac{1}{1-\left(-\frac{1}{2}\right)}\right)=\frac{1}{4\pi\,\varepsilon_0}\,\frac{2q}{3}$$

and the electric intensity at $x = 0$ is given by

$$E=\frac{1}{4\pi\,\varepsilon_0}\cdot q\left(\frac{1}{1^2}-\frac{1}{2^2}-\frac{1}{2^4}-\frac{1}{2^6}+\ldots..\right)=\frac{q}{4\pi\,\varepsilon_0}\left(\frac{1}{1-\left(-\frac{1}{4}\right)}\right)=\frac{1}{4\pi\,\varepsilon_0}\,\frac{4q}{5}$$

● **Ex. 4.19. The outer one of two concentric metallic spheres having radii *a* and *b* ($b > a$) has a charge *q*. If the inner sphere is earthed, what is the charge on it ?**

○ **Solution :** Here the inner sphere is earthed. So the charge q given to the outer sphere will be divided into two parts – one part q_1 (say) will remain on the outer surface of B and rest q_2 will remain on the inner surface of B. So $q = q_1 + q_2$.

Now, due to induction, the inner sphere will get a charge $-q_2$.

Potential of the sphere A is V_A = Potential due to its own charge + Potential due to the sphere B

$$\therefore\ V_A=\frac{1}{4\pi\,\varepsilon_0}\,\frac{-q_2}{a}+\frac{1}{4\pi\,\varepsilon_0}\,\frac{q}{b}$$

But $V_A = 0$ as it is connected to the earth.

$$\therefore\ \frac{1}{4\pi\,\varepsilon_0}\left(\frac{-q_2}{a}+\frac{q}{b}\right)=0$$

$$\therefore\ -q_2=-\frac{a}{b}q$$

So, the charge on the inner sphere is $\dfrac{-aq}{b}$.

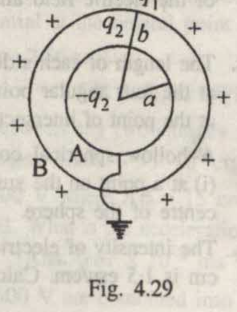


Fig. 4.29

• EXERCISE •

A. Short answer type questions :

1. Show graphically, how the potential at a point due to a point charge varies with distance.
2. What is meant by potential and potential energy of a charged conductor ?
3. What is meant by potential at a point is 1 volt ?
4. Intensity at a point is zero. But potential is not zero. Is it possible ? Give example.
5. What is meant by line integral of an electric field ? How it is related to potential difference ?
6. In an electric field potential V is zero, but intensity is not zero. Is it possible ? Give example.
7. How much work is done to transfer a positive charge from one point to another of an equipotential surface?
8. A charge Q is located at a point. A unit positive charge is taken round a circular path of radius r with the charge Q at the centre of the circle. How much work is done ?
9. What is meant by electric potential at a point ? Define its SI unit.
10. What is represented by line integral of electric field ?
11. Obtain a relation between electric potential and intensity.
12. When is an equipotential surface ? Why two equipotential surfaces do not intersect ?
13. Define 1 eV. Obtain its relation with joule.
14. Draw equipotential surfaces for the following cases :
    (i) due to a point charge (ii) due to an electric dipole.

B. Essay type questions :

1. Define electric potential at a point in the electric field. **[W. B. H. S. 2001, 2005]**
2. Find the expression for electric potential at a point due to a point charge. **[W. B. H. S. 2001]**
3. Derive an expression for the Kinetic energy acquired by a charged body in an electric field. Define one electric-volt. What quantity does it represent ?
4. What is meant by equipotential surface ? Draw equipotential surfaces for a uniform electric field. Show that lines of force are always perpendicular to the equipotential surfaces.
5. Prove the electrical potential difference between two points is independent of path followed in moving the unit positive charge from an point to another.
6. Deduce an expression for the mutual potential energy of a system of three charges.

C. Simple numerical problems :

1. At each of the four corners of a square of side 10 cm positive charge of 20 esu each are placed. Find the intensity of the electric field and potential at the point of intersection of the two diagonals.
   [**Ans.** $E = 0$; $V = 8\sqrt{2}$ stat volt] **[W. B. H. S. 2005, 1992]**
2. The length of each side of a square is 10 cm. Four point charges + 100, – 50, + 20 and – 30 esu are placed at the four angular points A, B, C and D of the square. Calculate the intensity of electric field and potential at the point of intersection of the two diagonals. [**Ans.** $E = 2{\cdot}26$ esu ; $V = 14{\cdot}14$ esu] **[W. B. H. S. 2000]**
3. A hollow spherical conductor of radius 10 cm is given a charge of +10 statcoulomb. Calculate potential (i) at a point on the surface of the sphere (ii) at a point inside the sphere and (iii) at a point 25 cm from the centre of the sphere. [**Ans.** (i) 1 esu (ii) 1 esu (iii) 0·4 esu.]
4. The intensity of electric field at a point between two parallel conducting plates separated by a distance of 2 cm is 1·5 esu/cm. Calculate difference of potential between the plates in volt.
   [**Ans.** 900 volt] **[W. B. H. S. 1980]**

5. A point charge of $\frac{1}{3}\times10^{-7}$ C is located at a point. How much work is done to bring a charge +1C from infinity upto a point distant 20 cm from the point charge. [**Ans.** 1500 J]

6. Potential at a point situated at a distance of 20 cm from a point charge is 100 V. What is the electric field at the point ? [**Ans.** 500 Vm^{-1}]

7. The separation between two parallel plates is 2 cm and potential difference between the plates is 200 V. Calculate the intensity of the electric field at a point between the plates. [**Ans.** $1{\cdot}0 \times 10^4$ Vm^{-1}]

8. The intensity of a uniform electric field is 10^7 Vm^{-1}. Calculate the work done to displace a charge of 10^{-9} C through a distance of 20 cm against the electric field. [**Ans.** 2×10^{-3} J]

9. The potential at a point in an electric field is 30 volt. Calculate the work done to bring a charge + 5μ*c* from infinity upto the point. [**Ans.** 15×10^{-5} J]

10. To transfer a charge of 2×10^{-9}C from a point P at potential – 3000V to another point Q, 5×10^{-5} J of work is done. Calculate the potential at Q. [**Ans.** 22000 volt]

11. A particle of mass 50 mg and carrying + 5 stat coulomb charge is moving with a velocity of 10 cm/s. It is moving towards a fixed charge of + 10 stat coulomb. Calculate its nearest distance of approach. [**Ans.** 0·2 m]

12. 8 identical spherical oil drops each of diameter 2 mm and each carrying a charge 5 microstatcoulomb are combined to form a larger drop. Express the potential on the surface of the larger drop in volt.
[**Ans.** 0·06 volts] [**J. E. E. 1987**]

13. Two parallel metal plates are kept at a separation of 1·5 cm. One plate is at 3000 volt and the other one is maintained at 300 volt. Calculate the electric field intensity at a point between the plates. [**Ans.** $1{\cdot}8 \times 10^5$ Vm^{-1}]

14. A charge + 10 μ*c* is placed at a point. What is the potential at a distance of 10 cm from the charge ? How much work is required to displace a charge from an initial distance of 25 cm to a distance of 5 cm from the charge ? [**Ans.** 9×10^5V ; 18×10^5 J]

15. At the angular points of a cube of side 4 cm charges each equal to 5 μ*c* is placed. Calculate the potential at the centre of the cube. [**Ans.** $10{\cdot}4 \times 10^6$ V]

16. At the point A, B and C of the triangle ABC, the charges 5 μ*c*, 6 μ*c* and 9 μ*c* are situated. Calculate the potential at the centre of the triangle. [**Ans.** $31{\cdot}17 \times 10^5$ V]

17. Two charges –20 esu and +180 esu are placed at a separation of 50 cm. At which point joining the charges (i) electric intensity is zero and (ii) electric potential is zero.

[**Ans.** (i) 50 cm from the second charge on the opposite side of the first charge electric intensity is zero.
(ii) In between the two charges at a distance of 10 cm from the first charge electric potential is zero.]

18. An electron initially at rest is allowed to pass through a potential difference of 180 volt. Calculate the maximum velocity acquired by the electron. Specific charge on electron is $1{\cdot}77\times10^{11}$ C/kg.

[**Ans.** 8×10^6 m/s] [**W. B. H. S. 1991**]

19. Two charges + 44 esu and + 81 esu are at a separation of 100 cm in air. Obtain the position of the neutral point in the electric field produced by the two charges. What is the electric potential at the neutral point ?
[**Ans.** On the line joining the two charges at a distance of 43·75 cm from the first charge ; 2·56 esu]

[**W. B. H. S. 1995**]

20. Three point charges q_1, $2q$ and $-4q$ are placed in air and distance of separation between any two charges is 10 cm. Here q = 300 esu. What is the mutual potential energy of the system of charges ?[**Ans.** -9×10^4 erg]

21. In a uniform electric field the potentials at two point A and B are 2000 V and 500 V where AB = 15 cm. A particle of mass 0·01 kg and having charge $0{\cdot}67 \times 10^{-8}$C starts from A towards B. What is the acceleration of the particle ? [**Ans.** $0{\cdot}67 \times 10^{-2}$ ms^{-2}]

22. Two identical drops of water each having charges 3×10^{-9}C and surface potential 500 V are combined into a single drop. What is the surface potential of the larger drop ? [**Ans.** 794 V]

23. The potential in a region follows the equation $V = 2x + 3y - z$. Calculate potential gradient and electric field intensity. **[Ans.** gradient $2\hat{i}+3\hat{j}-\hat{k}$; $E = -2\hat{i}-3\hat{j}+\hat{k}$]

24. Two positive charges 10 μc and 15 μc are placed in air at a distance of 10 cm. What is the work done to bring them 5 cm closer ? **[Ans.** 13·5 J] **[J. E. E. 2005]**

25. The radius of a charged water drop is 1 mm and its charge 10^{-10}C. When eight such water drops are made to coahesce, what will be its surface potential ? **[Ans.** 3600 V]

26. Four point charges 2 μc, 4 μc, – 6 μc and 8 μc are placed at the angular points of a square of side 5 cm. Calculate mutual potential energy of the system. **[Ans.** – 6·10 J]

27. Two positive charges $Q_1 = 4$ μc and $Q_2 = 2$ μc are placed in air at a distance of 20 cm. How much work will be done to bring them 10 cm closer ? **[Ans.** 0·36 J]

28. A particle of mass 5 g and carrying a charge 12×10^{-9}C travels a distance of 300 cm under a potential difference of 500 V. If the initial velocity of the particle be zero, calculate its acceleration and final velocity. **[Ans.** 4×10^{-4} m/s^2 ; 4·9 m/s]

29. The point charges 6, 12 and 24 esu are placed at the three angular points A, B and C of a square ABCD of side 20 cm. How much work is done to transfer unit positive charge from the point D to the centre of the square ? **[Ans.** 1·055 erg]

30. Two charges + q and + $4q$ are placed at a separation of x. A third charge is placed so that the system remains in equilibrium. What is the position, nature and magnitude of the third charge ?

[Ans. $Q = -\frac{4q}{9}$, at a distance of $x/3$ from the smaller charge]

◢ D. Harder numerical problems :

1. A particle of mass 1 gm and carrying 1 esu charge is released on the perpendicular bisector of AB, where AB is the line joining the two charges each 5 esu. AB = 8 cm. Initially the particle was at a distance of 3 cm from AB and velocity towards AB was $\frac{2}{3}$ cm/s. What is its nearest distance of approach ? What will happen after that ? **[Ans.** 2·06 cm ; then it will return along the same path] **[J. E. E. 2002]**

2. Two point charges each 1000 esu are placed at two points A and B at a separation of 200 cm. From the midpoint C of AB a particle carrying a charge –1000 esu is projected with a kinetic energy 10^4 erg. How far it can go before it returns ? **[Ans.** 1000 $\sqrt{3}$ cm]

3. At the point A of a square of side 10 cm a charge $\frac{2}{3}\times10^{-7}$ C and at the centre another charge $-\frac{1}{3}\times10^{-7}$ are placed. How much work has to be done to transfer a charge 5×10^{-19}C from the point C to the point B. **[Ans.** $87{\cdot}9 \times 10^{-7}$ J] **[I. I. T.]**

4. Two particles each carrying equal amount of charge first pass through equal potential difference and then enters a uniform magnetic field. If the two particles describe circles of radii R_1 and R_2 what is the ratio of the mass of two particles. **[Ans.** $M_A / M_B = (R_1 / R_2)^2$] **[I. I. T.]**

5. Two particles each with a charge $+ 5 \times 10^{-5}$C are located at the points A and B at a separation of 6 m [Fig. 4.30]. Another particle carrying equal and opposite charge is released from C along COD with a kinetic energy 4J and OC = 4m. The third particle goes upto D and then returns. Calculate OD. **[Ans.** OD = 8·485 m]

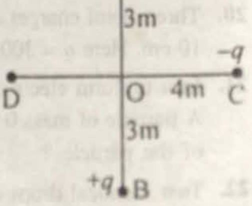


Fig. 4.30

6. Four charges + q, + q, – q and – q are placed at the angular points of a square ABCD of side 'a'. Calculate potential and intensity at the centre O of the square. E and F are the midpoints of the sides BC and CD. How much work is done to transfer a charge e from O to E and from O to F ?

[Ans. (i) V = 0, (ii) $4\sqrt{2q}/a^2$ along OF, (iii) O, $4eq(1-\sqrt{5})/\sqrt{5a}$] **[I. I. T. 1985]**

7. An inclined plane of inclination 30° is placed in a horizontal electric field of intensity $E = 100\ Vm^{-1}$. A body of mass 1 kg and carrying a charge 0·01 C is dropped from a height of 1 m along the plane. If the coefficient of friction between the body and the plane be 0·2, after what time, it will reach the bottom of the plane ? [**Ans.** $t = 1{\cdot}345$ sec]

8. ABCD is a rhombus. Length of each side is 'a' and angle D is 60°. Three charges each equal to q are placed at the points A, B and C. What is the potential at D ? Show it is equal half of the potential at the junction of two diagonals. [**Ans.** $V_D = q\left(\frac{2\sqrt{3}+1}{\sqrt{3}a}\right)$; $V_0 = 2q\frac{2\sqrt{3}+1}{\sqrt{3}a}$]

9. In a hydorgen atom the distance between proton and electron is 0·53A°. (i) Calculate the potential energy of the system is eV. Assume that potential energy is zero when the proton and electron are at infinite distance from each other $e = 1{\cdot}6\times10^{-19}$C. (ii) Work done to free the electron. Given, Kinetic energy of the electron is half of the potential energy of case (i). [**Ans.** (i) 27·17 eV ; (ii) 13·585 eV]

10. Two vertical parallel plates are placed in air with the separation between the plates being 15 cm. The electric field between the plates is uniform having value 3000 NC^{-1}. If an electron is released from the negative plate, after what time it will reach the positive plate ? Calculate also the velocity when the electron reaches the positive plate. [**Ans.** $2{\cdot}4 \times 10^{-8}$ sec ; $1{\cdot}26 \times 10^{7}$ m/s]

11. The potential difference between the plates of a parallel plate capacitor is 600 volt. The plates are vertical and separation between the plates is 3 mm. An electron is projected vertically parallel to the plates with the velocity 2×10^{6} m/s. What magnetic field be applied in the space between the plates so that the electron goes undeviated charge of electron, $e = 1{\cdot}6 \times 10^{-19}$ C [**Hint :** $Bev = Ee$] [**Ans.** 0·1 Wb/m^2] **[I. I. T.]**

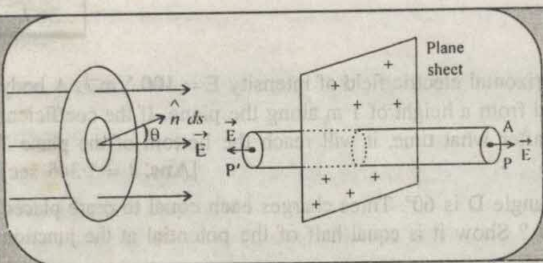
12. A charge 6×10^{-8}C is placed at A of a square of side 0·1 m and at the point of intersection of the two diagonals another charge -4×10^{-8} C is placed. How much work is done to transfer the charge $+5 \times 10^{-9}$C from the point C to B ? [**Ans.** $7{\cdot}9 \times 10^{-6}$ J]

13. The radii of three concentric hollow spheres at a, b and c ($a < b < c$). Surface density of charges of the spheres is $+\sigma$, $-\sigma$ and $+\sigma$. (i) Calculate the potential of the spheres. (ii) If the potentials of A and C are equal, obtain a relation between a, b and c.

[**Ans.** (i) $V_a = \frac{\sigma}{\epsilon_0}(a-b+c)$, $V_b = \frac{\sigma}{\epsilon_0}\left(\frac{a^2}{b}-b+c\right)$, $V_c = \frac{\sigma}{\epsilon_0}\left(\frac{a^2}{c}-\frac{b^2}{c}+c\right)$ (ii) $a + b = c$.] **[I. I. T. 1990]**

14. The radii of three concentre hollow spheres are a, b and c, when $a < b < c$. The spheres are given charges q_1, q_2 and q_3. Calculate the potential of three spheres.

[**Ans.** $\frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{a}+\frac{q_2}{b}+\frac{q_3}{c}\right)$, $\frac{1}{4\pi\epsilon_0}\left(\frac{q_1+q_2}{b}+\frac{q_3}{c}\right)$, $\frac{1}{4\pi\epsilon_0}\left(\frac{q_1+q_2+q_3}{c}\right)$]



CHAPTER 5

GAUSS'S THEOREM AND ITS APPLICATION

● **TOPICS :** ○ Area as a vector; ○ Electric flux, φ_E; ○ Gauss's theorem; ○ Coulomb's law from Gauss's theorem; ○ Application of Gauss's theorem; ○ Short answer type questions (with answers); ○ Exercise.

✪ 5.1. Area as a vector ✪

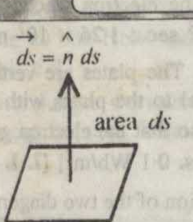


Fig. 5.1

In most cases area of a surface is treated as a vector. So any area element *ds* is a vector $\vec{ds}$. The arrow representing the area vector $\vec{ds}$ is perpendicular to the area element *ds*. If $\hat{n}$ is a unit vector along the normal to the area element *ds*, then $\vec{ds} = \hat{n}\, ds$ . . . . (5.1)

✪ 5.2. Electric flux, φ_E ✪

Flux is a property of any vetor field. For a fluid flow, flux is measured by the number of steamlines that pass through the surface. For an electric field, **the flux φ_E is measured by the number of lines of force passing normally through a surface in the electric field.** Hence, the surface integral of the electric field is the measure of the electric flux that pass through the surface.

Consider a surface of area S inside an electric field of intensity $\vec{E}$ [Fig. 5.2]. The electric field $\vec{E}$ makes an angle θ with the area vector $\vec{ds}$. Then the component of the electric field along the normal to the area element *ds* i.e. along the area vector $\vec{ds}$ is given by $E_n = E \cos\theta$.

Hence the electric flux crossing the area element *ds* along the normal to it is given by

$$d\varphi_E = E_n\, ds = E \cos\theta\, ds$$

So, the electric flux passing through a small area is equal to the product of the area and the normal component of the electric field.

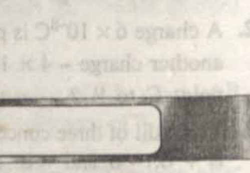


Fig. 5.2

Therefore, the total electric flux through the surface S is

$$\varphi_E = \int_S E \cos\theta\, ds = \int_S \vec{E}\cdot\vec{ds} = \int_S \vec{E}\cdot\hat{n}\, ds \, . \, . \, . \, . \, . \, . \, . \, . \, (5.2)$$

Thus, **electric flux cutting a surface in an electric field may be defined as the surface integral of the electric field over the surface.**

✪ 5.3. Gauss's theorem ✪

Gauss's theorem which applies to any closed hypothetical surface (called a Gaussian surface) gives a relation between φ_E for the surface and the net charge q enclosed by the surface. **The theorem states that the total normal electric flux through a closed surface enclosing a charge is equal to $\frac{1}{\varepsilon_0}$ times the magnitude of the charge enclosed.**

Let a closed surface 'S' encloses a charge q, then according to Gauss's theorem, the total electric flux passing through the closed surface is given by $\varphi = \frac{q}{\varepsilon_0}$ . . . . (5.3)

By definition, the total electric flux through the closed surface S is given by

$$\varphi_E = \int_S \vec{E}\cdot\vec{ds}$$

Therefore, the Gauss's theorem may be expressed as $\int_S \vec{E}\cdot\vec{ds} = \frac{q}{\varepsilon_0}$ . . . . (5.4)

Hence, Gauss's theorem may be states as –

The surface integral of an electric field due to a charge enclosed by the surface is equal to $\frac{1}{\varepsilon_0}$ times the charge enclosed by the surface.

- **Proof of the theorem :**

To prove Gauss's theorem we consider a spherically symmetric closed surface.

Let a point charge q is placed at the centre of a sphere of radius r and $\vec{E}$ be the electric field intensity at a point P on the surface [Fig. 5.3]. Thus by Coulomb's law.

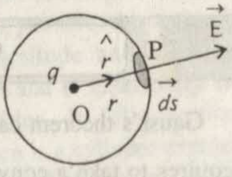


Fig. 5.3

$$\vec{E} = \frac{1}{4\pi\,\varepsilon_0}\,\frac{q}{r^2}\,\hat{r}$$

There the flux passing through the area element $\vec{ds}$ is

$$d\varphi_E = \vec{E}\cdot\vec{ds} = \frac{1}{4\pi\,\varepsilon_0}\,\frac{q}{r^2}\,\hat{r}\cdot\vec{ds} = \frac{1}{4\pi\,\varepsilon_0}\,\frac{q}{r^2}\,ds \quad \left[\because \hat{r}\cdot\vec{ds} = ds\right]$$

So, the total electric flux through the closed surface of the sphere is

$$\varphi = \int_S d\varphi = \int_S \frac{1}{4\pi\,\varepsilon_0}\,\frac{q}{r^2}\,ds$$

$$\therefore\ \varphi = \frac{1}{4\pi\,\varepsilon_0}\,\frac{q}{r^2}\cdot 4\pi r^2 = \frac{q}{\varepsilon_0}$$

It proves the Gauss's theorem in electrostatics.

5.4. Coulomb's law from Gauss's theorem

Coulomb's law can be deduced from Gauss's law and symmetry consideration. Here we apply Gauss's theorem to an isolated point charge q as the fig. 5.4. Although Gauss's law holds for a surface of any shape, here we consider spherical surface. The advantage of such a surface is that, from symmetry $\vec{E}$ is normal to it and has the same magnitude for all the points on the surface.

In fig. 5.4 $\vec{E}$ and $\vec{ds}$ on the Gaussian surface are directed radially outwards. So $\vec{E}\cdot\vec{ds} = E\,ds$. Hence from equation (5.4)

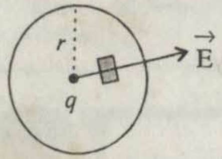


Fig. 5.4

$$\int_S \vec{E}\cdot\vec{ds} = \int_S E\,ds = \frac{q}{\varepsilon_0} \qquad \therefore\ E\int_S ds = \frac{q}{\varepsilon_0}$$

[As E is constant]

$$\therefore\ E\times 4\pi r^2 = \frac{q}{\varepsilon_0}$$

$$\therefore\ E = \frac{1}{4\pi\,\varepsilon_0}\,\frac{q}{r^2} \quad \ldots\ldots\ldots\ldots (5.5)$$

This equation (5.5) gives the magnitude of the electric field strength $\vec{E}$ at any point at a distance r from an isolated point charge q. It is directed radially outwards.

Now, we put a second point charge q_0 at the point at which $\vec{E}$ is calculated. The magnitude of the force that acts on its is $F = Eq_0$

Hence by equation 5.5, $F = \frac{1}{4\pi\,\varepsilon_0}\,\frac{q\,q_0}{r^2}$ ……………… (5.6)

It is Coulomb's law. Thus we have deduced Coulomb's law from Gauss's law and consideration of symmetry.

5.5. Applications of Gauss's theorem

Gauss's theorem can be used to evaluate electric field $\vec{E}$ due to a charge distribution. For this, one requires to take a convenient Gaussian surface to evaluate surface integral. Conversely, if $\vec{E}$ is known for all points on a closed surface, the theorem can be employed to obtain the charge inside.

If $\vec{E}$ has an **outward** component for every point of a closed surface then the surface integral will be **positive** and net charge inside the surface is **positive**. On the other hand, $\vec{E}$ has an inward component for every point of a closed surface, then the net charge within the surface is **negative**. Also, if the surface does not enclose any charge and the field is due to charge outside the surface, then by Gauss's theorem, the surface integral will be zero.

Here we consider some applications of Gauss's theorem. In each example, we require to choose a suitable Gaussian surface. The choice depends on the symmetry of the electric field so that the surface integral is easily evaluated.

➤ 5.5.1. Electric field due to a line charge :

We consider a thin infinitely long straight rod with a uniform linear charge density λ cm^{-1} placed along XY [Fig. 5.5]. We require to calculate electric field at a distance r from the thin rod.

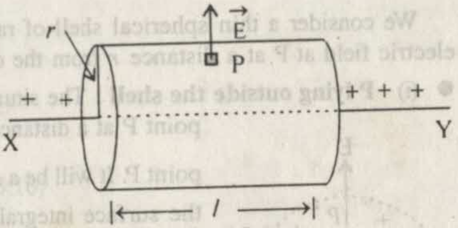


Fig. 5.5

From symmetry, it follows that electric field $\vec{E}$ due to uniform linear charge is radially directed. $\vec{E}$ at any point P will lie in the plane containing P and its direction will be radially outward (for positive charge) or inwards (for negative charge). Evidently, the magnitude of $\vec{E}$ depends only on r. So, we should choose the Gaussian surface to be a circular cylinder of radius r and length l (say).

$\vec{E}$ is a constant over the cylindrical surface. The electric flux only crosses through the curved surface of the cylinder

$$\therefore \int_S \vec{E} \cdot \vec{ds} = E \times 2\pi r l$$

Hence by Gauss's theorem $E \times 2\pi r l = \dfrac{q}{\epsilon_0}$ where $q = \lambda l$

$$\therefore \quad E = \frac{\lambda}{2\pi \epsilon_0 r} = \frac{1}{4\pi \epsilon_0} \cdot \left(\frac{2\lambda}{r}\right) \quad \ldots\ldots\ldots\ldots (5.7)$$

It gives the magnitude of the electric field $\vec{E}$ at a distance r from the line.

➤ 5.5.2. Electric field due to an infinite plane sheet of charge :

Here we consider an infinite thin plane sheet of positive charge having a uniform surface charge density σ m^{-2}. We shall evaluate $\vec{E}$ at a distance r in front of the sheet [Fig. 5.6].

By symmetry, it follows that the electric field is perpendicular to the plane sheet of charge and directed outward when the charge is positive. Also, the field must have the same magnitude and oppositely directed at the two parts P and P′ equidistant from the sheet on its both sides. It follows that a convenient Gaussian surface is a cylinder extending upto P and P′ on both sides of the sheet. The axis of the cylinder is perpendicular to the sheet.

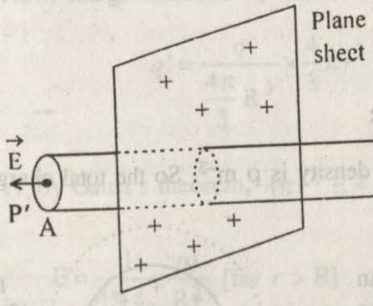


Fig. 5.6

Here $\vec{E}$ is perpendicular to the plane end faces of the cylinder. So, the electric flux crossing through the Gaussian surface is $\varphi = E \times$ area of the end faces $= E \times 2A$

Then, by Gauss's theorem, $\varphi = \dfrac{q}{\epsilon_0} = \dfrac{\sigma A}{\epsilon_0}$

$$\therefore \; 2AE = \frac{\sigma A}{\epsilon_0} \qquad \therefore \; E = \frac{\sigma A}{2\epsilon_0}$$

Thus we find that the magnitude of the electric field at a point due to an infinite plane sheet of charge is independent of its distance from the sheet of charge.

➤ 5.5.3. *Electric field due to a thin sheet of charge* :

We consider a thin spherical shell of radius R with centre at O. Its charge is q. We shall evaluate electric field at P at a distance r from the centre of the sphere. Here the field is purely radial.

- (i) **P lying outside the shell :** The situation is shown in fig. 5.7. We require to calculate field at the point P at a distance r ($r > \text{R}$) from O. We draw a Gaussian surface through the point P. It will be a spherical shell of radius r. If $\vec{\text{E}}$ be the electric field at P, then the surface integral

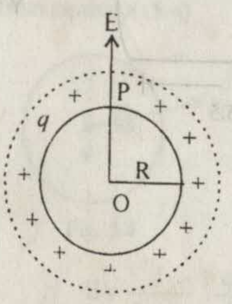


Fig. 5.7

$$\int_S \vec{\text{E}}\cdot\vec{ds} = \int_S \text{E}\,ds = \text{E}\times 4\pi r^2$$

By Gauss's theorem, $\text{E}\times 4\pi r^2 = \dfrac{q}{\varepsilon_0}$

$$\therefore\ \text{E} = \frac{1}{4\pi\,\varepsilon_0}\,\frac{q}{r^2}\ (r > \text{R})\ \ldots\ldots\ldots\ldots\ldots\ (5.8)$$

It follows that for a point outside the charged spherical shell, the shell behaves as if whole charge of the shell is concentrated at the centre.

- (ii) **P lying on the thin shell :** In this case, Gaussian surface through the point P will just enclose the charge on the shell. Then according to Gauss's theorem, $\text{E}\times 4\pi r^2 = \dfrac{q}{\varepsilon_0}$

$$\therefore\ \text{E} = \frac{1}{4\pi\,\varepsilon}\,\frac{q}{\text{R}^2}\ (r = \text{R})\ \ldots\ldots\ldots\ldots\ldots\ (5.9)$$

- (iii) **P lying inside the shell :** In such a case, the Gaussian surface through the point P does not enclose any charge. Hence, by Gauss's theorem, $\text{E}\times 2\pi r^2 = 0$

$$\therefore\ \text{E} = 0 \quad (r < \text{R})$$

➤ 5.5.4. *Electric field due to a solid sphere of charge* :

Fig. 5.8. shows a solid sphere of radius R. Its uniform charge density is ρ m^{-3}. So the total charge on the sphere is $q = \dfrac{4}{3}\pi\,\text{R}^3\,\rho$

Here the electric field is radial. So we choose a spherical Gaussian surface. We require to calculate electric field at a point P in the following cases.

- (i) **When the point P lies outside the sphere of charge :** The distance of the point P from the centre of the sphere is r ($r > \text{R}$). We draw a Gaussian surface passing through P. So it is a sphere of radius r with O as centre. It is evident that the field due to charged sphere

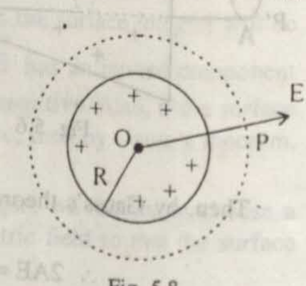


Fig. 5.8

is spherically symmetric. At every point on the surface of the sphere, the field has the same magnitude and is along the normal to the surface.

So, the total flux through the Gaussian surface is

$$\varphi = \int_S \overrightarrow{E} \cdot \overrightarrow{ds} = E \int_S ds = 4\pi r^2 \cdot E$$

By Gauss's theorem, $4\pi r^2 \cdot E = \frac{q}{\varepsilon_0}$

$$\therefore E = \frac{1}{4\pi\,\varepsilon_0} \frac{q}{r^2} \quad (\text{for } r > R) \ldots\ldots\ldots\ldots\ldots\ldots (5.10)$$

- (ii) **P lying on the surface of the sphere of charge :** Here the area of the Gaussian surface is equal to surface area of the sphere of charge.

$\therefore$ Area of the Gaussian surface = $4\pi R^2$

$$\therefore \int_S \overrightarrow{E} \cdot \overrightarrow{ds} = E \int_S ds = 4\pi r^2 \cdot E$$

$\therefore$ By Gauss's theorem, $4\pi r^2 \cdot E = \frac{q}{\varepsilon_0}$

$$\therefore E = \frac{1}{4\pi\,\varepsilon_0} \frac{q}{R^2} \quad (\text{for } r = R) \ldots\ldots\ldots\ldots\ldots\ldots (5.11)$$

- (iii) **P lying inside the sphere of charge :** Here P is situated at a distance r ($r > R$) for the centre O. Area of the Gaussian surface = $4\pi r^2$

$\therefore$ Surface integral of the electric field is $\int_S \overrightarrow{E} \cdot \overrightarrow{ds} = 4\pi r^2 E$

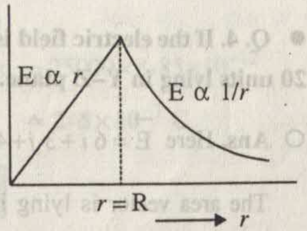


Fig. 5.9

Now, charge enclosed by the surface,

$$q' = \frac{q}{\frac{4\pi}{3} R^3} \times \frac{4}{3} \pi r^3 = \frac{r^3}{R^3} \cdot q$$

$\therefore$ By Gauss's theorem, $4\pi r^2 \ E = \frac{1}{\varepsilon_0} \frac{r^3 q}{R^3}$

$$E = \frac{1}{4\pi\,\varepsilon_0} \frac{rq}{R^3} \ [\text{for } r > R] \ldots\ldots\ldots\ldots\ldots (5.12)$$

The variation of electric field intensity with distance from the centre of a uniformly charged sphere is shown graphically in fig. 5.9 :

(i) E α r (for $r < R$)

(ii) E α 1/r (for $r > R$)

● ● Short Answer Type Questions (with answers) ● ●

● **Question 1. Define electric flux. Is it a vector or a scalar ? Give its SI unit.**

○ **Ans.** Electric flux is associated with electric field. Electric flux over an area in an electric field represents the total number of electric lines of force crossing the area normal to it.

$$\varphi = \int_S \vec{E} \cdot \vec{ds}$$

2nd part : It is a scalar quantity

3rd part : Its SI unit is $Nm^2\ C^{-1}$

● **Q. 2. Define electric intensity at a point in terms of electric flux.**

○ **Ans.** Electric intensity at a point in the electric field is defined as the number of lines of force passing normally through a unit area drawn around the point

$$\therefore\ E = \frac{\varphi}{A}$$

● **Q. 3. A spherical Gaussian surface encloses a charge $8{\cdot}85\times10^{-8}$C.**

(i) Calculate the electric flux passing through the surface.

(ii) If the radius of the Gaussian surface is doubled, how would the flux change.

○ **Ans.** (i) Electric flux through the surface, $\varphi = \dfrac{q}{\varepsilon_0}$

$$\therefore\ \varphi = \frac{8{\cdot}85\times10^{-8}}{8{\cdot}85\times10^{-12}} = 10^4\ Nm^2\ C^{-1}$$

(ii) If the radius of the Gaussian surface is made double, total flux passing through the surface will not change as the charge enclosed by the surface is same.

● **Q. 4. If the electric field is given by $6\hat{i}+3\hat{j}+4\hat{k}$. Calculate electric flux through a surface of area 20 units lying in Y–Z plane.**

○ **Ans.** Here $\vec{E} = 6\hat{i}+3\hat{j}+4\hat{k}$

The area vector is lying in the Y–Z plane. So $\vec{S} = 20\hat{i}$

$\therefore$ Electric flux through the surface is $\varphi = \vec{E}\cdot\vec{S}$

$$\therefore\ \varphi = \left(6\hat{i}+3\hat{j}+4\hat{k}\right)\cdot 20\hat{i} = 120\ \text{unit}.$$

● **Q. 5. A point charge produces an electric flux of $-1{\cdot}0\times10^3\ Nm^2C^{-1}$ to pass through a spherical Gaussian surface of radius 10 cm centered on the charge. What is the charge ?**

○ **Ans.** Here $\varphi = -1{\cdot}0\times10^3\ Nm^2C^{-1}$

Then the charge enclosed by the Gaussian surface is $q = \varphi\, \epsilon_0$

$\therefore\ q = -1{\cdot}0\times10^3\times8{\cdot}85\times10^{-12} = \mathbf{-8{\cdot}85\times10^{-9}C}$

● **Q. 6. S_1 and S_2 are two parallel concentric spheres enclosing charges Q and 2Q respective as shown in fig. 5.10.**

(a) What is the ratio of the electric flux through S_1 and S_2 ?

(b) How will be electric flux through the surface S_1 change if a medium of dielectric constant 5 is introduced in the sphere S_1 in place of air ? **[C. B. S. E. 2002]**

○ **Ans.** (a) Let φ_1 and φ_2 be the electric flux through sphere S_1 and S_2 respectively. Then

$$\varphi_1 = \frac{Q}{\varepsilon_0}, \quad \varphi_2 = \frac{Q+2Q}{\varepsilon_0} = \frac{3Q}{\varepsilon_0} \quad \therefore \frac{\varphi_1}{\varphi_2} = \frac{1}{3}$$

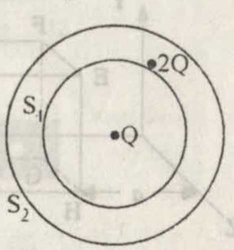


Fig. 5.10

(b) Let $\vec{E}$ be the electric field intensity on the surface of sphere S_1 due to the charge Q placed inside the sphere. Then by Gauss's theorem,

$$\therefore \varphi_1 = \int_S \vec{E} \cdot \vec{ds} = \frac{Q}{\varepsilon_0}$$

When a medium of dielectric constant K is introduced inside the sphere S_1, the electric field on the surface of the sphere will be $\vec{E}' = \frac{1}{K}\vec{E}$

Then the flux through the sphere is $\varphi' = \frac{\varphi_1}{K} = \frac{1}{K}\frac{Q}{\varepsilon_0} = \frac{Q}{5\varepsilon_0}$

● **Q. 7. If the number of electric lines entering a closed surface is 20,000 and emerging out 45,000, then what is the total charge enclosed by the closed surface ?**

○ **Ans.** The required relation : $\varphi = \varphi_2 + \varphi_1$

Hence φ_1 = flux entering the closed surface = –20,000

and φ_2 = flux leaving the closed surface = + 45000

∴ Net flux leaving the closed surface, φ = 45,000 – 20,000 = 25,000

∴ By Gauss's theorem, the charge enclosed by the surface $q = \varphi\,\varepsilon_0 = 25000 \times 8{\cdot}85 \times 10^{-12}$

$$= \mathbf{2{\cdot}2 \times 10^{-7}\ C}$$

●●Numerical Examples●●

● **Example 5.1. A charge 10^{-6}C is placed at the centre of cube of side 10 cm. How much flux passes through each surface of the cube ?**

○ **Solution :** Total flux produced by the charge $q = 10^{-6}$C is $\varphi = \frac{q}{\varepsilon_0}$

∴ Flux passing each surface, $\varphi_1 = \frac{\varphi}{6} = \frac{1}{6}\frac{q}{\varepsilon_0}$

$$\therefore \varphi_1 = \frac{1}{6} \times \frac{10^{-6}}{8{\cdot}85 \times 10^{-12}} = \frac{100}{6 \times 8{\cdot}85} \times 10^4 = 1{\cdot}88 \times 10^4\ \text{Nm}^2\text{C}^{-1}$$

- **Ex. 5.2. Calculate the amount of electric flux through a cube of side 10 cm [Fig. 5.11]. Given $E_x = b\sqrt{x}$, $E_z = E_y = 0$, $b = 800$ N / Coul – m$^{1/2}$.**

○ **Solution :** Here the electric field acts along *x*-axis. Its Y- and Z–components are zero. Hence total electric flux :

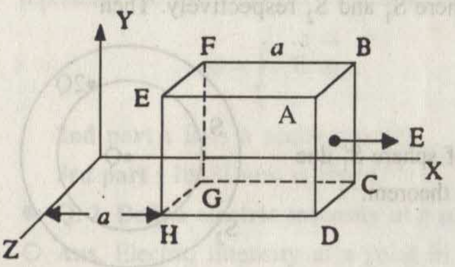


Fig. 5.11

$$\varphi = \varphi_{ABCD} - \varphi_{EFGH} = \int \vec{E_1}\cdot\vec{ds} - \int \vec{E_2}\cdot\vec{ds}$$

$$= \left(E_x a^2\right)_{ABCD} - \left(E_x a^2\right)_{EFGH}$$

[*a* = length of each side]

$$= a^2\left[b\sqrt{2a} - b\sqrt{a}\right] = ba^{5/2}\left(\sqrt{2}-1\right)$$

$$= 800\times(0\cdot1)^{5/2}\left(\sqrt{2}-1\right)$$

$$= \mathbf{1\cdot05\ N-m^2C^{-1}}$$

- **Ex. 5.3. A thin straight wire of length 30 cm is given a charge of 15 μc. Calculate electric field and its direction at a distance of 10 cm from the wire.**

○ **Solution :** Linear charge density of the wire, $\lambda = \dfrac{q}{l} = \dfrac{15\times10^{-6}}{30\times10^{-2}} = 5\times10^{-5}$ C / m

∴ So, the electric field at a distance *r* from the wire is $E(r) = \dfrac{1}{4\pi\,\varepsilon_0}\dfrac{2\lambda}{r} = 9\times10^9\times\dfrac{2\times5\times10^{-5}}{0\cdot1}$

$$\therefore\ E(r) = 9\times10^6\ \text{N}\cdot\text{C}^{-1}$$

- **Ex. 5.4. A spherical shell of radius 15 cm is given a charge of 10 μc. Calculate electric field in the following cases (i) at a distance of 10 cm from the centre of the shell (ii) at a distance of 30 cm from the centre of the shell.**

○ **Solution :** (i) The point is considered to be at a distance of r = 10 cm which is less then the radius R = 15 cm of the spherical shell, the electric field at this point E (r) = 0 as the Gaussian surface of radius 10 cm does not enclose any charge.

(ii) In this case r > R. Hence the electric field at a distance of 30 cm from the centre of the shell is

$$E(r) = \frac{1}{4\pi\,\varepsilon_0}\frac{q}{r^2} = 9\times10^9\times\frac{10\times10^{-6}}{(0\cdot3)^2}$$

$$\therefore\ E(r) = \mathbf{1\times10^6\ NC^{-1}}$$

- **Ex. 5.5. A sphere of radius 1 cm is uniformly charged with 4×10^{-8}C of charge. It is enclosed by a concentric sphere of radius 5 cm. Calculate the electric field at a distance of 2 cm from the centre.**

○ **Solution :** We require to calculate the electric field at P [Fig. 5.12]. We draw the Gaussian surface through the point P. If E be the electric field at P, the surface integral of the electric field is

$$\varphi = \int_S \vec{E}\cdot\vec{ds} = 4\pi\, r_1^2 E \qquad [r_1 = OP]$$

Then, by Gauss's theorem, $4\pi\, r_1^2 \mathrm{E} = \dfrac{q}{\in_0}$

$$\therefore\ \mathrm{E} = \frac{1}{4\pi \in_0} \frac{q}{r_1^2}$$

$$\therefore\ \mathrm{E} = 9\times10^9 \times \frac{4\times10^{-8}}{(0\cdot02)^2}$$

$$= \mathbf{9\times10^5\ N\cdot C^{-1}}$$

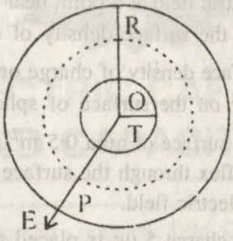


Fig. 5.12

• E ◇ X ◇ E ◇ R ◇ C ◇ I ◇ S ◇ E •

◢ A. Short answer type questions :

1. What is electric field ? Define electric field in terms of electric flux. Give its SI unit.
2. How can you measure electric flux passing through a surface ?
3. What is surface integral of electric field ?
4. State Gauss's theorem.
5. What is the electric field inside a charged hollow sphere ? Explain your answer.
6. What surface density of charge ? Give its SI unit.
7. A boy encloses an electric dipole consisting of charge 5 μc and –5 μc and of length 5 cm. What is the total electric flux through the box ? **[Ans.** Zero]
8. What is the Gaussian surface ? Mention its use.
9. If Coulomb's law involves $1/r^3$ dependence instead of $1/r^2$, would Gauss's law be still true ?

◢ B. Essay type questions :

1. (a) What is surface integral of electric field ? What does it represent ?

   (b) State and prove Gauss's theorem in electrostatics. Deduce Coulomb's law from Gauss's theorem.
2. State Gauss's theorem. Give its mathematical expression. Derive an expression for the electric field intensity at any point due to an infinite plane sheet of charge of density σ c/m². **[C.B.S.E. 2001]**
3. Apply Gauss's theorem to calculate electric field due to a point charge and due to a charged hollow conductor.
4. Derive an expression for the electric field due to a uniformly charged spherical shell at a point (a) inside and (b) outside the shell.
5. Apply Gauss's theorem to calculate the electric field due to a solid sphere of charge at a point (a) outside the sphere (b) inside the sphere.

◢ C. Numerical problems :

1. Five thousand lines of force enter a certain volume of space and three thousand lines emerge from it. What is the total charge in Coulomb enclosed by the volume ? **[Ans.** $- 1{\cdot}77 \times 10^{-8}$ C]
2. An infinite line charge produces a field of 9×10^4 NC^{-1} at a distance of 4 cm. Calculate linear charge density. **[Ans.** 2×10^{-7} C.m.$^{-1}$]
3. A spherical conductor of radius 12 cm has a charge $1{\cdot}6\times10^{-7}$C distributed uniformly on the surface. Calculate the electric field (i) inside the sphere (ii) just outside the sphere (iii) at a point 18 cm from the centre of sphere. **[Ans.** (i) Zero, (ii) 10^5 NC^{-1}, (iii) $4{\cdot}44 \times 10^4$ N/C.]

4. The electric field at a point near the earth's surface is 330 $V.m^{-1}$ and it is directed normal to the earth's surface. What is the surface density of charge on the earth's surface. **[Ans.** $2{\cdot}92 \times 10^{-9}$ $C.m^{-2}$]

5. The surface density of charge on a charged sphere of radius 40 cm is $18{\times}10^{-6}$ $C.m^{-2}$. What is the electric field intensity on the surface of sphere ? **[Ans.** $2{\cdot}033 \times 10^{6}$ Vm^{-1}]

6. A plane surface of area $0{\cdot}5$ m^2 is placed inside a uniform electric field of strength $5{\times}10^3$ NC^{-1}. Calculate the electric flux through the surface when normal to the plane makes an angle of (i) 30° (ii) 60° with the direction of the electric field. **[Ans.** (i) $2165{\cdot}1$ Nm^2C^{-1} (ii) 1250 Nm^2C^{-1}]

7. A point charge 5 μc is placed at the centre of sphere of radius 0·2 m. Calculate (i) electric flux through the surface and (ii) electric field at a point on its surface. $\epsilon_0 = 8{\cdot}854 \times 10^{-12} C^2N^{-1}m^{-2}$.

[Ans. (i) $5{\cdot}65 \times 10^5$ Nm^2C^{-1} (ii) $11{\cdot}25 \times 10^5$ NC^{-1}]

8. A charged particle having charge -3 μ*c* is placed close to a sheet of charge having a surface charge density $5{\times}10^{-6}$ cm^{-2}. Find the force of attraction between the particle and the sheet of charge. **[Hint.** $F = Eq = \frac{\sigma}{\epsilon_0} q$]

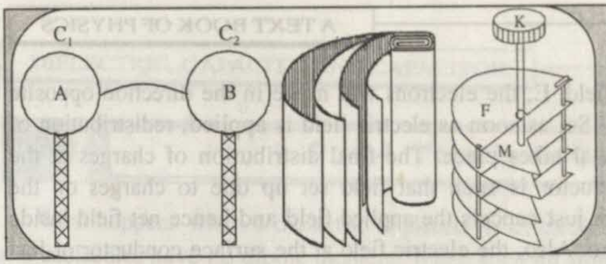
[Ans. 0·847 N]

9. A ball of mass 1g carrying a charge 2×10^{-8}C hangs from a thread when a large conducting plate of charge is brought near the ball, the thread makes an angle of 30° with the plate. What is the surface charge density of the plate ? **[Ans.** $2{\cdot}505 \times 10^{-9}$ $C.m^{-2}$]

10. The electric potential V(*x*) in a region along *x*-axis varies with distance *x* (in meters) according to the relation $V(x) = 4x^2$. Calculate the force experienced by 1 μ*c* charge placed at a point $x = 1$m.

[Hint : $|E| = \frac{dv}{dx} = 8x$; $|E|_{x=1m} = 8NC^{-1}$; $F = |\vec{E}|q$] **[Ans.** 8×10^{-6} N]

11. The hollow concentric conducting spheres of radii 6 cm and 12 cm are insulated from each other. The outer sphere is given a charge 20 esu and inner sphere is given a charge of 5 esu. Calculate the potential at points 5 cm, 10 cm and 15 cm from the centre. **[Ans.** 2·5 statvolt ; 2·17 statvolt ; 1·67 statvolt]



CHAPTER 6

DIELECTRIC, CAPACITY AND CAPACITOR

● **TOPICS :** ○ Conductors and insulators, ○ Behaviour of conductors in a magnetic field, ○ Dielectric, ○ Polarisation of matter, ○ Gauss's law in dielectric, ○ Electric Displacement, ○ Capacitance, ○ Potential energy of a charged conductor, ○ Capacity of an isolated conducting sphere, ○ Distribution of charge between connected conductors, ○ Sharing of charges between two conductors at different potentials, ○ Energy loss on sharing of charges, ○ Capacitor and its principle, ○ Calculation of capacitance of different types of capacitor, ○ Combination of Capacitors, ○ Parallel plate capacitor with dielectric slab, ○ Capacitance of parallel plate capacitor with a conducting slab between the plates, ○ Practical form of capacitors, ○ The Van de Graaff Generator, ○ Short answer type questions (with answers), ○ Exercise.

6.1. Conductors and insulators

In the previous chapters we have studied electrostatic fields due to different kinds of charge distribution in which charges were assumed to be situated at different points and were held there at rest by some means. From these discussions we have learnt some basic properties of electrostatic fields. Here we are going to discuss the electrical properties of matter.

Depending on the ability to conduct electricity, materials are broadly put into two categories, **Conductors and insulators**—The basis of distinction being free and bound electrons.

In conductors which are mainly metals, there are large number of free electrons. The elements in which the valence shell is filled less than half, are found to be good conductors. In metals, such as copper, aluminium, silver etc the valence shell contains three or less electrons. Now, the valence electrons in the atoms of these metal leave the atom and are free to move through the lattice in a random manner. They constantly collide among themselves and also with the positive ions in the metal lattice. The average velocity of free electrons in a metal is zero. When an electric field is applied across the two ends of a metal, the free electrons are acted upon by a force and is accelerated. There is a net flow of electrons which constitute electric current through a metal. As such, metals are termed as conductors for electricity.

On the other hand, the materials which do not have free electrons in them are unable to conduct electricity and are termed as **insulators**. Non-metals are in general insulators.

The valence electrons in atoms of an insulator are tightly band to their nuclei and usually cannot detach themselves.

An insulator may behave in the following two ways :

(i) It does not conduct electricity even when a potential difference is applied across it. It is called an insulator.

(ii) It may not conduct electricity through it but on applying electric field, induced charges are produced on its faces. Such an insulator is called **dielectric.**

6.2. Behaviour of conductors in a magnetic field

The general features of the behaviour of conductors in an electrostatic field are very different from those of dielectrics. The reason is that conductors have free charges but dielectrics do not.

When a conductor is placed in an electric field, E, the electrons will move in the direction opposite to the electric field. So, as soon as electric field is applied, redistribution of electrons in the metal takes place. The final distribution of charges at the surface of the conductor is such that field set up due to charges on the surface of conductor just cancels the applied field and hence net field inside the conductor is zero. Also, the electric field at the surface conductor or just outside it cannot have tangential component. Had it been so, it would lead to the flow of surface current. Thus, when a conductor is placed in an electric field, the field will be of the form as shown in fig. 6.2. Evidently, the electric field lines will enter or leave the conductor at right angles to the surface of the conductor.

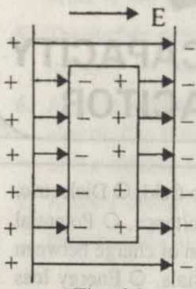


Fig. 6.1

Electric field inside a conductor is zero. It implies that, there is no net charge in the interior of the conductor. It can be proved by Gauss's theorem. For this, we draw a Gaussian surface just inside the surface of the conductor. As there is no electric field inside the conductor, the electric flux through the Gaussian surface is zero. It implies that there is no charge in the interior of the conductor.

So, when a conductor is placed in an electric field, conditions apply to it :

(i) Inside the conductor, both electric field and charge is zero.

(ii) Charge resides on the surface of the conductor.

(iii) Just outside the surface of the conductor, the electric field is perpendicular to the surface.

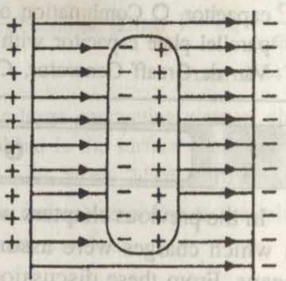


Fig. 6.2

❂ 6.3. Dielectric ❂

In a dielectric (or insulator) there is no free electrons. Monatomic substance consists of atoms. It consists of central nucleus (positively charged) around which electrons revolves. In general, the centre of positive and negative charges coincide. As a result an atom is electrically neutral.

On the other hand, a polyatomic substance coinsists of two or more atoms. In the case of a molecule, the centre of positive charges or protons may or may not coincide with the centre of negative charges or electrons.

❖ **Definition of Non-polar dielectric : A dielectric, in the atoms and molecules of which, the centre of positive and negative charges coincide, is called Non-polar dielectric.**

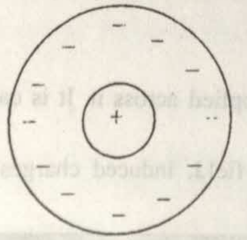


Fig. 6.3

Evidently in a non-polar dielectric, the positive and negative charges of atoms and molecules have a symmetrical charge distribution about their centre [fig. 6.3]. So electric dipole moment of such atoms or molecules is zero.

❖ **Definition Polar dielectric : A dielectric, in the atoms and molecules of which, the centre of gravity of positive and negative charges do not coincide, is called a polar dielectric.**

So, in this case, the positive and negative charges of atoms and molecules have an asymmetrical charge distribution about their centre [fig. 6.4]. So polar dielectric molecules possess a finite electric dipole moment. The molecules such as NH_3, HCl, H_2O, CO_2, N_2O etc. are the example of polar molecules.

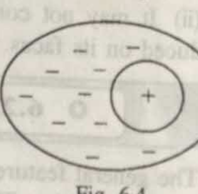


Fig. 6.4

✪ 6.4. Polarisation of matter ✪

What happens when a dielectric is placed in an electric field ? In their normal state non-polar atoms and molecules have no dipole moment. When such an atom or molecule is placed in an electric field, the positive nucleus is pulled in one direction and the negative charges (i.e. electrons) in the opposite direction. The atom is said to be **polarised.** Thus two centres of charge are created and the atom acquires a dipole moment proportional to the applied field $\vec{E}_0$. The induced dipole moment acquired by an atom is found to be

$$p = \alpha\,\varepsilon_0\,E_0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (6.1)$$

where α = atomic polarizability and ε_0 = permittivity of vacuum. α has the dimension of volume. For most atoms α is of the order of 10^{-20} to 10^{-30} m^3, which is the same order as atomic volume. The order of induced dipole moment is 10^{-34} C.m. when the applied electric field is about 10^6 Vm^{-1}.

On the other hand, polar molecules are permanently polarised even in the absence of an external field. They have permanent dipole moment in the range of about 10^{-30} C.m. For example, the water molecule consists of two hydrogen atoms and an oxygen atom making an angle of about 105° as in fig. 6.5. Its parmanent dipole moment is about ten thousand times more than the induced moments in nonpolar molecules.

When a polar molecule placed in an electric field, the creation of additional dipole moment is negligible compared to the aligning of the existing moment along the direction of the field.

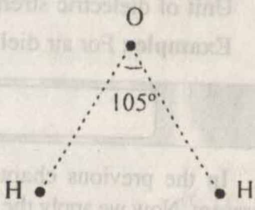
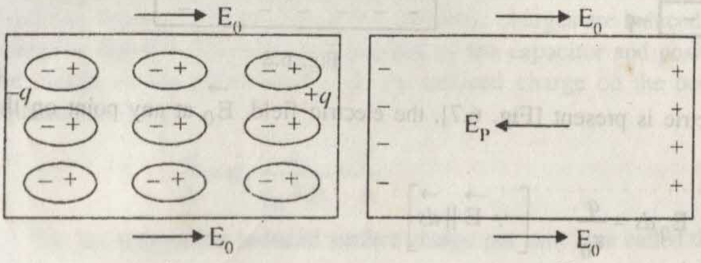


Fig. 6.5

Electric polarisation :

Assume that a rectangular slab of a non-polar dielectric is placed in a uniform electric field $\vec{E}_0$. Immediately each molecule of the dielectric gets polarised. It means that the centres of positive and negative charges are diplaced from each other [Fig. 6.6(i)]. On the left face of the slab, a net negative charge, $-q$ and on the right face a net positive charge $+q$ appears. there is no net charge in the interior of the dielectric [Fig. 6.6 (ii)]. The charges $-q$ and $+q$ on the two surfaces of the dielectric slab are called **induced charges.** The induced charges set up an electric field E_P inside the dielectric. It is called **electric field due to polarisation.** $\vec{E}_P$ is opposite to the applied field $\vec{E}_0$. So, the resultant electric field within the dielectric is



(i) (ii)

Fig. 6.6

$$\vec{E} = \vec{E}_0 - \vec{E}_P \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (6.2)$$

Hence, on placing a dielectric slab inside an electric field, the strength of the field is reduced. E is called the reduced value of electric field.

- **Dielectric constant, K :** From the equation (6.2) we see that $|\vec{E}| < |\vec{E_0}|$. We write it as

$$|\vec{E}| = \frac{|\vec{E_0}|}{K} \quad \therefore \quad K = \frac{E_0}{E}$$

The value of K is always greater than 1.

Example : For water K = 78 and for mica K = 5·4

Dielectric strength :

Assume that a dielectric slab is placed in a uniform electric field. As the electric field is gradually increased, the molecules of the dielectric undergo more and more stretching.

A stage will come when electrons break away from the molecules of the dielectric. The phenomena is called dielectric break down. Now the dielectric becomes conducting. The maximum electric field when break down occurs is expressed by **dielectric strength.**

❖ **Definition of Dielectric strength : The dielectric strength of a dielectric is defined as the maximum value of the electric field that can be applied to the dielectric without its electric break down.**

Unit of dielectric strength is Vm^{-1} and its practical unit KV $(mm)^{-1}$.

Example : For air dielectric strength is 0·8 KV/mm and for mica it is 160 KV/mm.

6.5. Gauss's law in dielectric

In the previous chapter we have applied Gauss's law to the problems in which no dielectric was present. Now we apply the law to a parallel plate capacitor filled with a dielectric. Let $+q$ and $-q$ be the charges on the plates of this capacitor in both the cases with and without dielectric [Fig.. 6.7 and 6.8].

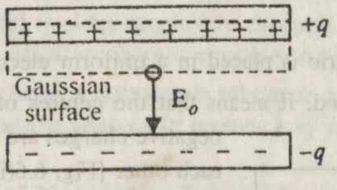


Fig. 6.7

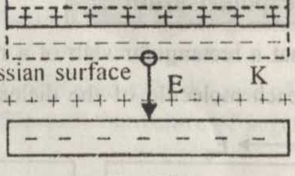


Fig. 6.8

In the first case when no dielectric is present [Fig. 6.7], the electric field $\vec{E_0}$ at any point on the Gaussian surface is given by

$$\int_S \vec{E_0}\cdot\vec{ds} = \frac{q}{\varepsilon_0} \quad \text{or,} \quad \int_S E_0\, ds = \frac{q}{\varepsilon_0} \qquad \left[\because \vec{E} \parallel \vec{ds}\right]$$

$$\therefore E_0\, A = \frac{q}{\varepsilon_0} \quad \therefore E_0 = \frac{q}{\varepsilon_0 A} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(6.3)}$$

where A is the area of each plate.

In the second case when dielectric is present between the plates [Fig. 6.8], the net charge within the Gaussian surface is $(q - q')$, where q' is the induced charge. The electric field E is given by

$$\int_S \vec{E_0}\cdot\vec{ds} = \frac{(q-q')}{\varepsilon_0} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(6.3a)}$$

$$\therefore\ EA=\frac{q-q'}{\varepsilon_0}\qquad \therefore E=\frac{q}{A\varepsilon_0}-\frac{q'}{A\varepsilon_0}\quad\ldots\ldots\ldots\ldots(6.4)$$

Thus the induced charge tends to weaken the original field (i.e. $E < E_0$) which reveals itself as a reduction in potential difference between the plates : If V_a and V_0 are the potential differences with and without the dielectric respectively, then,

$$\frac{E_0}{E}=\frac{V_0}{V_a}=K=\text{dielectric constant}$$

$$\therefore\ E=\frac{E_0}{K}=\frac{1}{K}\,\frac{q}{\varepsilon_0 A}\ \text{[From 6.3]}\ldots\ldots\ldots\ldots(6.5)$$

Combining Equations (6.4) and (6.5), $\frac{1}{K}\,\frac{q}{\varepsilon_0 A}=\frac{q}{\varepsilon_0 A}-\frac{q'}{\varepsilon_0 A}$

$$\therefore\ q'=q\left(1-\frac{1}{K}\right)\ldots\ldots\ldots\ldots(6.6)$$

Combining (6.3a) and (6.6) $\int_S \vec{E}\cdot\vec{ds}=\frac{q}{\varepsilon_0 K}$

$$\therefore\ \varepsilon_0\int_S K\vec{E}\cdot\vec{ds}=q\quad\ldots\ldots\ldots\ldots(6.7)$$

It is the Gauss's law in presence of dielectric.

❂ 6.6. Electric Displacement ❂

Suppose a dielectric slab is placed in the electric field set up between the plates of a parallel plate capacitor. Owing to polarisation of the dielectric charges are induced at the surfaces of the dielectric. The charge is negative near the positive plate of the capacitor and positive near the negative plate. If q be the charge on the plates and q' be the induced charge on the boundary of the dielectric, then from equation (6.6),

$$\frac{q}{A}=\varepsilon_0\frac{q}{\varepsilon_0 KA}+\frac{q'}{A}\quad\ldots\ldots\ldots\ldots(6.8)$$

The last term is the induced surface charge per unit area called the **electric polarisation P**. Thus we have

$$\frac{q}{A}=\frac{\varepsilon_0 q}{\varepsilon_0 KA}+P\quad\ldots\ldots\ldots\ldots(6.9)$$

$$\frac{q}{A}=\varepsilon_0 E+P\quad\left[E=\frac{q}{\varepsilon_0 KA}\right]\ldots\ldots\ldots\ldots(6.10)$$

The quantity on right hand side is given *a* name **electric displacement D**. Thus

$$\vec{D}=\varepsilon_0\vec{E}+\vec{P}\quad\ldots\ldots\ldots\ldots(6.11)$$

Evidently, D depends only on free charge density $\left(\frac{q}{A}\right)$, whereas E depends on both free and polarisation charges.

A general form of Gauss's law for $\vec{E}$ is written as $\varepsilon_0 \int_S \vec{E} \cdot \vec{ds} = q - q' = q - \int_S \vec{P} \cdot \vec{ds}$

$$\therefore \int \left(\varepsilon_0 \vec{E} + \vec{P}\right) \cdot \vec{ds} = q \quad \therefore \int_S \vec{D} \cdot \vec{ds} = q \qquad \ldots (6.12)$$

Thus the surface integral of normal component of D over any charged surface, the flux of $\vec{D}$, is equal to the total free charge only within its surface.

- **A special discussion :** $\vec{E}$, $\vec{P}$ and $\vec{D}$ are three important vector quantities in electrostatics and they are related by the equation (6.11).

(i) Electric displacement vector $\vec{D}$ is related to free charge only. The flux of $\vec{D}$ equals the free charge.

(ii) Polarisation vector $\vec{P}$ is connected to the induced charge on the surface. The flux of $\vec{P}$ is equal to the negative of the bound (free) charge.

The electric field intensity $\vec{E}$ is connected with the charge actually present (free and bound charges). The flux of $\vec{E}$ equals the total enclosed charge divided by ε_0.

The units of $\vec{D}$ and $\vec{P}$ is C/m^2 and that of $\vec{E}$ is NC^{-1}

In free space where there is no dielectric and thus $\vec{P} = 0$, we get $\vec{D} = \varepsilon_0 \vec{E}$

In the presence of dielectric, vector $\vec{D}$ and $\vec{P}$ both can be expressed in terms of $\vec{E}$ alone. So

$$D = \frac{q}{A} = K\varepsilon_0 \left(\frac{q}{K\varepsilon_0 A}\right) = K\varepsilon_0 E \qquad \ldots (6.13)$$

$$\therefore D = \varepsilon E \qquad \ldots (6.14)$$

where $\varepsilon = \varepsilon_0 K$ is called absolute permittivity of the medium. For vacuum K = 1, then $\varepsilon = \varepsilon_0$. The quantity $K = \frac{\varepsilon}{\varepsilon_0}$ is also called relative permittivity. Using relations (6.10) and (6.12), we write

$$\varepsilon_0 E + P = K\varepsilon_0 E$$

$$\therefore P = \varepsilon_0 (K-1) E \qquad \ldots (6.15)$$

ε_0 and K are constant for the medium. The term (K–1) is also replaced by another property, called the **susceptibility,** χ of the dielectric medium.

$$\therefore \vec{P} = \varepsilon_0 \chi \vec{E} \qquad \ldots (6.16)$$

The three quantity ε, K and χ are actually different ways of describing the same property of the dielectric. These are related as $\chi = K - 1 = \frac{\varepsilon}{\varepsilon_0} - 1$.

● **Example. The two plates of a parallel plate capacitor are identical and carry equal amount of opposite charges. The separation between the plates is 5 mm and the space between the plates is filled with a dielectric of dielectric constant 3. The electric field within the dielectric is 10^6 Vm^{-1} and $\varepsilon_0 = 8{\cdot}85 \times 10^{-12}$. Calculate (i) polarisation vector $\vec{P}$ and (ii) displacement vector $\vec{D}$.**

○ **Solution :** (i) The magnitude of the polarisation vector is $P = \varepsilon_0 (K-1)\ E = 8 \cdot 85 \times 10^{-12}\,(3-1) \times 10^6$

$$\therefore\ P = 17 \cdot 7\ \mu C/m^2$$

(ii) The magnitude of displacement vector, $D = K\varepsilon_0 E$

$$\therefore\ D = 3 \times 8 \cdot 85 \times 10^{-12} \times 10^6 = \mathbf{26 \cdot 55 \times 110^{-6}\ C.m^{-2}}$$

✪ 6.7. Capacitance ✪

Capacitance or capacity of a conductor denotes its ability to hold charge. This ability of a conductor depends on its size, shape, medium and presence of other conductor near it. When a conductor is given a charge its potential rises. It is found that rise of potential is proportional to charge imperted. So if V be the rise of potential of a conductor due to a charge q given to it, then $q \propto V$

$$\therefore\ q = CV \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6.17)$$

The constant of proportionality is called the **capacitance** of the conductor.

❖ **Definition of C :** If in relation (6.17) V = 1, then q = C

Hence, the capacitance of a conductor is numerically equal to electric charge required to raise the potential by one unit.

Units of capacitance :

(i) In SI system, unit of capacitance is Farad (F). It is also taken as practical unit of capacitance. A conductor is said to have capacitance of one Farad, if one coulomb charge raises its electric potential by one volt.

$$\therefore\ 1\ F = \frac{1\ \text{Coulomb}}{1\ \text{volt}} = 1\ C.V^{-1}$$

(ii) In c.g.s. system, unit of capacitance is statfarad. A conductor is said to have capacitance of one statfarad if one stat-coulomb charge raises its potential by one statvolt.

$$\therefore\ 1\ \text{Statfarad} = \frac{1\ \text{Stat Coulomb}}{1\ \text{Stat volt}}$$

● **Relation :** $1F = \frac{1\ C}{1\ V} = \frac{3 \times 10^9\ \text{Stat Coulomb}}{1/300\ \text{Stat volt}} = \mathbf{9 \times 10^{11}\ Statfarad}$

In practice, the following smaller units of capacitance are also used :

1 microfarad (μF) = 10^{-6} F ; 1 picofarad = 10^{-12} farad.

- **Capacitance of a conductor depends on the following factors :**

(i) **Surface area of the conductor :** If the surface area of the conductor is large, for a given charge, its potential will be smaller. As $C \alpha \frac{1}{V}$ (fixed charge), so the capacitance increases.

(ii) **Proximity of other conductor :** If another conductor is kept near the given conductor, its rise of potential is less due to a charge given to it. So it capacitance increases.

(iii) **Nature of the surrounding medium :** The capacitance of a conductor depends on dielectric constant $K\left(K = \frac{\varepsilon}{\varepsilon_0}\right)$ of the medium. Higher the value of K higher is capacitance of the conductor.

6.8. Potential energy of a charged conductor

When a conductor is given a charge, its potential rises and some work is done to charge a conductor. This work is stored in the conductor in the form of potential energy.

Let during charging process, at any instant charge delivered is q and corresponding rise of potential is v. Then

$q = Cv$ where C is the capacitance of the conductor.

Next an infinitesimal amount of charge dq is given to the conductor so that potential v remains practically unchanged. Then the work done to give this small charge is $dW = v \cdot dq = \frac{q}{C} dq$

Hence the total work done in delivering the maximum amount of charge Q is $W = \int_0^Q \frac{q}{C} dq = \frac{1}{2}\frac{Q^2}{C}$

$\therefore$ Stored energy of the conductor or potential energy is $E_P = \frac{1}{2}\frac{Q^2}{C}$ . . . . (6.18)

Now, V be the potential of the conductor due to the charge Q given to it. Then Q = CV

$\therefore E_P = \frac{1}{2} CV^2$ . . . . (6.19)

- **A discussion :** During charging a conductor, some work is done and at the same time an electric field is produced around it. To obtain a relation between energy stored in the conductor with the intensity of the electric field we consider a parallel plate capacitor. Let the area of each of its plate be A and separation between the plates be d. We assume that the electric field between the plates is uniform. The energy stored in the capacitor is

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{(\sigma A)^2}{\varepsilon_0 A/d} = \frac{1}{2}\frac{\sigma A d}{\varepsilon_0} \quad [\sigma = \text{surface density of charge, } C = \varepsilon_0 A/d]$$

So, energy density or the energy stored per unit volume is

$$U' = \frac{1}{2}\frac{\sigma^2}{\varepsilon_0} = \frac{1}{2}\varepsilon_0 E^2 \quad \left[\because E = \frac{\sigma}{\varepsilon_0}\right]$$

If the dielectric constant of the medium between the plates be K, then $C = \dfrac{K\,\varepsilon_0 A}{d}$

then $U' = \dfrac{1}{2} K\,\varepsilon_0\,E^2$ . . . . (6.20)

It is to be noted that this equation (6.19) for energy density is applicable to any type of capacitor.

6.9. Capacity of an isolated conducting sphere

A conducting sphere of radius *r* is given a charge Q [Fig. 6.9]. As a result, its potential will rise. To calculate the rise of potential it will be assumed that the whole charge is concentrated at its centre. So the potential at a point on the surface of the sphere is

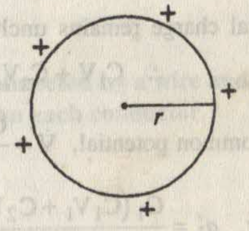


Fig. 6.9

$$V = \frac{1}{4\pi\,\varepsilon_0}\,\frac{Q}{r} \quad \text{[If the surrounding medium be air]}$$

$$\therefore \text{ Capacity of the sphere, } C = \frac{Q}{V} = (4\pi\varepsilon_0)\,r$$

$$\therefore C = (4\pi\,\varepsilon_0)\,r \text{ Farad} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (6.21)$$

If the surrounding medium has dielectric constant K then

$$C = (4\pi\,\varepsilon_0)\,Kr \text{ Farad} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (6.22)$$

In c.g.s. system C = K*r* Statfarad . . . . (6.23)

6.10. Distribution of charge between connected conductors

Two conductor A and B are initially uncharged and they are connected by a conducting wire. Let their capacities be C_1 and C_2 [Fig. 6.10]. If a charge is given to one conductor, immediately it will be distributed between the conductors. Amount of charge received by one conductor depends on its capacitance. But their final potential will be same.

Let a charge Q is given to them. Conductor A receives q_1 and conductor B receives q_2. So $Q = q_1 + q_2$. Let the final common potential be V.

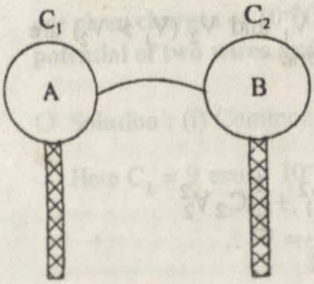


Fig. 6.10

$$\therefore V = \frac{q_1}{C_1} = \frac{q_2}{C_2} = \frac{q_1+q_2}{C_1+C_2} = \frac{Q}{C_1+C_2} \quad \ldots\ldots\ldots \quad \text{(i)}$$

∴ Charge received by the conductor A of capacity C_1 is

$$q_1 = C_1 V = C_1 \frac{Q}{C_1+C_2}$$

and charge received by the second conductor B of capacity C_2 is

$$q_2 = C_2 V = C_2 \frac{Q}{C_1+C_2}$$

$$\therefore \frac{q_1}{q_2} = \frac{C_1}{C_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad \text{(ii)}$$

So, charge of one conductor is proportional to its capacitance.

6.11. Sharing of charges between two conductors at different potentials

Let the capacity of two insulated conductors A and B be C_1 and C_2. Initially they have charges q_1 and q_2 and their potentials V_1 and V_2. We assume $V_1 > V_2$.

$\therefore\ q_1 = C_1V_1$ and $q_1 = C_2V_2$

Now, the conductors are connected by a wire. Immediately charge will flow from A to B until their potentials be equal.

Let the common potential be V.

Final charge of A, $q_1' = C_1V$ and that of B is $q_2' = C_2V$

As total charge remains unchanged, $q_1' + q_2' = q_1 + q_2$

$$\therefore\ C_1V + C_2V = C_1V_1 + C_2V_2$$

$$\therefore\ \text{Common potential, } V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(6.24)}$$

$$\therefore\ q_1' = \frac{C_1(C_1V_1 + C_2V_2)}{C_1 + C_2} \text{ and } q_2' = \frac{C_2(C_1V_1 + C_2V_2)}{C_1 + C_2}$$

$\therefore$ Charge given by the conductor A, $\Delta q_1 = q_1 - q_1' = C_1V_1 - C_1V$

$$\therefore\ \Delta q_1 = C_1\left[V_1 - \frac{C_1V_1 + C_2V_2}{C_1 + C_2}\right] = \frac{C_1C_2(V_1 - V_2)}{C_1 + C_2}$$

and charge received by the conductor B, $\Delta q_2 = q_2' - q_2 = C_2V + C_2V_2$

$$\therefore\ \Delta q_2 = \frac{C_1C_2(V_1 - V_2)}{C_1 + C_2} \qquad \therefore\ \Delta q_1 = \Delta q_2$$

6.12. Energy loss on sharing of charges

When two conductors of capacitances C_1 and C_2 initially at potentials V_1 and V_2 ($V_1 > V_2$) are connected by wire, there is a redistribution of charge between the conductors.

The final common potential is $V = \dfrac{C_1V_1 + C_2V_2}{C_1 + C_2}$

Before sharing of charges, energy of the two conductors is $E_i = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2$

After charge share, final energy of the two conductors is

$$E_f = \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2 = \frac{1}{2}(C_1 + C_2)V^2 = \frac{1}{2}\frac{(C_1V_1 + C_2V_2)^2}{C_1 + C_2}$$

$$\therefore\ \text{Loss of energy, } \Delta E = E_i - E_f = \frac{1}{2}(C_1V_1^2 + C_2V_2^2) - \frac{1}{2}\frac{(C_1V_1 + C_2V_2)^2}{C_1 + C_2}$$

$$\therefore\ \Delta E = \frac{1}{2}\frac{C_1C_2}{C_1+C_2}(V_1-V_2)^2 \quad \ldots\ldots\ldots\ldots (6.25)$$

Now, ΔE is a positive quantity. So there is a definite loss of energy during sharing of charge between two conductors.

Energy loss is due to heating of the connecting wire. If there is sparking during connection, then energy is lost due to light, heat, sound etc.

● **Example 6.1. The radius of a metal sphere is 10 cm. What will be its capacitance when the medium is air ?**

○ **Solution :** Capacitance is $C = (4\pi\varepsilon_0)\,r$

$$\therefore\ C = \frac{1}{9\times10^9}\times 0{\cdot}1 \quad [r = 0{\cdot}1 \text{ m}]$$

$$\therefore\ \mathbf{C = 1{\cdot}11\times10^{-11}\ F}$$

● **Ex. 6.2. Two conductors having capacitance 40 and 60 esu are connected by a wire and a charge 10^{-6}C is given to them. What will be the final potential and charge an each conductor ?**

○ **Solution :** Common potential, $V = \dfrac{Q}{C_1+C_2}$

$$\therefore\ V = \frac{10^{-6}\times3\times10^9}{40+60} \quad [Q = 10^{-6}\times3\times10^9 \text{ esu},\ C_1 = 40 \text{ esu},\ C_2 = 60 \text{ esu}]$$

$\therefore$ Common potential, V = 30 esu.

Charge received by first conductor, $q_1 = C_1V = 40\times30 = 1200$ esu

and charge received by the second conductor, $q_2 = C_2V = 60\times30 = 1800$ esu

● **Ex. 6.3. 0·2 μF capacitor is charged to a potential of 50V. Calculate its charge and stored energy.**

○ **Solution :** Charge on the capacitor $q = CV$ $[C = 0{\cdot}2\times10^{-6}\text{ F},\ V = 50 \text{ volt}]$

$$\therefore\ q = 0{\cdot}2\times10^{-6}\times50 = \mathbf{10^{-5}\ C}$$

Stored energy, $E = \dfrac{1}{2}\dfrac{q^2}{C} = \dfrac{1}{2}\times\dfrac{10^{-10}}{0{\cdot}2\times10^{-6}} = \mathbf{2{\cdot}5\times10^{-4}\ J}$

● **Ex. 6.4. Two insulated metal spheres having radii 9 cm and 18 cm respectively are placed in air. They are given charges of 10^{-8}C and 3×10^{-8}C. If the spheres are connected by a wire, calculate (i) common potential of two wires and (ii) loss of energy.**

○ **Solution :** (i) Common potential, $V = \dfrac{q_1+q_2}{C_1+C_2}$

Here $C_1 = 9$ esu $= 10^{-11}$ F, $C_2 = 18$ esu $= 2\times10^{-11}$ F $\qquad q_1 = 10^{-8}$C, $q_2 = 3\times10^{-8}$C.

$$\therefore\ V = \frac{10^{-8}(1+3)}{10^{-11}(1+2)} = 1{\cdot}33\times10^3 \text{ volt}$$

Second part : The potential of the first sphere, $V_1 = \dfrac{q_1}{C_1} = \dfrac{10^{-8}}{10^{-11}} = 1000\text{ V}$

and potential of the second sphere, $V_2 = \dfrac{q_2}{C_2} = \dfrac{3\times10^{-8}}{2\times10^{-11}} = 1500\text{ volt}$

$$\therefore \text{ Loss of energy, } \Delta E = \frac{1}{2}\frac{C_1 C_2}{C_1 + C_2}(V_1 - V_2)^2$$

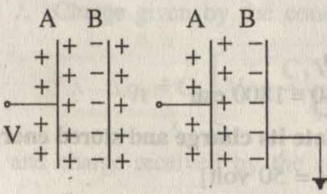
$$\therefore \Delta E = \frac{1}{2} \times \frac{2 \times 10^{-22}}{3 \times 10^{-11}}(1000 - 1500)^2 = \mathbf{8 \cdot 33 \times 10^{-7}\ J}$$

6.13. Capacitor and its principle

A capacitor is a combination of two conductors placed close to each other. Its one conductor is insulated and the other is connected to the earth. The space between the conductors is filled with air or by a dielectric medium. Different types of practical capacitors are (i) parallel plate capacitor (ii) spherical capacitor and (iii) cylindrical capacitor.

Principle : We know that capacitance of a conductor depends not only on its surface area and medium, but also on the presence of another conductor near it. It is found that when a conductor is placed near an insulated conductor, its ability to hold charge i.e. its capacitance increases.

Now, considering the fig. 6.11. The plate A is insulated and it is connected to an electrostatic machine so that it is charged to a potential + V. Now an identical plate B is placed near A. Due to induction inner surface of B gets negative charge and outer surface equal amount of positive charge. The negative charge of B will reduce the positive potential of A. On the other hand, positive charge of B will increase the positive potential of A by a small amount.



(i) (ii)

Fig. 6.11

Now, the initial potential of A was $V_{AO} = V$

Final potential of plate A is $V_{AB} = V_{AO} + V_{AB}- + V_{AB}+$

$$\therefore V_{AB} = V + V_{AB}- + V_{AB}+$$

Here $V_{AB}-$ = potential of A due to negative charge of B

and $V_{AB}+$ = potential of A due to positive charge of B

Since the negative charge is closer to A, $|V_{AB}-| > |V_{AB}+|$

$V_{AB}-$ being negative, $V_{AB} < V_{AO}$

Now, $C_{AB} = \dfrac{Q}{V_{AB}}$, $C_{AO} = \dfrac{Q}{V_{AO}}$ Hence $C_{AB} > C_{AO}$

It means that capacitance of plate A in presence of plate B (C_{AB}) is higher that the capacitance of plate A (C_{AO}) when it is kept isolated.

Next the plate B is connected to the earth. Immediately its free positive charge goes to the earth. As a result $V_{AB}+ = 0$. Hence V_{AB} will increase further. Evidently to raise the potential A, more charge has to be given to it. Thus the capacitance of a conductor can be raised by placing an earthed conductor near it.

6.14. Calculation of capacitance of different types of capacitor

The procedure to calculate the capacitance of a given capacitor is : A charge + Q is given the insulated plate of the capacitor. – Q is induced on the other plate. Next we calculate the electric field between the plates and from this potential difference 'V' between the plates. The capacitance is then obtained using the relation $C = \dfrac{Q}{V}$.

▲ (A) Parallel plate capacitor :

A parallel plate capacitor consists of two large plane plates parallel to each other with a small separation between them and the space between the plates is filled with air or any other dielectric medium [Fig. 6.12].

Let area of each of the facing surfaces is A and the separation between the plates is *d*. The space between the plates is filled with air.

Let + Q be the charge on insulated plate P_1. The charges will appear on the facing surfaces. The charge density on each of these surfaces has a magnitude $\sigma = \frac{Q}{A}$.

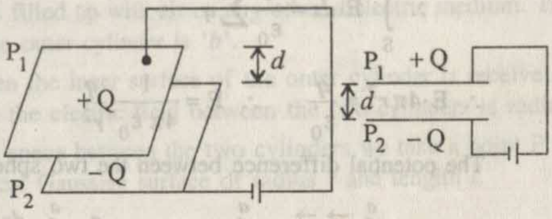


Fig. 6.12

Here we assume that the plates are large compared to the separation between the plates. Then the electric field between the plates is uniform and perpendicular to the plates except for a small region near the edge [Fig. 6.13].

The intensity of the electric field at any point between the plates is

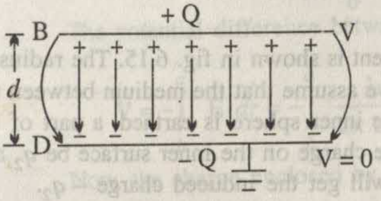


Fig. 6.13

$$E = \frac{\sigma}{\varepsilon_0} \quad \text{[Here medium is taken to be air]}$$

If V be the potential of plate B then,

$$V = -\int_D^B \vec{E}\cdot \vec{dr} = \int_0^d \frac{\sigma}{\varepsilon_0} dr = \frac{\sigma d}{\varepsilon_0} = \frac{Q\,d}{A\,\varepsilon_0}$$

$$\therefore \text{ Capacitance, } C_0 = \frac{q}{V} = \frac{\varepsilon_0 A}{d} \qquad \therefore C_0 = \frac{\varepsilon_0 A}{d} \quad \ldots\ldots\ldots\ldots\ldots (6.23)$$

$$\text{In C. G. S. system, } C_0 = \frac{1}{4\pi\,\varepsilon_0}\frac{\varepsilon_0 A}{d} = \frac{A}{4\pi d} \quad \ldots\ldots\ldots\ldots\ldots\ldots (6.23a)$$

▲ (B) Spherical capacitor :

A spherical capacitor consists of two concentric spherical shells, such that one shell is earthed and the other is used to store charge on it. The two spherical shells have a very small spacing between them. Let the space between the two shells be filled with air.

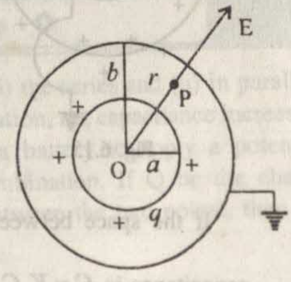


Fig. 6.14

● **Case I : When the outer shell is earthed :** The spherical capacitor with the outer sphere earthed is shown in fig. 6.14. Radius of the inner sphere is '*a*' and that of the outer sphere is '*b*'.

Now, the inner sphere is given a charge + *q*. Immediately inner surface of the outer sphere will receive an induced charge − *q*.

The space inside the charged hollow inner sphere is an equipotential region. So, electric intensity in this region is zero.

Hence in the space between the two spherical shells, electric intensity is due to inner shell only. The electric intensity at the point P at a distance

r $(a < r < b)$ from the centre O can be calculated by Gauss's theorem. We draw a Gaussian spherical surface of radius OP = r.

$$\therefore \int_S \vec{E}\cdot\vec{ds} = \frac{1}{\varepsilon_0}\sum q$$

$$\therefore \ E\cdot 4\pi r^2 = \frac{q}{\varepsilon_0} \qquad \therefore \ E = \frac{1}{4\pi\,\varepsilon_0}\,\frac{q}{r^2}$$

The potential difference between the two spherical shells is

$$V = -\int_b^a \vec{E}\cdot\vec{dr} = -\int_b^a E\,dr = -\frac{q}{4\pi\,\varepsilon_0}\int_b^a \frac{dr}{r^2} = -\frac{q}{4\pi\,\varepsilon_0}\left[\frac{1}{a}-\frac{1}{b}\right] = \frac{1}{4\pi\,\varepsilon_0}\,\frac{q\,(b-a)}{ab}$$

$$\therefore \ \text{Capacitance of the capacitor, } C_0 = \frac{q}{V} = 4\pi\,\varepsilon_0\,\frac{ab}{b-a} \quad \ldots\ldots\ldots\ldots\ldots\ldots (6.27)$$

If the space between the two shell be filled with a medium of dielectric constant K, then capacitance is $C = KC_0 = 4\pi\,\varepsilon_0\left(\dfrac{Kab}{b-a}\right)$

If $b = \infty$, $C_0 = 4\pi\,\varepsilon_0\,a$

● **Case II : When the inner shell is earthed :** The arrangement is shown in fig. 6.15. The radius of the inner sphere is 'a' and that of the outer sphere is 'b'. First we assume that the medium between the two spheres is air. The outer sphere is given a charge q. As the inner sphere is earthed, a part of the charge q will go to the inner surface of the outer sphere. Let the charge on the inner surface be q_2 and charge on the outer surface, $q_1 = q - q_2$. So the inner sphere will get the induced charge $-\,q_2$.

The potential of inner sphere is zero and potential at infinity is also zero. So, the potential of the outer sphere is $V = -\int_\infty^b \vec{E}\cdot\vec{dr} = -\int_a^b \vec{E}\cdot\vec{dr}$

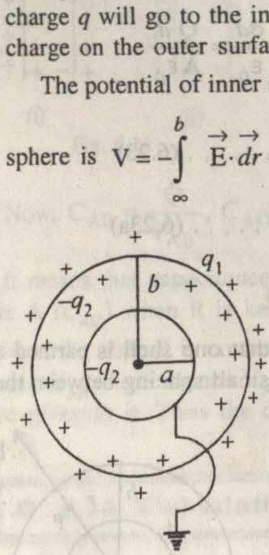


Fig. 6.15

$$\text{or, } V = -\int_\infty^b \frac{1}{4\pi\,\varepsilon_0}\,\frac{q_1}{r^2}\,dr = -\int_a^b \frac{1}{4\pi\,\varepsilon_0}\,\frac{q_2}{r^2}\,dr$$

$$= \frac{1}{4\pi\,\varepsilon_0}\,\frac{q_1}{b} = \frac{q_2}{4\pi\,\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)$$

$$\therefore \ q_1 = 4\pi\,\varepsilon_0\,b\,V, \quad q_2 = 4\pi\,\varepsilon_0\,V\,\frac{ab}{b-a}$$

$$\therefore \ q = q_1 + q_2 = 4\pi\,\varepsilon_0\,V\left[\frac{ab}{b-a}+b\right] = 4\pi\,\varepsilon_0\,V\,\frac{b^2}{b-a}$$

$$\therefore \ \text{Capacitance, } C_0 = \frac{q}{V} = 4\pi\,\varepsilon_0\,\frac{b^2}{b-a} \quad \ldots\ldots\ldots\ldots (6.28)$$

If the space between the two spheres be filled with a medium of dielectric constant K, then its capacitance is $C = K\,C_0 = 4\pi\,\varepsilon_0\,\dfrac{K\,b^2}{b-a}$ $\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$ (6.29)

▲ (C) Cylindrical capacitor :

A cylindrical capacitor consists of a solid or a hollow cylindrical conductor surrounded by another co-axial hollow cylindrical conductor [Fig. 6.16]. The outer cylinder is earthed and the inner sphere is given a charge. The space between the two cylinders is filled up with air or any other dielectric medium. The radius of the inner cylinder is '*a*' and that of the outer cylinder is '*b*'.

The inner cylinder is given a charge $+q$. Then the inner surface of the outer cylinder is receive an induced charge $-q$. We assume $l >> b$, so that the electric field between the two cylinders is radial.

To calculate the electric field intensity in the space between the two cylinders we take a point P so that $OP = r$ $(a < r < b)$. We consider a cylindrical Gaussian surface of radius r and lenglth l.

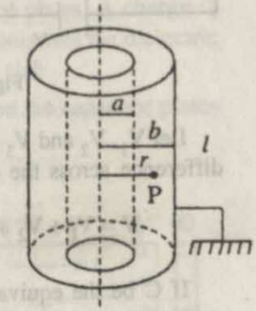


Fig. 6.16

$$\therefore \int_S \vec{E}\cdot\vec{ds} = \int_S E\,ds = E\times 2\pi rl$$

Hence by Gauss's theorem,

$$E\times 2\pi rl = \frac{q}{\varepsilon_0} = \frac{\lambda l}{\varepsilon_0} \quad [\lambda = \text{linear charge density}]$$

$$\therefore\ E = \frac{1}{4\pi\,\varepsilon_0}\frac{\lambda}{r}$$

The potential difference between the two cylinders is

$$V = -\int_b^a \vec{E}\cdot\vec{dr} = -\int_b^a \frac{\lambda}{2\pi\,\varepsilon_0 r}dr = \frac{\lambda}{2\pi\,\varepsilon_0}\log_e\frac{b}{a}$$

Now, the charge enclosed by the Gaussian surface, $q = l\lambda$

$\therefore$ Capacitance of the cylindrical capacitor with air as dielectric is

$$C_0 = \frac{q}{V} = \frac{l\lambda}{\frac{\lambda}{2\pi\,\varepsilon_0}\log_e\frac{b}{a}} = \frac{2\pi\,\varepsilon_0\,l}{\log_e\frac{b}{a}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6.30)$$

If the space between the two cylinder be filled with medium of dielectric constant K, then its capacitance will be

$$C = K\,C_0 = \frac{2\pi\,\varepsilon_0\,Kl}{\log_e\frac{b}{a}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6.31)$$

✪ 6.15. Combination of Capacitors ✪

Two or more number of capacitors may be connected in two ways – (i) the series and (ii) in parallel. In series combination net capacitance decreases and in the parallel combination, net capacitance increases. The combination must have two points which may be connected to a battery to apply a potential difference. The battery supplied the required charge to charge the combination. If Q be the charge supplied to the combination when a potential difference V is applied between the two points, then the equivalent capacitance of the combination is $C = \frac{Q}{V}$.

If the combination is replaced by a single capacitor of the capacitance, it will store same amount of charge for the same potential difference.

▲ (A) Series combination :

Fig. 6.17 shows the method of connecting three capacitors of capacitances C_1, C_2 and C_3 in series. The battery supplies a charge $+q$ to the plate A of capacitor C_1. It induces charge $-q$ to the left face and $+q$ to the right force of the plate B. This $+q$ charge goes to plate C of the capacitor C_2. Plate D gets induced charge $-q$ on its left face and $+q$ on its right face and it flows to the plate E of capacitor C_3. The charge induced on the right face of C_3 goes to earth. Thus the three capacitors get the same amount of charge $+q$, when the charge $+q$ is given to the first capacitor.

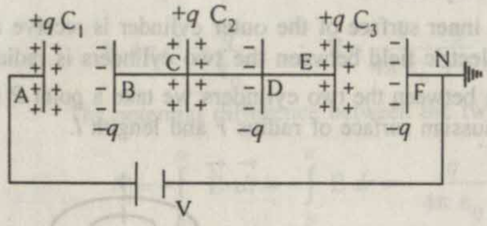


Fig. 6.17

Let V_1, V_2 and V_3 be the potential difference across the plates of the three capacitors. So, the potential difference across the combination is

$$V = V_1 + V_2 + V_3 = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3} = q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right) \quad \ldots\ldots\ldots\ldots\ldots \text{(i)}$$

If C be the equivalent capacitance of the series combination. So, when a potential difference V is applied across a capacitor of capacitance C, it will also store equal amount of charge.

$$\therefore \; V = \frac{q}{C} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(ii)}$$

From the relations (i) and (ii), $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$ ………………… (6.32)

- **Example 6.5. Calculate the charge and potential difference of each capacitor of fig. 6.18.**

○ **Solution :** The equivalent capacitance of series combination of C_1 and C_2 is

$$C = \frac{C_1 C_2}{C_1 + C_2} = \frac{10 \times 20}{30} = \frac{20}{3} \mu F$$

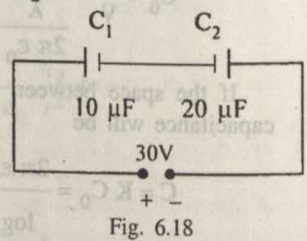


Fig. 6.18

The charge supplied by the battery, $q = CV = \frac{20}{3} \times 30 = \mathbf{200\,\mu c}$

$\therefore$ Potential difference of C_1, $V_1 = \frac{q}{C_1} = \frac{200}{10} = \mathbf{20\ volt}$

and potential difference of C_2, $V_2 = \frac{q}{C_2} = \frac{200}{200} = \mathbf{10\ volt}$

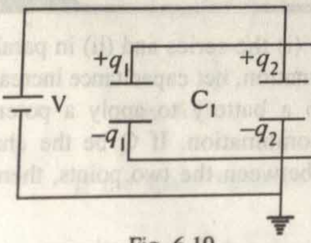


Fig. 6.19

▲ (B) Parallel combination :

Fig. 6.19 shows the parallel combination of three capacitors C_1, C_2 and C_3. One plate of each capacitor is connected to the source of emf and the other to the earth. So all the capacitors are directly connected between the source of emf and the earth. Thus potential differences across all capacitors are equal. The capacitors will draw different amount of charge from the source of emf.

Let the common potential difference be V. The amount of charges drawn by the capacitors C_1, C_2 and C_3 are respectively $q_1 = C_1V$, $q_2 = C_2V$ and $q_3 = C_3V$.

Total charge supplied by the source is $q = q_1 + q_2 + q_3 = V(C_1 + C_2 + C_3)$ . . . . (i)

If C be the equivalent capacitance of the combination of the capacitors, then $q = CV$ . . . . (ii)

From (i) and (ii) we get $CV = V(C_1 + C_2 + C_3)$ . . . .(6.33)

$\therefore C = C_1 + C_2 + C_3$

6.16. Parallel plate capacitor with dielectric slab

We consider a parallel plate capacitor with plate area A and separation *d* between the plates [Fig. 6.20]. A dielectric slab of dielectric constant K is inserted in the space between the plates. A charge Q is given to the positive plate and – Q to the negative plate. The applied electric field polarises the dielectric so that induced charges + Q′ and – Q′ appear on the two faces of the dielectric slab.

The electric field at a point between the plates due to the charges + Q and – Q on the capacitor plates is $E_0 = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$ . . . . (i)

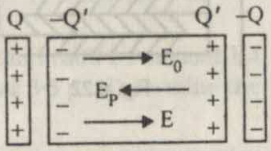


Fig. 6.20

It is directed from left to right as shown in fig. 6.20.

If K be the dielectric constant of the medium, then

$K = \frac{E_0}{E}$ where $E = E_0 - \frac{P}{\varepsilon_0}$

$\therefore E = \frac{E_0}{K} = \frac{Q}{A\varepsilon_0 K}$ . . . . (ii)

The potential difference between the plates is $V = Ed$

$$\therefore V = \frac{Qd}{\varepsilon_0 AK}$$

So, the capacitance of the capacitor with the dielectric is

$C = \frac{Q}{V} = \frac{K\varepsilon_0 A}{d} = KC_0$ . . . . (iii)

Here, C_0 is the capacitance without dielectric. Here, the capacity of the capacitor increases K times when the space between the plates is filled with a material of dielectric constant K.

● **Example 6.6. A parallel plate capacitor has plate area A and separation between the plates is '*d*'. The space between the plates is partly filled up by a dielectric of thickness *x* (*x* < *d*) and of dielectric constant K. Calculate capacitance of the system.**

○ **Solution :** The arrangement is shown in fig. 6.21. The given system is equivalent to a series combination of two capacitors— one between P and R and the other between R and Q. Here R represents the upper surface of the dielectric.

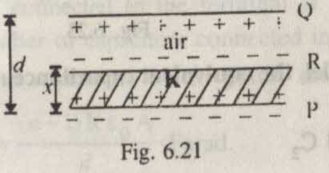


Fig. 6.21

Now, the capacitance between P and R is $C_1 = \frac{K\varepsilon_0 A}{x}$

and that between R and Q is $C_2 = \frac{\varepsilon_0 A}{d - x}$

∴ The equivalent capacitance is given by $\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}$

$$\therefore \frac{1}{C}=\frac{x}{K\varepsilon_0 A}+\frac{d-x}{\varepsilon_0 A}$$

$$\therefore C=\frac{\varepsilon_0 A}{\frac{x}{K}+\left(\frac{d-x}{1}\right)}=\frac{K\varepsilon_0 A}{Kd-x(K-1)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6.34)$$

- **A discussion :** (i) In c.g.s. system $=\frac{1}{4\pi\varepsilon_0}\frac{\varepsilon_0 A}{\frac{x}{K}+(d-x)}=\frac{A}{4\pi\left[\frac{x}{K}+(d-x)\right]}$

(ii) Instead of two plates we can also consider a capacitor provided with n number of plates and separation between any two consecutive plates is always same. Let alternate plates are connected to a common point as in fig. 6.22. So here we get $(n-1)$ number of identical capacitors connected in parallel. Hence total capacitance is

$$C'=(n-1)\,C=(n-1)\frac{K\varepsilon_0 A}{d}$$

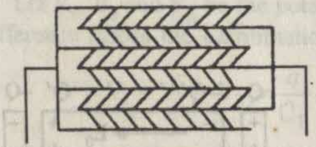


Fig. 6.22

6.17. Capacitance of parallel plate capacitor with a conducting slab between the plates

Consider a parallel plate capacitor having plate area A and separation d. The capacitance of the air capacitor is $C_0=\frac{\varepsilon_0 A}{d}$

When the capacitor is connected to a battery, the electric field between the plates is E_0. Now a conducting slab of thickness t $(t<d)$ is introduced between the plates. The electric field inside the plate is zero. So, electric field exists over the region $x=d-t$. Hence the potential difference between the plates will be reduced to $V=E_0\,(d-t)$

$$\therefore V=\frac{q}{\varepsilon_0 A}(d-t) \quad \left[\because E_0=\frac{q}{\varepsilon_0 A}\right]$$

So, the capacitance of the capacitor with the metal slab is

$$C=\frac{q}{V}=\frac{q}{\frac{q}{\varepsilon_0 A}(d-t)}=\frac{\varepsilon_0 A}{d-t}$$

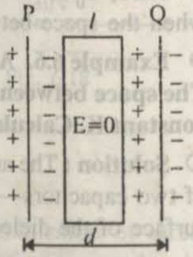


Fig. 6.23

- **Ex. 6.7. When two capacitors are connected in series and in parallel, the equivalent capacitance are 1·2 μF and 5 μF. Find the value of capacitance.**

○ **Solution :** Let the capacitances of the two capacitors are C_1 and C_2

$\therefore\ C_1 + C_2 = 5\ .\ .\ .\ .\ .\ .\ .\ .\ .\ (i)$ and $\dfrac{C_1 C_2}{C_1 + C_2} = 1{\cdot}2 \quad \therefore\ C_1 C_2 = 6(\mu F)^2$

$$\therefore\ C_1 - C_2 = \sqrt{(C_1 + C_2)^2 - 4C_1C_2} = \sqrt{25-24} = 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(ii)}$$

From (i) and (ii), $C_1 = 3\mu F$, $C_2 = 2\mu F$

- **Ex. 6.8. For a parallel plate capacitor, plate area 100 cm² and the plate separation 1 mm. When connected to 120 V supply, it holds a charge of 0·12 μC. What is the dielectric constant of the material between the plates ?**

○ **Solution :** Capacity of the parallel plate capacitor $C = \dfrac{q}{V} = \dfrac{0{\cdot}12 \times 10^{-6}}{120} = 10^{-9}\,F$

Now, $C = \dfrac{\varepsilon_0 KA}{d}$

$\therefore\ K = \dfrac{d \cdot C}{\varepsilon_0 A} \quad \therefore\ K = \dfrac{10^{-3} \times 10^{-9}}{8{\cdot}85 \times 10^{-12} \times 10^{-2}} \quad \therefore\ K = 11{\cdot}3$

$d = 1 \times 10^{-3}$ m

$C = 10^{-9}$ F

$\varepsilon_0 = 8{\cdot}85 \times 10^{-12}$

$A = 100 \times 10^{-4}\ m^2$

$\therefore$ Dielectric constant, **K = 11·3**

- **Ex. 6.9. A cylindrical capacitor is made of two cylinders of lengths 15 cm and radii 1·5 cm and 1·4 cm. The outer cylinder is earthed and the inner cylinder is given a charge of 3·5 μc. Determine the capacitance of the system and potential of the inner cylinder.**

○ **Solution :** Expression of capacitance, $C = \dfrac{2\pi\varepsilon_0 l}{\log_e b/a}$

$$\therefore\ C = \frac{2\pi \times 8{\cdot}85 \times 10^{-12} \times 0{\cdot}15}{2{\cdot}303 \times \log_{10}(15/14)} = 1{\cdot}21 \times 10^{-10}\,F$$

$\varepsilon_0 = 8{\cdot}85 \times 10^{-12}$

$l = 0{\cdot}15\,m$

$\dfrac{b}{a} = \dfrac{15}{14}$

Since the outer cylinder is earthed, the potential of the inner cylinder will be equal to the potential difference between them.

$\therefore$ Potential of the inner cylinder, $V = \dfrac{q}{C} = \dfrac{3{\cdot}5 \times 10^{-6}}{1{\cdot}21 \times 10^{-10}} = \mathbf{2{\cdot}89 \times 10^4}$ **volt**

✪ 6.18. Practical form of capacitors ✪

- **(A) Mica capacitor :** It is a type of parallel plate capacitor provided with large number of parallel plates each of equal area. This is done to increase the capacitance of the capacitor. It consists of two sets of metal foils insudated from each other with mica filling the space between them. [Fig. 6.24]. Here odd foils are connected together and then to a common terminal A. Similarly even foils are connected to the terminal B. The system this consists of a large number of capacitors connected in parallel.

If n number of tin plates are used, the total capacitance will be

$$C = \frac{(n-1)K\varepsilon_0 A}{d} \cdot \text{Farad.}$$

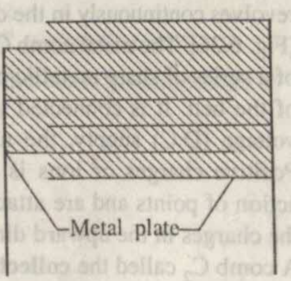


Fig. 6.24

● **(B) Paper capacitor :** It is a most common type of capacitor widely used in practical circuits. The capacitor is constructed by interleaving two long strips of aluminium or tin foils with two sheets of thin waxed or oiled paper [Fig. 6.25]. These are then rolled up into a tight cylindrical form. Each sheet of tin foil becomes one plate of the capacitor and the paper becomes the dielectric separating them.

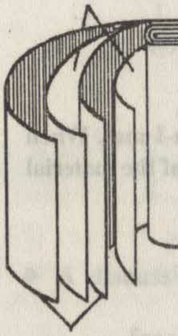


Fig. 6.25

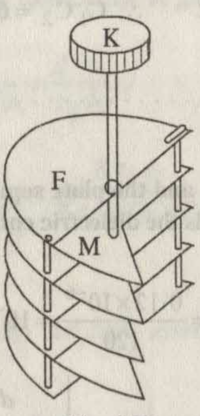


Fig. 6.26

● **(C) Variable air capacitor :** It is a modified parallel plate capacitor. Its capacitance can be varied at will. It consists of two sets (F and M) of semi circular aluminium or brass plates separated from each other (fig. 6.26). Air between the plates acts as the dielectric. The two sets of plates must remain separate. They must not touch each ohter. One set of plate (F) is fixed while the other set M can be rotated between F by a knob K. So the area of overlab between the plates can be varied. As a result, the capacitance changes. So, here capacitance can be varies from its minimum value to maximum.

This type of capacitor is used in tuning radio sets.

✪ 6.19. The Van de Graaff Generator ✪

R. J. Van de Graaff designed an electrostatic generator capable of generating very high potential of the order of 10^6 volt. It is to be noted that Van de Graaff generator is a collector of charge rather than a generator. Its performance depends on the following electrostatic phenomenon.

If a hollow conductor is in contact with another conductor, as charge is supplied to the conductor, the hollow conductor accepts the charge irrespective of the fact, however large its potential may grow. The charge immediately shifts to the outer surface of the hollow conductor.

Construction and working :

A simplified diagram of the Van de Graaff generator is shown in fig. 6.27. It consists of a large smooth metallic sphere G mounted on two insulating column C and C′. An endless belt made of some insulating material is made to run on two pulleys A and B with the help of some electric motor, of which the upper one a lies inside the hollow sphere.

The lower pulley B is driven by a motor M, the belt revolves continuously in the direction as shown in the diagram [Fig. 6.26]. The metal comb C_1, called **spray comb** (it consists of a series of sharp metallic points) is held near the lower end of the belt. It is connected to the positive terminal of high voltage (D.C) supply, the negative terminal being earthed. Positive charges or ions is sprayed from the comb C_1 by action of points and are attached to the belt. The belt carries the charges in the upward direction inside the hollow sphere. A comb C_2 called the **collection comb** is positioned near the upper end of the belt, such that the pointed ends touch the

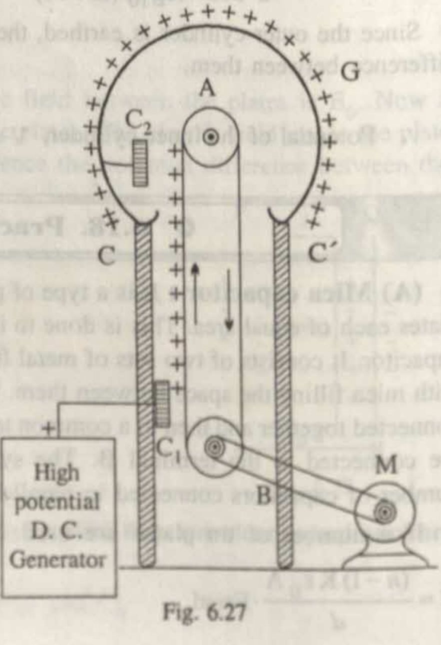


Fig. 6.27

belt and the other end is in contact with the inner surface of the metallic sphere G. The comb C_2 collects the positive ions and transfer them to metallic sphere. The charge immediately distributes itself over the outer surface of the sphere.

The uncharged belt moves round to C_1 again when the action is repeated. Thus as the belt rotates, the hollow sphere acquires more and more positive charges and its potential grows up. However, the potential of the shell is limited by the breakdown voltage of the shell. Then the charges pass to the surrounding atmosphere. The maximum potential to which the sphere can be raised is reached when the rate of loss of charge due to leakage becomes equal to the rate at which the charge is transferred to the sphere.

● ● Short Answer Type Questions (with answers) ● ●

● **Question 1. What is an dielectric ?**

○ **Ans.** Dielectric is a material in which there is no free electrons. All the electrons are bound to the atom.

● **2. Define dielectric constant of a dielectric.**

○ **Ans.** The dielectric constant of a dielectric is the ratio of the capacitance of a capacitor with the dielectric as the medium to the capacity of the same capacitor with air as dielectric. So, dielectric constant, $K = \dfrac{C}{C_0} = \dfrac{\text{Capacitance of the capacitor with the medium}}{\text{Capacitance of the capacitor with air as medium}}$

● **Q. 3. What we mean by 'dielectric constant of glass is 8·5' ?**

○ **Ans.** 'Dielectric constant of glass is 8·5' means that the ratio of the capacitance of a capacitor with glass as dielectric to the capacity of the capacitor with air as dielectric is 8·5.

● **Q. 4. What is meant by capacitance of a capacitor is 1μF ?**

○ **Ans.** Capacitance of a capacitor is 1μF means that to raise the potential of the capacitor by 1 volt, it requires a charge of 1 μc.

● **Q. 5. A capacitor is charged by a battery and then the battery is disconnected. Now a dielectric slab is introduced between the plates of the capacitor. What will be the change in charge, capacitance and potential difference of the capacitor ?**

○ **Ans.** (i) The battery is disconnected from the capacitor. So its charge will remain same. So $\Delta Q = 0$.

(ii) When the dielectric slab is introduced, the capacity will increase K times where K is the dielectric constant.

Initial capacitance = C_0, final capacitance $C = KC_0$

∴ Change is capacitance $\Delta C = C - C_0 = C_0 (K - 1)$

(iii) Potential difference between the plates will increase.

Potential difference across the capacitor, $V = q/C$ when q = charge and C = capacitance.

If C_0 be the capacitance with air as the medium then, $V = q/C_0$ when the dielectric is introduced, $V' = q/KC_0$

$$\therefore \; V' = \frac{V}{K}$$

∴ Charge in potential difference, $\Delta V = V' - V = \dfrac{V}{K} - V = V\left(\dfrac{1}{K} - 1\right)$

$$\therefore \Delta V = -V\left(\frac{K-1}{K}\right)$$

● **Q. 6. Two conductors carry equal amount of similar charge. Can there be any potential difference between the two conductors ? [J. E. E. 1988]**

○ **Ans.** Although the conductors carry equal amount of similar charge, still there may be a potential difference between the two conductors.

Reason : For a given charge, potential of a conductor depends on its capacitance. Potential $V = \frac{\text{Charge Q}}{\text{Capacitance C}}$. So if Q is fixed, VC is constant. So, the conductor whose capacitance is large, potential will be less.

● **Q. 7. A parallel plate capacitor is charged to a potential V. When the separation between the plates is *d*, the electric field intensity at a point between the plates is E. What will be the electric field if the separation is increased to 2*d* ?**

○ **Ans.** In the first case, the electric field between the plates is $E = V/d$. When the separation between the plates is made double, capacitance of the capacitor will be half $C_1 = C/2$.

But the charge on each plate remains same, as the capacitor is not connected to source of emf.

$$\therefore \text{ Potential on the capacitor, } V_1 = \frac{Q}{C_1} = \frac{2Q}{C} = 2V$$

$$\therefore \text{ The electric field between the plates, } E_1 = \frac{V_1}{2d} = \frac{2V}{2d} = \frac{V}{d} = E$$

∴ The electric field between the plates remains unchanged.

● **Q. 8. N identical capacitors are connected in parallel and potential difference 'V' is applied to the combination. Now, the capacitors are connected in parallel ; what will the potential difference across the combination.**

○ **Ans.** Let the capacity of each capacitor is C.

Charge on each capacitor is $Q = CV$

When the capacitors are connected in series equivalent capacitance is $C_S = C/N$ and total charge $= Q$.

$$\therefore \text{ Potential difference across the combination is } V_1 = \frac{Q}{C_S} = \frac{Q}{C/N} = N\frac{Q}{C} = NV$$

● **Q. 9. A mica sheet of width equal to the separation between the plates of a parallel plate capacitor is introduced between the plates of the capacitor. Obtain the change in capacitance in the following cases :**

(i) When the sheet is partly introduced and (ii) When the sheet occupies whole space between the plates.

○ **Ans.** (i) The situation is shown in fig. 6.28 (i) Capacitance will increase Dielectric constant of the mica sheet is K. Area of plate is A. Area of the mica sheet within the capacitor in A_1 $(A_1 < A)$.

Total capacitance C = Capacitance of the part of the parallel plates filled by mica sheet + capacitance of the rest portion of the capacitor.

$$\therefore C = \frac{K\varepsilon_0 A_1}{d} + \frac{\varepsilon_0 (A - A_1)}{d} \quad [d = \text{separation between the plates}]$$

$$= \frac{\varepsilon_0}{d}\left[KA_1 + (A - A_1)\right] = \frac{\varepsilon_0}{d}\left[A + (K-1)A_1\right]$$

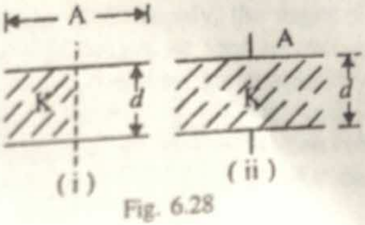


Fig. 6.28

(ii) Consider the fig. 6.28(ii).

Here whole space is filled with dielectric. So its capacity will increase K times where K is the dielectric constant of the medium. Hence total capacitance $= KC_0 = \frac{K\varepsilon_0 A}{d}$

Q. 10. Capacity of a conducting sphere is C. It is placed inside a hollow conducting sphere concentrically and it is earth connected. Show that the capacity of the conducting sphere will increase *n* times where the diameters of the two sphere are in the ratio $(n-1):n$.

○ **Ans.** Capacitance of the spherical conductor is $C = (4\pi\varepsilon_0)a$ where '*a*' is the radius of inner sphere.

When the conducting sphere is placed inside the hollow sphere which is earthed, then the capacity is $C_1 = \frac{ab}{b-a}$ [*b* = radius of the hollow sphere]

Now, $\frac{b}{a} = \frac{n}{n-1}$ $\therefore$ $\frac{C_1}{C} = \frac{ab}{a(b-a)} = \frac{b}{b-a}$

$$\therefore \quad \frac{C_1}{C} = \frac{1}{1-\frac{a}{b}} = \frac{1}{1-\frac{n-1}{n}} = n \qquad \therefore C_1 = nC$$

Q. 11. Two identical metal plates are given charges q_1 and q_2 $(q_2 < q_1)$ respectively. If they are now brought close together to form a parallel plate capacitor with capacitance C, what will be the potential difference between the plates. [I. I. T. 1999]

○ **Ans.** Let area of each plate is A. When they are placed at a distance '*d*', the capacitance of parallel plate capacitor is $C = \frac{\varepsilon_0 A}{d}$.

Let E_1 and E_2 are electric fields due to the two plates, then net electric field between the two plates of the capacitor is $E = E_1 - E_2 = \frac{q_1/A}{2\varepsilon_0} - \frac{q_2/A}{2\varepsilon_0} = \frac{q_1 - q_2}{2A\varepsilon_0}$

$\therefore$ Potential difference between the two plates of the capacitor is

$$V = Ed = \frac{1}{2\varepsilon_0 A}(q_1 - q_2) \times d = \frac{d}{2\varepsilon_0 A}(q_1 - q_2) = \frac{q_1 - q_2}{C}$$

●● Numerical Examples ●●

Example 6.10. Area of each plate of a parallel plate air capacitor is 200 cm² and plate separation 2 mm. One plate is earthed and the other is at potential 300 volts. Calculate the charge on each plate. [W. B. H. S. 2001]

○ **Solution :** Capacitance of the given parallel plate capacitor is $C = \frac{\varepsilon_0 KA}{d}$

$\varepsilon_0 = 8{\cdot}85 \times 10^{-12}$
$K = 1$
$A = 2 \times 10^{-2}\ \text{m}^2$
$d = 2 \times 10^{-3}\ \text{m}$
$V = 300\ \text{volt}$

$$C = \frac{8{\cdot}85 \times 10^{-12} \times 1 \times 2 \times 10^{-2}}{2 \times 10^{-3}}$$

$$= 8{\cdot}85 \times 10^{-11}\ \text{F}$$

$\therefore$ Charge on the capacitor, $Q - CV = 8{\cdot}85 \times 10^{-11} \times 300 = 26{\cdot}55 \times 10^{-9}$ coulomb.

- **Ex. 6.11. In a spherical capacitor, radius of the inner sphere is 9 cm and that of the outer sphere is 10 cm. Calculate its capacitance when (i) outer shell is earth connected (ii) inner sphere is earth connected.**

○ **Solution :** (i) $C = 4\pi\varepsilon_0 \dfrac{ab}{b-a}$. Here $a = 9$ cm $= 0{\cdot}09$ m, $b = 0{\cdot}1$ m, $\varepsilon_0 = 8{\cdot}85 \times 10^{-12}$

$$\therefore\ C = \frac{4 \times 3{\cdot}14 \times 8{\cdot}85 \times 10^{-12} \times 0{\cdot}09 \times 0{\cdot}1}{0{\cdot}10 - 0{\cdot}09} = \mathbf{100{\cdot}5 \times 10^{-12}\ F}$$

$$\text{(ii) } C = 4\pi\varepsilon_0 \frac{b^2}{b-a} = \frac{4 \times 3{\cdot}14 \times 8{\cdot}85 \times 10^{-12} \times (0{\cdot}1)^2}{0{\cdot}10 - 0{\cdot}09} = \mathbf{11{\cdot}75 \times 10^{-12}\ F}$$

- **Ex. 6.12. A parallel plate capacitor is made of 21 circular metal plates each of diameter 10 cm. Th plates are separated by mica sheet of thickness 0·2 mm. If alternate plates are connected to two commo points, calculate the capacity of the capacitor in microfarad. Dielectric constant of mica = 6.** **[H.S. 199]**

○ **Solution :** Total capacitance, $C = (n-1)\dfrac{\varepsilon_0 KA}{d}$

$n = 21$
$\varepsilon_0 = 8{\cdot}85 \times 10^{-12}$
$K = 6$
$d = 2 \times 10^{-4}$ cm
$A = \pi \times (0{\cdot}05)^2 = 25\pi \times 10^{-4}\ \text{m}^2$

$$\therefore\ C = \frac{(21-1) \times 8{\cdot}85 \times 10^{-12} \times 6 \times 25\pi \times 10^{-4}}{2 \times 10^{-4}} = 4{\cdot}17 \times 10^{-2}\,\mu\text{F}$$

- **Ex. 6.13. Two concentric metal spheres of inner radius 4 cm and enter radius 6 cm are taken. Outer sphere is given a charge of + 30 esu and inner sphere is earthed. What is the charge on the inner sphere ? Calculate capacity of the system.** **[J.E.E. 1994]**

○ **Solution :** If the radius of the inner sphere is a, radius of the outer sphere is b and if the outer sphere is given a charge Q, then the induced charge on the inner sphere is $Q_1 = -Q\dfrac{a}{b} = -30 \times \dfrac{4}{6} = -20$ esu

2nd part : Capacitance of the system, $C = \dfrac{b^2}{b-a} = \dfrac{6^2}{6-4}$

$$\therefore\ C = 18 \text{ esu} = \mathbf{2 \times 10^{-11}\ F}$$

- **Ex. 6.14. Two spheres having radii 4 cm and 6 cm are given charge of 20 and 42 esu. Now they are connected by a conducting wire. Calculate (i) common potential and surface density of charge. Also calculate loss of energy.**

○ **Solution :** (i) Capacitance of first sphere $C_1 = 4$ esu and that of the second sphere $= 6$ esu

$\therefore$ Potential of the first sphere, $V_1 = \dfrac{20}{4} = 5$ esu

Potential of the second sphere, $V_2 = \dfrac{42}{6} = 7$ esu

When the spheres are connected by a wire, the common potential is

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = \frac{4 \times 5 + 6 \times 7}{4 + 6} = 6{\cdot}2 \text{ esu}$$

2nd part : Charge on the first sphere, $q_1 = C_1V = 4 \times 6 \cdot 2 = 24 \cdot 8$ esu

$\therefore$ Surface density of charge of the first sphere, $\sigma_1 = \dfrac{24 \cdot 8}{4\pi \times (4)^2} = 0 \cdot 12 \text{ esu/cm}^2$

Similarly, charge on the second sphere $q_2 = C_2V = 6 \times 6 \cdot 2 = 37 \cdot 2$ esu

Surface density of charge on the second sphere, $\sigma_2 = \dfrac{37 \cdot 2}{4\pi \times (6)^2} = 0 \cdot 082 \text{ esu/cm}^2$

3rd part : Loss of energy, $E = \dfrac{1}{2}\dfrac{C_1C_2}{C_1 + C_2}(V_1 - V_2)^2$

$\therefore \; E = \dfrac{1}{2} \times \dfrac{4 \times 6}{4+6}(5-7)^2 = \mathbf{4 \cdot 8 \text{ erg}}$

○ **Ex. 6.15. 64 identical water drops coalesce to form a larger drop. If nature and amount of charge be same for all the drops, calculate potential and capacitance of larger drop. Also calculate increase of stored energy.**

○ **Solution :** Let radius of each drop be r and charge $+q$

$\therefore$ Capacitance of each drop is $C_1 = r$ and potential $V_1 = \dfrac{q}{C_1} = \dfrac{q}{r}$

initial energy of 64 drops, $E_1 = 64 \times \dfrac{1}{2} C_1 V_1^2$

$\therefore \; E_1 = 64 \times \dfrac{1}{2} \times \dfrac{q^2}{r} = 32\dfrac{q^2}{r}$

Now, let the radius of the larger drop be R when 64 drops coalesce, then

$\dfrac{4}{3}\pi R^3 = \dfrac{4}{3}\pi R^3 \times 64 \qquad \therefore \; R = 4r$

So, the capacitance of the larger capacitor is $C = 4r$

Its charge $Q = 64\, q$

$\therefore$ Potential of the bigger drop, $V = \dfrac{Q}{C} = \dfrac{64q}{4r} = \mathbf{16\, V_1}$

and its energy $= \dfrac{1}{2}CV^2 = \dfrac{1}{2} \times 4r \times \left(16\dfrac{q}{r}\right)^2 = 512\dfrac{q^2}{r} = \mathbf{16\, E_1}$

So, the potential will increase 4 times and energy 16 times.

- **Ex. 6.16. When a dielectric slab of thickness 4 mm is introduced in the space between two parallel plates of a capacitor, it is found that the value of the capacitance of the capacitor will be equal to its initial value if the separation between the plates is increased by 3·5 mm. What is the dielectric constant of the material of the slab ?**

○ **Solution :** The capacitance of parallel plate air capacitor is $C = \dfrac{\varepsilon_0 A}{d}$. When the slab is introduced, to keep the value of capacitance same the separation between the plates becomes $d_1 = d + 3 \cdot 5$ mm.

It capacitance, $C_1 = \dfrac{\varepsilon_0 A}{d_1 - t + \dfrac{t}{K}}$ [t = thickness of the slab]

But equation $C = C_1$

$$\therefore\ \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 A}{d_1 - t + \dfrac{t}{K}} \qquad \therefore\ d = d_1 - t + \frac{t}{K}$$

$$\therefore\ d = d + 3{\cdot}5\times10^{-3} - t + \frac{t}{K}$$

$$\therefore\ t - \frac{t}{K} = 3{\cdot}5\times10^{-3}\ \text{m} = \mathbf{3{\cdot}5\ mm}$$

$$\therefore\ 1 - \frac{1}{K} = 3{\cdot}5\times10^{-3}\ \text{m} = \frac{3{\cdot}5}{4} \qquad \therefore\ \mathbf{K = 8}$$

So, dielectric constant of the table is K = 8

- **Ex. 6.17. A capacitor of capacitance $10^{-4}\ \mu$F is charged to a potential of 100 V. After disconnecting the battery, the capacitor is connected to another capacitor in parallel. If the final potential be 50V, calculate the capacitance of the second capacitor.**

○ **Solution :** Assume $C_1 = 10^{-10}$ F, $V_1 = 10$V

So, its charge, $Q_1 = C_1 V_1 = 10^{-10} \times 100 = 10^{-8}$C

If the capacitance of the second capacitor be C_2, then total capacitance $C = C_1 + C_2$

$$\text{Now, } C = \frac{Q_1}{V} = \frac{10^{-8}}{50}$$

$$\therefore\ C_1 + C_2 = \frac{1}{5}\times10^{-9}$$

$$\therefore\ C_2 = \frac{1}{5}\times10^{-9} - 10^{-10} = 10^{-10}\text{F}$$

∴ Capacitance of the second capacitor = $\mathbf{10^{-10}}$**F.**

- **Ex. 6.18. Two charged bubbles of equal volume combine to form a large bubble. If the potential of each of smaller bubble be V, what is the potential of the larger bubble ? [J. E. E. 2001]**

○ **Solution :** Assume that radius of the smaller bubble is r.

So, its capacitance is $C_1 = r$ and charge $q_1 = C_1 V = rV$

If R be the radius of the larger bubble then, $\dfrac{4}{3}\pi R^3 = 2\times\dfrac{4}{3}\pi r^3 \quad \therefore\ R = \sqrt[3]{2}\cdot r$

∴ Capacitance of the larger bubble $C = \sqrt[3]{2}\cdot r$ and its total charge, $Q = 2rV$

∴ Potential of the larger bubble, $V_1 = \dfrac{Q}{C} = \dfrac{2rV}{\sqrt[3]{2}r} = (2)^{2/3}\cdot V$

● **Ex. 6.19. A parallel plate capacitor of plate area 50 cm^2 and plate separation 2 mm is connected to 100 V supply. The space between the plates is filled up with a dielectric of dielectric constant K = 5. Calculate charge and potential in the following cases.**

(i) when the source remains connected (ii) when the source is disconnected. [J. E. E. 2001]

○ **Solution :** Capacity of the air capacitor, $C_0 = \frac{\varepsilon_0 A}{d}$

$$\therefore \quad C_0 = \frac{8\cdot 85\times 10^{-12}\times 50\times 10^{-4}}{2\times 10^{-3}} = \frac{8\cdot 85\times 5}{2}\times 10^{-12}\ \text{F}$$

Its charge, $q_0 = C_0 V = \frac{8\cdot 85\times 5}{2}\times 100\times 10^{-12} = \frac{8\cdot 85\times 5}{2}\times 10^{-10}\text{C}$

Now, if a dielectric is inserted, its capacitance is $C = KC_0$

$$C = \frac{8\cdot 85\times 25}{2}\times 10^{-12}\ \text{F}$$

(i) When the source of emf remains connected, voltage remains constant. So, change of potential, $\Delta V = 0$,

Charge on the capacitor, $q_1 = \frac{8\cdot 85\times 25}{2}\times 10^{-10}\ \text{C}$

$\therefore$ Charge of charge, $\Delta q = q_1 - q = (25-5)\times \frac{8\cdot 85}{2}\times 10^{-10}$

$$\therefore \Delta q = 8\cdot 85\times 10^{-9}\ \text{C}$$

(ii) When the source is disconnected before the dielectric is inserted, then charge remains constant. So $\Delta q = 0$.

Initial potential $V = 100$ V

$\therefore$ Final potential, $V_1 = \frac{q_0}{C} = \frac{5}{25}\times 10^2 = \mathbf{20\ volt}$

$\therefore$ Change of potential, $\Delta V = V_1 - V = 20 - 100 = \mathbf{-80\ volt}$

● **Ex. 6.20. The capacitances of two capacitors A and B are 1 μF and 2μF respectively. Both are charged to a potential of 12V. They are connected in parallel so that (i) positive plate of one capacitor is connected to the positive plate of the other, (ii) positive plate of one is connected to the negative plate of the other. [I. I. T.]**

○ **Solution :** Initial charge on one capacitor, $q_1 = C_1 V_1 = 1\times 12 = 12\ \mu\text{C}$

and initial charge of second capacitor, $q_2 = C_2 V_2 = 2\times 12 = 24\ \mu\text{C}$

(i) In the first case, both the positive plates and both the negative plates are connected together. As the capacitors are at the same potential, no transference of charge takes place. So there will be no change of potential.

$\therefore$ Final potential of the combination is $V = 12$ volt.

(ii) In the second case, the positive plate of one capacitor is connected to the negative plate of the other. So due to redistribution of charge, final charge, $q' = 24 - 12 = 12\ \mu\text{C}$

$\therefore$ Difference of potential between the two plates of the capacitor is now, $V' = \frac{q'}{C} = \frac{12}{3} = \mathbf{4\ volt}$

- **Ex. 6.21. Between two spherical capacitors, volume of one is double the other. Both are given same amount of charge of same nature. If they are connected by a wire, what will be the final charge of each sphere ?**

○ **Solution :** Let the volume of the first sphere be r and that of the other is R.

Then, $\frac{4}{3}\pi R^3 = 2\times\frac{4}{3}\pi r^3 \quad \therefore \ R = \sqrt[3]{2}\cdot r$

$\therefore$ Capacitance of the first sphere $C_1 = r$ and that of the second sphere is $C_2 = \sqrt[3]{2}r$

$\therefore$ Potential of the first sphere, $V_1 = \frac{Q}{r}$ and that of the second, $V_2 = \frac{Q}{\sqrt[3]{2}r}$

$\therefore \ V_2 = \frac{Q}{1{\cdot}26r}$

When they are connected by a conducting wire, their common potential is

$$V = \frac{Q_1+Q_2}{C_1+C_2} = \frac{2Q}{r+1{\cdot}6r} = \frac{Q}{1{\cdot}13r}$$

Final charge on the first sphere, $q_1 = C_1V = r\cdot\frac{Q}{1{\cdot}13r} = 0{\cdot}885\ Q/\text{Coulomb}$

and Final charge on the second sphere, $q_2 = C_2V = 1{\cdot}26r = r\cdot\frac{Q}{1{\cdot}13r} = \mathbf{1{\cdot}115\ Q}$

- **6.22. Four capacitors each 12μF are connected as in fig. 6.29. (i) What is the equivalent capacitance between the points A and B ? (ii) If a potential difference of 300 V is applied across A and B, calculate the charge of each capacitor. (iii) What charge is supplied by the source ?**

○ **Solution :** (i) Here C_2, C_3 and C_4 are connected in series. Their equivalent capacitance C_5 is given by

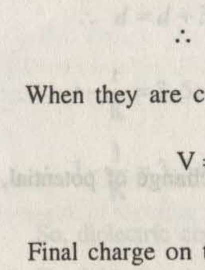


Fig. 6.29

$$\frac{1}{C_5} = \frac{1}{C_2}+\frac{1}{C_3}+\frac{1}{C_4} = \frac{3}{12} \quad \therefore \ C_5 = 4\mu F$$

Equivalent capacitance between the points A and B is

$$C = C_1 + C_5 = 12 + 4 = 16\ \mu F$$

(ii) Charge across C_1 is Q_1 and that across C_5 is Q_2

$\therefore \quad Q_1 = C_1V = 12 \times 300 = 3600\ \mu C$

and $Q_5 = C_3V = V \times 300 = 1200\ \mu C$

As, C_2, C_3, C_4 are in series, each has the charge $Q_2 = 1200\ \mu C$

(iii) Charge supplied by the source is $Q = Q_1 + Q_2 = \mathbf{4800\ \mu C}$

- **Ex. 6.23. Three capacitors C_1 = 2μF, C_2 = 2μF and C_3 = 1μF are connected as in fig. 6.30. A potential difference of 100 V is applied across the combination. Calculate : (i) Equivalent capcitance, (ii) Potential of each capacitor and (iii) charge of each capacitor.**

○ **Solution :** (i) The equivalent capacity of the combination of capacitors is given by

$$\frac{1}{C} = \frac{1}{C_1}+\frac{1}{C_2+C_3} \ [\because C_2 \text{ and } C_3 \text{ are in parallel}]$$

$$= \frac{1}{2}+\frac{1}{3} \quad \therefore \ C = \frac{6}{5} = 1{\cdot}2\ \mu F$$

(ii) Total charge supplied by the battery, $q = CV = 1{\cdot}2 \times 100 = 120\ \mu C$

$\therefore$ Charge across C_1, $q_1 = q = 120\ \mu C$

$\therefore$ Potential across C_1, $V_1 = \dfrac{q_1}{C_1} = \dfrac{120}{2} = 60\ V$

$\therefore$ Potential across C_2 or C_3, $V' = 100 - 60 = 40$ volt

(iii) Charge across C_2, $q_2 = C_2V_2 = 2 \times 40 = 80\ \mu C$

and charge across C_3, $q_3 = C_3V_3 = 1 \times 40 = 40\ \mu C$

$\therefore$ Charges across C_1, C_2 and C_3 are 120 μC, 80 μC and 40 μC respectively.

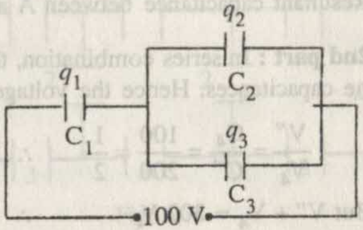


Fig. 6.30

● **Ex. 6.24. The two plates of a capacitor are square of side l. The angle between the p;ates is θ [Fig. 6.31]. If the minimum separation between the plates is t, calculate the capacitance of the capacitor.**

○ **Solution :** The arrangement of the plates is shown in fig. 6.31.

The average distance between the plates is $d = \dfrac{t + t + l \sin\theta}{2} = t + \dfrac{l\theta}{2}$ $[\because \sin\theta = \theta]$

$\therefore$ Capacity of the capacitor, $C = \dfrac{\varepsilon_0 KA}{d}$

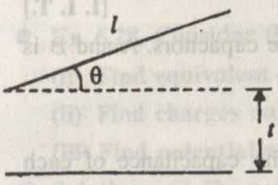


Fig. 6.31

$$C = \frac{\varepsilon_0 KA}{d} = \frac{\varepsilon_0 l^2}{t + \dfrac{l\theta}{2}} \quad \left[K = 1,\ A = l^2\right]$$

$$= \frac{\varepsilon_0 l^2}{t\left(t + \dfrac{l\theta}{2t}\right)} = \frac{\varepsilon_0 l^2}{t}\left(t - \frac{l\theta}{2t}\right)$$

● **Ex. 6.25. Find the equivalent capacitance of the network shown in fig. 6.32. For a 300 V supply determine the voltage across each capacitor.**

○ **Solution :** The equivalent capacitance of C_2 and C_3 is $C' = \dfrac{C_2C_3}{C_2 + C_3} = \dfrac{200 \times 200}{400} = 100\,pF$

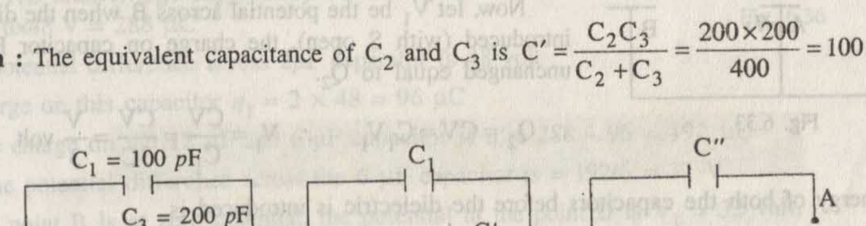


Fig. 6.32

The given circuit reduces to fig. 6.32 (ii). The equivalent capacitance between A and B is

$C'' = C' + C_1 = 100 + 100 = 200\ pF$

The final simplified circuit is shown in fig. (iii),

Resultant capacitance between A and D is $C = \dfrac{C'' \times C_4}{C'' + C_4} = \dfrac{200 \times 100}{300} = \dfrac{200}{3}\,pF$

2nd part : In series combination, the voltage divides among the capacitors in the inverse proportion of the capacitances. Hence the voltages across C″ and C_4 [Fig. 6.32 (ii)] are in the ratio

$$\frac{V''}{V_4} = \frac{C_4}{C''} = \frac{100}{200} = \frac{1}{2} \qquad \therefore\ V_4 = 2V''$$

But $V'' + V_4 = 300$ V $\qquad \therefore$ **$V_4 = 200$ V and $V'' = 100$ V.**

Evidently voltage across C′ and C_1 = voltage across C″

$\therefore$ Voltage across $C_1 = 100$ V and voltage across $C' = 100$ V.

$$\text{And } \frac{\text{voltage across } C_2\,(V_2)}{\text{voltage across } C_3\,(V_3)} = \frac{C_3}{C_2} = \frac{200}{200} = 1$$

But $V_2 + V_3 = 100$ V. Hence $V_2 = 50$ V and $V_3 = 50$ V

Then, $V_1 = 100$ V, $V_2 = 50$ V, $V_3 = 50$ V and $V_4 = 200$ V.

- **Ex. 6.26. Fig. 6.33 shows two identical parallel plate capacitors connected by a battery with the switch 'S' closed. The switch is now open and the space between the plates of the capacitor is filled with a dielectric of dielectric constant 3. Find the ratio of the total electrostatic energy stored in both capacitors before and after the introduction of the dielectric. [I. I. T.]**

○ **Solution :** When the switch 'S' is closed, the potential difference across the capacitors. A and B is the same = V volt.

$\therefore$ Charges on A and B are $q_1 = q_2 = CV$

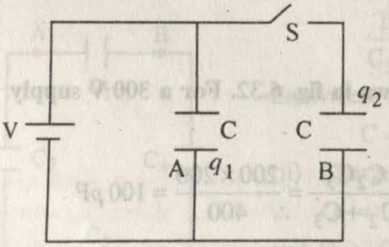


Fig. 6.33

When a dielectric is intorduced the capacitance of each capacitor will be $C_1 = KC = 3C$

After the switch is open, the potential across A remain unchanged equal to V.

Now, let V_1 be the potential across B when the dielectric is introduced (with S open), the charge on capacitor B remains unchanged equal to Q_2.

$$\therefore\ Q_2 = CV = C_1V_1 \quad \therefore\ V_1 = \frac{CV}{C_1} = \frac{CV}{3C} = \frac{V}{3} \text{ volt}$$

Next, energy of both the capacitors before the dielectric is introduced is

$$U = \frac{1}{2}CV^2 + \frac{1}{2}CV^2 = CV^2$$

Also energy of the capacitors when the dielectric is introduced is $U_1 = \dfrac{1}{2}C_1V^2 + \dfrac{1}{2}C_1V_1^2$

$$= \frac{1}{2}3C \times V^2 + \frac{1}{2} \times 3C \times \left(\frac{V}{3}\right)^2 = \frac{5}{3}CV^2 \quad \therefore\ \frac{U}{U_1} = \frac{3}{5}$$

● **Ex. 6.27. In fig. 6.34 capacitances are in μF units. Calculate (i) total capacitance between the point A and B (ii) charges on the capacitors near the point A and B when V_{AB} = 900 V (iii) when the potential difference between A and B is 900 V, the potential difference across C and D.**

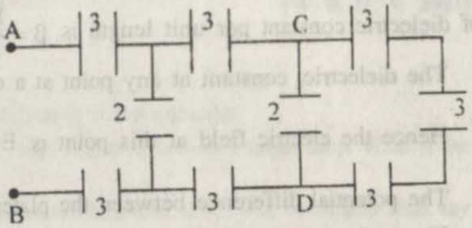


Fig. 6.34

○ **Solution :** (i) Equivalent capacitance of three capacitors on the right of C and D is C_1 = 1 μF (as each of 3 μF).

So, capacitance between C and D = Parallel combination of 1 μF and 2μF = 3 μF

In this way we can show that equivalent capacitance between A and B is C = 1 μF

(ii) Three capacitors each of 3 μF are near to the points A and B (Fig. 6.35]. If q be the charge across each capacitor, potential difference across each is $V = \frac{q}{C}$ [C = 3μF]

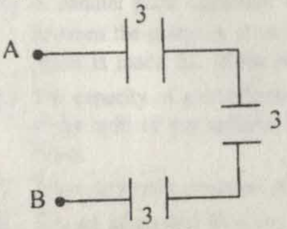


Fig. 6.35

$$\therefore \quad \frac{3q}{C} = 900 \qquad \therefore \; q = 300\,\mu C$$

∴ Each capacitor near A and B carries a charge 300 μC.

(iii) Potential difference across the points C and D is

$$V' = 900 - 300 - 300 = \mathbf{300\ volt}$$

● **Ex. 6.28. Consider the connection shown in fig. 6.36.**

(i) Find equivalent capacitance across A and B.

(ii) Find charges on three capacitors.

(iii) Find potential at D, if that at B is zero.

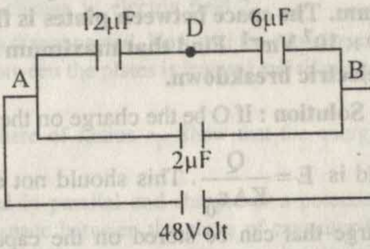


Fig. 6.36

○ **Solution :** (i) The equivalent capacitance between A and B is

$$C = 2 + \frac{12 \times 6}{18} = 6\mu F$$

(ii) The total charge supplied by the battery is $q = CV = 6 \times 48\ V = 288\ \mu C$

Now, potential difference across 2μF capacitor is 48 volt

∴ Charge on this capacitor $q_1 = 2 \times 48 = 96\ \mu C$

So, the charge on the 12 μF and 6 μF capacitor is $q' = 288 - 96 = 192\ \mu C$

(iii) The potential difference across the 6 μF capacitor is = 192/6 = 32 V

As the point B is at zero potential, the potential at the point D is V_1 = **32 volt.**

● **Ex. 6.29. The gap between the plates of a parallel plate capacitor is filled with an isotropic dielectric whose dielectric constant varies linearly from K_1 and K_2 (> K_1) in the direction perpendicular to the plates.**

The area of each plate A and the separation between the plates is *d*. Find the expression for the capacitance per unit area of the plates.

○ **Solution :** If σ be the surface density of charge on the inner surface of the plates of capacitor and ε_0 the permittivity of free space, then electric field intensity is $E = \frac{\sigma}{\varepsilon_0 K}$

It is given that dielectric constant K_1 varies linearly with distance from K_1 to K_2. So, the increase of dielectric constant per unit length is $\beta = \frac{K_2 - K_1}{d}$

The dielectric, constant at any point at a distance x from the first plate is $K = K_1 + \beta x$.

Hence the electric field at this point is $E = \frac{\sigma}{\varepsilon_0 (K_1 + \beta x)}$

The potential difference between the plates of elementary capacitor of thickness dx is $dV = E\, dx$. Hence the potential difference between the plates

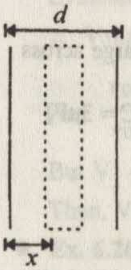


Fig. 6.37

$$V = \int dV = \int_0^d E\, dx = \frac{\sigma}{\varepsilon_0} \int_0^d \frac{dx}{K_1 + \beta x}$$

$$= \frac{\sigma}{\varepsilon_0} \times \frac{1}{\beta} \log_e \left(\frac{K_1 + \beta d}{K_1} \right) = \frac{\sigma d}{(K_2 - K_1)\, \varepsilon_0} \log_e \frac{K_2}{K_1}$$

Hence capacity per unit area $C = \frac{\sigma}{V}$

$$C = \frac{\varepsilon_0}{d} \cdot \frac{K_2 - K_1}{\log_e K_2 / K_1}$$

- **Ex. 6.30. A parallel plate capacitor is formed by two plates each of area $100\ cm^2$, separated by 1 mm. The space between plates is filled up with a dielectric of dielectric constant 5·0 and strength $1{\cdot}9 \times 10^7\ Vm^{-1}$. Find that maximum charge that can be stored on the capacitor without causing any dielectric breakdown.**

○ **Solution :** If O be the charge on the capacitor, the surface density of charge is $\sigma = Q/A$ and the electric field is $E = \frac{Q}{KA\varepsilon_0}$. This should not exceed the dielectric strength $1{\cdot}9 \times 10^7$ V/m. Then the maximum charge that can be stored on the capacitor is given by the relation : $\frac{Q}{KA\varepsilon_0} = 1{\cdot}9 \times 10^7$

$$Q = KA\varepsilon_0 \times 1{\cdot}9 \times 10^7 = 5 \times 10^{-3} \times 8{\cdot}85 \times 10^{-12} \times 1{\cdot}9 \times 10^7 = 8{\cdot}4 \times 10^{-6} C$$

• E ◈ X ◈ E ◈ R ◈ C ◈ I ◈ S ◈ E •

◢ A. Short answer type questions :

1. What is meant by capacitance of a capacitor ?
2. The capacitance of a conductor is 9 μF – State what it means in terms of its charge and potential. [W. B. H. S. 1995]
3. Two conductors carry equal amount of charge of same nature. Can there be potential difference between the conductors ?
4. What is a capacitor ? What is its principle ?
5. Define dielectric constant of a medium. [W. B. H. S. 1998]
6. The capacitance of a metal sphere is 10 cm – What does it mean ? Express it in SI unit.

7. Dielectric constant of water is 80 – What does it mean ?
8. The capacitance of a capacitor is 1 μF. Explain it meaning. **[W. B. H. S. 2001]**
9. What is the dielectric constant of a metal ?
10. On which factors capacitance of a capacitor depends ?
11. Define dielectric constant of a medium in terms of the capacitance of the capacitor.
12. N identical mercury drops are charged to a potential V. The drops coalesce to form a larger drop. What is the potential of the larger drop ?
13. Show that the equivalent capacitance of a number of capacitors connected in series will be smaller than any of the capacitors of the combination.
14. N identical capacitors are connected in parallel and a potential difference V is applied to the combination. Next the capacitors are connected in series. If the charges on the capacitors remain unchanged, what will be the difference of potential now ? **[Ans. NV]**
15. A parallel plate capacitor is charged to a potential V by connecting it to a source of emf. The separation between the plates is *d* and electric field intensity E_0. Without removing the source the separate between the plates is made 2*d*. What is the electric field between the plates ?
16. The capacity of a conducting sphere is C. It is placed concentrically inside a hollow sphere which is earthed. If the radii of the spheres be in the ratio $n : (n-1)$, show that the capacitance of the system will increase n times.
17. What dielectric constant of water is greater than that of mica ?
18. You are given two identical capacitors. How they should be connected so that for the same voltage, the system will acquire maximum energy ?
19. A parallel plate capacitor is charged to a potential and then the battery is disconnected. Now a dielectric slab is introduced between the plates of the capacitor. What will be the change in electric field ?
20. A capacitor is charged by connecting it to a battery. Then the battery is disconnected. How will the capacitance and potential difference change in the following cases : (i) separation between the plates is lowered and (ii)plates are connected by a wire ?
21. A charged sphere of radius r shares it charge with an uncharged sphere of radius r_1. Show that the energy of the spheres before and after connection is the ratio $(r + r_1) : r$.
22. Two parallel plate capacitors of capacitance C and 2C are connected in parallel and charged to a potential difference V by a battery. The battery is then disconnected and the space between the plates of capacitor C is filled with a dielectric of dielectric constant K. What is the potential difference across the capacitors now? **[Ans.** $\frac{3V}{K+2}$ **]**
23. If n drops, each of capacitance C, coalesce to form a single drop, what will be the capacitance of big drop? **[Ans.** $\sqrt[3]{n} \cdot C$ **]**

B. Essay type questions :

1. (a) What is capacitance of a conductor ? Give the principle of capacitor. What is meant by capacitance of capacitor 1·5 F ?

   (b) Obtain an expression for capacitance of an isolated sphere.

   (c) What is meant by capacitance of a conductor 10 cm ?

2. (a) On which factors capacitance of a capacitor depends ?

   (b) Obtain an expression for the energy of a charged conductor ?

3. Deduce an expression for the capacitance of a parallel plate capacitor (i) with a single dielectric and (ii) with compound dielectric.

   Hence define dielectric constant of a dielectric.

4. Obtain the expression for the capacitance of a spherical capacitor when (i) outer sphere is earthed and (ii) inner sphere is earthed.
5. Obtain an expression for capacitance of a cylindrical capacitor.
6. Two capacitors of capacities C_1 and C_2 have been charged to potentials V_1 and V_2 ($V_1 > V_2$). Now they are connected by a wire. What will be their common potential ? Calculate also loss of energy in the process. Account for this loss.
7. Describe a Van de Graaff generator. Explain its principle of action and working.

C. Numerical problems :

1. Two capacitors 20 μF and 60 μF are connected in series. A potential difference of 40 V is applied across the combination. Calculate charge and potential of each conductor. **[Ans.** 600 μC ; 30 V ; 10 V]
[W. B. H. S. 1998]
2. The capacitance of a conducting sphere is 1 μF. If it is given a charge of -10C, what will be its potential? **[Ans.** -10^7V] **[W. B. H. S. 1994]**
3. The capacitance of a conductor is 2×10^{-4} μF. It is given a charge of 5×10^{-8}C. Calculate its increase of potential. **[Ans.** 250 V]
4. A parallel air capacitor consists of two circular plates each of radius 5 cm. Its capacitance is equal to that of a spherical conductor of radius 50 cm. Obtain the separation between the plates. **[Ans.** 1·25 mm]
5. When a conductor is given a charge of 10 μC, its potential becomes 50V. What is its capacitance ? **[Ans.** 2×10^{-5}F]
6. The potential of a sphere of radius 30 cm is raised by 150V. What is its electrical energy ? **[Ans.** $3{\cdot}750 \times 10^{-7}$J]
7. A capacitor of capacitance 75 μF is raised to a potential of 20V. It is then connected to a uncharged conductor of capacity 25 μF. Calculate (i) their common potential and (ii) charge on each capacitor. **[Ans.** (i) 15V (ii) $1{\cdot}15 \times 10^{-3}$C, $0{\cdot}375 \times 10^{-3}$C]
8. Three capacitors 2μF, 3μF and 5μF are first connected is series and then in parallel. But the equivalent capacitance in both the cases. **[Ans.** 0·96μF ; 10μF]
9. A parallel plate air capacitor has a capacitance of 2μF. What will be its capacitance when it is half filled with a dielectric of dielectric constant K = 3. **[Ans.** 4μF]
10. A capacitor of capacitance $10^{-6}\mu$F is charged by a 100V battery. After disconnecting the battery, it is connected in parallel to a uncharged capacitor. If the final potential be 50V, what is the capacitance of the second sphere ? **[Ans.** 10^{-12}F]
11. The radii of two concentric spheres are 9 cm and 10 cm respectively. The space between the spheres is filled with air. Calculate the capacitance of the spherical capacitor. **[Ans.** 10^{-10}F]

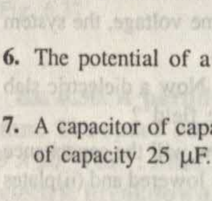


Fig. 6.38

12. In fig. 6.38 C_1 = 100 pF, C_2 = 200 pF, C_3 = 200 pF and C_4 = 100 pF. What is the equivalent capacitance of the system ? Also find potential of different capacitors.
[Ans. C = 200/3 pF ; V_1 = 100 V, V_2 = V_3 = 50 V ; V_4 = 200 V]
13. 8 water drops each of radius 1·0 mm and charge 10^{-4} μc combine to form a larger drop. Calculate the potential of the larger drop. **[Ans.** 3600 V]
14. Two capacitors of capacities 3μF and 6μF are connected in parallel and potential difference of 300V is applied across the combination.

MAGNETISM

• TOPICS DISCUSSED •

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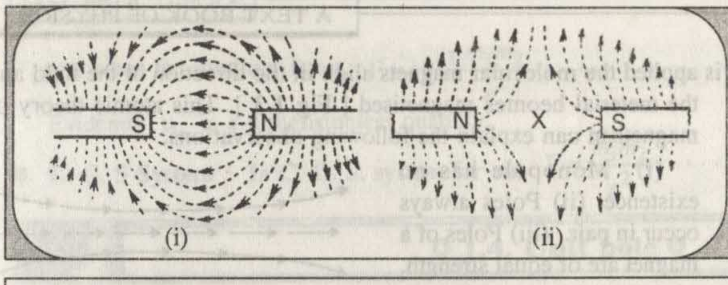


CHAPTER 1

MAGNETIC DIPOLE

● **TOPICS :** ○ Introduction; ○ Atomic theory of magnetism; ○ Interaction between magnetic poles : Coulomb law; ○ Unit pole; ○ Magnetic field; ○ Magnetic field intensity due to an isoluted pole; ○ Magnetic moment; ○ Torque acting on a magnetic in a magnetic field; ○ Work done in deflecting a magnet in a magnetic field : Potential energy of a magnet; ○ Magentic intensity at a point due to a magnet; ○ Magnetic intensity at a point due to short bar magnet or dipole; ○ Magnetic lines of force; ○ Properties of magnetic lines of force; ○ Mapping of lines of force; ○ Picture of lines of force in same cases; ○ Effect of earth's magnetic field on the arrangement of lines of force due to a bear magnet : Neutral points; ○ Current loop as a magnetic dipole; ○ Magnetic dipole moment of a revolving electron; ○ A solenoid as an equivalent bar magnet; ○ Short answer type questions (with answers); ○ Exercise.

✪ 1.1. Introduction ✪

The existence of natural magnet was known to the Greeks as early as 600 B. C. They found that a certain kind of rock has the ability to attract pieces of iron. This special kind of rock was called **lodestone** or **magnetite** which is now called magnet. The basic properties of a magnet are : (i) **Attractive property** : A magnet can attract some metals like iron, nickel and cobalt. (ii) **Directive property :** When a magnet is freely suspended, it aligns itself in the north-south direction. (iii) A magnet has two poles. The pole of a freely suspended magnet which points towards the geographical north is called the north-seeking pole or simply **north pole** (N). The other pole which points towards the geographical route is called the south is called the south-seeking or **south pole** (S).

When a magent is placed over a heap of iron filings, the ends of the magnet shows greater attraction [Fig 1.1]. Maximum amount of filings cling together at the two ends and almost no filing collect at the middle portion PQ of the bar. These points are called poles. These points are situated near the ends and not exactly at the ends. It is found from experiment that the distance between the two poles called magnetic length is about 0.86 times the geometrical length of the magnet.

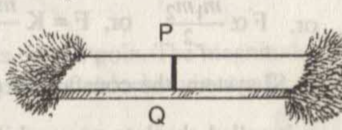


Fig. 1·1

(iv) Like poles repel and unlike pole attract.

(v) Poles always occur in pairs. It is not possible to isolate a single magnetic pole.

● **N. B.** When compared with electrostatic charge, north pole is equivalent to positive charge and south pole to negative charge. So N pole is called positive (+) pole and S- pole is negative (–) pole.

✪ 1.2. Atomic theory of magnetism ✪

A current carrying coil behaves like a magnetic dipole. Similarly any atom of a magnetic material also behaves as a magnetic dipole. The reason is : electron orbit around a nucleus is equivalent to a current carrying coil. So any atom of a magnetic material can be treated as a permanent magnetic dipole.

Hence, from magnetic point of view, a magnet or a magnetic substance consists of large number of molecular magnets. These molecular magnets form closed chain of poles in a magnetic material

[Fig 1.2]. When a magnetic field is applied the molecular magnets align in the direction of the field and the meterial beomes magnetised [Fig 1.3]. This atomic theory of magnetism can explain the following observations. (i) Monopole has no existence. (ii) Poles always occur in pair. (iii) Poles of a magnet are of equal strength. (iv) When a magnet is gradually heated, it loses its magnetism. At Curie point, it becomes non-magnetic.

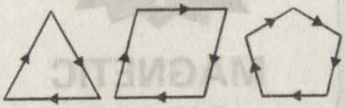


Fig. 1·2

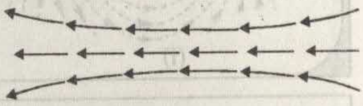


Fig. 1·3

In electrically, an isolated positive or negative charge can exist. Two equal charges of opposite sign kept at a small separation form an **electric dipole**, characterised by an electric dipole moment $\vec{P}$. In magnetism, the simplest magnetic structure is the **magnetic dipole**, characterised by a magnetic dipole moment $\vec{M}$.

✪ 1.3. Interaction between magnetic poles : Coulomb's law ✪

Coulomb was the first to determine experimentally the quantitative expression for the force between two **isolated** magnetic poles. The concept of isolated pole is purely theoretical. However, poles of a thin but long magnet may be assumed to be point poles for all practical purposes.

He found that the force between two magnetic poles is (i) proportional to the product of the pole strengths and (ii) inversely proportional to the square of the distance between them. This force acts along the line joining the two poles.

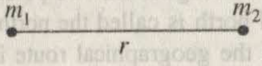


Fig. 1·4

Let two magnetic poles of strength m_1 and m_2 are kept at a separation r. [Fig 1.4] Then the mutual force F between the poles is given by

$$F \alpha m_1 m_2 \text{ and } F = \alpha \frac{1}{r^2}$$

$$\text{or, } F \alpha \frac{m_1 m_2}{r^2} \quad \text{or, } F = K \frac{m_1 m_2}{r_2}$$

In SI system, the constant of proportionally is $K = \frac{\mu}{4\pi}$ Wb $A^{-1} m^{-1}$

μ is called absolute permeability of the medium in which the poles are situated.

$$\therefore \quad F = \frac{\mu}{4\pi} \cdot \frac{m_1 m_2}{r_2} \quad \ldots\ldots (1.1)$$

For free space or air, $F = \frac{\mu_0}{4\pi} \frac{m_1 m_2}{r^2}$ ……(1.2)

μ_0 is the absolute permeability of air or free space.

- **Relative permeability :** Permeability of a medium with respect to air is called its relative permeability. It is denoted by μ_r

$$\therefore \quad \mu_r = \frac{\mu}{\mu_0} \quad \text{Or, } \mu = \mu_r \mu_0$$

In S I, $\mu_0 = 4\pi \times 10^{-7}$ Wb / A. m (or H. m^{-1}). Hence when the medium is other than air,

$$F = 10^{-7} \mu_r \frac{m_1 m_2}{r^2} \quad \ldots\ldots (1.3)$$

and when the medium is air, $F = 10^{-7}\frac{m_1 m_2}{r^2}$ . . . . (1.4)

Evidently μ_r is a dimensionless quantity.

- **C. G. S System :** In C. G. S. system for air, $F = \frac{m_1 m_2}{r^2}$ . . . . (1.5)

✪ 1.4. Unit pole ✪

(i) **In SI :** In equation 1.4, if $m_1 = m_2 = 1$ SI, $r = 1m$, then $F = \mathbf{10^{-7}}$ **N**. Hence if two identical poles placed at a separation of 1 m in air, repel each other with a force of 10^{-7} N then, each pole is a unit SI pole.

(ii) **In C. G. S system :** In equation 1.5, if $m_1 = m_2 = 1$ c. g. s., $r = 1$ cm then $F = 1$ dyne. So, if two identical poles placed at a separation of 1 cm in air repel each other with a force of 1 dyne.

- **Unit of pole strength :** In S I, the unit of pole strength is **A- m** and in c. g. s. the unit is **emu**. The relation between the units may be obtained as follows :

$$\frac{10^{-7}}{1\,\text{dyne}} = \frac{(1\,\text{A-m})^2}{1\,\text{m}^2} \times \frac{1\,\text{cm}^2}{(1\,\text{emu})^2} \quad \text{or,} \quad \frac{10^{-7}}{10^{-5}} = \frac{(1\,\text{A-m})^2}{(1\,\text{emu})^2} \times \frac{1\,\text{cm}^2}{10^4\,\text{cm}^2}$$

$\therefore$ **1 A-m = 10 emu**

✪ 1.5. Magnetic field ✪

❖ **Definition : The space around a magnet (or a conductor carrying a current) in which magnetic effect can be experienced, is called the magnetic field.**

Theoretically, the magnetic field is extended upto infinity. Because by Coulomb's law, the force between magnetic poles is zero only when the poles are at infinite separation. But in practice it is found that with distance the magnetic force decreases rapidly and is not appreciable after a certain distance. For this reason, it is assumed that magnetic field exists upto a certain distance around the magnet. This distance is more higher is the pole strength.

▲ Intensity of the magnetic field, B :

When a magnetic pole is placed at a point in the field, a force acts on the pole. The magnitude of the force depends on a particular property of the field called the intensity of the field which is usually denoted by B.

❖ **Definition : The intensity of the magnetic field at a point in the field is defined as the force on a unit north pole placed at the point.**

So, the intensity of the magnetic field is a force. The direction of the field is same as the direction of the force on a north the pole in the field.

$\therefore$ intensity of the field, $B = \frac{F}{m}$ $\therefore$ Force $F = Bm$ .... (1.6)

$\therefore$ Force on a magnetic pole, F = intensity of the field, B × Pole strength, m

- **Unit of B :** (i) In SI, the unit of B is **Tesla** (T) or **Wb / m^2**

(ii) In C. G. S. system its unit is **Gauss** (G)

The field intensity is 1 Tesla if a north pole of strength 1 SI kept at the point experiences a force of 1 N. On the other hand, the field intensity is 1 G if a north pole of strength 1 cgs experiences a force of 1 dyne.

$$\therefore \quad 1\,\text{T} = \frac{1\,\text{N}}{1\,\text{A-m}} = \frac{10^5\,\text{dyne}}{10\,\text{emu}} = \mathbf{10^4\,G}$$

- **Example 1.1. Two magnetic poles of equal strength exert a force of 1.6 N on each other when they are at a separation of 1.0 m. What is the strength of each pole ?**

○ **Solution :** $F = 10^{-7}\dfrac{m_1 m_2}{r^2}$

$$m_1 = m_2 = m$$
$$r = 1\cdot0m$$
$$F = 1\cdot6\,N$$

$$\therefore \quad 1\cdot6 = 10^{-7} \times \frac{m^2}{(1)^2}$$

$$\therefore \quad m^2 = 16 \times 10^6$$

$\therefore$ $m = \pm 4 \times 10^3 NT^{-1}$ [$\because$ Unit of pole strength is $NT^{-1} = A\,.\,m$]

$\therefore$ Strength of each pole is **$\pm 4 \times 10^3$ A.m**

✪ 1.6. Magnetic field intensity due to an isolated pole ✪

Assume that a north pole of strength *m* is situated in a medium of absolute permeability of μ. We require to calculate intensity of the magnetic field at a distance *r* from the pole [Fig 1.5]

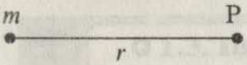


Fig. 1·5

Now, according to Coulomb's law the force on a unit north pole placed at P is

$$F = \frac{\mu}{4\pi}\cdot\frac{m \times 1}{r^2} = \frac{\mu}{4\pi}\frac{m}{r^2}$$

$\therefore$ Intensity of magnetic field at P is, $B = \dfrac{\mu}{4\pi}\cdot\dfrac{m}{r^2}$ . . . . (1.7)

and if the medium be air, $B = \dfrac{\mu_0}{4\pi}\cdot\dfrac{m}{r^2}$ . . . . (1.8)

- **Example. 1.2. Two magnetic poles of strength 16 A. *m.* and 25 A. *m.* are situated at a separation of 9 *m* in air. At which point joining the two pole, field intensity is zero ?**

○ **Solution :** Let the field intensity be zero on the line joining the poles at a distance *r* from the weaker pole. Then,

$$\frac{\mu_0}{4\pi}\cdot\frac{m_1}{r^2} = \frac{\mu_0}{4\pi}\frac{m_2}{(9-r)^2} \quad \text{or,} \quad \frac{16}{r^2} = \frac{25}{(9-r)^2} \Rightarrow \frac{4}{r} = \frac{5}{9-r} \quad \text{or,} \quad r = 4 \text{ meter.}$$

$\therefore$ The field intensity is zero at a distance of 4*m* from the first pole.

✪ 1.7. Magnetic moment ✪

The magnetic moment of a magnet is measured by the product of its pole strength and separation between the poles [Fig 1.6]

$\therefore$ Magnetic moment, M = Pole strength, *m* × magnetic length, NS

$\therefore$ $M = 2\,l.\,m$ [$\because$ $NS = 2\,l$] . . . . (1.9)

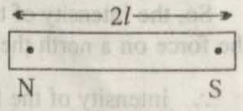


Fig. 1·6

M is a vector quantity. It is directed from the S- pole towards the N-pole of the magnet. Its SI unit is **A. m²**.

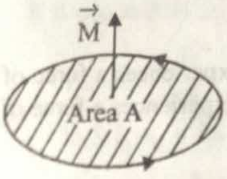


Fig. 1·7

- **A discussion :** We shall see in the chapter on Electromagnetism in current electricity that a current carrying circular coil behaves like a magnet. Its magnetic moment is perpendicular to the plane of the coil as shown in fig 1.7. The direction of $\vec{M}$ depends the direction of current in the coil.

Magnetic moment of the coil, M = area of the coil × current

$\therefore$ $M = A\,i$ . . . . (1.10)

Alternative definition of magnetic moment is also given in terms of torque acting on a magnet in a magnetic field.

✪ 1.8. Torque acting on a magnet in a magnetic field ✪

The bar magnet NS of magnetic length $2l$ and pole strength m is placed in a uniform magnetic field of intensity $\vec{B}$ making an angle θ with the direction of the field [Fig 1.8]. The length $2l$ is treated as a vector acting from south pole to north pole. So, the magnetic moment of the magnet is $\vec{M} = m(2\vec{l})$

The force acting on n-pole of the magnet is $\vec{F} = m\vec{B}$ (in the direction of the field) and the force on the south pole is $\vec{F} = m\vec{B}$ (opposite to the direction of the field).

These two forces are equal in magnitude, but oppositely directed. So a couple acts on the magnet. The moment of the couple or torque is

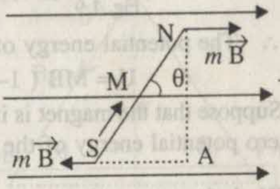


Fig. 1·8

$$\begin{aligned}\tau &= \text{either force} \times \text{perpendicular distance between the forces}\\ &= m\,\mathrm{B} \times (\mathrm{AN}) = m\,\mathrm{B}\ (2l\sin\theta)\\ &= \mathrm{MB}\sin\theta\ [\ \mathrm{M} = 2lm\ ] \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.11)\end{aligned}$$

In vector notation $\vec{\tau} = \vec{M} \times \vec{B}$

Due to the torque the magnet tends to align in the direction of the field.

❖ **Definition of M :** If in equation (1.11) $B = 1\ wb/m^2$ and $\theta = 90^\circ$, then $\tau = M$.

Hence the magnetic moment of a magnet is numeric called equal to the torque required to put the magnet perpendicular to the magnetic field of unit intensity.

- **Unit of dipole moment, M :** From the equation (1.11), $M = \dfrac{\tau}{B\sin\theta}$.

$\therefore$ SI unit of M is $\dfrac{N.m}{\text{Tesla}} = N.mT^{-1} = \mathbf{JT^{-1}}$

Also from the equation 1.10, unit of M is $A.m^2$.

- **Relation between c. g. s and SI unit of dipole moment :**

$$1\,N.mT^{-1} = \frac{10^5\,\text{dyne} \times 10^2\,\text{cm}}{10^4\,G} = 10^3\,\text{cgs moment}$$

- **Example 1.3. A bar magnet has pole strength 48 A-m and magnetic length 25 cm. What torque is required to hold the magnet at an angle 30° with a uniform magnetic field of flux density 0.15 T ?**

○ **Solution :** Required torque, $\tau = MB\sin\theta$

Here $M = 2\,l.\,m = 0{\cdot}25 \times 48 = 12\ A\text{-}m^2$, $B = 0{\cdot}15\ T$, $\theta = 30^\circ$

$\therefore\ \ \tau = 12 \times 0{\cdot}15 \times \sin 30^\circ = \mathbf{0{\cdot}9\ N\text{-}m}$

✪ 1.9. Work done in deflecting a magnet in a magnetic field : Potential energy of a magnet ✪

When a magnet is put at an angle with respect to a magnetic field, a torque tending to restore the magnet in the direction of the field acts on the magnet. So a work has to be done to deflect the magnet. This work remains stored in the magnet in the form of potential energy.

In fig 1.9, the bar magnet NS is inclined at any distant by an angle α with respect to a magnetic field of intensity $\vec{B}$. The torque acting on the magnet at this instant is $\tau = MB \sin\alpha$

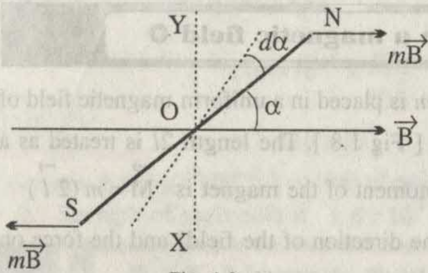


Fig. 1·9

Where M = magnetic moment of the magnet. Then work done to produce an additional small deflection $d\alpha$ is $dw = MB \sin\alpha.\, d\alpha$

Hence total work done on the magnet to deflect the magnet from $\alpha = 0^\circ$ to $\alpha = \theta^\circ$ is

$$W = \int_0^\theta MB\sin\alpha.d\alpha = MB\,(1-\cos\theta) \ldots\ldots\ldots\ldots (1.12)$$

$\therefore$ The potential energy of the magnet in the deflected position is

$$U = MB\,(\,1-\cos\theta) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.13)$$

Suppose that the magnet is intially perpendicular to the direction of the magnetic field and it corresponds to zero potential energy of the magnet.

Work done to produce a deflection θ from this position is $W = MB\displaystyle\int_{\frac{\pi}{2}}^{\theta} \sin\alpha\, d\alpha = -MB\cos\theta$

$\therefore$ Potential energy of the magnet, $U = -MB\cos\theta$

$$\therefore \quad U = -(\vec{M}\,.\,\vec{B}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.14)$$

● **Example 1.4. A magnet of magnetic moment 5 A.m^2 is freely suspended in a magnetic field of flux density 0·8 Wb / m^2. How much work is done to deflect it by 60°. Calculate the torque acting on the magnet in the deflected position.**

○ **Solution :** Work to be done, $W = MB\,(\,1-\cos\theta)$

$\therefore$ $W = 5 \times 0{\cdot}8 \times (\,1-\cos 60^\circ\,) = \mathbf{2{\cdot}0\ J}$

Torque, $\tau = MB\sin\theta = 5 \times 0{\cdot}8 \times \sin 60^\circ = 3{\cdot}86$ J

$M = 5$ A-m^2
$B = 0.8$ Wb / m^2
$\theta = 60^\circ$

✪ 1.10. Magnetic intensity at a point due to a magnet ✪

▲ (A) First Case : When the point is situated on the end on position :

By end-on position we mean that the point is on the axis of the magnet. In fig 1.10, P is a point on the axis of the bar magnet NS. The length of the magnet is $2l$ and pole strength m. The distance of the point P from the centre of the magnet is r.

The field intensity at P due to n-pole of the magnet is

$$B_N = \frac{\mu_0}{4\pi}.\frac{m}{(r-l)^2} \quad \text{along } \overline{PQ}$$

and field intensity at P due to s-pole of the magnet is

$$B_s = \frac{\mu_0}{4\pi}.\frac{\mu}{(r+l)^2} \quad \text{along } \overline{PN}$$

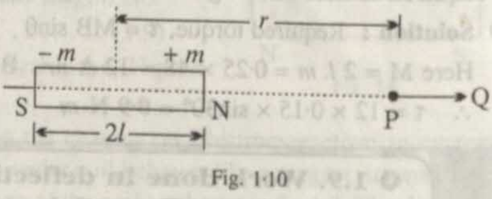


Fig. 1·10

As $B_N > B_S$, the resultant intensity at P due to the bar magnet is

$$B = B_N - B_S = \frac{\mu_0}{4\pi}.m\left[\frac{1}{(r-l)^2} - \frac{1}{(r+l)^2}\right] = \frac{\mu_0}{4\pi}.\frac{4mlr}{(r^2-l^2)^2} = \frac{\mu_0}{4\pi}.\frac{2Mr}{(r^2-l^2)^2} \quad [M = 2lm]$$

If the magnet is small, $r^2 >> l^2$, In that case $B = \frac{\mu_0}{4\pi}\cdot\frac{2M}{r^3}$ . . . . (1.14)

▲ (B) Second Case : The point is on the broadside-on position :

By broadside-on position we mean that the point P is situated on the perpendicular bisector of the magnet.

The point P is situated on the perpendicular bisector of the magnet at a distance r from the middle of the magnet NS. The length of the magnet is $2l$ and pole strength m [Fig 1.11]

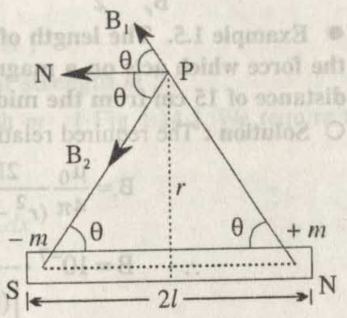


Fig. 1·11

From the figure $\cos\theta = \frac{1}{\sqrt{r^2+l^2}}$, $PN = PS = \sqrt{r^2+l^2}$

The field intensity at the point P due to N- pole of the magnet,

$$B_N = \frac{\mu_0}{4\pi}\cdot\frac{m}{(PN)^2} = \frac{\mu_0}{4\pi}\cdot\frac{m}{r^2+l^2} \text{ along } \overline{NP}$$

and field intensity at P due to S- pole

$$B_S = \frac{\mu_0}{4\pi}\cdot\frac{m}{(PS)^2} = \frac{\mu_0}{4\pi}\cdot\frac{m}{r^2+l^2} \text{ along } \overline{PS}$$

The resultant intensity at the point P due to bar magnet is

$$B = \sqrt{B_N{}^2 + B_S{}^2 + 2B_N B_S \cos 2\theta}$$

$$= \left[2B_N{}^2(1+\cos 2\theta)\right]^{\frac{1}{2}} = B_N\left[4.\cos^2\theta\right]^{\frac{1}{2}} \quad \left[\because \overrightarrow{B_N} = \overrightarrow{B_S} = B_N\right]$$

$$\therefore \ B = \frac{\mu_0}{4\pi}\cdot\frac{m}{r^2+l^2}.2\cos\theta = \frac{\mu_0}{4\pi}\cdot\frac{2l.m}{(r^2+l^2)^{\frac{3}{2}}} = \frac{\mu_0}{4\pi}\cdot\frac{M}{(r^2+l^2)^{\frac{3}{2}}}$$

When $r^2 >> l^2$, $B = \frac{\mu_0}{4\pi}\cdot\frac{M}{r^3}$ . . . . (1.15)

❂ 1.11. Magnetic intensity at a point due to short bar magnet or dipole ❂

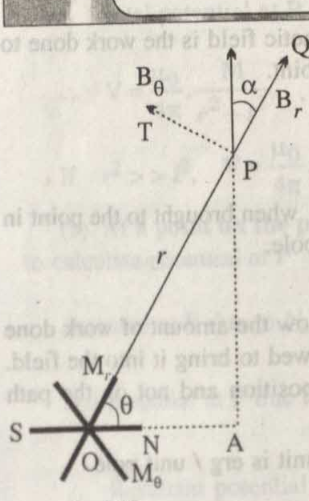


Fig. 1·12

As magnetic moment is a vector quantity, the intensity of the magnetic field at any point (r, θ) with respect to the centre of the magnet can be obtained by utilising the results of article 1.7.

In fig 1.12, NS is a magnetic dipole and the co-ordinates of P with the origin at O, the centre of the magnet is (r, θ).

So, OP = r and $\angle$ NOP = θ.

To calculate potential at the point P, moment of the magnet M is resolved into two components

$M_r = M\cos\theta$ acting along $\overline{OP}$ and $M_\theta = M\sin\theta$ acting at right angle to OP

Evidently P is an axial point with respect to M_r and with respect to M_θ component P is a point on broad-side-on position.

$\therefore$ Magnetic intensity at P due to M_r is $B_r = \frac{\mu_0}{4\pi}\cdot\frac{2M\cos\theta}{r^3}$ along PQ and Magnetic intensity at P due to M_θ is $B_\theta = \frac{\mu_0}{4\pi}.\frac{M\sin\theta}{r^3}$ along PT

$\therefore$ Resultant field at P is $B = \sqrt{B_r{}^2 + B_\theta{}^2}$

$$\therefore \quad B = \frac{\mu_0}{4\pi}.\sqrt{\left(\frac{2M\cos\theta}{r^3}\right)^2 + \left(\frac{M\sin\theta}{r^3}\right)^2} = \frac{\mu_0}{4\pi}.\frac{M}{r^3}\sqrt{1+3\cos^2\theta} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1.16)$$

and $\tan\alpha = \frac{B_\theta}{B_r} = \frac{1}{2}\tan\theta$

- **Example 1.5. The length of a bar magnet is 10 cm and its magnetic moment 2 A-*m*^{2}. Calculate the force which acts on a magnetic pole of strength 15 A-*m* placed on the axis of the magnet at a distance of 15 cm from the middle of the magnet.**

○ **Solution :** The required relation for the field at a point on the axis is

$$B = \frac{\mu_0}{4\pi}\frac{2Mr}{(r^2 - l^2)^2}$$

$$\therefore \quad B = 10^{-7}\cdot\frac{2\times2\times0\cdot15}{\left[(0\cdot15)^2 - (0\cdot05)^2\right]^2}$$

$$= \mathbf{1\cdot5\times10^{-4}\,wb/m^{-2}}$$

$\mu_0 = 4\pi\times10^{-7}$ H/m^{-1}

M = 2 A-m^2

r = 0·15 m

l = 0·05 m

Hence the force on the magnetic pole is F = mB

$\therefore$ F = 15 × 1·5 × 10^{-4} = **22·5 × 10^{-4} N**

- **Ex. 1.6. The moment of a magnetic dipole is 2 A- m^2. Calculate the magnetic field at a point on its perpendicular bisector at a distance 10 cm from the mid point of the magnet.**

○ **Solution :** The required relation : $B = \frac{\mu_0}{4\pi}.\frac{M}{r^3}$

$$\therefore \quad B = 10^{-7}\times\frac{2}{(0\cdot1)^3} = 2\times10^{-4}\,wb/m^2$$

$\therefore$ The required magnetic moment = **2 × 10^{-4} T**

- **A discussion :**

● (i) **Magnetic potential :** The magnetic potential at a point in the magnetic field is the work done to bring a unit n-pole from infinity (*i.e.* from outside the field) upto to the point.

$$\therefore \text{ Magnetic potential, } \quad V = \frac{\text{Work, W}}{\text{Pole strength, } m} = \frac{W}{m}$$

$$\therefore \quad W = V\,m \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.17)$$

Magnetic potential energy : It is the energy gained by a magnetic pole when brought to the point in the field from outside the field. Evidently, it is equal to work done on the pole.

$\therefore$ **Magnetic potential energy = potential × pole strength**

Magnetic potential is a scalar and is measured in terms of work done. Now the amount of work done on the pole to bring it to a point in the field does not depend on path followed to bring it into the field. Hence the potential energy of a pole in the electric field depends on its position and not on the path followed to bring it to the point.

- **Units :** Unit of magnetic potential in S I is J. (A- m)$^{-1}$ and in c. g. s unit is erg / unit pole or, erg / (emu). One J. (A- m)$^{-1}$ = 10^6 cgs potential

● (ii) **Relation between magnetic potential and intensity :** Magnetic potential is related to work done and intensity is related to force on a magnetic pole. Let a unit n- pole is shifted from the point P to Q against the magnetic field $\vec{B}$ [Fig 1.13]

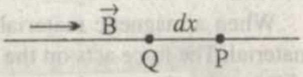


Fig. 1·13

$\therefore$ Work done against the filled, $dw = -B\,dx$ [Where PQ = dx]

Let potential at P is V and that at Q is $V + dV$

Also work done to bring the unit pole from P to Q is $(V + dV) - V = dV$

$\therefore\ -B\,dx = dV \quad \therefore\ B = -\dfrac{dV}{dx}$

$\therefore$ Intensity of magnetic field, B = – (rate of change of Potential)

● (iii) **Magnetic potential at a distance r from a magnetic pole of strength m :**

The point P is situated at a distance r from the n-pole of strength m [Fig 1.14]. We require to calculate the potential at P.

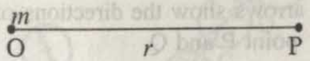


Fig. 1·14

Now, $B = -\dfrac{dV}{dx} \quad \therefore\ V = -\displaystyle\int_{\infty}^{r} B.dx$

Field intensity at any distance x is $B = -\dfrac{\mu_0}{4\pi}.\dfrac{m}{x^2}$

$$\therefore\ V = -\frac{\mu_0}{4\pi}.m\int_{\infty}^{r}\frac{dx}{x^2} = \frac{\mu_0}{4\pi}.\frac{m}{r} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1.18)$$

● (iv) **Magnetic potential at a point due to a bar magnet**

(a) **At a point on the axis of the magnet :** Consider the fig 1.10, we require to calculate potential at the point P.

Now, potential at P due to n- pole, $V_1 = \dfrac{\mu_0}{4\pi}\dfrac{m}{r-l}$

and potential at P due to S- pole, $V_2 = \dfrac{\mu_0}{4\pi}\dfrac{-m}{r+l}$

$\therefore$ Total potential at P, $V = V_1 + V_2 = \dfrac{\mu_0}{4\pi}m\left[\dfrac{1}{r-l} - \dfrac{1}{r+l}\right] = \dfrac{\mu_0}{4\pi}\dfrac{2l.m}{r^2 - l^2}$

$$\therefore\ V = \frac{\mu_0}{4\pi}.\frac{M}{r^2 - l^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1.19)$$

If $r^2 >> l^2$, $V = \dfrac{\mu_0}{4\pi}.\dfrac{M}{r^2}$

(b) **At a point on the perpendicular bisector of the magnet :** Now consider the fig 1.11. We require to calculate potential at P which is situated on the perpendicular bisector of the magnet NS.

Potential at P due to N - pole of the magnet, $V_1 = \dfrac{\mu_0}{4\pi}.\dfrac{m}{PN}$

and potential at P due to S pole of the magnet, $V_2 = \dfrac{\mu_0}{4\pi}.\dfrac{-m}{PS}$

$\therefore$ Resultant potential at P, $V = V_1 + V_2 = \dfrac{\mu_0}{4\pi}m\left[\dfrac{1}{PN} - \dfrac{1}{PS}\right] = 0$ $[\because PN = PS]$

✪ 1.12. Magnetic lines of force ✪

When a magnetic material (iron, nickel, cobalt etc) is brought near a magnet, a force acts on the material. The force acts on the magnetic material although they are not in contact. To explain the existence of force at a distance due to a magnet we introduce the concept of line of force. It is a line along which magnetic force operates.

Suppose an isolated magnetic n- pole is placed at a point in a magnet field. It experiences a force. The magnitude of the force and its direction depends on the position of the pole. In generally, both the magnitude and direction of the force change from point to point in the field. Hence if the pole is free to move it will move along a curve. This locus of the isolated north pole in the magnetic field is called a line of force.

❖ **Definition : A magnetic line of force is the locus of an isolated n-pole in the field and the tangent drawn at any point on the curve gives the direction of resultant magnetic field at the point.**

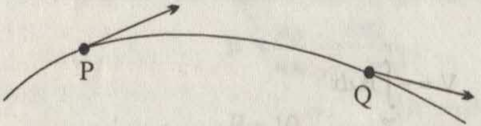


Fig. 1·15

The fig 1.15 shows a part of a magnetic line of force in a magnetic field. The arrows show the directions of the magnetic field at the point P and Q.

It must be mentioned that the concept of lines of force is imaginary. They have no real existence.

✪ 1.13. Properties of magnetic lines of force ✪

To explain different magnetic phenomena Michel Faraday attributed some properties on the lines of force. The properties are :

(i) A magnetic line of force starts normally from the north pole and enters also normally into the south pole of the magnet.

(ii) Lines of force behave like stretched elastic threads. They always try to contract along the length. This property is attributed to explain the attraction between opposite magnetic poles.

(iii) Any two lines of force exert lateral force on each other. This property of lines of force helps to explain repulsion between similar poles.

(iv) Any two lines of force never intersect each other. If they intersect two tangents can be drawn on the two curves at the point of interaction. It means that at a point in the field, it has two directions which is not possible. In fig 1.16, two lines of force PQ and P_1Q_1 are assumed to intersect at 0. Two tangents are drawn at O on the two lines of force. It implies the resultant field at a point has two direction. It cannot occur. Hence, **any two lines of force cannot intersect.**

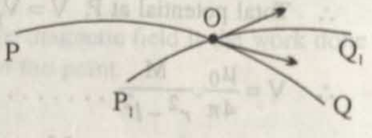


Fig. 1·16

✪ 1.14. Mapping of lines of force ✪

For mapping magnetic lines of force following the definition of lines of force we require an isolated north pole. But it is not possible to have an isolated pole. For this reason, lines are drawn by the help of a magnetic needle. The picture of lines of force gives the visual representation of the nature of the magnetic field around a magnet.

Magnetic lines of force may be mapped in two ways : (A) By iron filings and (B) By compass needle.

- **(A) Iron filing method :** Mapping of lines of force by this method can be done only in the case of strong field. It is not applicable to a weak field.

(i) A thin sheet of paper is first fixed on a drawing board by means of board pins and a bar magnet is placed at the centre of the paper [Fig 1.17].

(ii) Now sprinkle some iron filings on the paper around the magnet and slightly tap the board. Iron filing will align in a pattern. Alignment of filings along different curves shows the nature of the magnetic field [Fig 1.17].

- **(B) Compass needle method :** The field lines around a magnet can be drawn by compass needle. It is a small magnet whose two ends are made pointed. It is kept pivoted and can rotate in a horizontal plane. The bar magnet NS placed on a sheet of paper and a compass needle is placed close to one of its poles, say the north pole N. The position of the ends *n* and *s* of the needle are marked by pencil dots. Let the two dots be A and B. The compass is shifted so that its south pole points towards B and the new position C of its north pole is marked. The process is repeated to get a series of dots which are then joined by smooth curve. In this way other lines are drawn and each line is labelled with an arrow to indicate the direction which is from N to S.

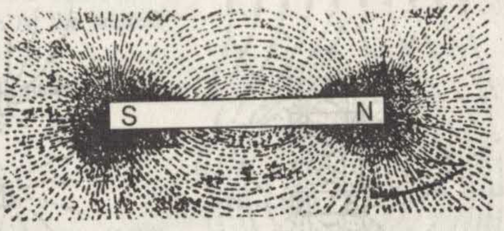


Fig. 1·17

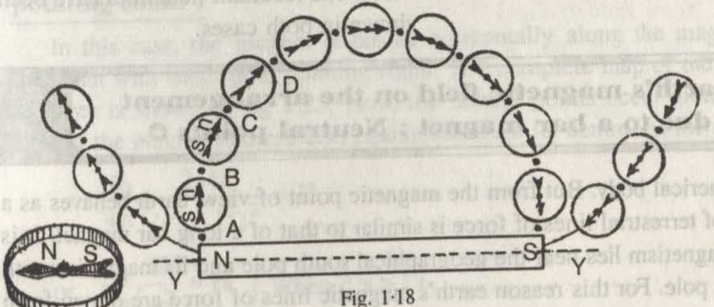
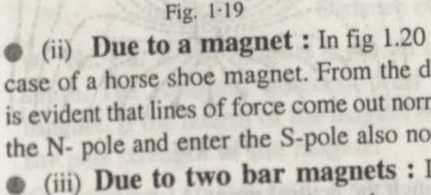


Fig. 1·18

1.15. Picture of lines of force in same cases

Here we draw field lines due to some permanent magnetic poles or magnets.

- (i) **Due to an isolated poles :** In figure 1.19 (i) and (ii), lines of force due to an isolated north and due to an isolated south pole are shown. The lines of force emerge normally from the N - pole and also enter normally to the S- pole.



(i) (ii)

Fig. 1·19

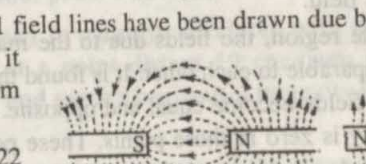


Fig. 1·20

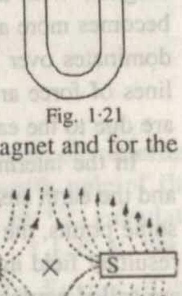
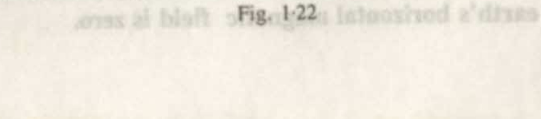


Fig. 1·21

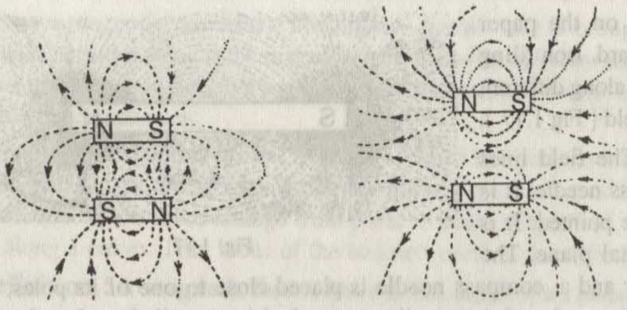
- (ii) **Due to a magnet :** In fig 1.20 and 1.21 field lines have been drawn due bar magnet and for the case of a horse shoe magnet. From the diagrams it is evident that lines of force come out normally from the N- pole and enter the S-pole also normally.
- (iii) **Due to two bar magnets :** In fig 1.22 (i) and 1.22 (ii) field lines have drawn when two bar magnets are situated close to each other. In fig 1.22 (i) the bar magnets are placed co-axially with their opposite poles facing each other.



(i) (ii)

Fig. 1·22

In fig 1.22 (ii) the magnets are also placed co-axially with their similar poles facing each other. Since lines of force always try to contract along the length, the oppose poles attract each other. On the other hand, two similar poles repel because lines of force repel each other laterally.



(i) (ii)

Fig. 1·23

In fig 1.23, the bar magnets are placed parallel to each other i.e. their axes are parallel. In fig 1.23 (i) opposite poles are placed side by side and in fig 1.23 (ii) similar poles are placed side by sides. The resultant field lines have been drawn in both cases.

✪ 1.16. Effect of earth's magnetic field on the arrangement of lines of force due to a bar magnet : Neutral points ✪

Earth is an enormously large spherical body. But from the magnetic point of view, earth behaves as a large bar magnet. Hence the nature of terrestrial lines of force is similar to that of a long bar magnet. It is assumed that north pole of earth's magnetism lies near the geographical south pole and its magnetic south pole lies near the geographical north pole. For this reason earth's magnetic lines of force are drawn from geographical south to north direction. As earth's magnetic field over a region of earth's surface is uniform, earth's magnetic lines of force are shown as equidistant parallel lines.

The pattern of lines of force produced by a magnet changes on account of earth's magnetism. The resultant pattern of lines of force when a magnet's field is added to that of earth depends on the direction in which the magnet is placed. Here we consider the arrangement of lines of force due to bar magnet for its two standard positions.

▲ (A) The bar magnet in magnetic meridian with n-pole pointing North :

As shown in fig 1.24, the magnet is placed in magnetic meridian with **north pole pointing north**. The resultant pattern of lines of force is as shown in fig, Near the magnet, the earth's field is negligible in relation to that of the magnet. With distance from the magnet, the earth's field becomes more and more and more important and ultimately dominates over the field due to the magnet. So the distant lines of force are due to earth's field while the nearer ones are due to the earth's field.

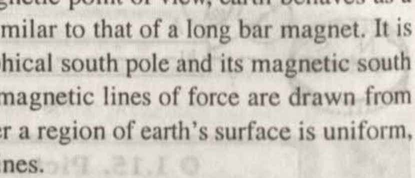


Fig. 1·24

In the intermediate region, the fields due to the magnet and the earth are comparable to each other. It is found that at some points, the two fields become equal and opposite. The resultant field intensity is zero at those points. These points are called **neutral points** obviously, no lines of force passes through the neutral point. A compass needle placed at these points would set itself in any direction.

So, **A neutral point is defined as a point at which resultant magnetic field due to a magnet and earth's horizontal magnetic field is zero.**

When the magnet is placed with its north pole pointing north, the two neutral points will be formed on the perpendicular bisector of the magnet. These points are equidistant from the centre of the magnet. It is shown in fig 1.25. At the neutral point.

Field intensity due to the magnet = Earth's horizontal intensity

$$\therefore \quad \frac{\mu_0}{4\pi}.\frac{M}{(r^2+l^2)^{3/2}} = H$$

$$\text{or,} \quad \frac{\mu_0}{4\pi}.\frac{M}{r^3} = H \qquad [\text{ When } r >> l\ ] \ldots\ldots\ldots\ldots\ldots (1.20)$$

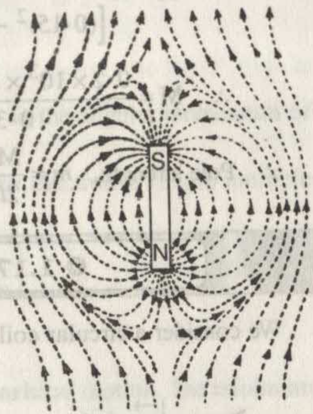


Fig. 1·25

▲ (B) The bar magnet is in magnetic meridian with north pole pointing south :

In this case, the magnet is placed horizontally along the magnetic meridian with north pole pointing south. The complete map of the lines of force is shown in fig 1.25. Now, the neutral points occur along the axis of the magnet and are also equidistant from its centre. At the neutral point

$$\frac{\mu_0}{4\pi}.\frac{2Mr}{(r^2-l^2)^2} = H$$

$$\text{or,} \quad \frac{\mu_0}{4\pi}.\frac{2M}{r^3} = H \quad [\text{ When } r >> l\ ] \ldots\ldots\ldots\ldots\ldots (1.21)$$

● **Example 1.7. The magnetic moment of a bar magnet of length 12 cm is 2.0 A-m^2. It is placed in magnetic meridian with north pole pointing north. If the horizontal intensity of earth's magnetism be 0·35 × 10^{-4} T, obtain the positions of the neutral points.**

○ **Solution :** In this case, the two neutral points are situated on the perpendicular bisector of the magnet. If r be the distance of the neutral points from the middle of the magnet be r, then

$$\frac{\mu_0}{4\pi}.\frac{M}{(r^2+l^2)^{3/2}} = H$$

$$\therefore \quad 10^{-7} \times \frac{2}{(r^2+l^2)^{3/2}} = 0\cdot35 \times 10^{-4}$$

$$\therefore \quad (r^2+l^2)^{3/2} = 5\cdot71 \times 10^{-3}$$

$$\therefore \quad r^2+l^2 = (5\cdot71)^{2/3} \times 10^{-2} = 3\cdot19 \times 10^{-2}$$

$$\therefore \quad r^2 = 3\cdot19 \times 10^{-2} - (0\cdot06)^2 = (3\cdot19 - 0\cdot36) \times 10^{-2} = 2\cdot83 \times 10^{-2}$$

$\therefore \; r = 0{\cdot}168\ m$ $\quad\therefore$ Distance of the neutral point = **16·8 cm**

$M = 2{\cdot}0\ A\text{-}m^2$

$\mu_0 = 4\pi \times 10^{-7}\ H\ m^{-1}$

$l = 0{\cdot}06\ m$

$H = 0{\cdot}35 \times 10^{-4}\ T$

● **Ex. 1.8. A bar magnet is placed horizontally in magnetic meridian with the north pole pointing south. The resultant magnetic field is zero at a point distant 10 cm from the south pole of the magnet. If the length of the magnet be 10 cm and earth's horizontal intensity 0·2 × 10^{-4} T, calculate the pole strength of the magnet. [J. E. E. '99]**

○ **Solution :** In this case, neutral points are formed on the axis of the magnet. If the distance of the neutral point from the middle of the magnet be r, then

$$10^{-7}\frac{2Mr}{(r^2-l^2)} = H$$

$$\therefore \quad 10^{-7} \times \frac{2M \times 0.15}{\left[(0.15)^2 - (0.05)^2\right]^2} = 0.2 \times 10^{-4}$$

$m = ?$
$l = 0.05$ m
$H = 0.2 \times 10^{-4}$ T
$r = 0.1 + 0.05 = \mathbf{0.15}\ \boldsymbol{m}$

$$\therefore \quad M = \frac{0.2 \times 10^{3} \times 4 \times 10^{-4}}{0.3} = \frac{0.8}{3}$$

$$\therefore \quad \text{Pole strength, } m = \frac{M}{2l} = \frac{0.8}{3 \times 0.1} = \mathbf{2.67\ A\text{-}}\boldsymbol{m}$$

✪ 1.17. Current loop as a magnetic dipole ✪

We consider a circular coil kept horizontally. It carries a current I which is anti-clockwise when seen from above [Fig 1.26]. The nature of the magnetic field due to the circular coil is shown in fig. The lines of force are parallel along the axis of the coil. But away from the axis the field lines are circular. Evidently upper surface of the coil develop north pole and lower surface south pole. So the current carrying coil behaves as a magnetic dipole. Its magnetic moment $\vec{M}$ is perpendicular to the plane of the coil and is directed upwards as shown in fig. The magnetic moment of the coil depends on :

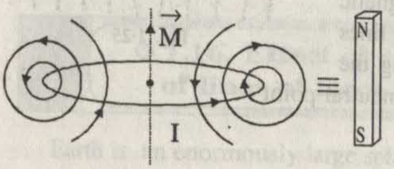


Fig. 1·26

(i) Magnetic moment is proportional to current i.e., $M \propto I$

(ii) Magnetic moment is proportional to area of the coil i.e. $M \propto A$

$\therefore$ $M \propto IA$ or, $M = k\,IA$

In SI system when current is measured in Amp unit and area in m^2 then $k = 1$

$\therefore$ $M = IA$

If the coil has n no. of terms, its total magnetic moment is $M = n\,IA$. Its SI, unit is $\text{A-}m^2$

✪ 1.18. Magnetic dipole moment of a revolving electron (Atom as a magnetic dipole) ✪

We assume that an elctron revolves around the nucleus in a circular orbit to radius r (Fig 1.27). Electron orbits may be considered as small current loops. As a result, an atom possesses magnetic dipole moment and hence behave as a magnetic dipole.

Expression for magnetic moment :

The electron of mass m and charge e is revolving in the orbit of radius r is anticlockwise direction. So the equivalent current is clockwise.

It is given by $I = \frac{e}{T}$ where T = time period.

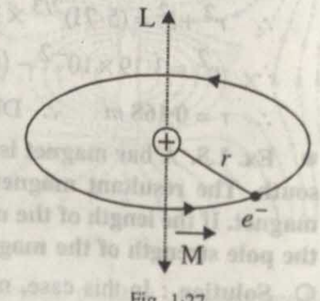


Fig. 1·27

$$\therefore \quad I = \frac{e}{2\pi/\omega} = \frac{e\omega}{2\pi} \qquad [\ \omega = \text{angular velocity of the electron}\ ]$$

$\therefore$ Magnetic moment of the electron orbit, $M = IA = I\,\pi r^2$

$$\therefore \quad M = \frac{e\omega}{2\pi}.\pi r^2 = \frac{1}{2}e\omega r^2 = \frac{e}{2m}(mr^2\omega)$$

Now, angular momentum of the electron, $L = mvr = mr^2\omega$

$$\therefore \quad M = \frac{e}{2m}.L \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.22)$$

In vector form, $\vec{M} = -\frac{e}{2m}\vec{L} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 1.22\ (a)$

The negative sign implies that magnetic moment of the orbit is opposite to the angular momentum of the electron.

Now, according Bohr's theory, in a stable orbit angular momentum of the electron is simple multiple of $h / 2\pi$, Where h = Planck's constant.

$$\therefore \quad mr^2\omega = n\frac{h}{2\pi}, \quad n = 1, 2, 3 \ldots\ldots \quad \therefore \quad M = \frac{e}{2}.\frac{nh}{2\pi m} = n\left(\frac{eh}{4\pi m}\right) \ldots\ldots\ldots (1.23)$$

$\therefore$ Minimum value of magnetic moment, $M = \frac{eh}{4\pi m} \quad (\because n = 1)$

The equation (1.23) gives the magnetic moment of the electron due to its **orbital motion**. The minimum value of the orbital magnetic moment is called Bohr magneton.

- **Bohr magneton :** It is defined as the magnetic dipole moment associated with an atom due to orbital motion of an electron in the first Bohr orbit of hydrogen atom.

$$1 \text{ Bohr magneton} = \frac{1\cdot6\times10^{-19}\times6\cdot62\times10^{-34}}{4\pi\times9\cdot1\times10^{-31}} = 9\cdot27\times10^{-24}\ \text{A-}m^2$$

If there are more than one electron in an atom, the total magnetic moment of the atom will be equal to the vector sum of all the orbital magnetic moments.

- **Example 1.9. A circular coil of radius 0·2 m carries 50 turns. If a current of 12 A passes through it, calculate : (i) magnetic field at the centre of the coil and (ii) magnetic moment of the coil.**

○ **Solution :** (i) Magnetic induction at the centre of the coil, $B = \frac{\mu_0}{4\pi}.\frac{2\pi ni}{r}$

$$\therefore \quad B = 10^{-7}\times\frac{2\times3\cdot14\times50\times12}{0\cdot2} = 1\cdot885\times10^{-3}\,T$$

(ii) Magnetic moment of the coil, $M = niA$

$\therefore$ $M = 50 \times 12 \times \pi\,(0\cdot2)^2 = $ **75- 4 Am²**

✪ 1.19. A solenoid as an equivalent bar magnet ✪

We consider a solenoid of length L, turns n, area of cross section A and it carries a current I [Fig 1.28]. Each turn of the solinoid is a circular coil having magnetic dipole moment AI. The equivalent magnetic moment of the solenoid having n turns is $M = nAI$.

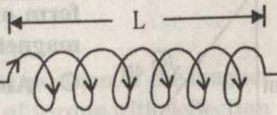


Fig. 1·28

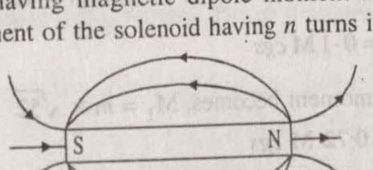


Fig. 1·29

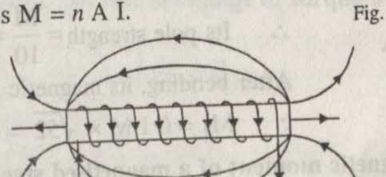


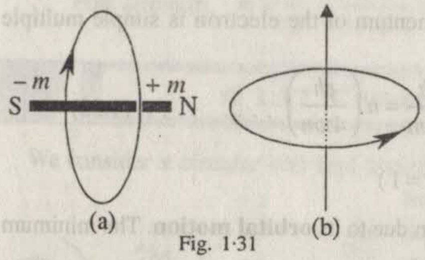
Fig. 1·30

Fig 1.29 shows the magnetic field due to magnet while the fig 1.30 shows the magnetic field produced by a solenoid. It follows that the magnetic field produced by a current-carrying solenoid is identical to that produced by a bar magnet.

• • Short Answer Type Questions (with answers) • •

- **Q 1. What is a magnetic dipole ? How its moment is measured ?**

○ **Ans.** A small magnet or a *n*-pole and a *s*-pole of equal strength separated by a small distance form a dipole. If its pole strength be m and separation be r then, dipole moment, $M = mr$



(a) (b)

Fig. 1·31

- **Q 2. A circular coil of area A and *n* turns carries a current *i*. What is its magnetic dipole moment ?**

○ **Ans.** The current carrying coil behaves as a magnetic dipole. The surface in which current flows anti-clockwise will develope north pole and the other surface will develope south pole [Fig 1.31 (a)]. If the pole strength be m, then dipole moment, $M = ml = n\,Ai$; M acts from south pole to north pole.

- **Q 3. Under what condition, the magnetic lines of force are straight and parallel to each other ?**

○ **Ans.** In the case of uniform magnetic field, the lines of force are straight and parallel to each other.

- **Q 4. A magnet of length 2*l* and pole strength *m* is equally divided into two parts perpendicular to its length. What is the magnetic moment of each part.**

○ **Ans.** Each part retains same pole strength m and its length l now. So its magnetic moment, $M = ml$.

- **Q 5. A magnetic needle pivoted horizontally is placed in a magnetic field of strength 1T. Now another magnetic field of intensity $\sqrt{3}$ T is applied perpendicular to the initial field. What is the deflection of the needle ?**

○ **Ans.** Initially the needle was along the field $B_1 = 1T$. Then the field $B_2 = \sqrt{3}$ T is applied perpendicular to B_1. Immediately the needle will be deflected by an angle θ as shown in fig 1.32.

From the diagram, $\tan\theta = \frac{B_2}{B_1} = \frac{\sqrt{3}}{1} = \tan 60^\circ \quad \therefore\ \theta = 60^\circ$

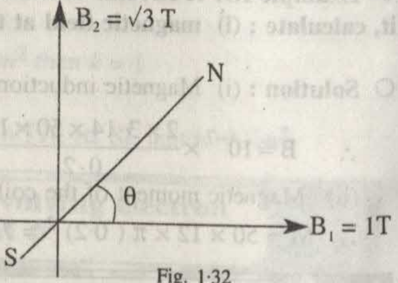


Fig. 1·32

- **Q 6. The magnetic moment of a magnetised wire is M. It is bent in the form of L. The length of the two sides are 6 cm and 4 cm. What is the new magnetic moment.**

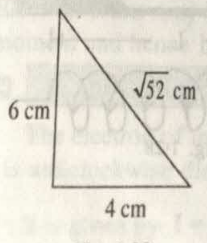


Fig. 1·33

○ **Ans.** The length of the magnetised wire is = (6 + 4) cm = 10 cm

$\therefore$ Its pole strength $= \frac{M}{10} = 0 \cdot 1\,M$ *cgs*

After bending, its magnetic moment becomes, $M_1 = m \times \sqrt{52}$

$\therefore\ M_1 = 0{\cdot}1\,M \times \sqrt{52} = 0{\cdot}72\,M$ ***cgs***

- **Q 7. The magnetic moment of a magnetised steel wire of length *l* cm is M cgs. If it is bent in a semi circle what will be its magnetic moment ?**

○ **Ans.** Let the diameter of the semi-circle be D. Then $\frac{\pi D}{2} = l \quad \therefore\ D = \frac{2l}{\pi}$

Pole strength of the wire, $m = \frac{M}{l}$ So, the new magnetic moment is $M' = mD = \frac{M}{l} \times \frac{2l}{\pi} = \frac{2M}{\pi}\,cgs$

● **Q 8. A magnet has magnetic moment is M. What will its changed magnetic moment in the following cases ?**

(i) When the magnet is divided two parts along its length.

(ii) When the magnet is divided into parts perpendicular to its length.

○ **Ans.** (i) $M' = (2l)\dfrac{m}{2} = \dfrac{M}{2}$ [Pole strength becomes $m / 2$]

(ii) $M' = l.m = \dfrac{M}{2}$ [Pole strength does not change]

● **Q 9. What is meant by pole strength of magnet is 1 A-m ?**

○ **Ans.** 'Pole strength of a magnet 1 A-*m*' means that if an identical pole is placed at a distance of 1 *m* in air from the given pole, it will experience a force of 10^{-7} N.

● **Q 10. How a magnetic dipole be placed with respect to a magnetic field so that its potential energy is maximum ?**

○ **Ans.** The potential energy of a magnet in a magnetic field will be maximum if it is so placed that its magnetic moment $\vec{M}$ and intensity of magnetic field $\vec{B}$ are opposite to each other. **Reason** : When the magnetic L deflected by an angle θ, the work done is W = M B (1– cos θ). If θ = 180°, then W = 2 MB. If is the maximum value.

● **Q 11. Define Bohr magneton. What is its value ?**

○ **Ans.** Bohr magneton is the magnetic moment of hydrogen atom when the electron revolves in the first Bohr orbit.

1 Bohr magneton, $\mu_B = \dfrac{eh}{4\pi m} = \dfrac{1\cdot6\times10^{-19}\times6\cdot62\times10^{-24}}{4\pi\times9\times10^{-31}} = 9\cdot27\times10^{-24}\,\text{A - }m^2$

● **Q 12. Define magnetic moment of a magnet. What is its SI unit ?**

○ **Ans.** Magnetic moment of a magnet is defined in terms of the torque acting on a magnet in a magnetic field.

Torque required to put a magnet of moment M at an angle θ with a magnetic field of intensity B is τ = MB sinθ.

∴ If B = 1 and θ = 90° then τ = M

Hence, magnetic moment of a magnet is the torque required keep the magnet perpendicular to a magnetic field of strength units.

Its SI unit : N*m* $T^{-1} = JT^{-1}$

● **Q 13. A magnetic dipole can rotate freely in a magnetic field. At what position of the magnet (i) torque is maximum (ii) potential energy is maximum (iii) rate of change of torque with deflection is maximum.**

○ **Ans.** (i) Torque acting on a magnet for its deflection θ is τ = MB sinθ

∴ When θ = 90°, torque is maximum. So $\tau_{max} = MB$

(ii) Work done to deflect the magnet through an angle θ is W = MB (1 – cos θ)

When θ = 180°, work done is maximum, $W_{max} = 2MB$

(iii) $\dfrac{d\tau}{d\theta} = \dfrac{d}{d\theta}(MB\sin\theta) = MB\cos\theta$

∴ When θ = 0°, rate of change of torque is maximum.

● Q 14. In an unexcited hydrogen atom, the electron revolves in a circular orbit in anti-clockwise direction. The atom is in a magnetic field of intensity $\vec{B}$. The normal to the electron orbit is inclined at 30° with the field direction. Calculate the torque acting with electron.

○ **Ans.** The orbital magnetic moment of the electron in the hydrogen atom is $M = \dfrac{eh}{4\pi m}$. It is directed along the axis of rotation which is perpendicular to the plane of the electron orbit.

In this case $\vec{M}$ and $\vec{B}$ are inclined at 30° with each other.

$$\therefore \quad \text{Torque on the electron,} \quad \tau = MB\sin 30^\circ = \frac{eh}{4\pi m}B.\times\frac{1}{2} = \frac{ehB}{8\pi m}$$

● Q 15. Two charged particle each of mass *m* and carrying a charge *q* are attached at the ends of a rigid light rod of length 2R. The rod is rotated about an axis passing through the centre of the rod and perpendicular to its length with a constant angular velocity. Calculate the ratio of magnetic moment of the system and its angular momentum about the axis.

○ **Ans.** The arrangement is shown in fig 1.34. The rod is rotated about the ZZ′ axis with angular velocity ω.

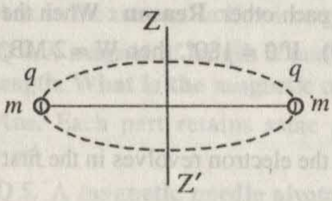


Fig. 1·34

Now, the angular momentum of the system about ZZ′ is

$$L = mR^2\omega + mR^2\omega = 2mR^2\omega$$

When the rod rotates, the charged particles describe circular orbit of radius R and with linear speed Rω. If the rod rotates with time period T, then the charge passes through a particular point of its path after a time $t = T/2$

$$\therefore \quad \text{Equivalent current} \quad i = \frac{q}{t} = \frac{2q}{T} = \frac{2q}{2\pi/\omega} = \frac{q\omega}{\pi}$$

$$\therefore \quad \text{Magnetic moment of the system,} \quad M = Ai = \frac{q\omega}{\pi}\times\pi R^2 = q\omega R^2$$

$$\therefore \quad \frac{M}{L} = \frac{q\omega R^2}{2mR^2\omega} = \frac{q}{2m}$$

●● Numerical Examples ●●

● Example. 1.10. The magnetic length of each of two magnetised wire is 20 cm and each of mass 4 gm. They are tied together at their two south poles and are suspended from a support. At equilibrium, the two north poles remain at a separation of 4 cm. What is the magnetic moment of the wire ?

○ **Solution :** The situation is shown in figure 1.35. Let the pole strength of each magnet is *m* A-*m*. Force of repulsion between N-poles is

$$F = 10^{-7}\times\frac{m^2}{16\times10^{-4}} = \frac{m^2}{16}\times10^{-3}\text{N}$$

The N-pole is in equilibrium. Taking moment of the forces acting on the N-pole about the point 'S' we get

$$F\times OS = W\times\frac{ON}{2} \quad [\,ON = 2\text{cm},\ OS = 19{\cdot}9\text{ cm}\,]$$

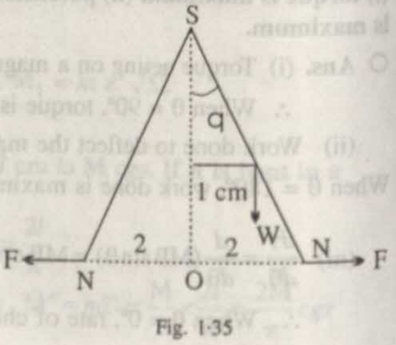


Fig. 1·35

$$\therefore \quad F = \frac{1}{2}W\frac{ON}{OS} = \frac{1}{2}W \times \frac{2}{19 \cdot 9}$$

$$\therefore \quad \frac{m^2}{16}10^{-3} = \frac{4 \times 10^{-3} \times 9 \cdot 8}{19 \cdot 9} \Rightarrow m = \mathbf{5 \cdot 614\ A \text{-} m}$$

- **Ex. 1.11. A bar magnet of length 4 cm is placed vertically on a horizontal table with its north pole touching the table. A neutral point is obtained on the table at a distance of 3 cm from the lower end of the magnet. Find the magnetic moment of the magnet. Horizontal intensity of earth's magnetism = 0·2 Oe.**

○ **Solution :** In fig 1.36, NS is the bar magnet. The point P is the position of the neutral point. If *m* be the pole strength of magnet, then the magnetic intensity at P due to N-pole of the magnet,

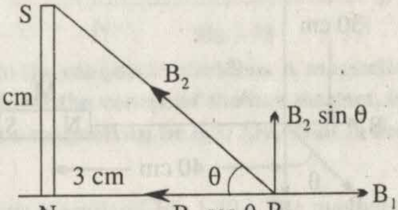


Fig. 1·36

$$B_1 = 10^{-7}\frac{m}{(NP)^2} \text{ along } \overline{NP}$$

Also magnetic field at P due to S-pole of the magnet

$$B_2 = 10^{-7} \times \frac{m}{(SP)^2} \text{ along } \overline{PS}$$

$\therefore$ Horizontal component of B_2 is $= B_2 \cos\theta$ along $\overline{PN}$

and vertical component = $B_2 \sin\theta$ (vertically upwards)

$$\therefore \quad B_1 - B_2 \cos\theta = H$$

$$\therefore \quad \frac{\mu_0}{4\pi}\left[\frac{m}{(NP)^2} - \frac{m\cos\theta}{(SP)^2}\right] = 0 \cdot 2 \times 10^{-4} \quad \text{or,} \quad \frac{m}{(0 \cdot 03)^2} - \frac{m}{(0 \cdot 05)^2} \times \frac{3}{5} = 0 \cdot 2 \times 10^3 \quad [\cos\theta = 3/5]$$

$$\text{or,} \quad m\left[\frac{1}{9} - \frac{3}{125}\right] = 0 \cdot 2 \times 10^3 \times 10^{-4} \quad \therefore \quad m = \mathbf{0.23\ A\text{-}}\ \boldsymbol{m}$$

$\therefore$ magnetic moment, $M = 0{\cdot}23 \times NS = 0{\cdot}23 \times 4 \times 10^{-2}$

$\therefore$ $M = 9{\cdot}2 \times 10^{-3}$ **A-** $\boldsymbol{m^2}$

- **Ex. 1.12. The force on a magnetic pole placed in a uniform magnetic field of strength 0·4 Oe is equal to the force exerted on the pole when placed at a distance of 10 cm from a unknown magnetic pole. Calculate the unknown pole strength.**

○ **Solution :** The force on the magnetic pole in the field, $F = 0{\cdot}4 \times 10^{-4}\, m$ (N)

If the strength of the unknown pole be m_1 then the force on the pole m is

$$F' = 10^{-7}\frac{mm_1}{(0 \cdot 1)^2} = 10^{-5}mm_1 \text{ (N)}$$

$\therefore$ From the question, $10^{-5}mm_1 = 0 \cdot 4 \times 10^{-4}m \Rightarrow m_1 = \mathbf{4\ A\text{-}m = 40\ cgs}$

- **Ex. 1.13. Two bar magnets A and B are placed co-axially with their north poles facing each other. The separation between the two N-poles is 10 cm. The magnetic moment of the magnet B is 1000 cgs units. If a neutral point is formed at a distance of 4 cm from the magnet A, calculate the magnetic moment of A.**

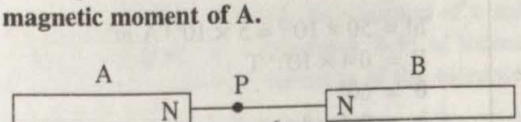


Fig. 1·37

○ **Solution :** The arrangement is shown in fig 1.37. The neutral point is formed on the common axis at P. Hence, the field intensity at P due to the magnet A = the field intensity at P due to the magnet B.

$$\therefore \quad \frac{2M_1 r_1}{(r_1^{\,2}-l^2)^2} = \frac{2M_1 r_2}{(r_2^{\,2}-l^2)^2} \qquad \therefore \quad M_1 = \frac{(r_1^{\,2}-l^2)^2}{(r_2^{\,2}-l^2)^2}\times\frac{r_2}{r_1}.M_2$$

$$= \frac{(9^2-5^2)^2}{(11^2-5^2)^2}\times\frac{11}{4}\times 1000 = \frac{(56)^2}{(96)^2}\times\frac{11000}{9} = \mathbf{415\cdot 87\ cgs}$$

$r_1 = 9$ cm
$r_2 = 11$cm
$l = 5$ cm
$M_2 = 1000$ cgs

- **Ex. 1.14. Two small bar magnets of magnetic moments 0·108 and 0·192 A-m^2 are placed on a horizontal plane so that their axes are mutually perpendicular and intersect at a point. The distances of the magnets from the point of intersection are 30 cm and 40 cm respectively. Calculate the magnitude and direction of the resultant magnetic field at the point of intersection.**

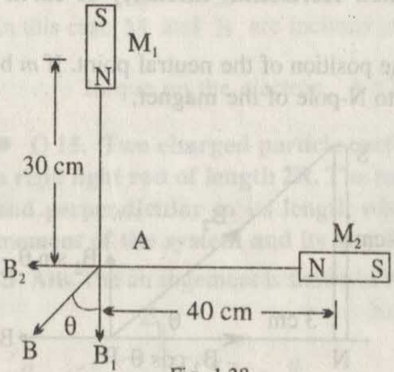


Fig. 1·38

○ **Solution :** The arrangement is shown in fig 1.38. The axes of the two magnets intersect at A. The magnetic field at A due to M_1 magnet is $B_1 = 10^{-7}\times\frac{2M_1}{r_1^{\,3}}$

$$= 10^{-7}\times\frac{2\times 0\cdot 108}{(0\cdot 3)^3} \quad [\,M_1 = 0\cdot 108\ A\text{-}m^2,\ r_1 = 0\cdot 3m\,]$$

$$= 8\times 10^{-7}\,T$$

Also magnetic field at A due to M_2 magnet is

$$B_1 = 10^{-7}\times\frac{2M_2}{r_2^{\,3}} = 10^{-7}\times\frac{2\times 0.192}{(0\cdot 4)^2}\ [\,M_2 = 0\cdot 192\ A\text{-}m^2,\ r_2 = 0\cdot 4m\,]$$

$$= \mathbf{8\times 10^{-7}}\,T$$

$\therefore$ Resultant field at A, $B = \sqrt{B_1^{\,2}+B_2^{\,2}} = \sqrt{2B_1^{\,2}} \quad [\because B_1 = B_2]$

$\therefore \quad B = \sqrt{2}\times 8\times 10^{-7} = 11\cdot 3\times 10^{-7}\,T$

The direction is $\theta = \tan^{-1}\frac{B_2}{B_1} = \tan^{-1}(1) = \mathbf{45^o}$

- **Ex. 1.15. A magnet of magnetic moment 50 cgs is suspended in a magnetic field of intensity 0.4 cgs. It is deflected by 60º and then released. It crosses the mean position with the angular velocity 0.4 rad / s, calculate its magnetic moment.**

○ **Solution :** Work done to produce a deflection θ is, W = MB (1− cos θ) . . . . (i)

When it is released, its potential energy (which is equal to work done on it) is converted to rotational kinetic energy at the mean position. If I be the moment of inertia and ω be the angular velocity at the mean position, then by the principle of conservation of energy.

$$\frac{1}{2}I\omega^2 = MB\,(1-\cos\theta)$$

$$\therefore \quad I = \frac{2\,MB\,(1-\cos\theta)}{\omega^2}$$

$$= \frac{2\times 5\times 10^{-2}\times 0\cdot 4\times 10^{-4}(1-\cos 60^o)}{(0\cdot 4)^2}$$

$$= \mathbf{125\times 10^{-7}\,kg.m^2}$$

$M = 50\times 10^{-3} = 5\times 10^{-2}$ A-m^2
$B = 0\cdot 4\times 10^{-4}$ T
$\theta = 60^o$
$\omega = 0\cdot 4$ rad / s

● Ex. 1.16. The length of the magnet AB is 20 cm and its pole strength 12 cgs. The triangle ABC is drawn on the magnet AB. What is the resultant intensity of the magnetic field at C ?

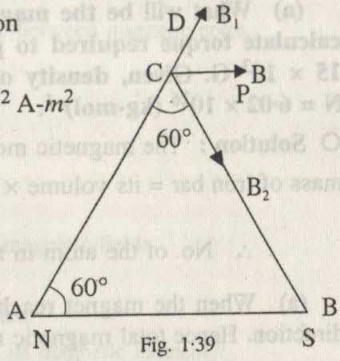


Fig. 1·39

○ **Solution :** Evidently the point C is a point on broad-side on position with respect to the magnet NS [Fig 1.39].

$$\therefore\ B = 10^{-7} \times \frac{M}{(r^2 + l^2)^{3/2}}$$

$$= 10^{-7} \times \frac{24 \times 10^{-2}}{(4 \times 10^{-2})^{3/2}}$$

$$= 10^{-7} \times \frac{24 \times 10^{-2}}{8 \times 10^{-3}} = \mathbf{3 \times 10^{-6}\ T}$$

$M = 2\,l.m = 0{\cdot}2 \times 1{\cdot}2 = 24 \times 10^{-2}$ A-m^2

$r = 10\sqrt{3}$ cm

$l = 10$ cm

$r^2 + l^2 = (300 + 100) \times 10^{-4}\ m^2$

$= 4 \times 10^{-2}\ m^2$

● Ex. 1.17. A magnet of length 10 cm is placed at right angle to the magnetic meridian. A magnetic needle placed at the equitorial position at a distance of 15 cm from the centre of the bar magnet, is found to be deflected by 45°. If the horizontal intensity of earth's magnetism be 0.37 Oe, what is the pole strength of the magnet ?

○ **Solution :** The magnet NS is placed at right angle to the magnetic meridian [Fig 1.40]. The magnetic needle *ns* is in earth's field and due to the field of the magnet NS. Now, the intensity of magnetic field due to NS at the site of *ns* is

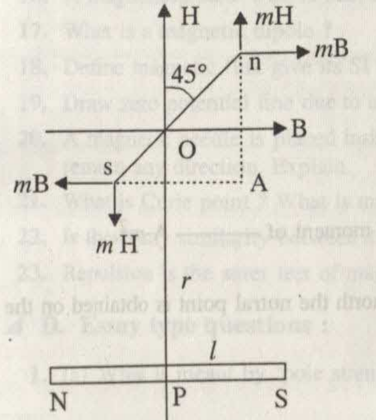


Fig. 1·40

$$B = 10^{-7} \frac{M}{(r^2 + l^2)^{3/2}}$$

[OP = r, M = magnetic moment of the magnet NS]

If m be the pole strength of the needle *ns*, then different forces on it are shown in fig. Considering its equilibrium we write $m\,B \times A\,n = m\,H \times (As)$

$$\therefore\ B = H \tan 45° = H$$

$$\therefore\ 10^{-7} \times \frac{M}{(r^2 + l^2)^{3/2}} = H$$

$H = 0.37 \times 10^{-4}$ T

$r = 0{\cdot}15\ m$

$l = 0{\cdot}05\ m$

$$\therefore\ M = 10^7 \times (r^2 + l^2)^{3/2} \times H$$

$$= 10^7 \times 0{\cdot}37 \times 10^{-4}[(0{\cdot}15)^2 + (0{\cdot}05)^2]^{3/2}$$

$$= 0{\cdot}37 \times 10^3 \times (250 \times 10^{-4})^{3/2}$$

$$= 0{\cdot}37 \times 10^{-3} \times (250)^{3/2}$$

$$\therefore\ \text{Pole strength,}\quad m = \frac{M}{2l} = \frac{0{\cdot}37 \times 10^{-2} \times (250)^{3/2}}{(0{\cdot}1)} = \mathbf{14{\cdot}62\ A\text{-}m}$$

● Ex. 1.18. The area of cross section of a solenoid is 2 × 10^{-4} m^2 and number of turns 3000. It is freely suspended in a magnetic field of intensity 0·5 T and a current of 5 A is passed through it. Calculate (i) magnetic moment of the solenoid and (ii) torque required to deflect it by 30°.

○ **Solution :** (i) Magnetic moment of the solenoid, $M = NiA = 1000 \times 5 \times 10^{-4} =$ **3 Am^2**

(ii) Torque, $\tau = MH \sin\theta = 3 \times 0.5 \times \sin 60° =$ **1·35 N-*m***

- **Ex. 1.19. The measurement of an iron bar is 5 cm × 1 cm × 1 cm. Magnetic moment of each atom is $1{\cdot}8 \times 10^{-23}$ Am^2.**

(a) What will be the magnetic moment when the bar reach to magnetic saturation. (b) Also calculate torque required to put the bar right angle to a magnetic field of magnetic induction 15×10^3 G. Given, density of iron 7800 kg. m^{-3}. At wt of iron = 56 and Avogadro's number, N = $6{\cdot}02 \times 10^{26}$ (kg-mol)$^{-1}$.

○ **Solution :** The magnetic moment of each atom, $M' = 1{\cdot}8 \times 10^{-23}\ Am^2$.

mass of iron bar = its volume × density = $5 \times 10^{-6} \times 7800 = 3{\cdot}9 \times 10^{-2}$ kg

$$\therefore \text{ No. of the atom in the bar } n = \frac{Nm}{A} = \frac{6{\cdot}02 \times 10^{26} \times 3{\cdot}9 \times 10^{-2}}{56} = 4{\cdot}19 \times 10^{23}$$

(a) When the magnet reaches the saturation, all the molecular magnets are arranged in the same direction. Hence total magnetic moment of the bar is

$$M = nM' = 4{\cdot}19 \times 10^{23} \times 1{\cdot}8 \times 10^{-23} = \mathbf{7{\cdot}54\ Am^2}$$

(b) Torque required to put the bar perpendicular to the magnetic field is

$$\tau = MB \sin\theta = 7{\cdot}54 \times 1{\cdot}5 \times \sin 90^\circ = \mathbf{11{\cdot}31\ N\text{-}m}$$

• EXERCISE •

A. Objective type questions :

- **Fill in the blanks :**

(i) SI unit of pole strength ———.

(ii) One SI pole strength is ——— emu.

(iii) Tesla is the unit of ———.

(iv) One Tesla is ——— Gauss.

(v) A single turn circular coil of area 100cm^2 carrying 2A has magnetic moment of ——— A-m^2.

(vi) Two lines of force ——— intersect.

(vii) For a magnet placed in magnetic meridian with north pole pointing north the nutral point is obtained on the ——— of the magnet.

(viii) One Bohr magneton is ——— A-m^2.

(ix) A bar magnet of length magnetic moment of each half is ———.

B. Very short answer type questions :

(i) What is the magnetic moment of a dipole ?

(ii) What is the SI unit of magnetic moment ?

(iii) What is the SI unit of pole strength ?

(iv) What force acts on a magnetic pole of strength m placed in a magnetic field of strength B ?

(v) What is the intensity of the magnetic field at a distance r from a pole of strength m ?

(vi) A bar magnet of moment M is suspended in a magnetic field of intensity B. How much work is to be done to deflect it by angle θ. ?

(vii) What is Bohr magneton ?

(viii) A magnet is placed in magnetic meridian with its north pole pointing south. At which position neutral point will be produced ?

C. Short answer type questions :

1. Define c. g. s. and SI unit of magnetic pole from Coulomb's law of force between magnetic poles.
2. Define pole strength of a magnet. What is its SI unit ?
3. Obtain the relation between SI and cgs unit of pole strength.
4. What is meant by intensity of magnetic field at a point ?
5. Which physical quantity has tesla as unit ? Give its definition.
6. The pole strength of a magnet is 2 A-m. Obtain its value in cgs system.
7. Define magnetic moment in terms of the torque acting on a magnet in a magnetic field.
8. What are magnetic lines of force ?
9. What is meant by uniform magnetic field ? How such a field is represented.
10. A circular coil of radius r and having n turns carries a current i. What is its magnetic moment ?
11. You are given three identical bars— one is a magnet, the second one is of iron and third is made of copper. How would you proceed to identify them ?
12. A magnetic north pole of strength 5 A-*m* is placed in a magnetic field of strength 0·4 oe. What force in Newton acts on it ? **[Ans : 2×10^{-4} N]**
13. Mention some properties of lines of force.
14. What is meant by neutral point ?
15. When is the neutral point produced at the axial point and at the equitorial point ?
16. A magnetised steel wire is bent into a semi-circle. How its moment changes ?
17. What is a magnetic dipole ?
18. Define magnetic flux give its SI unit.
19. Draw zero potential line due to a magnetic dipole.
20. A magnetic needle is placed inside an iron ring. It is now taken near a bar magnet. The needle is found to remain any direction. Explain.
21. What is Curie point ? What is meant by curie point of iron in 770°C ?
22. Is there any similarity between a bar magnet and a solenoid carrying current ?
23. Repulsion is the surer test of magnetisation. Explain. **[W. B. H. S. 2003]**

D. Essay type questions :

1. (a) What is meant by 'pole strength' of a magnet is 50 cgs ? What will be its value in SI system. **[W. B. H. S. 2001]**

   (b) Define intensity of magnetic field. How is a uniform magnetic field is represented by magnetic lines of force ? **[W. B. H. S. 2001]**

   (c) What are magnetic lines of force ? Mention the properties of lines of force. Why any two lines of force never intersect ? **[W. B. H. S. '98]**

2. (a) State and explain coulomb's law of force between two magnetic poles. How does the permeability of the medium affect this force.

   (b) Define the SI magnetic pole.

   (c) Draw the lines of force due to a bar magnet placed in magnetic meridian with north pole pointing north. What do the neutral points form ?

3. (a) What is a uniform magnetic field ? Give example. How a uniform magnetic field can be produced in the laboratory ?

   (b) Obtain the expression for the torque acting on a magnet in a magnetic field. Hence define magnetic moment of a magnet. What is the SI unit of magnetic moment ?

4. (a) A bar magnet is freely suspended in a magnetic field of intensity B. How much work is done to deflect it by an angle θ.

(b) A bar magnet is divided into two equal halves. How is the pole strength and magnetic moment change for each part.

5. (a) Deduce the expression for the intensity of the magnetic field at a distance r from the centre of a magnet of length $2l$ and pole strength m in the following cases : (i) the point on the axis and (ii) at a point on the perpendicular bisector of the magnet.

6. (a) What is meant by neutral point in a magnetic field ? Draw the lines of forces due to a bar magnet when it is in magnetic medium (i) with north pole pointing north and (ii) with north pole pointing south. Also locate the position of neutral points.

7. An electron is revolving around the nucleus in an atom in a circular orbit of radius r. Calculate the expression for the orbital magnetic moment of the electron. What one Bohr magneton.

8. (a) An electron of mass m and charge e is revolving in an hydrogen atom along a circular orbit of radius r with a velocity v. What is its magnetic moment ?

◢ E. Simple numerial problems :

1. (a) The pole strength of a magnet is 500 cgs. Obtain its value in SI.

(b) The magnetic moment of a bar magnet is 5 A-m^2. Obtain its value in cgs system.

[**Ans.** (a) 50 A-*m*. (b) 5000 cgs]

2. The area of a circular coil is 10cm^2 and number of turns 50. If a current of 1 A passes through it, calculate its magnetic moment. [**Ans.** 5×10^{-2} A- m^2]

3. Calculate the magnetic field intensity at a distance of 5 cm from a magnetic pole of strength 100 cgs.

[**Ans.** 0.4 × 10–3 T]

4. A magnetic pole of strength 40 cgs is placed on the axis of a bar magnet of 20 cm having pole strength 30 cgs at a distance of 30 cm from one end of the magnet. Calculate the force on the pole. Given, $\mu_0 = 4\pi \times 10^{-7}$ wb / A-*m*. [**Ans.** $0{\cdot}88 \times 10^{-5}$ N]

5. Two identical south poles when placed at a distance of 4.0 cm from each other mutual force between them is 0·04 N. Calculate pole strength. [**Ans.** 50·6 A-*m*]

6. Two poles of equal strength kept at a separation of 1m exert a force 1.6 N on each other. What is the strength of each pole ? [**Ans.** $\pm 4 \times 10^4 \text{N} - m$]

7. A magnetised steel wire of length d has a magnetic moment M. It is bent in a semi-circle. What will be its magnetic moment now ? [**Ans.** $\frac{2\text{M}}{\pi}$]

8. A magnetised steel wire has magnetic moment M. It is bent in the arc of a circle so that it sustained an angle of 60° at the centre. What is the moment of the magnet now ? [**Ans.** $\frac{3\text{M}}{\pi}$]

9. A magnetised steel wire of length 31·4 cm and pole strength 5 cgs is bent in the arc of a circle. What will be its magnetic moment now ? [**Ans.** 0.1 A-m^2]

10. How much torque is required to put a magnet of magnetic moment of 400 cgs at angle of 30° with respect to a uniform magnetic field of intensity 0·39 Oe ? [**Ans.** 7.8×10^{-6} N-*m*]

11. Two identical bar magnets each of length l and pole strength m are placed at right angle to each other such that north pole of one touches the south pole of the other. What will be the magnetic moment of the combination ? [**Ans.** $\sqrt{2}\, lm$]

12. Earth's magnetic field intensity at the north pole of the earth is 0·5 cgs. What will be the field intensity at a point on the equator ? [**Ans.** 0·25 cgs]

13. A magnet of length 10 cm is placed in magnetic meridian with north pole pointing north. Neutral point is obtained at a distance of 10 cm from the centre of the magnet. What is the pole strength of the magnet ? Earth's horizontal intensity 0·35 Oe. [**Ans.** 4·891 A-*m*]

14. The force of repulsion between two north poles of equal strength is 2·5 dyne when the separation is 2 cm. For what separation between the poles, the force of repulsion be 3·6 dyne ? [**W. B. H. S. 2003**] [**Ans.** 1·66 cm]

15. A bar magnet is placed in magnetic meridian horizontally with north pole pointing south. The resultant field intensity is zero at a distance of 10 cm from the south pole. If the distance between the two poles of the magnet be 10 cm and earth's horizontal intensity 0·2 cgs unit, what is the pole strength of the magnet ? [**Ans.** 2·67 A-*m*]

16. A bar magnet of length 12 cm and pole strength 25 cgs is placed in magnetic meridian with north pole pointing north. If the horizontal component of earth's magnetic field at the place be 0·3 cgs. Obtain the position of the neutral point. [**Ans.** 8 cm from the centre of the magnet] [**J E E**]

17. A bar magnet of length 8 cm and pole strength 25 cgs is suspended from its centre in a magnetic field of intensity 0·36 cgs. Calculate moment of couple required to keep the magnet at 30° with respect to the magnetic field. How much force acts on each pole of the magnet. [**W. B. H. S. '85**]
[**Ans.** 36×10^{-7} N-*m* ; 9×10^{-5} N]

18. A bar magnet of magnetic moment 0·5 A-m^2 is placed in magnetic meridian. How much work is required to deflect it by 60° ? Horizontal intensity of earth's field is 0·25 cgs. [**Ans.** $62{\cdot}5 \times 10^{-7}$ J]

19. A magnet of magnetic moment 0.5 A- m^2 is suspended in a magnetic field of intensity $0{\cdot}4 \times 10^{-4}$ T. It is deflected by 60° and then released. It passes through the mean position with angular velocity of 0·5 rad / sec. Calculate the moment of inertia of the magnet. [**Ans.** 8×10^{-5} kg- m^2]

20. A small bar magnet is placed horizontally in a magnetic meridian with north pointing south. A neutral is formed on the axis of the magnet at a distance of 24 cm from the south pole. If the horizontal intensity of earth's magnetism be $0{\cdot}18 \times 10^{-4}$ T, calculate field intensity at a distance of 20 cm north of the south pole. [**Ans.** 0.311×10^{-4} T]

21. The distance between two magnetic poles placed in air is 10 cm. The strength of one pole is 5 times the other. The effective force between the poles is 80 dyne. Calculate the strength of the poles.
[**Ans.** 4·0, 20·0 A-m] [**W. B. H. S. '86**]

22. Two magnetic poles of equal strength placed at a separation of 5 cm in air attract each other with a force of 14.4×10^{-4} N. For what separation the force of attraction between them be 1.6×10^{-4} N ? [**Ans.** 0.15 m]

23. A short bar magnet is placed with its axis at 30° to a uniform magnetic field of intensity 0·2 T experiences a torque of 0·06 N (a) Calculate the magnetic moment of the magnet. (b) Find out for what orientation of the magnet, it will be in stable equilibrium ? [**Ans.** (a) 0·6 A-m^2, (b) $\theta = 0°$ parallel to magnetic field]

24. A magnet placed in the magnetic meridian with north pole pointing north, neutralises the earth's field 27 cm from either pole. If the bar magnet is broken in three pieces perpendicular to the length and one such piece is similarly placed, find the position of the nutral point. [**Ans.** 18·62 cm]

25. A bar magnet 0·04 m long is held vertically with its north pole resting on a horizontal table. A neutral point is produced on the table 0·03 *m* from the lower end of the magnet What is the moment of the magnet ? Given, earth's horizontal intensity $0{\cdot}2 \times 10^{-4}$ T [**Ans.** $9{\cdot}2 \times 10^{-3}$ A-m^2]

26. The length of a bar magnet is 16 cm and pole strength 15 A-m. Calculate the intensity of magnetic field in the following cases : (i) at a point on the axis at a distance of 16 cm from the centre and (ii) at a point on the perpendicular bisector at a distance of 16 cm from the centre.[**Ans.** (i) $2{\cdot}08 \times 10^{-4}$ T; (ii) $0{\cdot}418 \times 10^{-4}$ T]

27. A bar magnet is placed along north-south with its north pole pointing north. If a south pole of strength 100 cgs units be placed at a point on its axis at a distance of 20 cm to the north of the north pole of the magnet, what is the magnitude and direction of the force that acts on it ?

Given, length of the magnet 6 cm, pole strength 1000 cgs units and earth's horizontal intensity 0·36 Oe. [**Ans.** 138·05 towards the magnet]

28. Two magnets having magnetic moments M and $\sqrt{3}$ M are placed at right to each other and fixed at the middle. When it is freely suspended in a uniform magnetic field, the first magnet makes an angle θ with the field. Find the value of θ. [**Ans.** 60°]

29. A circular coil of radius 0·01 m and having number of turns 100 carries a current of 0·7 A. It is suspended in a magnetic field of intensity 0·2 T. If the coil is free to rotate, at which position (i) it is in stable equation, (ii) it is unstable equilibrium. Calculate potential energy of the coil in both the positions.
[**Ans.** (i) $\theta = 0°$; zero (ii) 180°; 0.044 J]

30. A circular coil of radius 14 cm and having 300 turns is carrying a current of 15A. What is the magnetic moment of the coil ? **[Ans.** 69·3 A-m^2]

31. In an atom an electron is revolving in a circular orbit of radius 0·53Å. If the frequency of rotation be 10^{10} MHz, calculate equivalent orbital magnetic moment. **[Ans.** 1·26 × 10^{-23} A-m^2] **[C. B. S. E 1998]**

◢ E. Harder numerial problems :

1. The magnetic moment of a current carrying circular coil is 0·02 A-m^2. If its radius be 5 cm and number of turns 10. Calculate current in the coil. **[Ans.** 0.254 A]

2. A magnet freely suspended horizontally in a magnetic field of intensity 0·18 T requires a torque of 0·4 N-*m* to deflect it by 45º. (i) What is the magnetic moment of the magnet ? (ii) At what position, the magnet will remain equilibrium and calculate its potential energy at this position.

[Ans. (i) 3·14 A-m^2, (ii) θ = 0º, – 0·565 J]

3. A magnet of magnetic moment 4·0 A-m^2 is inclined at 30º with the horizontal component of earth's magnetism. How much work is required to be done to deflect it further by 30º ? Earth's horizontal intensity = 3·6 ×10^{-5} T. **[Ans.** 5·3 × 10^{-5} J]

4. Two bar magnets of equal mass are tied at the middle so that the magnets remain perpendicular to each other. Magnetic moment of N_1S_1 is three time that of the second magnet N_2S_2. The system is so pivoted so that it can rotate freely in horizontal plane. What angle the first magnet N_1S_1 will subtend with the magnetic meridian at its equilibrium position ? **[Ans.** $\theta = \tan^{-1}\left(\frac{1}{3}\right)$] **[I. I. T.]**

5. Two small bar magnets each of length 2 cm and each having pole strength 50 A-m are placed at the angular points A and C of the square ABCD of side 100 cm. Find the intensity of magnetic field at the points B and D. **[Ans.** 2·828 × 10^{-6} T, 1·424 × 10^{-5} T]

6. The area of cross section of a solenoid having 2000 turns is 1·6 × $10^{-4}m^2$. The solenoid is suspended from its centre so that it can rotate in a horizontal plane. If it carries a current of 4.0 A, calculate (i) magnetic moment of the solenoid (ii) torque required to keep the axis of the solenoid at angle 30º with the magnetic field of intensity 7·5 × 10^{-2} T. **[Ans.** (i) M = niA = 1·28 A-m^2 (ii) 0·048 J]

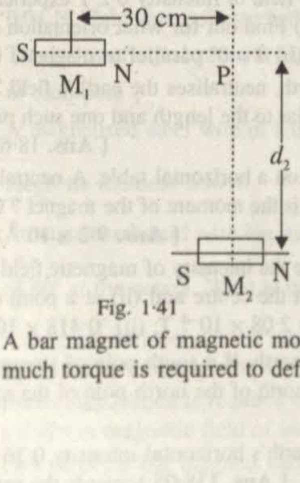


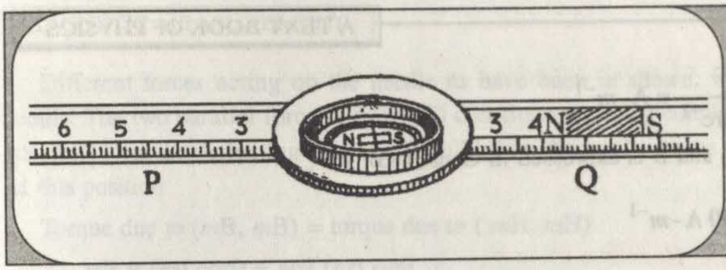
Fig. 1·41

7. Two small magnets M_1 and M_2 having magnetic moments 2·7 and 4·0 Am^2 are placed as shown in fig 1.41. If P be a neutral point, calculate the distance d_2. Earth's horizontal intensity = 0·3 G **[Ans.** d_2 = 20 cm]

8. The length of a long straight solenoid is 50 cm and its number of turns 1000 with area of cross section 2·0 × 10^{-4} m^2. It is placed at an angle of 30º with direction of a magnetic field of intensity 0·32 T. If the current in the solenoid be 0·32 A, how much torque acts on it ? **[Ans.** 0·064 N-*m*]

9. A bar magnet of magnetic moment 2·0 Am^2 is placed parallel to a magnetic field of intensity 0·25 T. How much torque is required to deflect it by 90º. What is the torque acting on it in this position ?

[Ans. 0·5 J ; 0·5 N-*m*]



CHAPTER 2

MAGNETIC MEASUREMENT

● **TOPICS :** ○ Magnetic field; ○ Couple acting on a magnetic dipole in cross magnetic fields : Tangent law; ○ Action of a magnetic needle in two mutually perpendicular magnetic fields; ○ Oscillation of a magnet in a uniform magnetic field; ○ Magnetometer; ○ Determination of magnetic moment and horizontal component of earth's magnetic field by magnetometer; ○ Oscillation of a magnet in earth's magnetic field; ○ Short answer type questions (with answers); ○ Exercise.

2.1. Magnetic field

There is a region around a magnet in which other magnet or any other magnetic material experiences a force. The region is called **magnetic field**. The magnitude and direction of the force on an isolated unit north pole placed in the field give the magnitude and direction of intensity of the magnetising field. It is also called $\vec{H}$ vector. The magnetic field produced by long straight conductor, circular coil or a solenoid placed in vacuum is denoted by $\vec{H}$ vector. It is to be mentioned that the magnitude of H - vector does not depend on the property of the medium. If these current carrying elements are in a magnetic medium, then the field produced in the medium is denoted by $\vec{B}$ vector. $\vec{B}$ is also called magnetic induction vector.

Intensity of the magnetic field can be described in terms of lines of force produced by a magnet. Lines of force passing normally through a unit area drawn around a point is also taken as a measure of intensity of the field at the point.

● (i) **Uniform magnetic field :** If the magnitude and direction of intensity of the magnetic field be same at all points in the field, it is called a uniform magnetic field. Such a field is represented by equidistant parallel lines [fig 2.1].

If a magnetic dipole is placed in a uniform magnetic field, two equal and opposite forces act on the two poles of the dipole. So the dipole is acted upon by a couple. Due to the couple rotational motion is produced but no translational motion.

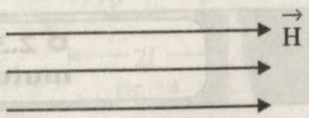


Fig. 2·1

● **Example :** Magnetic field produced in current carrying solenoid and earth's magnetic field over region.

● (ii) **Non-uniform magnetic field :** In such a field, magnitude of intensity and its direction are different at different points of the field. Lines of forces are not parallel. For example, magnetic field due to a bar magnet.

In a non-uniform field unequal force acts on a magnet. This unbalanced force may be treated as a combination of a couple and a force applied to the centre of mass. For this reason in a non-uniform magnetic field, the magnet has translational as well as rotational motion.

● **A discussion :** If B be the magnetic induction in a medium then magnetic intensity is defined by the relation.

$H = \frac{B}{\mu}$ μ is called absolute permeability of the medium unit of H is $= \dfrac{\text{Tesla}}{\text{tesla meter Ampere}^{-1}} = \text{A - } m^{-1}$

In cgs unit of H is Oersted (Oe) and B is expressed in Gauss (G)

$$1\,\text{Oe} = \frac{1\text{G}}{\mu_0} = \frac{10^{-4}}{4\pi \times 10^{-7}} = \mathbf{80\ A\text{ - }}m^{-1}$$

⚙ 2.2. Couple acting on a magnetic dipole in cross magnetic fields : Tangent law ⚙

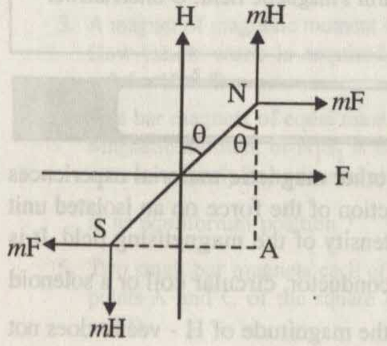


Fig. 2·2

The magnetic dipole NS is suspended in cross magnetic fields. The cross magnetic fields are produced by earth's horizontal intensity H and another magnetic field F which is perpendicular to H [Fig 2.2]. At equilibrium the magnetic dipole makes angle θ with the direction of H. Different forces acting on the magnet have been shown in fig. At its equilibrium, the deflecting torque produced by (mF, mF) = Restoring torque produced by (mH, mH). Here m = pole strength of the dipole.

$$\therefore\ m\text{F (NS)}\cos\theta = m\text{H (NS)}\sin\theta$$

$$\therefore\ \text{F} = \text{H}\tan\theta \text{ .... (2.1)}$$

So, the tangent of the angle of deflection is equal to the ratio between F and H. As H is constant, F α tanθ. This is tangent law.

● **Example 2.1. A magnetic needle is kept suspended in earth's magnetic field H = 0.3 × 10^{-4} T. Now a magnetic field F = 0.1 × 10^{-4} T is applied on the needle along East-West direction. Calculate its angle of deflection.**

○ **Solution :** From the equation (2.1), $\tan\theta = \dfrac{\text{F}}{\text{H}}$.

$\therefore\ \tan\theta = \dfrac{0\cdot1\times10^{-4}}{0\cdot3\times10^{-4}} = \dfrac{1}{3}$ $\therefore\ \theta = \tan^{-1}\left(\dfrac{1}{3}\right)$ It is the angle of deflection.

⚙ 2.3. Action of a magnetic needle in two mutually perpendicular magnetic fields ⚙

A pivoted magnetic needle is under two field acting in mutually perpendicular directions. One of the fields is earth's horizontal intensity H and the other one is the magnetic field B produced by a bar magnet NS placed in east-west direction. Here we consider two standard positions of the magnet NS.

▲ (A) Tangent-A position of Gauss :

In fig 2.3, the magnet NS is placed at right angle to earth's horizontal intensity. The small needle *ns* is so placed that its centre O′ lies on the axis of the bar magnet NS. The magnetometer is placed with its two arms in magnetic E-W direction. Evidently the magnetic needle is in the end-on position of the magnet NS. It is also called Tangent A position of Gauss.

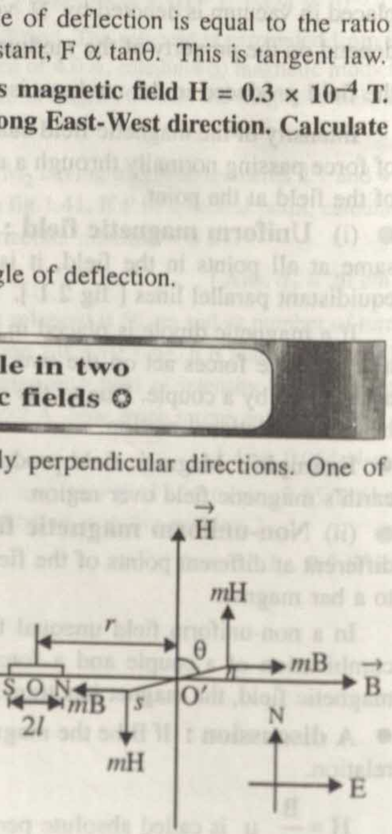


Fig. 2·3

Different forces acting on the needle *ns* have been in shown, where *m* is the pole strength of the needle. The two parallel forces (*m*H, *m*H) constitute restoring couple and the other two parallel forces (*m*B, *m*B) form the deflecting couple. At equilibrium the needle makes angle θ with the magnetic meridian. At this position

Torque due to (*m*B, *m*B) = torque due to (*m*H, *m*H)

$$\therefore \quad mB \times (ns)\cos\theta = mH\,(ns)\sin\theta$$

$$\therefore \quad B = H\tan\theta$$

$$\text{or,}\quad \frac{\mu_0}{4\pi}\frac{2Mr}{(r^2-l^2)^2} = H\tan\theta \left[\because\ B = \frac{\mu_0}{4\pi}\frac{2Mr}{(r^2-l^2)^2};\ M = \text{magnetic moment of NS}\right]$$

$$\therefore \quad \frac{M}{H} = \frac{4\pi}{\mu_0}\frac{(r^2-l^2)^2}{2r}.\tan\theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.2)$$

$$\text{If}\quad r^2 >> l^2,\ \frac{M}{H} = \frac{4\pi}{\mu_0}\frac{r^3}{2}\tan\theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.3)$$

▲ (B) Tangent-B position of Gauss :

The axis of the magnet NS is placed perpendicular to the magnetic meridian and the centre O of the needle *ns* is situated on the perpendicular bisector of the magnet NS. Now the magnetometer is so placed that its two arms are in magnetic N-S direction. This particular position of the needle is called Tangent-B position of Gauss.

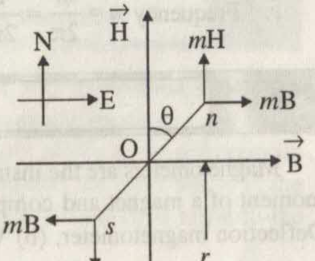


Fig. 2·4

As in Tangent- A position of Gauss, we can deduce that

$$B = H\tan\theta \quad\text{and}\quad B = \frac{\mu_0}{4\pi}.\frac{M}{(r^2+l^2)^{3/2}}$$

$$\therefore \quad \frac{\mu_0}{4\pi}.\frac{M}{(r^2+l^2)^{3/2}} = H\tan\theta$$

$$\therefore \quad \frac{M}{H} = \frac{4\pi}{\mu_0}(r^2+l^2)^{3/2}\tan\theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.4)$$

$$\text{If}\quad r^2 >> l^2\text{, then}\quad \frac{M}{H} = \frac{4\pi}{\mu_0}r^3\tan\theta \quad \ldots\ldots\ldots\ldots\ldots (2.5)$$

● **Example 2.2. A bar magnet is placed in East-West direction and a small magnetic needle is situated at a distance of 30 cm from the north pole of the magnet towards east. The deflection of the needle is 45°. If the length of the magnet NS be 6 cm, find magnetic moment and pole strength of the bar magnet. Earth's horizontal intensity H = 3·5 × 10⁻⁵ T.**

○ **Solution :** The required relation is

$$\frac{M}{H} = \frac{4\pi}{\mu_0}\frac{(r^2-l^2)^2}{2r}\tan\theta$$

$$\therefore \quad M = 3{\cdot}5\times10^{-5}\times10^{7}\times\frac{\left[(0{\cdot}33)^2-(0{\cdot}03)^2\right]^2}{2\times0{\cdot}33}$$

$$= \mathbf{6{\cdot}18\ A\text{-}m^2}$$

$\mu_0 = 4\pi\times10^{-7}$ H/m
$r =$ 33 cm = 0·33 m
$l =$ 0·03 m
H = 3·5 × 10^{-5} T

◎ 2.4. Oscillation of a magnet in a uniform magnetic field ◎

The bar magnet NS is suspended freely from its centre of mass in a horizontal magnetic field of intensity B. It remains at rest with its axis along the field [Fig 2.5]. Now it is deflected by a small angle θ from the equilibrium position and then released. The magnet will start oscillating about the mean position.

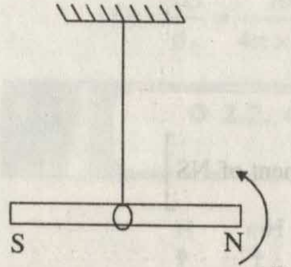


Fig. 2·5

Now, the restoring torque for the diflection θ is, $\tau = -\,MB\sin\theta$

When M = magnetic moment of the magnet.

Again $\tau = I\,\alpha$, When I = moment of inertia and α is the angular acceleration. Then,

$$\tau = I\,\alpha = -\,MB\sin\theta$$

$$\text{or,}\quad \alpha = -\frac{MB}{I}\sin\theta = \left(\frac{MB}{I}\right)\theta \qquad [\text{ As } \theta \text{ is small }]$$

So, the angular acceleration is proportional to deflection from the mean position and is always directed towards the mean position. Hence the motion of the magnet is simple harmonic.

$$\therefore\ \text{Angular velocity of the magnet,}\quad \omega = \sqrt{\frac{MB}{I}}$$

$$\therefore\ \text{Frequency } n = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{MB}{I}}$$

◎ 2.5. Magnetometer ◎

Magnetometers are the instruments employed for the measurement of magnetic quantities like magnetic moment of a magnet and components of earth's magnetism. There are two types of magnetometers : (a) Deflection magnetometer, (b) Vibration magnetometer.

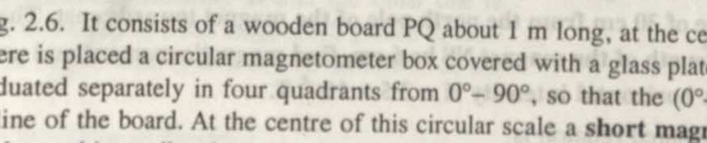


Fig. 2·6

(A) Deflection magnetometer :

It is shown in fig. 2.6. It consists of a wooden board PQ about 1 m long, at the centre of which there is a circular hole. Here is placed a circular magnetometer box covered with a glass plate. The box contains a circular scale graduated separately in four quadrants from 0°–90°, so that the (0°–0°) line is usually parallel to the axis line of the board. At the centre of this circular scale a **short magnetic needle** (*ns*) is pivoted. At right angles to this needle a long aluminium pointer is fixed so that the tips of the pointer may move over the circular scale. A plane mirror is fixed below the pointer which enables to avoid the parallax error during taking readings. Two metre scales are fixed on the board on the two sides of the magnetometer needle, so that the zeros of these two scales coincide with the centre of the needle. The magnet NS can be placed along or perpendicular to the length of the board. When the length of the wooden board is perpendicular to the magnetic meridian and the magnet is placed with its length along the board this position of the magnet is called the **Tangent A** position. If the board is placed along the magnetic meridian, the magnet should be placed with its length perpendicular to the board and this position of the magnet is called **Tangent B** position.

(B) The Vibration magnetometer :

One type of vibration galvanometer usually employed in the laboratory is shown in fig. 2.7. It consists of a hollow rectangular box A. The two longer sides of the box are made of glass. The top surface has two openings MM which are fitted with glass. A cylindrical glass tube G having a torsion head T at the top is fixed to the middle of the top surface of the box. A magnet NS can be placed on a light non-magnetic stirrup suspended by a torsionless fibre F from the torsion head. A rectangular piece of mirror R having longitudinal line PQ marked on it, is fixed on the inside surface of the base of the box. This line on the mirror helps us to find the time of transit of the magnet accurately during its oscillation. The magnet is to be placed on the stirrup (cradle) in such a way that the upper face of the magnet is horizontal. The axis of rotation of the system coincides with the vertical suspension fibre. During experiment, the box is rotated to bring the line MM parallel to the axis of the magnet.

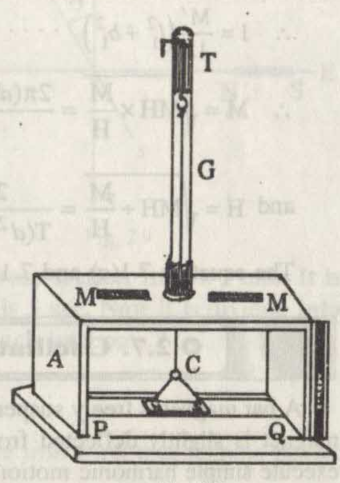


Fig. 2·7

◎ 2.6. Determination of magnetic moment and horizontal component of earth's magnetic field by magnetometer ◎

● **Theory :** Consider the fig. 2.8. The length of the bar magnet NS is $2l$ and its moment is M. It is placed on the deflection magnetometer with its axis perpendicular to the magnetic meridian (*i.e.* along east–west direction). The magnetic needle (*ns*) is in equilibrium under the action of two equal and opposite couples — one due to H, the horizontal component of earth's magnetic field and the other due to F, the magnetic field of the magnet NS which is placed in **Tan A position** (*i.e.* its axis is at right angle to the magnetic meridian.)

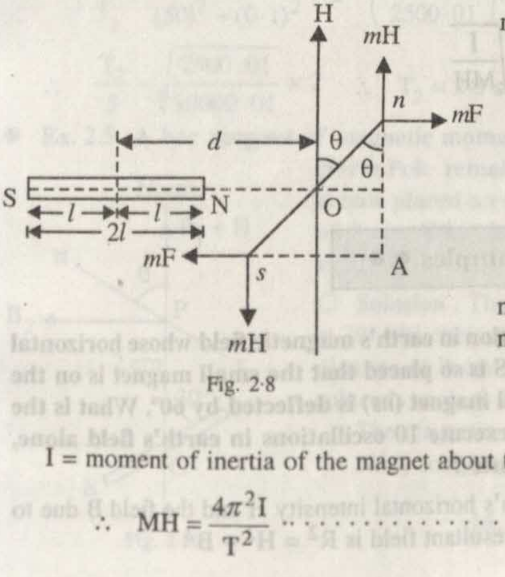


Fig. 2·8

If θ be the angle of deflection of the needle from the meridian then by tangent law : $F = H.\tan\theta$

$$\therefore \quad \frac{\mu_0}{4\pi}\cdot\frac{2Mr}{(d^2-l^2)^2} = H\tan\theta$$

$$\therefore \quad \frac{M}{H} = \frac{4\pi}{\mu_0}\cdot\frac{(d^2-l^2)^2}{2d} = \tan\theta \quad \ldots\ldots\ldots\ldots (2.6)$$

Here, M = magnetic moment of the magnet.

Again, the bar magnet is suspended in a vibration magnetometer and then its time period of oscillation T is measured. Then

$$T = 2\pi\sqrt{\frac{I}{MH}} \quad \ldots\ldots\ldots (2.7)$$

I = moment of inertia of the magnet about the axis of suspension.

$$\therefore \quad MH = \frac{4\pi^2 I}{T^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.8)$$

Here $I = \dfrac{\text{mass of the bar magnet } (M')}{12}\left[(\text{length})^2 + (\text{breadth})^2\right]$

$$\therefore \quad I = \frac{M'}{12}\left(l_1^2 + b_1^2\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.9)$$

$$\therefore \quad M = \sqrt{MH \times \frac{M}{H}} = \frac{2\pi(d^2 - l^2)}{T} \cdot \sqrt{\frac{4\pi}{\mu_o}\frac{I\tan\theta}{2d}} \ldots\ldots\ldots\ldots\ldots (2.10)$$

$$\text{and} \quad H = \sqrt{MH \div \frac{M}{H}} = \frac{2\pi}{T(d^2 - l^2)} \cdot \sqrt{\frac{\mu_o}{4\pi}\frac{2dI}{\tan\theta}} \ldots\ldots\ldots\ldots\ldots (2.11)$$

The equation 7.1(c) and 7.1(d) are the working formulae.

✪ 2.7. Oscillation of a magnet in earth's magnetic field ✪

A bar magnet is freely suspended by a strong and let at the place earth's horizontal intensity be H. The magnet is slightly deflected from its mean position and then released. The magnet will oscillate and execute simple harmonic motion.

When the deflection is θ, the restoring torque is $\tau_c = -MH\sin\theta$

If I be the moment of inertia and α be the angular acceleration, then deflecting torque is,

$$\tau_D = I\alpha = I\frac{d^2\theta}{dt^2}$$

$$\therefore \quad I\frac{d^2\theta}{dt^2} = -MH\sin\theta = -MH\theta \quad [\text{ as } \theta \text{ is small }]$$

$$\therefore \quad \frac{d^2\theta}{dt^2} = -\frac{MH}{I}\theta \qquad \therefore \quad \frac{d^2\theta}{dt^2} + \omega^2\theta = 0 \qquad \left[\omega = \sqrt{\frac{MH}{I}}\right]$$

It is the equation of simple harmonic motion.

$$\therefore \quad \text{Angular velocity,} \quad \omega = \frac{2\pi}{T} = \sqrt{\frac{MH}{I}} \qquad \therefore T = 2\pi\sqrt{\frac{I}{MH}}$$

$$\therefore \quad T \propto \frac{1}{\sqrt{H}}, \quad \text{But} \quad T = \frac{1}{n} \quad \therefore \quad n \propto \sqrt{H}$$

• • Numerical Examples • •

● **Example. 2.3. A small magnet *ns* is executing oscillation in earth's magnetic field whose horizontal intensity H = 0.3 cgs units. On its last a bar magnet NS is so placed that the small magnet is on the end-on position of the bar magnet. As a result the small magnet (*ns*) is deflected by 60°. What is the intensity of the resultant fixed ? If the small magnet execute 10 oscillations in earth's field alone, how many oscillations it will complete now in one minute ?**

○ **Solution.** The situation is shown in fig 2.9. Here earth's horizontal intensity H and the field B due to the bar magnet NS are mutually perpendicular. Here the resultant field is $R^2 = H^2 + B^2$

Here $H = 0{\cdot}3$ Oe, $B = H \tan 60° = 0{\cdot}3 \times \sqrt{3} = 0{\cdot}3\sqrt{3}$ Oe

$\therefore \quad R = (0{\cdot}3)^2 + (0{\cdot}3\sqrt{3})^2 = 2 \times 0{\cdot}3 = \mathbf{0{\cdot}6}$ **Oe**

Hence the intensity of the resultant field = **0·6 Oe**

2nd : Now, we know $H \propto n^2 \; \therefore \; \frac{R}{H} = \frac{n_1^{\,2}}{n^2}$

$$\therefore \; n_1^{\,2} = n^2 . \frac{R}{H} = 100 \times \frac{0{\cdot}6}{0{\cdot}3} = 200 \; \therefore \; n_1 = 10\sqrt{2} = \mathbf{14{\cdot}2}$$

$\therefore$ The small magnet will complete **14·2** oscillations in a minute.

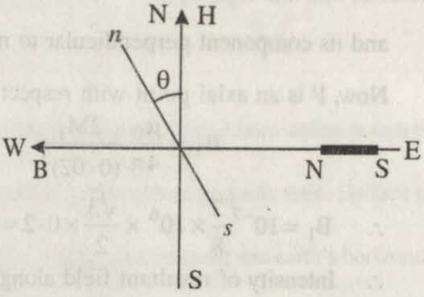


Fig. 2·9

● **Ex. 2.4. The length of a thin iron wire is 100 cm and its area of cross-section 1mm × 1 mm. It is uniformly magnetised. Its time period of oscillation in earth's field is 5 sec. Now it is divided into halves perpendicular to its length. What will be the time period of each half now ?**

○ **Solution :** The required relation $T = 2\pi\sqrt{\frac{I}{MH}}$

If I_1 be the moment of inertia and M_1 be the magnetic moment of the iron wire, then its time period of oscillation, $T_1 = 2\pi\sqrt{\frac{I_1}{M_1 H}}$

Now, let after division, moment of inertia of each half I_2 and magnetic moment M_2 then time period of oscillation $T_2 = 2\pi\sqrt{\frac{I_2}{M_2 H}}$

$$T_2 = 2\pi\sqrt{\frac{I_2}{M_2 H}} \quad \therefore \quad \frac{T_2}{T_1} = \sqrt{\frac{I_2 M_1}{I_1 M_2}}$$

Now, $I_1 = \frac{W}{12}(l^2 + b^2) = \frac{W}{12}\left((100)^2 + (0{\cdot}1)^2\right); \; I_2 = \frac{MW}{24}\left[(50)^2 + (0{\cdot}1)^2\right]$

$$\frac{I_1}{I_2} = \frac{100^2 + (0{\cdot}1)^2}{(50)^2 + (0{\cdot}1)^2} \times 2 = \left(\frac{10000{\cdot}01}{2500{\cdot}01}\right) \times 2 \quad \text{and} \quad \frac{M_1}{M_2} = 2$$

$\therefore \quad \frac{T_2}{5} = \sqrt{\frac{2500{\cdot}01}{10000{\cdot}01}} \times 2 \quad \therefore \quad T_2 = \mathbf{2{\cdot}5}$ **sec** $\quad \therefore$ Present time period = 2.5 sec.

● **Ex. 2.5 A bar magnet of magnetic moment 0·2 Am^2 is placed on a horizontal table so that its North Pole remain at 30º east of magnetic north pole. A magnetic needle is now placed on the table on the north of the bar magnet at a distance of 2 cm. What is the equilibrium position of the needle ? H = 0·336 × 10^{-4} T.**

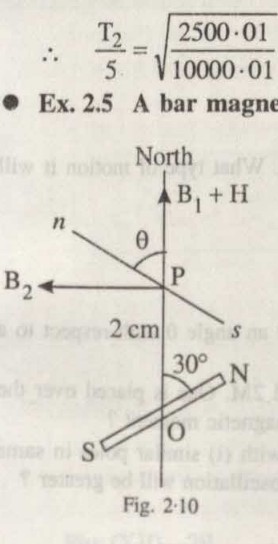


Fig. 2·10

○ **Solution :** The arrangement is shown in fig 2.10. The bar magnet NS is at 30º with the magnetic meridian. The magnetic needle *ns* is kept at P at a distance of 2 cm from the centre O of the magnet NS. At equilibrium *ns* subtends angle θ with magnetic meridian. We require the angle θ.

The magnetic moment of bar magnet NS is M.

Now, component of M along magnetic meridian,

$$M_1 = M\cos 30^{\circ} = \frac{\sqrt{3}}{2}M$$

and its component perpendicular to meridian, $M_2 = M \sin 30° = \dfrac{M}{2}$

Now, P is an axial point with respect to M_1. So, magnetic intensity at P due to M_1 is

$$B_1 = \frac{\mu_0}{4\pi} \frac{2M_1}{(0 \cdot 02)^3}$$

$$\therefore \quad B_1 = 10^{-7} \frac{2}{8} \times 10^6 \times \frac{\sqrt{3}}{2} \times 0 \cdot 2 = \mathbf{0 \cdot 433 \times 10^{-2} T}$$

∴ Intensity of resultant field along the meridian is

$$B' = B_1 + H = 4 \cdot 33 \times 10^{-3} + 3 \cdot 36 \times 10^{-5} = 4 \cdot 3636 \times 10^{-3} T$$

Again, P is an equiterial point with respect to M_2.

∴ Magnetic intensity at P due to M_2 is

$$B_2 = 10^{-7} \times \frac{M_2}{r_2^{\,3}} = 10^{-7} \times \frac{M}{2 \times (0 \cdot 02)^3} = 10^{-7} \frac{0 \cdot 2}{2 \times 8 \times 10^{-6}} = \frac{0 \cdot 2}{160} = \mathbf{1 \cdot 25 \times 10^{-3} T}$$

$$\therefore \quad \tan\theta = \frac{B_2}{B_1 + H} = \frac{1 \cdot 25 \times 10^{-3}}{4 \cdot 3636 \times 10^{-3}} = 0 \cdot 287$$

$$\therefore \quad \theta = \mathbf{\tan^{-1}(0 \cdot 287)} \text{ west of north}$$

• E ◇ X ◇ E ◇ R ◇ C ◇ I ◇ S ◇ E •

A. Objective type questions :

1. The magnetic field produced by a solenoid ———— .
2. The magnetic field due to a bar magnet is ———— .
3. A magnet is deflected by an angle θ from the direction of the magnetic field Torque on the magnet is ———— .
4. The time period of oscillation of a magnet in a magnetic field is ———— .

B. Very short answer type questions :

1. What is tangent law ?
2. What is a uniform magnetic field ?
3. Give an example of non-uniform magnetic field.
4. A magnet suspended in a magnetic field is slightly deflected and then released. What type of motion it will execute ?
5. What is a magnetometer ?

C. Short answer type questions :

1. What is meant by tangent- A and tangent - B position of a magnet ?
2. Write the expression for the torque acting on a magnet when it is deflected by an angle θ with respect to a magnetic field. Hence define moment of a magnet.
3. Two magnets of same length and cross-section have magnetic moments M and 2M. One is placed over the other such that opposite poles are in the same direction. What is the resultant magnetic moment ?
4. Two identical magnets of moment M and 4M are placed one above the other with (i) similar poles in same direction and (ii) opposite poles in one direction. In which case time-period of oscillation will be greater ?
5. What is a magnetometer ? Mention some of its uses.

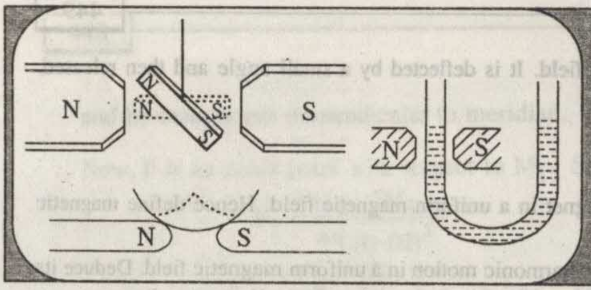
6. A bar magnet is suspended in a uniform magnetic field. It is deflected by a small angle and then released. Write its time period of oscillation.
7. Establish tangent law.

D. Essay type questions :

1. Obtain the expression for the torque acting on a magnet in a uniform magnetic field. Hence define magnetic moment.
2. A magnet of magnetic moment M is executing simple harmonic motion in a uniform magnetic field. Deduce its time period T.
3. What is a magnetometer ? Deduce the theory for the determination of magnetic moment and earth's horizontal intensity.

E. Numerical problems :

1. The time period of oscillation of a magnet in earth's magnetic field in 2 sec. Now it is divided into two equal parts perpendicular to its length. What will be the time period of one part ? **[Ans : T = 1 sec]**
2. A magnet of magnetic moment 0·05 Am^2 is suspended in a magnetic field of magnetic induction 4×10^{-5} T. It is deflected through 60° and then released. When it passes through the mean position, its angular velocity is 0·4 rad. s^{-1}. Calculate moment of inertia of the magnet. **[Ans : $1{\cdot}25 \times 10^{-5}$ kg-m^2]**
3. The magnetic moment of a small bar magnet is 2000 cgs unit. When it is placed on the magnetic meridian 20 cm from the centre of a vibration magnetometer, the time period of oscillation of the magnetometer is 1 sec. When the bar magnet is placed in the reversed position the time period becomes $\frac{1}{3}$ sec. Find horizontal intensity of earth's magnetism of the place. **[Ans : H = 2×10^{-5} T]**
4. In a magnetometer experiment, when a small magnet M_1 is placed on the tangent A position at a distance of 40 cm from the centre of the magnetometer, the magnetic needle is deflected by 30°. When another magnet M_2 is placed on the tangent-B position at a distance of 30 cm from the centre of the magnetometer, the needle produces 20° deflection. Compare the magnetic moments of the two magnets. **[Ans. 1.74]**
5. The horizontal intensity of earth's magnetism at a place is 0.2 Oe. A magnetic needle placed at a distance of 50 cm on the axis of a small bar magnet is deflected by 30°. Calculate the time period of oscillation of the magnet in earth's magnetic field. **[Ans. 1.261 sec]**
6. At a place when dip angle 45°, a magnetic needle completes 20 oscillations in one minute. Again at a place where dip angle 30°, a magnetic needle completes 30 oscillations in one minute. Compare the values of horizontal intensities of the two places. **[Ans. 1.83]**



CHAPTER
3

MOLECULAR THEORY OF MAGNETISM : MAGNETIC PROPERTIES OF MATERIALS

● **TOPICS :** ○ Isolation of single pole is impossible; ○ Molecular theory of magnetism; ○ Explanation of different magnetic phenomena by molecular theory; ○ Magnetic field inside a magnetic material; ○ Some quantities of classify different magnetic materials; ○ Relation between B, μ and χ; ○ Paramagnetic, Ferromagnetic and Diamagnetic substances; ○ I-H and B-H curves of a magnetic material; ○ Cycle of magnetisation : Hysteresis; ○ Discussion of magnetic properties of iron and steel by hysteresis loop; ○ Uses of ferromagnetic substances; ○ Short answer type questions (with answers); ○ Exercise.

○ 3.1. Isolation of single pole is impossible ○

In general, a magnet possesses two opposite poles of equal strength. The poles are located near the two ends of the magnet. At the middle of the magnet, magnetic property is negligible. So, it may be thought that if the magnet is broken at the middle, each part will be a magnet with one pole. But from experiment it is found that each part is again a complete magnet with two opposite poles at the ends [fig 3.1]. On further subdivision it is found that each and every part is a complete magnet with two opposite north pole and south pole. The process of breaking up may be continued still further till we reach to the molecule, the smallest part of a substance. So, we conclude that magnetic poles always occur in pair and a single north pole or south pole cannot be isolated. Even the molecules of a magnet or of a magnetic substance is a complete magnet.

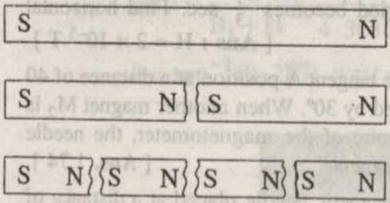


Fig. 3·1

○ 3.2. Molecular theory of magnetism ○

The concept of molecular magnet was first introduced by Weber. He assumed that **each molecule of a magnetic substance is itself a complete magnet having equal and opposite polarity at the two ends.** These tiny magnets are called **molecular magnets** or **Weber elements**. This theory of magnetism proposed by Weber is called molecular theory of magnetism . According to this theory—

A magnet or a magnetic substance consists of large number of molecular magnets. In a magnetic materials the molecular magnets are arranged indiscriminately in all directions [fig 3.2]. Due to irregular distribution of molecular magnets, the magnet effect of any one of them is completely nullifield by the neighbouring elements.

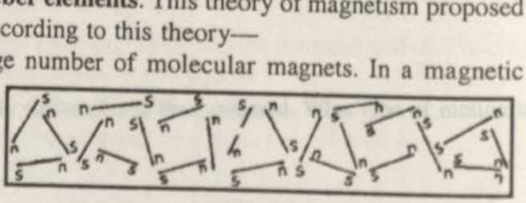


Fig. 3·2

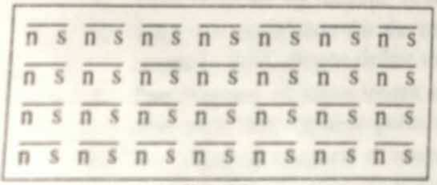

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Fig. 3·3

When the substance is magnetised by applying a suitable magnetising field, the elements arrange themselves along parallel lines along the direction of the applied field with their opposite poles facing each other—All north poles are in direction and all south poles in opposite direction. So the material develops magnetism [Fig 3.3]. When the alignment of the molecular magnets is complete, the state of magnetic saturation is reached.

Throughout the interior of the substance, opposite poles of two molecular magnets face each other. As the pole strength of each small magnet is equal, two adjacent opposite poles cancel the effect of each other. Thus the molecular magnets inside the specimen neutralise the effect of one another except at the two ends where only similar poles are arranged. For this reason, one end of the specimen exhibits north polarity and the other south polarity.

Higher the number of uncompensated magnetic poles at the ends, greater is the magnetism developed in the specimen. The amount of magnetism at the ends is the **pole strength**. Pole strength is maximum when the specimen is magnetically saturated.

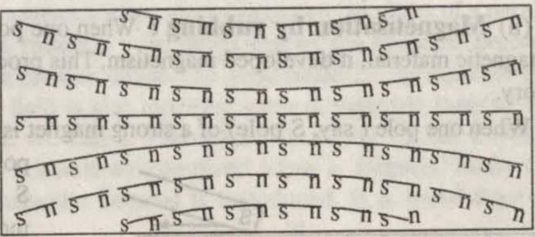


Fig. 3·4

Due to mutual repulsion between similar poles the molecular magnets actually arrange in curves and not in parallel straight lines as shown in fig 3.4. For this reason magnetic poles are situated near the ends and not at the ends.

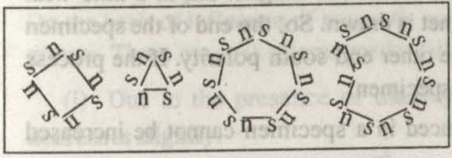


Fig. 3·5

Ewing put forward the hypothesis that the molecular magnets within an unmagnetised substance are arranged in 'closed chains' [Fig 3.5]. Such arrangement of molecular magnets is very stable. An unmagnetised specimen contains myriads of such chains. During magnetisation, the chains are broken and the molecular magnets gradually arrange in linear fashion.

- **Demonstration :** By a simple experiment we can demonstrate the validity of molecular theory of magnetism. A test tube filled with fresh iron filings is placed horizontally on a table. Now one pole (say, north pole) of a strong bar magnet is put near one end of the table [Fig 3.6]. The tube is gently tapped a number of times.

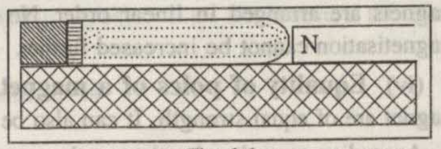


Fig. 3·6

The bar magnet is then removed without disturbing the test tube. Now a magnetic needle is brought near the same end of the test tube. It is found that the north of the needle is attracted. It shows that south pole is induced at this end. If the tube is now shaken, its magnetism disappear.

- **Explanation :** Each iron filing becomes a magnet by induction and behaves like molecular magnet. By the influence of the magnetic field of the bar magnet, they arrange themselves in linear pattern. The ends on the tube then behave like the poles of a magnet on shaking the tube this arrangement is destroyed leading to the disappearance of the polarities at the ends.

3.3. Explanation of different magnetic phenomena by molecular theory

Different common magnetic phenomena can be satisfactorily explained by the molecular theory of magnetism. Some of these phenomena are described below :

- (i) **Magnetic induction :** When one pole of a strong bar is brought near a magnetic material, its nearer and develope opposite polarity and further end similar polarity. This process of magnetic induction can be explained by molecular theory of magnetism.

Due to the presence of the inducing pole (say, N- pole), magnetic force acts on the molecular magnets. The south poles of molecular magnets being attracted by the strong N-pole of the magnet, turn towards the pole and the north poles turn in opposite direction. An opposite polarity is, therefore developed at the nearer end while the for end shows a similar polarity. Hence a free south pole is developed at the nearer end and free north pole at the furthest end.

● (ii) **Magnetisation by rubbing :** When one pole of a strong magnet is rubbed along the length of a magnetic material, it developed magnetism. This process of magnetisation can be explained by molecular theory.

When one pole (say, S pole) of a strong magnet is placed one one end of the magnetic material, north poles of molecular magnets are attracted by the striking S pole and at the same time south poles of the molecular magnets tend to move away [Fig 3.7]. As a result the molecular magnets rotate in such away that their north poles point towards the striking S pole of the bar magnet and their south poles point in opposite direction.

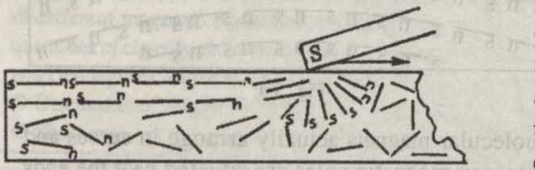


Fig. 3·7

As the magnet is drawn along the length of the specimen, the molecular magnet fall in a time with their opposite n- poles pointing to the direction in which the magnet is drawn. So, the end of the specimen where the striking s - pole leaves will develop north pole and the other end south polarity. If the process is repeated several times, strong magnetism is developed in the specimen.

● (iii) **Magnetic saturation :** When the magnetization induced in a specimen cannot be increased further, it is said to attain magnetic saturation.

According to molecular theory, the stage of magnetic saturation is reached when all the molecular magnets are arranged in linear order. No more of them are available for further alignment. So, the magnetisation cannot be increased further.

● (iv) **Equality of poles of a magnet :** From experiment it is known that the two free poles of a magnet are of equal strength. It can also be explained by molecular theory.

According to molecular theory, the process of magnetisation merely involves the arrangement of the molecular magnets in linear order. Hence on either side of the neutral region at the middle of the magnet, equal number of free poles of opposite nature will be present. Evidently, the two poles of a magnet are of equal strength.

● (v) **Curie point :** A magnet when heated above a particular temperature, it loses all its magnetism completely. This temperature is called **Curie point**. The existence of curie point can be explained by molecular theory of magnetism.

When a magnet is gradually heated, the vibration of the molecules becomes more and more violent. Consequently the linear arrangement of the molecular magnets is gradually destroyed. At temperature beyond the curie point, the molecular vibrations becomes so violent that the linear arrangement is completely destroyed. Under this condition the moleculer magnets again form closed chains. Thus the magnet loses its magnetism completely.

● (vi) **Magneto-striction :** When a long magnetic substance is suddenly magnetised by a strong electric current, a metallic sound is heard and the length of the rod increases slightly in the direction of magnetisation. The phenomenon is called **magneto striction.**

● **Explanation :** Due to rapid magnetisation, the closed chain of molecular magnets are suddenly broken and the molecules are set parallel to the length of the rod. This process produces a metallic sound. Also due to linear arrangement of the molecular magnets along the direction of magnetisation, it length increases slightly.

○ 3.4. Magnetic field inside a magnetic material ○

We have seen in electrostatics that if a dielectric material is introduced between the plates of a parallel plate capacitor, electric lines of force changes. The reason is : the molecules of the dielectric becomes electric dipoles and the electric field between the plates is modified. The capacitance of the capacitor also increases. Now we shall see whether the magnetic field is also modified when a magnetic material like iron is introduced in the field. For this experiment, we take a solenoid and a steady current is passed through it. The magnetic field intensities inside the solenoid are compared when a magnetic material is not introduced inside the solenoid and when a magnetic material is introduced. It is found from the experiment that when the magnetic material is present the magnetic induction $\vec{B}$ is largely modified. The same experiment with the solenoid can be performed with different magnetic materials.

It is found that when an aluminium bar is introduced inside the solenoid, the magnetic flux increases. But under the same condition, the magnetic flux dicreases when a bismuth bar is introduced. The magnetic flux enormously increases when an iron bar or steel bar is introduced inside the solenoid carrying current. On the basis of influence of magnetic materials on magnetic field, we divide magnetic materials into three groups. These are : (i) diamagnetic, (ii) paramagnetic and (iii) ferromagnetic.

(i) Due to the presence of **diamagnetic** material inside the solinoid, the magnetic induction $\vec{B}$ **decreases slightly**.

(ii) The magnetic induction $\vec{B}$ slightly increases due to the presence of **paramagnetic** materials.

(iii) The magnetic induction $\vec{B}$ inside the solenoid increases to a great extent (about several thousand times) due to the presence **ferro magnetic** materials.

When the solenoid is empty or air- core, the magnetic induction is

$B_0 = \mu_0 nI$ [μ_0 = permeability of air]

Here n is number of turns per unit length and I is the current in the solenoid. When the experiment is repeated with a specimen filling the solenoid, the magnetic induction is $B = \mu n I = B_0 + B_m$

Here μ = permeability of the material and B_m = magnetic induction due to magnetisation of the material.

Hence the magnetic effect of matter is measured by the difference of magnetic induction in the empty solenoid and when the solenoid is filled with a magnetic material.

To understand the difference between three types of magnetic materials quantitatively, let B_0 be the magnetic induction in the solenoid due to a fixed steady current when there is no magnetic material inside the solenoid. Also assume that B is the magnetic induction inside the solenoid filled with a magnetic material.

For diamagnetic materials $\frac{B}{B_0} < 1$

For paramagnetic materials $\frac{B}{B_0} > 1$ and for ferromagnetic materials $\frac{B}{B_0} >> 1$

Since the magnetic behaviour of a material is measured by the ratio $\frac{B}{B_0}$, we write $\frac{B}{B_0} = \mu_r$

$\therefore\ B = \mu_r B_0$

μ_r is called relative permeability of the material. In terms of relative permeability, the different between the three magnetic materials is :

in the case of diamagnetic materials $\mu_r < 1$

for the case of paramagnetic materials $\mu_r > 1$

and for the case of ferromagnetic materials $\mu_r >> 1$

For aluminium, relative permeability $\mu_r = 1 \cdot 000021 = 1 \cdot 000000 + 0 \cdot 000012$ and for bismuth

$\mu_r = 0 \cdot 99483 = 1 \cdot 00000 - 0 \cdot 00017$.

From the above discussion it is apparent that for paramagnetic and for diamagnetic materials relative permeability is slightly greater or less than 1·00000.

To express this difference in magnetic behaviour, a new magnetic quantity is introduced. It is called **magnetic susceptibility,** χ. It is measured by $\chi = \mu_r - 1$

Hence for aluminium $\chi = 0 \cdot 000021$ and for Bismuth, $\chi_r = -0 \cdot 00017$. Both μ_r and χ are dimensionless quantity. In terms of magnetic susceptibility, the difference between the three materials is

For diamagnetic materials χ is negative ($\chi < 0$)

For paramagnetic materials χ is positive ($\chi > 0$)

and for ferromagnetic materials χ is positive and $> > 1$

✪ 3.5. Some quantities to classify different magnetic materials ✪

In this article we define and explain the physical importance of some quantities. Some of these quantities help us to clearly distinguish between different magnetic materials.

- (i) **Lines of magnetisation :** When a magnetic material is placed in a magnetic field it developes magnetism due to magnetic induction process. Now, two types of lines of force passes through the material-lines of force due to applied magnetic field and lines of force due to its own magnetisation [Fig 3.8]. **Lines of force produced due to magnetisation of the material are called lines of magnetisation which are shown by dotted lines. Continuous lines** are the lines of force of the applied field.

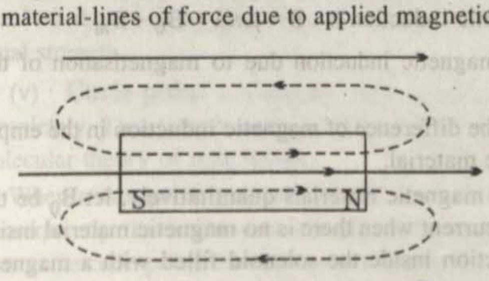


Fig. 3·8

- (ii) **Magnetic lines of induction :** Within the magnetic materials, lines of force due to applied field and the lines of magnetisation are parallel and all are in the same direction [Fig 3.8]. These two lines of force taken together are called **lines of induction**.

- (iii) **Magnetic induction $\vec{B}$:** The magnetic induction at a point inside a magnetic material is defined as the number of lines induction passing normally through a unit area drawn around the point. It is called $\vec{B}$ vector.

It is to be mentioned that magnetising field intensity is called $\vec{H}$ vector. Total magnetic intensity within the magnetic material is magnetic induction vector or $\vec{B}$ vector. $\vec{H}$ vector does not depend on medium. But $\vec{B}$ vector depends on magnetic properties of the medium.

In SI, unit of $\vec{B}$ is Wb / m^2 or Tesla (T). In C. G. S. system the unit is Gauss (G). Relation between the two units is **1T = 10^4 G**

- (iv) **Intensity of magnetisation, I :** When a magnetic material is placed in a magnetic field, it becames a magnet. It developes some magnetic moment. Intensity of magnetisation is defined as magnetic moment developed per unit volume of the material. It is denoted by I

$$\therefore \; I = \frac{\text{magnetic moment, } M}{\text{volume, } V} \quad \therefore \quad I = \frac{M}{V} \; \ldots\ldots\ldots\ldots\ldots (3.3)$$

The magnetic material SN of length $2l$ and being uniform cross-section are placed in a magnetic field [Fig 3.9]. Let its pole strength be m. Then $M = 2\,l.\,m$

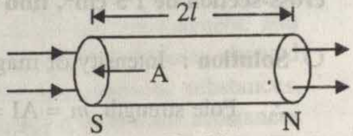


Fig. 3·9

$$\therefore \; I = \frac{2l.m}{2l.A} = \frac{m}{A} \; \ldots\ldots\ldots\ldots\ldots (3.4)$$

So, pole strength per unit area is also called known as intensity of magnetisation.

Its unit in SI is Am^{-1}

- (v) **Magnetising field intensity, H :** Let i be the current in the solenoid and its number of turns per unit length be n then, magnetising field intensity is $H = ni$

$$\therefore \quad B_0 = \mu_0 ni = \mu_0 H$$

Due to magnetisation the magnetic material within the solenoid produces additional magnetic induction $B_m = \mu_0 I$ where I = intensity of magnetisation. Hence total magnetic induction $B = B_0 + B_m = \mu_0 H + \mu_0 I = \mu_0 (H + I)$

- **Both H and I has the same unit Am^{-1}**

- (vi) **Magnetic permeability, μ :** It is a magnetic property of a material which determined how easily lines of force can pass through it. Higher the value of μ, greater number of lines of force with respect to air passes through the material.

Mathematically, magnetic permeability is defined us the ratio of magnetic lines of induction passing normally through a unit area of the material to the lines of force passing normally through unit area in air.

$$\therefore \quad \mu = \frac{\text{lines of induction per unit area, B}}{\text{lines of force per unit area, H}} \quad \therefore \mathbf{B = \mu\, H}$$

If the medium be air, $B_0 = \mu_0 H$

- (viii) **Magnetic Susceptibility, χ :** It is a magnetic property which determines how easily the material can be magnetised. It is defined as the ratio of intensity of magnetisation, I and magnetising field intensity, H. $\therefore \quad \chi = \dfrac{I}{H}$

As χ is the ratio of two similar quantities, it has no unit.

⭘ 3.6. Relation between B, μ and χ ⭘

Let the current in the solenoid is i. When it is empty, the magnetic induction inside the solenoid $B_0 = \mu_0 ni = \mu_0 H$.

If the solenoid is filled up with iron or any other magnetic material, due to magnetisation of the core, additional magnetic induction is produced. Hence total magnetic moment.

$$B = \mu_0 H + \mu_0 I = \mu_0 (H + I)$$

or, $\mu H = \mu_0 (H + I)$

or, $\dfrac{\mu}{\mu_0} = 1 + \dfrac{I}{H} \quad \therefore \ \boldsymbol{\mu_r = 1 + \chi}$

● **Example 3.1. The intensity of magnetisation in a magnetic material is 120 Am^{-1}. It is area of cross-section be 1·5 cm^2, find its pole strength.**

○ **Solution :** Intensity of magnetisation, $I = \dfrac{m}{A}$

$\therefore$ Pole strength, $m = AI = 1{\cdot}5 \times 10^{-4} \times 120 = \mathbf{0{\cdot}018\ Am^2}$

● **Ex. 3.2. The mean radius of a toriodal solenoid is 10 cm and total number of turus 3000. The core of the solenoid is soft iron of relative permeability 2000. If current in the solenoid be 1A. Calculate magnetic induction inside the core.**

○ **Solution :** The required relation to obtain magnetic induction inside the iron core is $B = \mu_0 \mu_r ni$

Here, $\mu_0 = 4\pi \times 10^{-7}\ H/m,\ \mu_r = 2000,\ n = \dfrac{3000}{2\pi r} = \dfrac{3000}{2\pi \times 0{\cdot}1} = \dfrac{1{\cdot}5}{\pi} \times 10^4$ and $i = 1A$

$\therefore \quad B = 4\pi \times 10^{-7} \times 2000 \times \dfrac{1{\cdot}5}{\pi} \times 10^4 \times 1{\cdot}0 \quad \therefore \ \mathbf{B = 12\ T}$

● **Ex. 3.3. A uniform bar of iron of length 50 cm and area of cross-section 20 mm^2 is introduced inside a long solenoid along its axis. The number of turns of the solenoid is 25 cm^{-1} and current 2A. If relative permeability of iron be 400 calculate magnetic moment of the iron bar.**

○ **Solution :** Magnetising field intensity produced by the solinoid is

$$H = ni = 2500 \times 2 = 5000\ A - m^{-1}$$

Now, $\mu_r = 1 + \chi$

$\therefore$ Susceptibility, $\chi = \mu_r - 1 = 400 - 1 = 399$

or, $\dfrac{I}{H} = 199 \quad \Rightarrow I = 399 \times 5000\ A\text{-}m^{-1}$

$n = 2500\ m^{-1}$

$i = 2\ A$

$\mu_r = 400$

$V = 20 \times 10^{-6} \times 0.5$

$= 10^{-5}\ m^3$

Also, $\dfrac{M}{V} = 399 \times 5 \times 10^3 \quad \therefore \ M = 399 \times 5 \times 10^3 \times 10^{-5} = \mathbf{19{\cdot}95\ A\text{-}m^2}$

$\therefore$ Magnetic moment of the iron bar = **19·95 A- m^2**

● **Ex. 3.4. Insulated copper wire is wound on a iron ring and a current of 1.5 A is passed through it. Number of turns per meter is 100. Magnetic induction inside iron 1.0 T. Calculate relative permeability of iron and its magnetic susceptibility.**

○ **Solution :** Magnetising field intensity due to the current in the wire is

$$H = ni = 100 \times 1{\cdot}5 = 150\ A\text{-}m^{-1}$$

Absolute permeability of iron, $\mu = \dfrac{B}{H} = \dfrac{1}{150}\ H - m^{-1}$

$\therefore \quad \mu_r \mu_0 = \dfrac{1}{150} \quad \therefore \ \mu_r = \dfrac{1}{150 \times 4\pi \times 10^{-7}} = \mathbf{5304}$

$\therefore$ Relative permeability, $\chi = \mu_r - 1 = \mathbf{5303}$

- **A discussion :** In all magnetic materials, the magnetic effects can be explained on the basis of behaviour of electrons in the atoms of the material. When an electron moves along a circle round the nucleus, it is effectively a current loop which is equivalent to a tiny magnetic dipole with a certain magnetic moment.

The magnetic moment of an atom may be due to (i) orbital motion of electron and (ii) electron spin. Due to these two factors, the magnetic field inside the magnetic material is modified.

3.7. Paramagnetic, Ferromagnetic and Diamagnetic substances

Primarily, the substances which are attracted by a magnet are termed as magnetic and those which are not attracted by a magnet are non-magnetic. On this basis iron, nickel, cobalt etc are magnetic and aluminium, copper etc are non-magnetic. Experimenting with strong magneting field, Faraday showed that all materials possess magnetic properties. In fact, in the case of so called non-magnetic substances, the magnetic properties are very feeble. It is found that in a non- uniform magnetic field some substances move from weaker to stronger part of the field i.e. they are attracted by a magnet. Also there are some other substances which move from stronger to weaker part of the field i.e. they are repelled by a magnet.

On the basis of the behaviour in a non-uniform magnetic field, magnetic materials are mainly divided into two groups. (i) Paramagnetic and (ii) Diamagnetic.

Some of the paramagnetic substances like iron, nickel, cobalt are highly magnetic and they are strongly attracted by a magnet. They are separately grouped as ferro-magnetic.

(i) Paramagnetic substances :

These substances are feebly attracted by a magnet and also feebly magnetised in the direction of the magnetising field. For these materials $\mu_r > 1$ and susceptibility $\chi > 0$. Their magnetic permeability and susceptibility are independent of magnitude of the applied field.

- **Example :** Aluminium, sodium, platinum, oxygen, copper sulphate, ferric chloride etc.

The properties of paramagnetic substances are :

(a) When a thin rod of a paramagnetic substance is suspended between the poles of a powerful horse shoe magnet, the rod remains parallel to the magnetic field as shown in fig 3.10 (a). The rod is also feebly magnetised with opposite poles facing N- S. A paramagnetic liquid is taken in a U-tube. Initially the liquid in the two arms of the tube remains at the same height. Now a strong magnetic field is established across the arm of the U- tube. It will be found that liquid level in that arm rises a little [Fig 3.10 (b)].

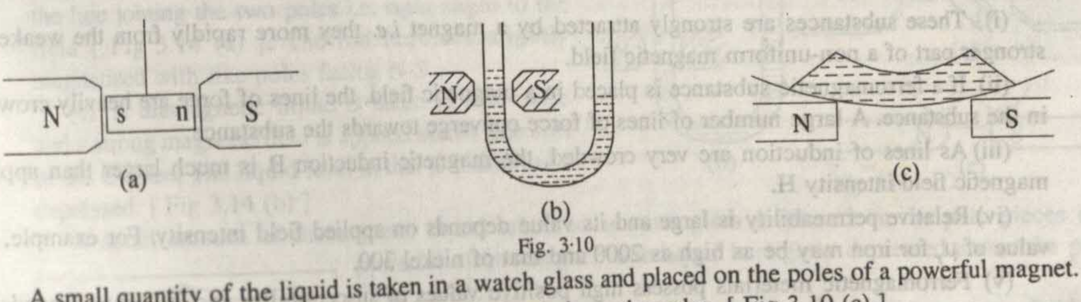


Fig. 3·10

A small quantity of the liquid is taken in a watch glass and placed on the poles of a powerful magnet. It will be observed that the liquid is slightly elevated above the poles [Fig 3.10 (c)]

(b) When a paramagnetic substance is placed in a magnetic field, the lines of force show a slightly greater tendency to crowed towards it. There is a greater concentration of lines of force through the material of the specimen. This is illustrated in fig 3.11 which shows the distribution of lines of force when a sphere of a paramagnetic substance is placed in an originally uniform magnetic field.

(c) Consequently the value of B is greater that the magnetising field H. Since $B = \mu_0 \mu_r H$, it shows for a paramagnetic substance $\mu_r > 1$. For example for aluminium and platinum the values of μ_r are 1.000022 and 1.000036 respectively.

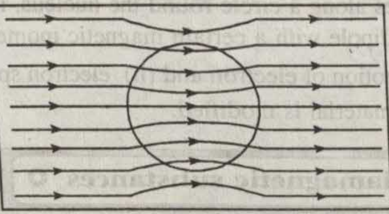


Fig. 3·11

(d) From the relation $\mu_r = 1 + \chi$, we see that magnetic susceptibility χ is positive and of small value. For example susceptibility of aluminium is $1{\cdot}76 \times 10^{-6}$ and that of platinum is $2{\cdot}88 \times 10^{-6}$.

(e) The susceptibility χ of paramagnetic substance is independent of magnetic field, but decreases with rise of temperature. It is found that paramagnetic susceptibility is inversely proportional to the absolute temperature of the substance. So,

$\chi = \frac{C}{T}$ where C is called Curie constant. It is known as **Curie's law**. This law holds only it the case of paramagnetic gases. For paramagnetic solids, however, the susceptibility obeys **Curie-Weiss law** which gives $\chi = \frac{C}{T - \theta}$ [θ is called Curie temperature]

- **A discussion :** Atoms of paramagnetic materials, in the normal state possess intrinsic non-zero magnetic moment, even in the absence of applied magnetic field. The atoms of a paramagnetic substance possesses magnetic moment due to orbital motion of the unpaired electron in the atom or due to spin motion of the electrons. The interaction between the atomic magnetic dipoles of the paramagnetic substance is very weak and so they may be regarded as independent of each other. Due to thermal motion, the atomic magnetic dipoles (also called weber elements) are randomly oriented. But when a strong magnetic fixed is applied the weber elements align in the direction of the applied field. The specimen becomes feebly magnetised.

For Para magneitc substance, $I \propto H$ and $I \propto \frac{I}{T}$ Where T is the absolute temperature of the specimen.

(ii) Ferro-magnetic substances :

Ferro-magnetism is a part of paramagnetism. So ferro-magnetic substances belong to the group of paramagnetic substances but are kept separate as they show extremely strong paramagnetic effects. Iron, nickel, cobalt, steel and some alloys are examples of ferro-magnetic substances. For ferromagnetic substances $\mu_r >> 1$ and susceptibility is positive and of high value. The general properties of these substances are :

(i) These substances are strongly attracted by a magnet *i.e.* they more rapidly from the weaker to stronger part of a non-uniform magnetic field.

(ii) If a ferromagnetic substance is placed in a magnetic field, the lines of force are heavily crowded in the substance. A large number of lines of force converge towards the substance.

(iii) As lines of induction are very crowded, the magnetic induction B is much larger than applied magnetic field intensity H.

(iv) Relative permeability is large and its value depends on applied field intensity. For example, the value of μ_r for iron may be as high as 2000 and that of nickel 300.

(v) Ferromagnetic meterials possess high positive values of susceptibility and its value depends on magnetising field.

(vi) If a ferromagnetic substance is gradually heated, it steadily loses its strong magnetic properties. At a critical temperature called curie point a ferromagnetic substance becomes paramagnetic curie point of iron is 785°C and cobalt it is 1100°C.

(vi) Ferromagnetic substances are solid crystalline. Liquid and gases do not exhibit ferro-magnetism.

- **A discussion :** Like paramagnetic substances, the atom of a ferromagnetic substance also possesses non-zero magnetic dipole moment. However, in such materials there are strong interactions between the individual magnetic moments of various atoms. Here an unpaired electron in one atom interact strongly with a unpaired electron of the next atom so that they align in common direction over a small volume. These small volumes of uniform magnetisation an called **domains**. Domain is a very small region ($\simeq 10^{-18}\, m^3$) but it contains about 10^{11} atomic dipoles [Fig. 3.12].

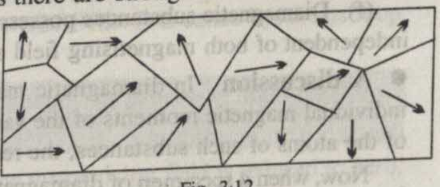
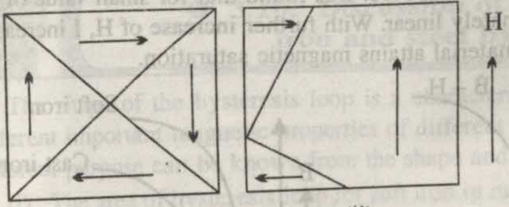


Fig. 3·12

The magnetic moments of the atoms in one domain are parallel to each other. So, a domain possesses a small net dipole moment. In the absence of external magnetic field the directions of magnetic moments in different domains are oriented randomly in different directions [3.12]. One domain cancels the effect of the other, so that the net magnetic moment of the material is zero. Evidently a ferro magnetic material appears unmagnetised in absence of an external magnetic field.

When a magnetic field is applied two types of changes occur within the specimen : (i) Different domains rotates as a whole and becomes parallel to the applied field and (ii) the domains whose axes are parallel to the field gradually grow in size and others gradually shrinks in size. In fig 3.13 shows the change in size of the domains parallel to the field, under the influence of the external field these two processes occur simultaneously and magnetisation of the material increases rapidly. Ultimate the whole of the substance turns into large domain whose axis remains parallel to the applied field.



(i) Fig. 3·13 (ii)

(iii) Diamagnetic substances :

These magnetic substances are repelled by a magnet *i.e.* they more from stronger to weaker part of the field. For these substances $\mu_r < 1$ and magnetic susceptibility is negative.

- **Examples :** Copper, Bismuth, antimony, gold, mercury, water, hydrogen etc.

The general properties of diamagnetic materials are (a) They are feebly repelled by a magnet. (b) When a thin and small rod of a solid diamagnetic substance is suspended between the poles of a powerful horse shoe magnet the rod will remain perpendicular to the line joining the two poles i.e. right angle to the field [Fig 3.14 (a)]. The rod becomes slightly magnetised with like poles facing N-S.

(c) A diamagnetic liquid is taken in a U- tube and a strong magnetic field is applied across one arm of the U- tube. The liquid level in that is found to be depressed. [Fig 3.14 (b)]

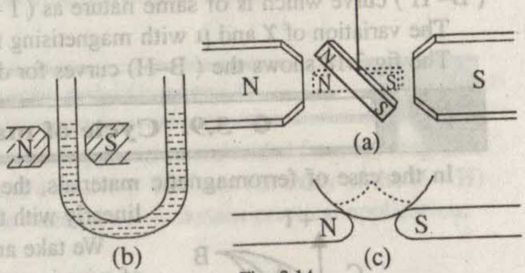


Fig. 3·14

A small amount of diamagnetic liquid taken on a water glass is placed on the pole pieces of a powerful electromagnet. The liquid is slightly depressed above the poles [Fig 3.14 (c)].

(d) When a diamagnetic substance is placed in a magnetic field, the lines of force tend to move away from the region, occupied by the substance.

This is shown in fig 3.15 where a sphere made of diamagnetic method is placed in an originally uniform magnetic field. Evidently, lines of force are less crowded inside the diamagnetic material compared to air.

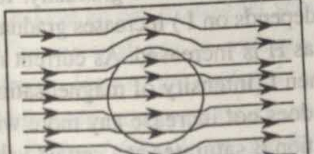


Fig. 3·15

(e) Hence the magnetic induction is a diamagnetic substance is slightly less than the applied field intensity H. So, relative permeability $\mu_r < 1$; for example, for bismuth $\mu_r = 0{\cdot}9999$.

(f) Diamagnetic substances possess small but negative susceptibility for bismuth $\chi = -0{\cdot}00015$. It is independent of both magnetising field as well as temperature.

- **A discussion** : In diamagnetic materials, there are no unpaired electrons. In such a material, the individual magnetic moments of the various atoms tend to cancel out completely. So, in the normal state of the atoms of such substances, the resultant magnetic dipole moment is zero.

Now, when a specimen of diamagnetic material is placed in a magnetic field B, a net magnetic moment is induced in it which is proportional to B but oppositely directed. The diamagnetic substance thus gets feebly magnetised along a direction opposite to that of the applied field.

❍ 3.8. I–H and B–H curves of a magnetic material ❍

A long solenoid carrying current *i* is taken. It produces magnetising field H = *ni* when *n* is the number of turns per unit length. The magnetic material placed inside the solenoid acquires magnetism. For different value of H, intensity of magnetisation can be determined. A graph is drawn with H plotted along X-axis and I along Y- axis. The nature of I - H graph is shown in fig 3.16. It is found that for small value of H, I is proportional to H. So the portion OA is approximately linear. With further increase of H, I increases slowly and ultimately I becomes constant. Now the material attains magnetic saturation.

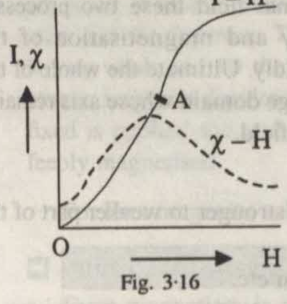


Fig. 3·16

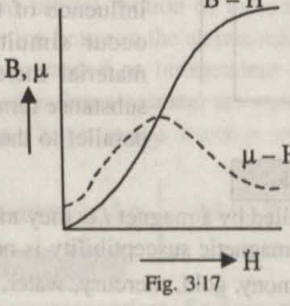


Fig. 3·17

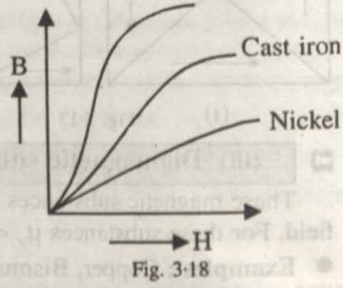


Fig. 3·18

Now, $B = \mu_0 H (1 + \chi) = \mu_0 (H + I)$. So, (B – H) curve can also be drawn Fig 3.17 shown the (B– H) curve which is of same nature as (I – H) graph.

The variation of χ and μ with magnetising field have been also shown. Here $\chi = I / H$ and $\mu = B / H$. The fig 3.18 shows the (B–H) curves for different materials.

❍ 3.9. Cycle of magnetisation : Hysteresis ❍

In the case of ferromagnetic materials, the magnetic induction B of the material does not increase linearly with the magnetising field H. The relation is quite complicated.

We take an unmagnetised soft iron rod and a large number of turns of wire is wound over it. It forms a straight solenoid with a soft iron core. Now a current is passed through the solenoid. A magnetising field H = *n i* is produced. The value of current is increased gradually. It is found that the magnetic field B (which depends on I) increases gradually as H is increased. B increases rapidly as H is increased. As current *i* is increased further, a stage is reached when I (intensity of magnetisation) saturates, *i.e.*, it becomes constant and does not increase any more with increase in H. Evidently magnetic induction B saturates at a certain value of H [Fig 3.19].

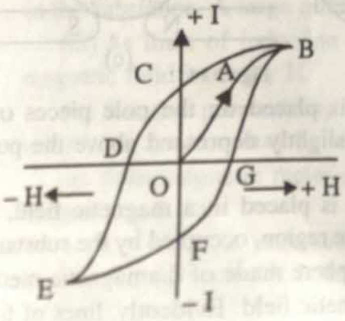


Fig. 3·19

Now H is decreased slowly to zero by gradually decreasing the current in the solenoid. It is found that some magnetisation I and also induction B is retained in iron. B does not decrease to zero. In one complete cycle of change of H, the magnetic induction follows a closed loop BDEFGB. It is called hysteresis loop. It is evident that magnetic induction B always lags behind H.

The lag of magnetic induction behind the magnetising field during the process of magnetisation of a ferromagnetic material is called hysteresis.

The value of intensity of magnetisation of the material, when the magnetising field is reduced to zero, is called **retentivity** or residual magnetism of the material. From the diagram OC gives the measure of retentivity or residual magnetism.

Now, to reduce residual magnetism to zero, the magnetising field has to be increased in reverse direction. When the reverse magnetising field is OD, the residual magnetism is completely destroyed. The demagnetising magnetic field gives the measure of **coercivity** of the material of the specimen.

The area of the hysteresis loop gives the measure of energy loss per unit volume per cycle of magnetisation.

3.10. Discussion of magnetic properties of iron and steel by hysteresis loop

The shape of the hysteresis loop is a characteristic of a ferro magnetic material. We can compare different important magnetic properties of different materials. The suitability of a magnetic material for different purpose can be known from the shape and area of the loop.

(i) The area of hysteresis loop for soft iron in much smaller than that for steel. It implies that the loss of energy per unit volume in the case of soft iron is smaller than in the case of steel, when they are taken over a complete cycle of magnetisation and demagnetisation.

(ii) Soft iron is easily magnetised. It acquires maximum intensity of magnetisation for comparatively much smaller value of magnetising field than in the case of steel. So soft iron is more susceptible to magnetisation than steel.

(iii) The retentivity of soft iron is higher than in the case of steel.

(iv) The coercivity of steel is much larger than that of soft iron. So residual magnetism of steel can not be easily destroyed.

3.11. Uses of ferromagnetic substances

The choice of a particular ferromagnetic material for a practical application is decided from the magnetic properties such as area of the hysteresis loop, retentivity and coercivity. So from the study of the (I – H) or (B – H) loop we can choose a particular ferro-magnetic material for a certain practical application.

- **(i) Construction of electromagnet :**

Electromagnet is a temporary magnet and its strength is required to be adjusted for different purposes. These magnets can be turned on and off by switching the current on and off. Evidently, the material used for making an electromagnet should possess high **retentivity** and **low coercivity**.

The material has to undergo magnetisation and demagnetisation again and again. The area of the hysteresis loop of the material used for construction of electromagnet should be small.

All these requirements are best fulfilled by soft iron. So soft iron is used for making electromagnet. Also Stalloy (5% silicon and 95% iron) and permalloy (50% iron and 50% nickel) are also suitable.

Electromagnets are used in electric bells, relays, telephones etc.

● (ii) Transformer core :

A transformer is an alternating current device. When alternating current is passed through a transformer, its core undergoes a large number of cycles of magnetisation and demagnetisation in a second. For example— if the supply frequency be 50 Hz, the core of the transformer will be taken through cycles of magnetisation 50 times in a second. Now, during each cycle, energy proportional to the area of the hysteresis loop for the core is wasted in the form of heat. If the area of the loop is large, greater amount of heat will be produced, temperature rise of the core will be high which is not desirable. So, for the construction of the core of the transformer, the material be so chosen that its hysteresis loop be narrow.

On account of this, soft iron is used to make the core of the transformer. Mumetal (76% nickel, 5% copper, 17% iron and 2% chromium) can also be used for making transformer core because mumetal possesses a narrow hysteresis loop.

● (iii) Permanent magnets :

Permanent magnets can retain their magnetism for a long time. So for the construction of permanent magnets the materials should possess the following special properties : (i) high retentivity (ii) high coercivity (iii) with rise of temperature, strength of magnetism should not change appreciably.

Soft iron has high retentivity, but its coercivity is low. So it is unsuitable for making permanent magnets. Steel possesses fairly larger value of retentivity and a high value of coercivity. The magnetisation of steel is not easily destroyed, The area of hysteresis loop is large for steel; but it is not a consideration for making permanent magnets. The reason is : a permanent magnet is never taken through cycle of magnetisation.

The suitable choice for making permanent magnet is cobalt steel (52% iron, 36% cobalt, 7% tungsten, 3.5 % chromium, 0.5% manganese and 0.7% carbon). The alloy alnico (55% iron, 10% aluminium, 17% nickel, 12% cobalt and 6% copper) is also very suitable for making permanent magnets. Alnico has the disadvantage that it is brittle.

The following table shows the values of μ_r and χ for a number of magnetic materials.

Material	μ_r	χ	Material	μ_r	χ
Vacuum	1·0000	0	Copper	0·99999904	–0·0000096
Air	1·00000	0·0000004	Iron	5000	4999
Aluminium	1·000021	0·000021	Nickel	600	599

● ● Short Answer Type Questions (with answers) ● ●

● **Q. 1. A ferromagnetic material in the form of a uniform bar is placed in a uniform magnetic field. Draw the lines of force, lines of magnetisation and lines of induction.**

○ **Ans.** As permeability of ferromagnetic material is extremely large, the lines of force are crowded in the material. It is shown in fig 3.20 south pole is induced near the end through lines of force are entering and at the other end north pole is induced. The lines of magnetisation is produced due to the magnetisation of the material itself. External lines of force and lines of magnetisation together are called lines of induction within the material.

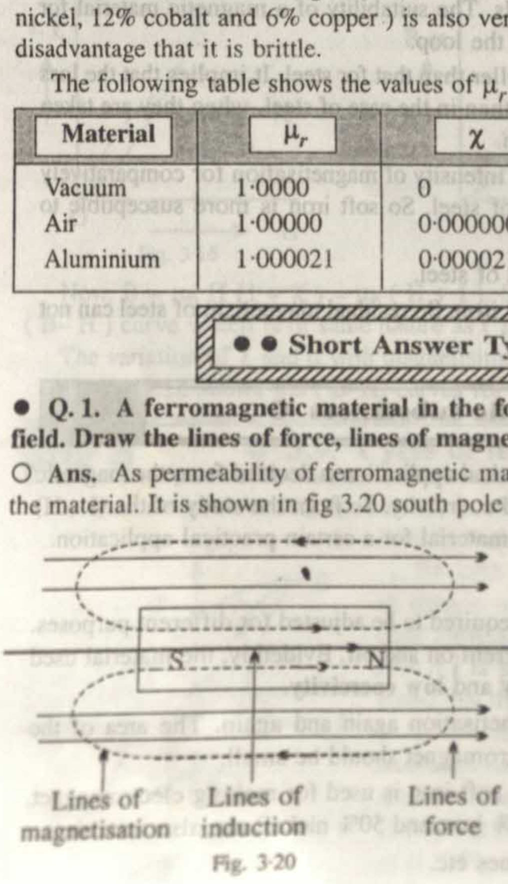


Fig. 3·20

● **Q. 2. What are the physical meaning of permeability and susceptibility ?**

○ **Ans.** Permeability is a magnetic property which determines the ability of material to conduct magnetic lines of force. Line of force are dense in the material having high value of permeability.

Susceptibility is another magnetic property which determines how easily a material can be magnetised. Soft iron is easily magnetised that steel. So for soft iron susceptibility is high.

- **Q. 3. What is the fundamental difference between the atoms of a paramagnetic and a ferro magnetic substance ?** **[J. E. E.]**

○ **Ans.** The fundamental difference between the atoms of a paramagnetic and a ferromagnetic substance is : In the case of paramagnetic substance atomic dipole moment is very feeble. But in the case of ferromagnetic substance, the outermost shell of the atom is not full and atomic dipole moment is high.

- **Q. 4. When is susceptibility is taken as positive and when it is negative ?**

○ **Ans.** Magnetic susceptibility, $\chi = \dfrac{\text{Intensity of magnetisation, I}}{\text{Magnetising field intensity, H}}$

If I and H are in the same direction, then χ is taken to be positive. For para - and ferromagnetic materials χ is positive.

On the other hand, when I and H are in opposite direction, χ is taken to be negative. It is the case for diamagnetic substance.

- **Q. 5. What is the nature of (B- H) curve for air ?**

○ **Ans.** Air is not a ferromagnetic material. In the case of air B α H. Hence its (B– H) curve will be straight line.

- **Q. 6. Mention c. g. s and SI units of magnetising field, H, magnetic induction, B and negative flux, φ. Also give the relation between the units.**

○ **Ans.**

Quantity	c. g. s. unit	SI units	relation
Magnetic induction, B	Gauss (G)	Tesla 1	$T = 10^4$ G
Magnetic field intensity, H	Oersted	Am^{-1}	$1\ Am^{-1} = 4\pi \times 10^{-3}$ Oe
Magnetic flux, φ	Maxwell	Weber	$1\ \text{Wb} = 10^8$ Maxwell

- **Q. 7. What is the importance of (B - H) loop of a ferro-magnetic meterial ?**

○ **Ans.** (B- H) loop is obtained only in the case of ferromagnetic material. The area of the (B- H) loop gives the measurement of energy loss per unit volume when the material is taken through one complete cycle magnetisation and demagnetisation.

- **Q. 8. Why the magnetic properly of a diamagnetic material does not depend on temperature of the material ?**

○ **Ans.** In a diamagnetic atom, there is no unpaired electron. Resultant magnetic dipole moment is zero. So it does not possess permanent magnetic moment. When a magnetic field is applied to the material, it developes small induced magnetism. No temperature has no effect on this induced magnetism. Hence magnetic properties of diamagnetic material does not depend on temperature.

- **Q. 9. Which atom or ion does not show paramagnetism ? Give example.**

○ **Ans.** The atom or ion in which all the shells are completely full, does not show paramagnetism. Example : Na^+, He, Ne

• • Numerical Examples • •

- **Ex. 3.5. An unmagnetised iron rod of volume 20 c.c. is placed inside a solenoid along its axis. Its length is 10 cm, number of turns 50 and it carries a current of 1.0 A. The magnetic moment of iron bar is 4500 cgs. Calculate :**

(i) Intensity of the magnetising field (ii) Magnetic induction before the introduction of iron bar (iii) Magnetic induction inside the iron, $\mu_0 = 4\pi \times 10^{-7}$ Hm^{-1}

○ **Solution :** (i) Magnetising field intensity, $H = ni$

$$\therefore \quad H = \frac{50}{0\cdot 10} \times 1\cdot 0 = 500 \text{ A}m^{-1}$$

(ii) Magnetic induction inside the solenoid when empty is

$$B_0 = \mu_0 H = 4\pi \times 10^{-7} \times 500 = 6\cdot 28 \times 10^{-4}\text{T}$$

(iii) Magnetic induction inside iron, $B = \mu_0(H + I)$

$$\text{Here } I = \frac{M}{V} = \frac{400 \times 10^{-3}}{20 \times 10^{-6}} = 225 \times 10^3 \text{ A}m^{-1}$$

$$\therefore \quad B = 4\pi \times 10^{-7} \times (500 + 225 \times 10^3) = 4\pi \times 10^{-4}(0\cdot 5 + 225)$$

$$= 2832\cdot 3 \times 10^{-4}\text{ T}$$

● **Ex. 3.6. A soft iron ring is of radius 10 cm and area of cross-section 5 cm^2 and its total number of turns 2000. If a current of 2A is passed through the wire, magnetic flux inside iron 8·0 × 10^{-3} Wb. What is the relative permeability of iron ?**

○ **Solution :** Magnetic flux, $\varphi = BA$ and $B = \mu_0 \mu_r H$

$$\therefore \quad B = \frac{\varphi}{A} = \frac{8 \times 10^{-3}}{5 \times 10^{-4}} = 16\text{T} \quad \therefore \quad \mu_0 \mu_r ni = 16$$

$$\mu_r = \frac{16}{\mu_0 ni} = \frac{16\pi}{4\pi \times 10^{-7} \times 10^4 \times 2} = 2000$$

$\therefore$ Relative permeability, μ_r = **2000**

$$\mu_0 = 4\pi \times 10^{-7} \text{ H}/m$$

$$n = \frac{2000}{2\pi \times 10 \times 10^{-2}} = \frac{1}{\pi} \times 10^4$$

$$i = 2\cdot 0 \text{ A}$$

● **Ex. 3.7. The magnetic moment of a magnetised steel rod is 3·6 A-m^2 and its mass 7·2 gm. If the density of steel be 8 × 10^3 kg / m^3, calculate intensity of magnetisation of the magnet.**

○ **Solution :** Volume of the magnet, $V = \frac{m}{\rho} = \frac{7\cdot 2 \times 10^{-3}}{8 \times 10^3} = 0\cdot 9 \times 10^{-6} m^3$

and its magnetic moment, M = 3·6 A- m^2.

Intensity of magnetisation, $I = \frac{M}{V} = \frac{3\cdot 6}{0\cdot 9 \times 10^{-6}} = \mathbf{4 \times 10^6 \text{ A - m}^{-1}}$

● **Ex. 3.8. An iron bar of length 0·1 m is placed with its length parallel to the field intensity 10Am^{-1}. The magnetic induction at a point on the axis of the magnet at a distance of 0.3m from the centre of the magnet is 0·25 × 10^{-6} T. If the area of cross-section of the bar be 0·3 × 10^{-4} m^2, calculate susceptibility of iron.**

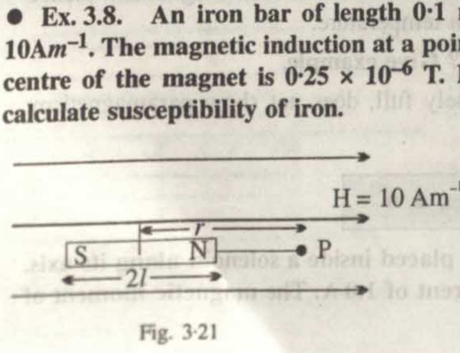


Fig. 3·21

○ **Solution :** The arrangement is shown in fig 3.21. Intensity of the magnetising field H = 10 Am^{-1}. P is a point at a distance of r = 0. 0·3 m from the centre of the magnet. Now, magnetic induction at B is

$$B = 10^{-7} \times \frac{2Mr}{(r^2 - l^2)^2}$$

$$\therefore \quad M = \frac{B(r^2 - l^2)^2}{2r} \times 10^7 = \frac{0\cdot25 \times 10^{-6} \times \left[(0\cdot3)^2 - (0\cdot05)^2\right]^2 \times 10^7}{2 \times 0\cdot3}$$

$$= \frac{2\cdot5}{6} \times 10 \times 76\cdot56 \times 10^{-4}$$

$$= 31\cdot9 \times 10^{-3}\,\text{A}m^2$$

$B = 0\cdot25 \times 10^{-6}\,\text{T}$

$r = 0\cdot3\ m$

$l = 0\cdot05\ m$

$\mu_0 = 4\pi \times 10^{-7}\,\text{H}.m^{-1}$

$$\therefore \quad \text{Susceptibility,} \quad \chi = \frac{I}{H} = \frac{M}{VH} \quad [V = 2lA]$$

$$\chi = \frac{31\cdot9 \times 10^{-3}}{0\cdot3 \times 10^{-4} \times 0\cdot1 \times 10} = \mathbf{1063\cdot3}$$

$A = 0\cdot3 \times 10^{-4}\,m^2$

$l = 0\cdot1\ m$

$H = 10\ \text{A}m^{-1}$

● **Ex. 3.9. When an iron bar is placed in a magnetic field of intensity 20 cgs and magnetic flux inside iron is 2400 cgs. If the area of cross-section of the bar is 0·2 cm^2, find the values of magnetic permeability and susceptibility of iron in SI unit.**

○ **Solution :** In this care, $H = \frac{20}{4\pi} \times 10^3 = \frac{5}{\pi} \times 10^3\,\text{A}m^{-1}$

$$\therefore \quad \varphi = 2400 \times 10^{-8} = 2\cdot4 \times 10^{-5}\,\text{Wb} \quad \text{and} \quad A = 0\cdot2 \times 10^{-4}\,m^2$$

$$\therefore \quad B = \frac{\varphi}{A} = \frac{2\cdot4 \times 10^{-5}}{0\cdot2 \times 10^{-4}} = 1\cdot2\,\text{T}$$

$$\therefore \quad \text{magnetic permeability,} \quad \mu = \frac{B}{H} = \frac{1\cdot2 \times \pi}{5 \times 10^3} = 7\cdot54 \times 10^4\,\text{H}/m$$

Now, $B = \mu_0(H + I)$ or, $\mu = \mu_0(I + \chi)$

$$\therefore \quad \chi = \frac{\mu}{\mu_0} - 1 = \frac{7\cdot54 \times 10^{-4}}{4\pi \times 10^{-7}} - 1 = 600 - 1 = 599$$

● **Ex. 3.10. The magnetic susceptibility of a medium is 9·48 × 10^{-9}. Calculate absolute permeability and relative permeability of the medium.** $\mu_0 = 4\pi \times 10^{-7}\,\text{H}m^{-1}$

○ **Solution :** $\chi = 9\cdot48 \times 10^{-9}$

$$\therefore \quad \text{Absolute permeability,} \quad \mu = \mu_0(1 + \chi)$$

$$\therefore \quad \mu = 4\pi \times 10^{-7}(1 + 9\cdot48 \times 10^{-9}) = 4\pi \times 10^{-7}\,\text{H}/m$$

$$\therefore \quad \text{Relative permability,} \quad \mu_r = \frac{\mu}{\mu_0} = 1$$

● **Ex. 3.11. Mean length of an iron ring 30 cm and area of cross-section 1 cm^2 and total number of turns 300. When a current of 0.032 A is passed a magnetic flux of 2 × 10^{-6} Wb is formed in iron. Calculate :**

(i) Flux density in iron. (ii) Absolute permeability of iron. (iii) Relative permeability of iron (iv) Susceptibility of iron. (v) Magnetised field intensity.

○ **Solution :** No. of turns per unit length, $n = \frac{300}{30 \times 10^{-2}} = 10^3\,m^{-1}$

Area of cross-section, $A = 10^{-4}\,m^2$, $i = 0\cdot032\,\text{A}$,

Magnetic flux, $\varphi = 2 \times 10^{-6}\,\text{Wb}$.

(i) Magnetic flux density in iron, $B = \dfrac{\varphi}{A} = \dfrac{2 \times 10^{-6}}{10^{-4}} = 0 \cdot 02\ \text{T}$

(ii) $B = \mu ni$ when μ = absolute permeability of iron

$\therefore \quad \mu = \dfrac{B}{ni} = \dfrac{0 \cdot 02}{10^3 \times 0 \cdot 032} = 6250 \times 10^{-7}\ \text{H}m^{-1}$

(iii) Relative permeability of iron, $\mu_r = \dfrac{\mu}{\mu_0} = \dfrac{6250 \times 10^{-7}}{4\pi \times 10^{-7}} = \mathbf{497}$

(iv) Magnetic susceptibility, $\chi = \mu_r - 1 = \mathbf{496}$

(v) Magnetising field intensity, $H = \dfrac{B}{\mu} = \dfrac{0 \cdot 02}{6250 \times 10^{-7}} = \mathbf{32\ A}\boldsymbol{m^{-1}}$

- **Ex. 3.12. Magnetic permeability of mumetal is 0·12 TA^{-1} *m*. Find the value of its relative permeability and susceptibility.**

○ **Solution :** $\mu = 0 \cdot 12\ \text{TA}^{-1}m$ and $\mu_0 = 4\pi \times 10^{-7}\ \text{TA}^{-1}m$

$\therefore$ Relative permeability, $\mu_r = \dfrac{\mu}{\mu_0} = \dfrac{0 \cdot 12}{4\pi \times 10^{-7}} = 9 \cdot 59 \times 10^4$

$\therefore$ Susceptibility, $\chi = \mu_r - 1 = 9 \cdot 59 \times 10^4$ (Approx)

- **Ex. 3.13. Magnetic susceptibility of magnesium at 300 K is 1·2 × 10^{-5}. At what temperature its value becomes 1·44 × 10^{-5} ?**

○ **Solution :** From Curie's law : $\chi \propto \dfrac{1}{T} \quad \therefore \quad \dfrac{\chi_1}{\chi_2} = \dfrac{T_2}{T_1}$

$\therefore \quad T_2 = \dfrac{T_1\chi_1}{\chi_2} = \dfrac{300 \times 1 \cdot 2 \times 10^{-5}}{1 \cdot 44 \times 10^{-5}} = 250\ \text{K}$

- **Ex. 3.14. The average radius of a metallic ring 15 cm and number of turns 3500. Relative permeability of the metal is 800. If a current of 1.2 A is passed, calculate magnetic flux density within the metal.**

○ **Solution :** The required relation, $B = \mu_0\mu_r ni$

Here $\mu_0 = 4\pi \times 10^{-7}\ \text{H}.m^{-1}$; $\mu_r = 800$, $n = \dfrac{3500}{2\pi r} = \dfrac{3500}{2\pi \times 0 \cdot 15} = 3715\ m^{-1}$ and $i = 1 \cdot 2$ A

$\therefore \quad B = 4\pi \times 10^{-7} \times 800 \times 3715 \times 1 \cdot 2 = \mathbf{4 \cdot 48\ T}$

- **Ex. 3.15. The pole strength of a bar magnet is 4·5 A- *m*, its magnetic length 12 cm and area of cross-section 0·9 cm^2. Calculate**

(i) Intensity of magnetisation, I (b) Magnetic moment, M

○ **Solution :** (i) Intensity of magnetisation, $I = \dfrac{m}{A} = \dfrac{4 \cdot 5}{0 \cdot 9 \times 10^{-4}} = 5 \times 10^4\ \text{A} - m^{-1}$

It is directed from south pole to north pole.

(ii) Magnetic moment, $M = 2lm = 0 \cdot 12 \times 4 \cdot 5\ \text{A}m^2 \quad [\,2l = 0 \cdot 12\ m\,]$

$\therefore \quad \mathbf{M = 0 \cdot 54\ m^2}$

• EXERCISE •

A. Objective type questions :

Fill up the blanks :

1. Each molecule of a magnetic meterial is ——— .
2. The moleculer magnets in a magnetic material are ——— ——— .
3. According to Ewing in an unmagnetised material the molecular magnets form ——— of poles.
4. For diamagnetic material susceptibility is ——— and paramagnetic material it is ——— .
5. In SI, the unit of intensity of magnetisation is ——— .
6. Aluminium is a ——— material.
7. When a ferromagnetic material is heated above its curie point it becomes ——— .

B. Very short answer type questions :

1. How do the molecular magnets remain inside a magnetic material ?
2. What is meant by intensity of magnetisation ?
3. Does the paramagnetic substances possess curie point ?
4. Does the susceptibility of diamagnetic substances depend on temperature ?
5. Which materials shows the property of hysteresis ?
6. Can a ferromagnetic material be a liquid ?
7. What is the importance of (B – H) loop of a ferromagnetic substance ?
8. What type of material is repelled by magnet ?

C. Short answer type questions :

1. What are molecular magnets ? Why are they called Weber elements ?
2. What is the basic difference between Weber's theory and Ewing's theory ?
3. What is Curie point ? Does paramagnetic materials possess Curie point ?
4. Why does a magnet loses magnetism completely when it is heated above Curie point ?
5. Why the poles of a bar magnet is situated near the ends and not exactly at the end ?
6. What is meant by magnetostriction ?
7. What is the measurement of dipole moment of an electric orbit in an atom ? What is its direction ?
8. How domains are formed in ferromagnetic materials ?
9. Give two examples for ferro-, Para- and diamagnetic materials.
10. Give the physical meaning of permeability and susceptibility.
11. Define magnetic permeability and susceptibility. How are they related ?
12. What is the difference between $\vec{B}$ and $\vec{H}$ vectors ? Mention their SI units ?
13. When is magnetic susceptibility is taken as positive and when negative ?
14. What is meant by intensity of magnetisation ? For which material it is negative ?
15. What is cycle of magnetisation ? What is the importance of (B – H) loop ?
16. Show (B – H) loop for soft iron and steel on the same diagram.
17. What we mean by retentivity and coercivity ?
18. What is hysteresis ? What type of material shows hysteresis ?
19. Which atoms or ions show paramagnetism ?
20. How can you convert a ferromagnetic material to paramagnetic one ?

D. Essay type questions :

1. (a) Why it is not possible to isolate a single pole ?
   (b) Discuss molecular theory of magnetism.
2. How can you explain the following phenomena by molecular theory of magnetism ?
   (a) magnetic induction. (b) magnetic saturation. (c) Curie point.
3. Define the following magnetic quantities.
   (a) magnetic permeability, (b) magnetic susceptibility, (c) retentivity (d) coercivity
4. What is the difference between paramagnetic and diamagnetic ? What is the primary difference between the atoms of paramagnetic and ferromagnetic materials ?

E. Simple numerical problems :

1. The measurement of a magnet is 10 cm × 2 cm × 1 cm. If its magnetic moment is $1{\cdot}0\ Am^2$, find its intensity of magnetisation ? [**Ans.** $5 \times 10^4\ Am^{-1}$]
2. A steel rod of measurement 23 cm × 1·2 cm × 0·5 cm is pland along a magnetic field of intensity 7·5 emu. If the relative permeability of steel be 640, what is the induced magnetic moment of the rod. [**Ans :** $5{\cdot}266\ \mathrm{Am}^2$] **[J. E. E. '91]**
3. The length of an iron rod is 50cm and area of cross-section 20 mm^2. The rod is kept inside a long solenoid along its length. The number of turns of the solenoid 25 / cm and current is 2A. If relative permeability of iron be 400, what is the magnetic moment of the magnet ? [**Ans.** $19{\cdot}95\ \mathrm{Am}^2$]
4. An iron rod of area of cross-section $4\mathrm{cm}^2$ is put parallel to a magnetic field intensity $1600\mathrm{Am}^{-1}$. The magnetic flux within iron is 4×10^{-4} Wb. Calculate absolute permeability of iron. If $\mu_0 = 4\pi \times 10^{-7}\ \mathrm{H}m^{-1}$ also calculate relative permeability of iron. [**Ans.** $6{\cdot}25 \times 10^{-4}\ \mathrm{H.m}^{-1}$; 500]
5. An iron rod is magnetised by placing it in a magnetising field of intensity 500 Am^{-1}. If magnetic induction inside iron be 0·2T, calculate relative permeability and susceptibility of iron. [**Ans.** 318·5; 317·5]
6. The pole strength of a magnet is '*m*' and area of cross-section '*a*', what is the intensity of magnetisation of the material ? [**Ans.** m / a]
7. The radius of a toroid is 25 cm and total number of turns 1000. If a current of 2A is passes through it, calculate magnetic induction inside the toroid ? If the toroid is filled with iron of $\mu_r = 100$, what be the value of magnetic induction inside iron ? [**Ans.** 16×10^{-4} T; 16×10^{-2} T]
8. An iron rod of area of cross-section 0·2 cm^2 is placed in a magnetic field of 20 Oe. The magnetic flux inside iron is 2400 emu. Obtain relative permeability of iron. [**Ans.** 600]
9. The radius of toroid is 15 cm and total number of turns 3500. The relative permeability of iron core is 800. If a current of 1·2 A is passed, find the value of magnetic induction produced inside iron. [**Ans.** 4.48 T]
10. The magnetic moment of magnetised steel rod is 2·5 Am^2 and its mass is 66 gm. If density of steel be 7·9 gm / c.c. calculate intensity of magnetisation of steel. [**Ans.** $3 \times 10^5\ \mathrm{Am}^{-1}$]
11. An iron rod of area of cross-section 0·2 cm^2 is placed in a magnetising field of intensity 1600 Am^{-1}. Magnetic flux inside iron is $2{\cdot}4 \times 10^{-5}$ Wb. Calculate (i) relative permeability of iron, (ii) intensity of magnetisation and (iii) susceptibility. [**Ans.** (i) 596·8, (ii) $9{\cdot}534 \times 10^5\ \mathrm{Am}^{-1}$, 595·8]
12. The coercivity of a bar magnet is $4 \times 10^3\ \mathrm{Am}^{-1}$. It is placed inside of solenoid of length 12 cm and having 60 turns. How much current has to be passed through the solenoid to demagnetise the magnet ? [**Ans.** 8.0 A]

    [**Hints :** $\tau = \dfrac{\mathrm{B}}{\mu_0 n} = \dfrac{\mathrm{H}}{n} = \dfrac{4 \times 10^3}{500}$]
13. Inside a magnetic material magnetic induction is 1·0 T and magnetising field intensity is $2 \times 10^3\ \mathrm{Am}^{-1}$. Calculate (i) absolute magnetic permeability (ii) relative permeability (iii) susceptibility and (iv) intensity of magnetisation. Given, $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{TA}^{-1} m$ [**Ans.** (i) $5 \times 10^{-4}\ \mathrm{Hm}^{-1}$ (ii) 397·9 (iii) 396·9 (iv) $7{\cdot}95 \times 10^5\ \mathrm{Am}^{-1}$]

E. Harder numerical problems :

1. Insulated copper wire is wound on an iron ring and a current of 0·15 A is passed through it. The number of turns over the ring is 1000 per m. If the magnetic induction iron is 1·0 T, calculate relative permeability of iron and its magnetic susceptibility. **[Ans. 5304, 5303]**

2. A straight solenoid of length 50 cm, number of turns 1000 and having cross sectional area $2{\cdot}0 \times 10^{-4}\ m^2$. It is placed in a magnetic field of magnetic induction 0·32 T making an angle of 30° with the magnetic field. How much torque will act on it ? Assume current through solenoid 2·0A. **[Ans : 0·064 N-*m*]**

3. A air core toroid carries a current. If now it is filled with an aluminium ring. Calculate percentage change in magnetic induction. Magnetic susceptibility of aluminium = $2{\cdot}1 \times 10^{-5}$ **[Ans : $2{\cdot}1 \times 10^{-3}$ %]**

4. The radius of a toroid is $\dfrac{20}{\pi}$ cm and total number of turns 400 and it carries a current of 1.0 A. (i) If the core of the toroid is an aluminium ring having intensity of magnetisation $4{\cdot}8 \times 10^{-2}\,\text{Am}^{-1}$ at 280 K, what will be its magnetic susceptibility at 280 K ? (ii) If the temperature of aluminium ring be raised to 320 K, what will be its intensity of magnetisation. **[Ans : (i) $2{\cdot}4 \times 10^{-5}$ (ii) $4{\cdot}2 \times 10^{-2}\ \text{Am}^{-1}$]**

   [**Hint :** $\dfrac{\chi_2}{\chi_1} = \dfrac{T_1}{T_2}$; $I = \chi_2 H$]

5. The length of a solenoid is 1 m and number of turns 600. A cylindrical iron rod of length 20 cm and diameter 1 cm and having relative permeability 1000 is inserted into the solenoid. If a current of 0·5 A is passed through the solenoid, what is the magnetic moment of the rod ? **[Ans : 4·71 Am^2]**

   [**Hint :** $B = \mu_0(H + I)$, $B = \mu H = \mu_0 \mu_r H$

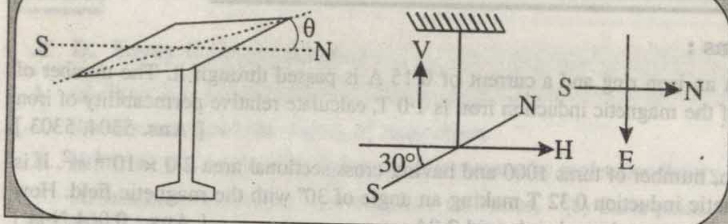
$$I = (\mu_r - 1)\,H = (\mu_r - 1)\,ni = (1000 - 1) \times 600 \times 0{\cdot}5 = 2{\cdot}997 \times 10^5\ Am^{-1}$$

$$M = IV = 2{\cdot}997 \times 10^5 \times 3{\cdot}14 \times (0{\cdot}5 \times 10^{-2})^2 \times 0{\cdot}2 = 4{\cdot}71\,Am^2$$

6. The magnetic induction at a point on the axis of a bar magnet at a distance of 15 cm from the centre is $1{\cdot}5 \times 10^{-4}$ T. The bar magnet is of length 1 cm with area of cross-section $1{\cdot}0\ \text{cm}^2$. Calculate (i) magnetic moment of the magnet (ii) intensity of magnetisation and (iii) magnetic induction at the centre of the magnet. **[Ans : (i) 2·5 Am^2 (ii) $2{\cdot}5 \times 10^6\ \text{Am}^{-1}$ (iii) 1·2 T]**

7. The volume of an iron rod is $10^{-4}\ m^3$ and relative permeability 1000. It is kept inside a long solenoid along the axis. If the solenoid has 5 turns / cm and carries 0·5 A current, what is the magnetic moment of the rod ? **[Ans : 25 Am^2]**

8. A paramagnetic sample has magnetic moment 5·0 JT^{-1} at a temperature of 4·0 K and magnetic induction 1·0 T. What will be the magnetic moment if the temperature is decreased to 3·0 K, and the magnetic field is increased by 1.5 T. **[Ans : 10 JT^{-1}]**



CHAPTER 4

TERRESTRIAL MAGNETISM

● **TOPICS :** ○ The earth is a large magnet : Earth's magnetic field; ○ Earth's magnetic elements; ○ Real and Apparent dip angle; ○ Variation of Earth's magnetic elements; ○ Magnetic Maps; ○ Short answer type questions (with answers); ○ Exercise.

✪ 4.1. The earth is a large magnet : Earth's magnetic field ✪

A magnet freely suspended from its centre of gravity or a compass needle always set itself approximately along a north-south direction. An iron rod placed in the north-south direction for a sufficiently long time, it developes feeble magnetism. As there is no magnet in the neighbourhood, it is natural to assume that the earth is a magnet.

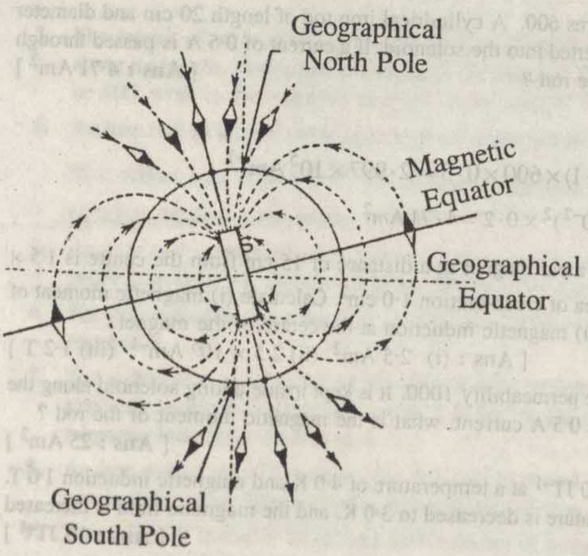


Fig. 4·1

Earth is a natural source of magnetic field. A magnetic field prevails throughout the whole surface of the earth. It is found that the magnetic field is perceptible even at a height of 66,000 miles from the earth's surface. The observed magnetic field can be roughly described by imagining a bar magnet within the earth, its length being much shorter than the earth's diameter [Fig 4.1]. It is extended along a diameter. The magnetic dipole moment of this terrestrial magnet is about 8.0×10^{22} Am^2. The axis of the dipole is inclined by a small angle of 11.5°. The south pole of the imaginary magnet points towards the earth's geographical north pole. The magnetic field of this imaginary magnet may be mapped by drawing the lines of force as shown in figure.

From the study of the nature of lines of force due to earth's magnetism we can draw three important conclusions.

(i) Since the tangent to the line of force gives the direction of magnetic field, it follows that **the actual direction of the magnetic field of the earth at most places on the earth is not horizontal.** Hence a freely suspended needle from its c.g. would not remain horizontal at most places of the earth. Only at places situated on magnetic equator, the needle will set itself in a perfectly horizontal position. At the magnetic north and south pole, it would remain at rest in a vertical position.

(ii) The lines of force of the earth's magnetism are directed from the geographical south pole towards geographical north pole of the earth. Hence arrow marks must be put on the lines of force from south to north.

(iii) The axis of earth's magnetism is inclined at a small angle with earth's geographical north-south direction.

✪ 4.2. Earth's magnetic elements ✪

In order to specify earth's magnetic field both in magnitude and direction at any place on the surface of the earth, we require to know the intensity of earth's magnetic field at that place in both magnitude and direction. For this we require to know three quantities. These quantities are known as Earth's magnetic elements. These are (i) the magnetic dip or inclination, θ (ii) the magnetic declination, δ and (iii) the horizontal component of earth's magnetic intensity, H.

➡ (i) The magnetic dip or inclination :

The earth's magnetic intensity at most places of the earth is not horizontal, but is inclined at an angle with the horizontal. So a freely suspended magnet from its centre of gravity will remain inclined with the horizontal, because it will align in the direction of earth's total magnetic intensity. The angle of inclination of the magnet with the horizontal is called dip or inclination.

❖ **Definition : The angle which the axis of a freely suspended magnet makes with the horizontal measured in the plane of the magnetic meridian, is called the dip angle of the place.**

In fig 4.2, a freely suspended magnetic needle has been shown. Its axis, which is along the earth's magnetic field make angle θ with the horizontal plane passing through the point of suspension. Hence θ is the dip of the place. It is taken to be positive when the north pole of the needle is downward.

The dip at a place is denoted as θ°N or θ°S. Dip at calcutta is 30°N means that the axis of a freely suspended needle at Calcutta will make angle of 30° with the horizontal and its pole will dip down.

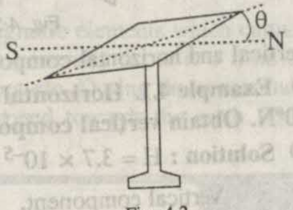


Fig. 4·2

Evidently dip at different places of the earth is different. At the magnetic equator dip is 0°. It means that at the equator a magnetic needle, when freely suspended will remain horizontal. At the north pole dip is 90°N and at the south pole it is 90°S. So at the poles the magnetic needle will remain vertical. Between the pole and the equator, dip will vary from 90° to 0°.

➡ (ii) Magnetic Declination :

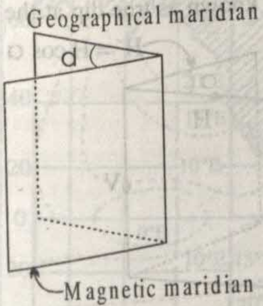


Fig. 4·3

It has been mentioned earlier that earth's magnetic poles do not coincide with earth's geographical poles. As a result, the magnetic meridian and geographical maridian at a place may not coincide. The angle between the two meridian planes at a place is called magnetic declination.

❖ **Definition : The declination at a place is defined as the angle between the magnetic meridian plane and geographical meridian plane of the place.**

The declination is measured from the geographical plane. It is denoted as δ ° E or δ ° W.

For example, declination at a place is 1.5° E means that the two meridian planes are inclined at 1.5° and the north pole of a freely suspended magnetic needle will remain east of geographical meridian. The places where the two meridian planes coincide, have zero declination.

➡ (iii) Horizontal component of earth's magnetic intensity, H :

A freely suspended magnetic needle remains in magnetic meridian and its axis makes an angle θ with the horizontal in the meridian plane. Here θ is the dip of the place. Evidently earth's total intensity of magnetisation I is inclined at the angle θ with the horizontal. Now, total intensity I may be resolved into two components— one horizontal and the other vertical. **The horizontal component denoted by H is known as earth's horizontal intensity.** The vertical component is denoted by V [Fig 4.4]

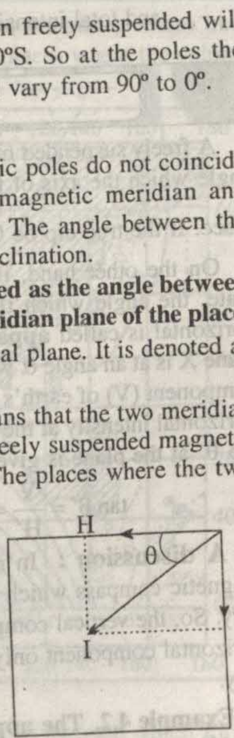


Fig. 4·4

Hence, **the horizontal component of earth's total magnetic intensity in the meridian plane is called earth's horizontal intensity.**

Now, $H = I\cos\theta$ and $V = I\sin\theta$

$$\therefore \quad \frac{V}{H} = \tan\theta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.1)$$

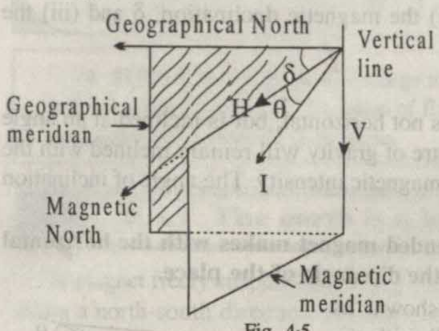


Fig. 4·5

The equation (4.1) gives the relation between the dip at the place and horizontal and vertical component of earth's magnetic intensity.

Again, $V^2 + H^2 = I^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.2)$

As θ is different at different place, H changes. He is maximum at the equator and minimum (zero) at the magnetic poles.

All three elements of earth's magnetism are shown in fig 4.5. The angle between the geographical and magnetic meridian denotes declination δ. Horizontal direction in the magnetic meridian denotes magnetic north V and H are the vertical and horizontal components of I. Evidently V is constant in all vertical planes.

- **Example 4.1. Horizontal intensity of earth's magnetism is $3{\cdot}6 \times 10^{-5}$ T and dip of the place is 60ºN. Obtain vertical component and total intensity of earth's magnetism.**

○ **Solution :** $H = 3.7 \times 10^{-5}$ T and $\theta = 60^\circ$N

$\therefore$ Vertical component, $V = H\tan\theta = 3{\cdot}7\times10^{-5}\tan 60^\circ = \mathbf{6{\cdot}41\times10^{-5}\ T}$

and total intensity, $I = \dfrac{H}{\cos\theta} = \dfrac{3{\cdot}7\times10^{-5}}{\cos 60^\circ} = \mathbf{7{\cdot}4\times10^{-5}\,T}$

✪ 4.3. Real and Apparent dip angle ✪

A freely suspended magnetic needle will remain in equilibrium in magnetic meridian. The dip (i.e, the angle which the axis of the needle make with the horizontal) at this position is known as true dip at the place. If the true dip be θ then $\dfrac{V}{H} = \tan\theta$

On the other hand, if the needle is in a plane other than meridian plate, the angle which the axis of the magnetic needle make with the horizontal is called **apparent dip at the place**. In fig 4.6, the vertical plane X is at an angle α with the magnetic meridian. The value of vertical component (V) of earth's magnetism V remains unchanged. But value of horizontal intensity at the X- plane is H′= H cos α. Hence the θ apparent dip θ′ at the plane is given by

$$\tan\theta' = \frac{V}{H'} = \frac{V}{H\cos\alpha} = \tan\theta.\sec\alpha$$

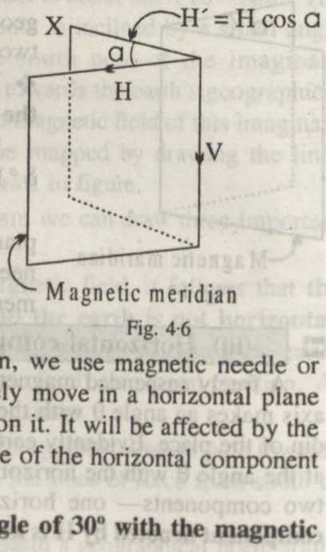


Fig. 4·6

- **A discussion :** In most of the laboratory experiment on magnetism, we use magnetic needle or magnetic compass which are pivoted in such a manner that they can freely move in a horizontal plane only. So, the vertical component of earth's magnetism will have no effect on it. It will be affected by the horizontal component only. For this reason, we require in general the value of the horizontal component only.

- **Example 4.2. The apparent dip at a plane which is inclined at angle of 30º with the magnetic meridian by 30º is 45º. What is the true dip at the place ?**

○ **Solution :** The required relation, $\tan\theta' = \tan\theta \sec\alpha$ Here, $\theta' = 45^\circ$, $\alpha = 30^\circ$

$$\therefore \quad \tan\theta = \tan\theta'.\cos\alpha = \tan 45^\circ.\cos 30^\circ = \frac{\sqrt{3}}{2} \quad \therefore \ \theta = \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

✪ 4.4. Variation of Earth's magnetic elements ✪

The values of earth's magnetic elements are different at different places of the earth. Also at a particular place on earth, a systematic recording of the magnetic elements shows that they are not constant, but undergo variations. The variations are :

(i) **Daily Variations :** The magnetic elements undergo fairly regular daily variations. These variations are, however, not the same from day to day.

(ii) **Annual Variations :** The magnetic elements also have periodic yearly changes. They undergo cyclic changes during the year.

(iii) **Secular Variations :** From the study of variation of earth's magnetic elements it is found that these elements return to their initial values after about 960 years.

● **Magnetic storms :** Sudden and violent changes in the values of the magnetic elements which cannot be predicted before hand, is known as **magnetic storm**. Magnetic storms generally occur at a time of natural calamity like volcanic eruptions, earthquakes, aurora borealis sunspot etc. During the appearance of aurora borealis and sunspot, charged particles are thrown at a great speed towards the earth. This phenomena cause sudden change in the values of magnetic elements.

✪ 4.5. Magnetic Maps ✪

The three earth's magnetic elements are different at different places. But there are places where one particular element has the same value. On the geographical map of the earth, the places where the values of a particular element are constant are joined by a line. Many lines can be draw on the map which join the different places having equal values of the magnetic elements. These are known as magnetic maps. On the magnetic maps three types of lines can be drawn for three different elements.

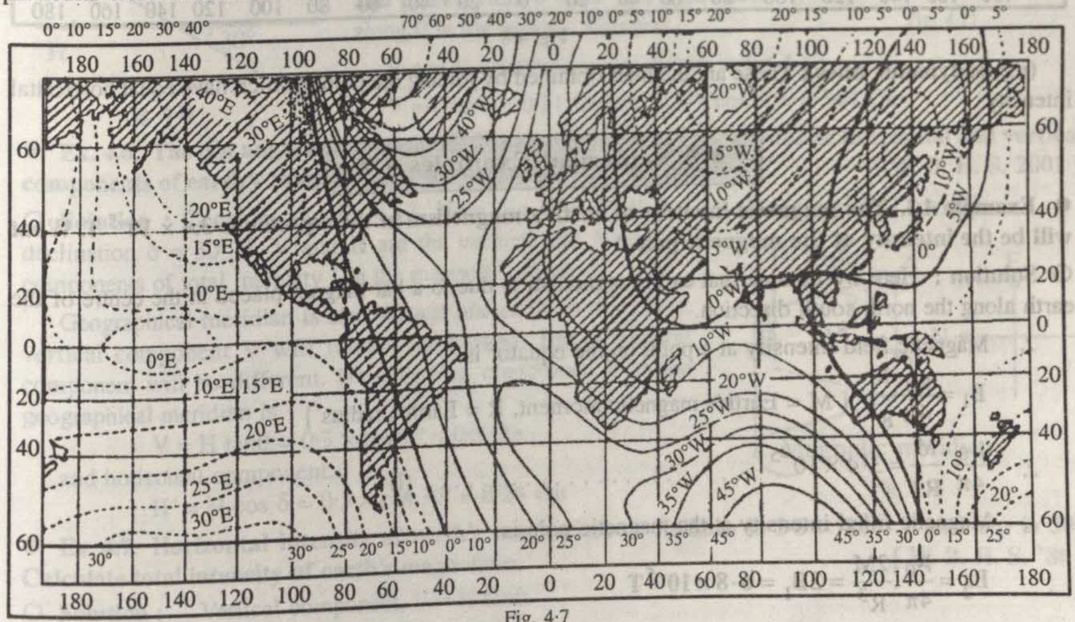


Fig. 4·7

(i) **Isogonic lines :** Places on the surface of the earth, which have equal declination are joined by a line. These are known as isogonic lines. These are shown in fig 4.7. The lines obtained by joining planes of zero declination are called agonic lines.

(ii) **Isoclinic lines :** These are the lines obtained by joining the places having equal dip or inclination. These are shown in fig 4.8.

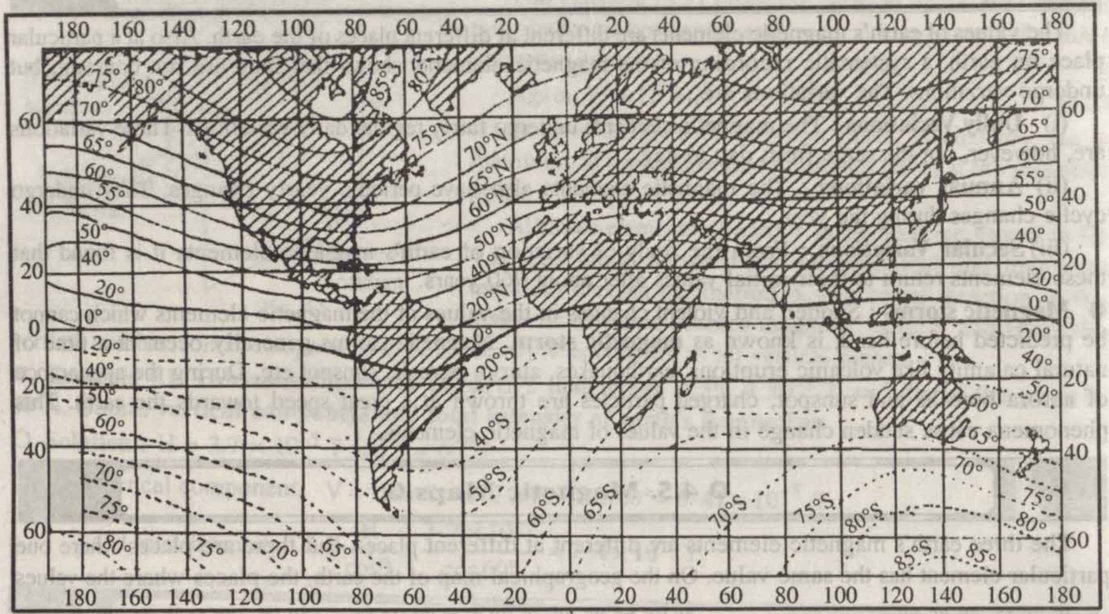


Fig. 4·8

(iii) **Isodynamic lines :** These are the lines obtained by joining the points which have equal horizontal intensity.

• • Numerical Examples • •

● **Example 4.3. The magnetic intensity at earth's magnetism at the equator is 3.4×10^{-5} T. What will be the intensity at the magnetic pole ?**

○ **Solution :** Here we assume that earth magnetism is due to a bar magnet placed at the centre of the earth along the north south direction.

$\therefore$ Magnetic field intensity at a point on the equator is

$$B_1 = \frac{\mu_0}{4\pi}\frac{M}{R^3} \quad [\text{ M = Earth's magnetic moment, R = Earth's radius }]$$

$$\therefore \quad \frac{\mu_0}{4\pi}.\frac{M}{R^3} = 3 \cdot 4 \times 10^{-5} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

$\therefore$ Magnetic filled intensity at the magnetic pole is

$$B_2 = \frac{\mu_0}{4\pi}.\frac{2M}{R^3} = 2B_1 = \mathbf{6 \cdot 8 \times 10^{-5}\,T}$$

● **Ex. 4.4. At a place on the earth's surface dip is 30°S; horizontal intensity is 0.35 Oe. Find the magnitude and direction vertical component of earth's magnetism. Show by diagram.**

[W. B. H. S. 1993]

○ **Solution :** From the fig 4.9 dip is 30°S. So at equilibrium the south pole of the magnetic needle will remain below the horizontal.

$H = 0{\cdot}35$ Oe $\therefore V = H\tan\theta = 0{\cdot}35 \times \tan 30^\circ = \mathbf{0{\cdot}202}$ **Oe**

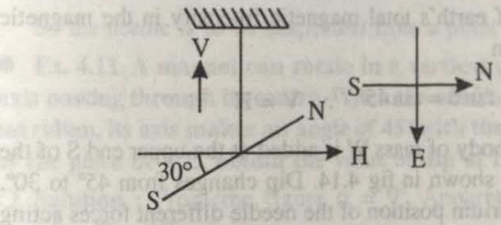


Fig. 4·9

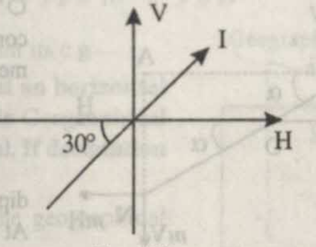


Fig. 4·10

The direction of total intensity of earth's magnetism is shown fig 4.10. Evidently, the vertical component V is directed upwards.

● **Ex. 4.5. Dips at two plates are 30°N and 30°S and total magnetic intensity 0.42 Oe. Calculate vertical and horizontal components at the two places. Also shows their directions in diagrams.** **[W. B. H. S '93]**

○ **Solution : 1 st place :** Dip is θ = 30°N [Fig 4.11]. At this place total magnetic intensity acts along θ is the downward direction as shown in fig.

V is in the upward direction and H horizontal.

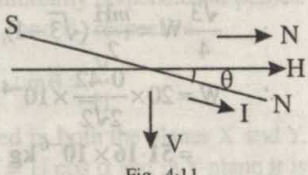


Fig. 4·11

$V = I\sin\theta = 0{\cdot}42 \sin 30^\circ = \mathbf{0{\cdot}21}$ **Oe**

$V = I\cos\theta = 0{\cdot}42 \cos 30^\circ = \mathbf{0{\cdot}364}$ **oe**

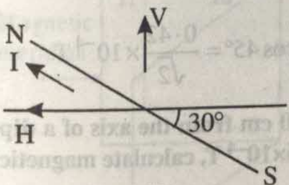


Fig. 4·12

2nd place : Here θ = 30° S, I is directed along θ in the upward direction.

V and H components are shown in fig 4.12.

$\therefore V = I\sin\theta = 0{\cdot}21$ Oe and $H = I\cos\theta = \mathbf{0{\cdot}364}$ **Oe**

● **Ex. 4.6. The dip angle at a place is 45° N and declination 30°E. Calculate horizontal and vertical components of earth's magnetism in the geographical meridian.** **[W. B. H. S. 2001]**

○ **Solution :** Consider the fig 4.13. Here dip θ = 45°N and declination δ = 30° E. V and H are the vertical and horizontal components of total intensity I in the magnetic meridian.

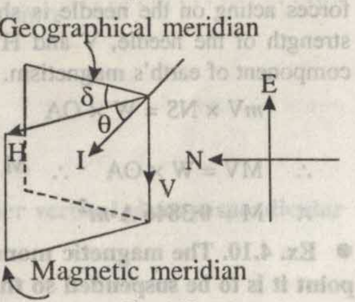


Fig. 4·13

Geographical meridian is also vertical plane. So in this plane, vertical component V will remain unchanged. But horizontal component will be different. Now, vertical component V in the geographical meridian is

$V = H\tan\theta = 0{\cdot}3 \times \tan 45^\circ = \mathbf{0{\cdot}3}$ **Oe**

and horizontal component,

$H' = H\cos\delta = 0{\cdot}3 \times \cos 30^\circ = \mathbf{0{\cdot}26}$ **Oe**

● **Ex. 4.7. Horizontal intensity of earth's magnetism at Kolkata is $3{\cdot}5 \times 10^{-5}$ T and dip is 30°. Calculate total intensity of earth's magnetism.** **[W. B. H. S. '86]**

○ **Solution :** Vertical component, $V = H\tan\theta$

$$\therefore \quad V = 3{\cdot}5\times10^{-5}\times\tan 30^\circ = 2{\cdot}02\times10^{-5}\,\mathrm{T}$$

and total intensity, $I = \dfrac{H}{\cos\theta} = \dfrac{3{\cdot}5\times10^{-5}}{\cos 30^\circ} = \mathbf{4{\cdot}04\times10^{-5}\,T}$

- **Ex. 4.8. When a small body is added on the upper end of a dip needle, the value of dip at a place changes from 45°N to 30°N. Total intensity of earth's magnetism at the place is $0{\cdot}42\times10^{-4}$ T and pole strength 20 A-*m*. Calculate the mass of the body.**

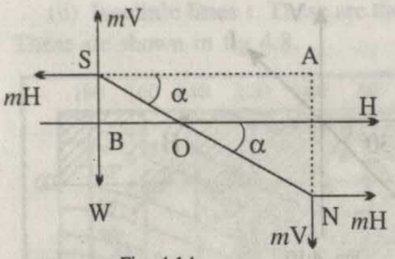


Fig. 4·14

○ **Solution :** If V and H be the vertical and horizontal component of earth's total magnetic intensity in the magnetic meridian, then

$$\frac{V}{H} = \tan\theta = \tan 45° \quad \therefore \quad V = H$$

A small body of mass W is added at the upper end S of the dip needle as shown in fig 4.14. Dip changes from 45° to 30°. At the equilibrium position of the needle different forces acting on the needle have been shown.

Taking moment of all the forces about the point 0, we get

$$m\,H \times (AN) + W \times (OB) = mV \times (AS) \quad [\ m = \text{pole strength of the needle}\ ]$$

$$\therefore \quad mH \times NS\sin\alpha + W\frac{NS}{2}\cos\alpha = mV \times NS\cos\alpha$$

$$\therefore \quad \frac{mH}{2} + \frac{\sqrt{3}W}{4} = \frac{\sqrt{3}mV}{2} = \frac{\sqrt{3}mH}{2} \quad [\text{As } \alpha = 30°]$$

$$\therefore \quad \frac{\sqrt{3}}{4}W = \frac{mH}{2}(\sqrt{3}-1)$$

$$\therefore \quad W = 20\times\frac{0{\cdot}42}{2\sqrt{2}}\times10^{-4}(\sqrt{3}-1)\ \frac{4}{\sqrt{3}}\ \text{Newton}$$

$$= \mathbf{51{\cdot}16\times10^{-6}\,kg}$$

m = 20 A-m

$$H = 0{\cdot}42\times10^{-4}\cos 45° = \frac{0{\cdot}42}{\sqrt{2}}\times10^{-4}\,T$$

- **Ex. 4.9. When a weight 0.01 gm is added at a point at a distance of 10 cm from the axis of a dip needle, it becomes horizontal. If the vertical component at the place be $0{\cdot}26\times10^{-4}$ T, calculate magnetic moment of the dip needle. $g = 10\ m/s^2$**

○ **Solution :** The situation is shown in fig 4.15. A weight W= 0.01 gm is added at A on the needle. So OA = 10 cm. Different forces acting on the needle is shown in fig. Here *m* is the pole strength of the needle, V and H are the vertical and horizontal component of earth's magnetism. For equilibrium of the needle

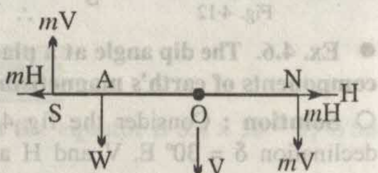


Fig. 4·15

$$mV \times NS = W \times OA$$

$$\therefore \quad MV = W \times OA \quad \therefore \quad M = \frac{10^{-4}\times0{\cdot}1}{0{\cdot}26\times10^{-4}}$$

$$\therefore \quad M = \mathbf{0{\cdot}3846\ A\text{-}m^2}$$

$V = 0{\cdot}26\times10^{-4}$ T
$W = 10^{-4}$ N
$OA = 0{\cdot}1\ m$

- **Ex. 4.10. The magnetic moment of a magnetic needle of mass 3.2 g is 980 cgs unit. For which point it is to be suspended so that it remains horizontal in the magnetic meridian. $g = 980\ cm/s^2$**

H = 0.32 Oe; dip = 45° N **[J. E. E. '86]**

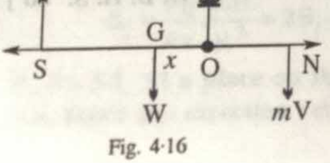


Fig. 4·16

○ **Solution :** In fig NS is the magnetic needle having centre of gravity G. Let when the needle is suspended from O, the needle will remain horizontal, where GO = *x* cm

For the equilibrium of the needle is

$$mV \times NS = W.\,x \quad [\ m = \text{pole strength of the needle}\ ]$$

$\therefore$ MV = Wx [M = m . (NS)]

$$\therefore \quad x = \frac{MV}{W} = \frac{0 \cdot 98 \times 0 \cdot 32 \times 10^{-4}}{3 \cdot 2 \times 10^{-3} \times 9 \cdot 8} = 0 \cdot 1 \text{ cm}$$

M = 980 cgs = 0·98 Am^2

V = 0·32 × 10^{-4} tan 45° = 0·32 × 10^{-4} T

W = 3·2 × 10^{-3} × 9·8 N

So the needle is to be suspended from a point 0.1 cm from its c.g

● **Ex. 4.11. A magnet can rotate in a vertical plane about an horizontal axis passing through its centre. When the needle is placed in Geographical meridian, its axis makes an angle of 45° with the horizontal. If declination of the place be 9°E, obtain the value of dip at the place.**

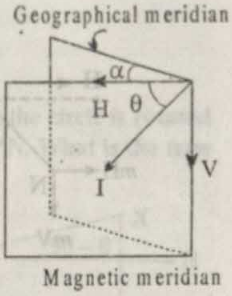


Fig. 4·17

○ **Solution :** From the figure $\alpha = 9^\circ$. Apparent dip at the geographical meridian $\theta' = 45^\circ$.

If the true dip at the place be θ then $\tan\theta' = \tan\theta . \sec\alpha$

$\therefore \quad \tan\theta = \tan\theta' . \cos\alpha = \tan 45^\circ . \cos 9^\circ$

$\therefore \quad \theta = \tan^{-1}(\cos 9^\circ) = \mathbf{44^\circ\ 40'}$

● **Ex. 4.12. The apparent dips at a place in two mutually perpendicular planes are θ_1 and θ_2. If true dip at the place be θ, show that $\cot^2\theta = \cot^2\theta_1 + \cot^2\theta_2$** **[J. E. E. '95]**

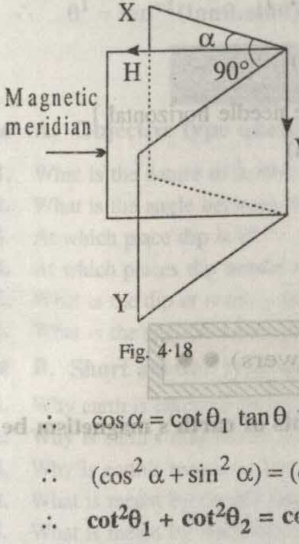


Fig. 4·18

○ **Solution :** In fig 4.18, X and Y are two mutually perpendicular planes. The apparent dips at there two plane are θ_1 and θ_2 respective.

Now, true dip at the place is given by $\tan\theta = \dfrac{V}{H}$

Vertical component V remain unchanged in both the planes X and Y. Now, horizontal component in X- plane, $H' = H\cos\alpha$ and in Y-plane it is

$$H'' = H\cos(90^\circ - \alpha) = H\sin\alpha$$

$$\therefore \quad \text{for the X-plane,} \quad \tan\theta_1 = \frac{V}{H\cos\alpha} = \frac{\tan\theta}{\cos\alpha} \quad \ldots\ldots\ldots\ldots \text{(i)}$$

$$\text{and for the Y-plane,} \quad \tan\theta_2 = \frac{V}{H\sin\alpha} = \frac{\tan\theta}{\sin\alpha} \quad \ldots\ldots\ldots\ldots \text{(ii)}$$

$\therefore \quad \cos\alpha = \cot\theta_1 \tan\theta$ and $\sin\alpha = \cot\theta_2 \tan\theta$

$\therefore \quad (\cos^2\alpha + \sin^2\alpha) = (\cot^2\theta_1 + \cot^2\theta_2)\tan^2\theta$

$\therefore \quad \mathbf{\cot^2\theta_1 + \cot^2\theta_2 = \cot^2\theta}$

● **Ex. 4.13. In a vertical plane, the dip circle reads 40° and in another vertical plane perpendicular to the first plane, the dip is 30°. What is the true dip at the place ?**

○ **Solution :** The required relation : $\cot^2\theta = \cot^2\theta_1 + \cot^2\theta_2$

$\therefore \quad \cot^2\theta = \cot^2 40^\circ + \cot^2 30^\circ \quad [\theta_1 = 40^\circ, \theta_2 = 30^\circ]$

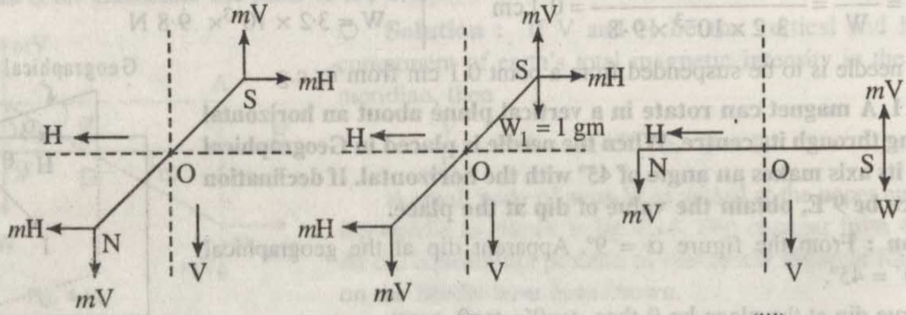
$\qquad = \cot^2 40^\circ + 3 = 4 \cdot 42$

$\therefore \quad \cot\theta = 2 \cdot 1024 \Rightarrow \mathbf{\theta = 25^\circ 27'}$

$\therefore$ true dip at the place is $\mathbf{25^\circ 27'}$

● **Ex. 4.14. The dip at a place is 60°N. When a mass of 1 gm is placed on the upper end of the needle, the dip decreases to 30°N. What mass will make the needle horizontal ?**

○ **Solution :** In fig 4.19 we see the needle in three different equilibrium position. H and V are the horizontal and vertical components of earth's total intensity in meridian plane '*m*' is the pole strength of the needle.



(i) (ii) (iii)

Fig. 4·19

For fig 4.19 (i), $\frac{V}{H} = \tan 60^\circ$ ∴ $V = \sqrt{3}H$ . . . . (i)

From the fig 4.19 (ii) , $mV \times 2l \cos 30^\circ = mH \times 2l \sin 30^\circ + W_1 \times l \cos 30^\circ$

or, $2mV \times \frac{\sqrt{3}}{2} = mH \times 2 \times \frac{1}{2} + \frac{\sqrt{3}W_1}{2}$ or, $mH = \sqrt{3}W_1$ [$V = \sqrt{3}H$]

∴ $4mV = \frac{3}{4}W_1$ . . . . (ii)

Next from 4.19 (ii), $mV \times 2l = W \times l$ [W = mass required to keep the needle horizontal]

$2mV = W$ . . . . (iii)

from the equations (ii) and (iii) $\frac{W}{2} = \frac{3}{4}W_1$

∴ $W = \frac{3}{2} \times W = \frac{3}{2} \times 2 = 1{\cdot}5$ **gm-wt**

●● Short Answer Type Questions (with answers) ●●

● **Q. 1. At which place on the earth, vertical and horizontal components of earth's magnetism be equal in magnetic meridian ?**

○ **Ans.** Here V = H. Now, $\frac{V}{H} = \tan\theta$ ∴ $\tan\theta = 1$ ∴ $\theta = 45^\circ$

Hence the two components are equal at a place where dip is 45°.

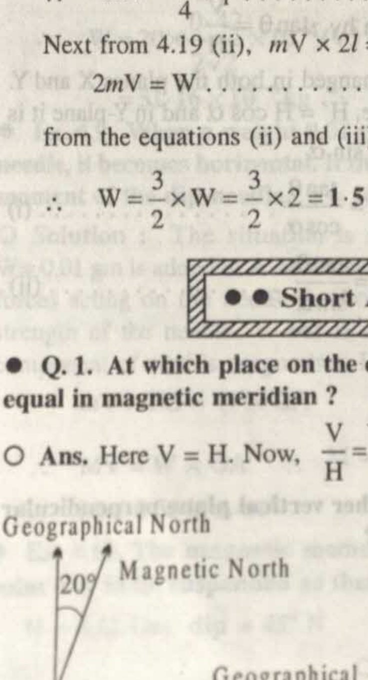


Fig. 4·20

● **Q. 2. According to Mariner's compass, a ship is moving due cost. If the declination of the place be 20°E, what is the correct direction of motion of the ship ?**

○ **Ans.** The declination at the place is 20°E. So, north pole of the magnetic needle will remain 20°E of geographical north, [Fig 4.20]. Mariner's compass shows that the ship is moving due east. Hence the ship is actually moving 20° south of east.

● **Q. 3. A dip needle is in magnetic meridian. It is rotated by an angle α with respect to the vertical axis. Show that the tangent of apparent dip increases in the ratio sec α : 1**

○ **Ans.** If the trup dip be θ, then tan θ = V / H. If the needle is rotates by angle α, it will come in a vertical plane which is inclined at α with the magnetic meridian. In this vertical place, the vertical component of earth's magnetic intensity will remain unchanged. But horizontal component will be now H′ = H cos α.

Hence, if the apparent dip angle be θ′, then $\tan\theta' = \dfrac{V}{H\cos\alpha} = \dfrac{V}{H}\sec\alpha$

$\therefore \quad \dfrac{\tan\theta'}{\tan\theta} = \sec\alpha \qquad \therefore$ **tan θ′ : tan θ′ = sec α : 1**

● **Q. 4. A dip circle is placed in such a position that the needle is vertical. Now, the circle is rotated by angle θ with respect to the vertical axis and the needle shows dip equal to 30ºN. What is the true dip at the place ?**

○ **Ans.** When the dip circle is rotated by angle θ, the needle comes in the X- plane. The horizontal component of earth's magnetic intensity in this plane is $H^1 = H\cos(90^\circ - \theta) = H\sin\theta$

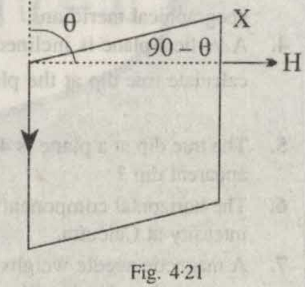


Fig. 4·21

If the apparent dip in this plane be θ then $\tan\theta = \dfrac{V}{H.\sin\theta}$

or, $\dfrac{V}{H} = \tan\theta.\sin\theta$

$\therefore \quad \tan\theta^1 = \tan\theta.\sin\theta$ where θ^1 = true dip at the place.

$\therefore \quad \boldsymbol{\theta^1 = \tan^{-1}(\tan\theta.\sin\theta)}$

EXERCISE

A. Objective type questions :

1. What is the nature of earth's magnetism ?

2. What is the angle between the earth's magnetic axis and geographical axis ?

3. At which place dip is 0º ?

4. At which places dip needle remain's vertical ?

5. What is the dip at south pole ?

6. What is the angle between magnetic meridian and geographical meridian at a place.

B. Short answer type questions :

1. Why earth is taken to be large magnet ?

2. Why is earth's magnetism is represented by a bar magnet ?

3. Why is earth's magnetic lines of force are taken to directed from geographical south pole.

4. What is meant by earth's magnetic elements ? Name these elements.

5. What is meant by declination at a place 10ºE ?

6. What is meant by dip at a place is 30ºN ?

7. At which place on earth horizontal component is zero and at which place vertical component is zero ?

8. What we mean by true dip and apparent dip ? How are they related ?

9. What is magnetic storm ? Why does it happen ?

10. The dip at a place θº N and declination δº E. If the dip needle is placed in geographical meridian, what is the apparent dip at the place ?

11. At iron bar is kept in vertical position for sufficiently long time. What will be your observation?

12. What is magnetic map ? Define isogonic and isodimic lines .

C. Essay type questions :

1. (a) What is the nature of earth's magnetism ? Draw lines of force due to earth's magnetism.

(b) What are elements of earth's magnetism ? Explain their meaning by a diagram.

2. (a) Define all the elements of earth's magnetism. How does dip vary at different places on the earth ? Mention its maximum and minimum values ?

(b) A magnet is freely suspended so that it can rotate in vertical plane. At which place, it will remain horizontal and at which place vertical ?

D. Simple numerical problems :

1. The apparent dip angle at a place when the needle is in a vertical plane inclined at 30° with the magnetic meridian is 45°N. What is the true dip at the place ? [**Ans.** 40.9° N]
2. The horizontal component of earth's magnetic field at a place is $0·75 \times 10^{-4}$ T and vertical component is $0·25 \times 10^{-4}$ T. Calculate dip at the place and total magnetic intensity of earth. [**Ans.** 18·9°; $0·8 \times 10^{-4}$ T]
3. The horizontal component of earth's magnetism at a place is 2×10^{-5} T, dip is 60°N and declination 30°E. Obtain the values of horizontal and vertical components of magnetic intensity of earth's magnetism in the geographical meridian. [**Ans :** $H = 0·188 \times 10^{-4}$ T; $V = 0·346 \times 10^{-4}$ T]
4. A vertical plane is inclined at 45° with the magnetic meridian. If the apparent dip angle in this plane is 30°N, calculate true dip at the place. [**Ans :** $\theta = \tan^{-1}\left(\frac{1}{\sqrt{6}}\right)$]
5. The true dip at a plane is 45°N. When the dip circle is rotated by 60° from the meridian plane, what will be the apparent dip ? [**Ans.** 63° 26′ N]
6. The horizontal component of earth's magnetic intensity at Calcutta is 0·35 Oe and dip is 30°N. Calculate total intensity at Calcutta. [**Ans :** 0·404 Oe]
7. A magnetic needle weighs 8 gm and its magnetic moment is $9·8 \times 10^{-2}$ Am^2. From which point, the needle be suspended so that it will remain horizontal at the place on northern hemisphere. Given, vertical component of earth's magnetism is $0·2 \times 10^{-5}$ T, $g = 9·8$ m / s^2. [**Ans :** at a distance of 0·025 cm from c.g.]
8. At a place on the earth's horizontal intensity $0·4 \times 10^{-4}$ T, declination 30°E, dip 45°N. Calculate vertical and horizontal component of intensity in the geographical meridian. [**Ans :** $0·4 \times 10^{-4}$ T; $0·3464 \times 10^{-4}$ T]
9. The dip angle at a place 30°S and earth's horizontal intensity 0·35 Oe. Calculate magnitude and direction of vertical component. [**Ans :** $0·202 \times 10^{-4}$ T (upwards)] [**W. B. H. S. '94**]

E. Harder numerical problems :

1. Mass of magnetic needle is 7.5 g and its magnetic moment 90 cgs units. From which point it is to be suspended so that it will remain horizontal in Northern hemisphere. Given, H = 0.25 Oe. [**J. E. E '80**]
[**Ans :** $3·33 \times 10^{-3}$ cm towards north pole]
2. The pole strength of a magnetic needle 5 Am and length 20 cm. It is pivoted from the mid point. What mass has to be added to its upper end so that it will remain horizontal at a place where dip is 45°N and $H = 0·2 \times 10^{-4}$ T. $g = 10$ m / s^2 [**Ans.** $m = 0·02$ gm]
3. When a small mass is added at the upper end magnetic needle the dip angle changes from 45°N to 30°N. Given total intensity, $I = 0·42 \times 10^{-4}$ T and pole strength of the needle 20 Am, What is the mass added ? [**Ans :** $5·1 \times 10^{-5}$ kg]
4. When a mass of 1 gm is added at the upper tip of magnetic needle dip angles decreases from 60° to 30°. Total intensity of earth's magnetism = $0·42 \times 10^{-4}$ T; What is the pole strength of the needle ?]
G = 10 m / s^2 [**Ans :** 205·2 A-m]
5. The dip at a place is 60°. When a mass of 1 gm is added to the upper end, dip decreases to 30°. What mass will make it horizontal ? [**Ans :** $2·5 \times 10^{-3}$ kg]
6. A magnetic needle of length 10 cm is so pivoted that it can rotate in a horizontal plane. Horizontal intensity $H = 3·6 \times 10^{-5}$ T. One end of the needle is pulled by a string with a force of 10 mg-wt and the needle is deflected by 60°. Calculate magnetic moment and pole strength of the needle. [**Ans :** 0·0786 Am^2; 0·786 Am]
7. The earth's magnetism may be supposed to be due to a small magnet of moment M existing at its centre with its north pointing geographical south. Calculate horizontal and vertical components of earth's magnetism at a place of latitude 60°. Given, Radius of the earth = 6400 km and magnetic moment = $6·4 \times 10^{22}$ A-m^2
[**Ans :** $1·22 \times 10^{-5}$ T; $4·2 \times 10^{-5}$ T]
8. Earth's magnetism is due to a small magnet existing at the centre of the earth and its north pole is towards geographical south. If the dip angle at a place of latitude $\lambda°$ be θ_1. Show that $\tan\theta_1 = 2\tan\lambda$

CURRENT ELECTRICITY – II

• TOPICS DISCUSSED •

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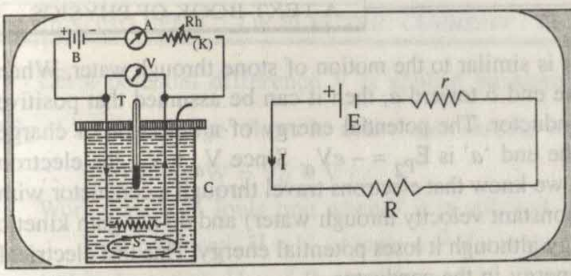


CHAPTER 1

HEATING EFFECT OF ELECTRIC CURRENT : THERMOELECTRICITY

▲ HEATING EFFECT OF ELECTRIC CURRENT ▲

● **TOPICS :** ○ Introduction; ○ Heating effect of electric current; ○ Joule's Law; ○ Deduction of Joule's law; ○ Experimental verification of Joule's law; ○ Determination of J by electrical method; ○ Principle of least heat; ○ Power of electrical appliances connected parallel; ○ Power electrical appliances connected in series; ○ Efficiency of a machine; ○ Practical application of heating effect of current; ○ Short answer type questions (with answers); ○ Exercise.

❂ 1.1. Introduction ❂

An electric current or charge in motion can produce a number of easily observable effects : (i) heating effect, (ii) chemical effect and (iii) magnetic effect. Of all these effects, magnetic effect is most fundamental because unlike other effects of electric current, magnetic effect does not depend on any external condition. In this chapter we shall discuss only heating effect of electric current.

When a current is passed through a conductor, it become hot and its temperature rises. It is called **heating effect** of electric current. The heating effect primarily depends on resistance of the conductor. The moving electrons as they pass through molecules or atoms of that substance, collide with other electrons. This electronic collision results in opposition to flow of electrons. The external source of emf has to perform work to keep the electrons moving. This work done by the source is converted to heat. It is the explanation of heating effect of electric current.

❂ 1.2. Heating effect of electric current ❂

When a current is passed through a conductor having some resistance, a heat is produced. Now, we consider a simple circuit consisting of a battery B connected in series with a conductor *ab*. A fixed potential difference V acts between the points *a* and *b* of the current and a constant current I flows through it. Here the end '*a*' is connected to positive terminal and other end '*b*' to negative terminal. So the end '*a*' is at higher potential them the end '*b*' [fig. 1·1].

Let a charge *q* flows through the conduction under the potential difference V. As the potential difference between two points in the electric field is the work done to transfer unit positive charge from one point to another, then work done to transfer the charge *q* through the potential difference V is

$$W = Vq \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1.1)$$

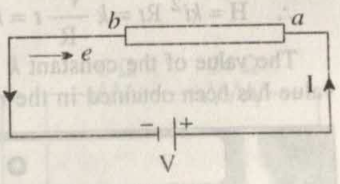


Fig. 1·1

This work is manifested as heat. Now we consider the following simple example :

Let a stone of mass is falling through air from a certain height *h*. Its loss of potential energy is converted to kinetic energy and it acquires some velocity. Now, let the stone enters into water. Its motion is progressively retarded as it falls in water. Ultimately its speed becomes constant. The potential energy that is steadily being made available as the stone descends then appears in the form of heat in the surrounding water. The heat is produced due to friction between the stone and water.

The motion of an electron through a conductor is similar to the motion of stone through water. When the electron passes through the conductor from the end *b* toward *a*, then it can be assumed that positive charge moves from the end *a* toward *b* of the conductor. The potential energy of an electron of charge $-e$ at *b* is $E_{P_1} = -eV_b$ and potential energy at the end '*a*' is $E_{P_2} = -eV_a$. Since $V_a > V_b$, the electron loses potential energy as it moves from *b* to *a*. But we know that electrons travel through a conductor with constant drift velocity v_d (as the stone falls with constant velocity through water) and do not gain kinetic energy. Thus the electrons do not gain kinetic energy although it loses potential energy. This lost electrical potential energy of electron is converted to heat energy in the conductor.

The phenomenon of conversion of potential energy to heat can be explained in terms of collision of electrons with atoms of the conductor. When the electron moves in a certain direction in the electric field, electrons suffer collision with the atoms of the conductor. The motion of the electrons is opposed i.e. the atoms offer resistance to motion of electrons. In between collisions, the electrons acquire kinetic energy. During collision, this energy is transferred to the atoms of the conductor. Consequently, the amplitude of vibration of the atoms increases. So the average kinetic energy of the atoms increases. It results a rise in temperature of the conductor. It is the reason why the conductor gets heated by the flow of current through the conductor.

✪ 1.3. Joule's law ✪

The law associated with the measurement of heat produced in a conductor due to flow of electric current is known as **Joule's law of heating**. The English physicist J. P. Joule expressed the results of his experiment on heating effect of electric current in the form of a law. This law is known as Joule's law.

If a current flows through a conductor resistance R for a time *t*, Joule's law states that—

(i) the heat produced is proportional to the square of the current when the resistance of the conductor and the time of flow remain constant.

i.e., $H \propto i^2$ [when R and *t* are constant]

(ii) the heat produced is proportional to the resistance of the conductor when current and time of flow remains constant.

i.e., $H \propto R$ [when *i* and *t* are constant]

(iii) the heat produced is proportional to the time of flow of the current when the current and resistance of the conductor remains constant.

i.e., $H \propto t$ [when *i* and R are constant]

So, the mathematical representation of Joule's law is $\mathbf{H \propto i^2 Rt}$

$$\therefore \quad H \propto \frac{V^2}{R} t \quad \left(\because \; i = \frac{V}{R}\right) \quad \text{or,} \quad H \propto V i t \quad (V = iR)$$

$$\therefore \quad H = ki^2 Rt = k\frac{V^2}{R} t = kVit \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1.2)$$

The value of the constant *k* does not depend on the units of different quantities of this equation. Its value has been obtained in the next article.

✪ 1.4. Deduction of Joule's law ✪

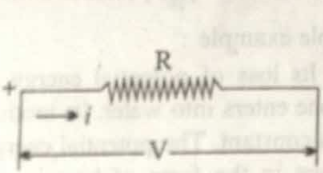


Fig. 1·2

Joule's law can be established from the definition of potential difference. The potential difference between two points in the electric field is defined as the amount of work done in moving a unit positive charge from one point to another. If *q* amount of charge is transferred under a potential difference V, then the work done is

$$W = Vq$$

Let a potential difference V is applied between the two ends of the conductor of resistance R [fig. 1·2]. If q charge flows in time t, then electric current, $i = \frac{q}{t}$

$\therefore$ work done, $W = Vq = V\,i\,t$

Work done is in Joule unit when V is in volt and current i is in ampere

$\therefore$ Heat produced, $H = V\,i\,t$ joule $= i^2\,R\,t$ joule $\quad [\because\ V = iR]$

and in c.g.s. unit, $H = i^2\,R\,t \times 10^7$ erg

If the heat is expressed in calorie, $H = \frac{1}{J} i^2 Rt$ cal or, $H = 0{\cdot}24\ Vit$ . . . . (1.3)

$\therefore$ $H \propto i^2 Rt$ — is known as Joule's law.

From the equations (1.2) and (1.3), $k = 0{\cdot}24$ cal / Joule.

● **A discussion :** (i) Heat produced in a current carrying conductor is proportional to the square of the current. So, the amount of heat produced does not depend on the direction of current. So, the Joule's heating proces is irreversible.

(ii) $H \propto R$ or, $H \propto \rho \cdot \frac{l}{A}$ $\quad [\, i = \text{constant} \,]$

$\therefore$ $H \propto l$ or, $H \propto \frac{l}{A}$

Hence at constant current, heat produced is proportional to the length of the conductor and inversely proportional to its area of cross-section.

Evidently these rules are applicable in the case of series combination of resistors when current is constant.

(iii) Again, $H = \frac{V^2}{R} t$ $\quad \therefore$ $H \propto \frac{l}{R}$ $\quad$ [when V is constant]

So, in this case, $H \propto \frac{A}{\rho l}$ $\quad \therefore$ $H \propto \frac{1}{l}$ and $H \propto A$

These rules are applicable when the voltage is held constant. When a number of conductors are connected in parallel and a fixed voltage is applied across the combination, then the heat produced in any conductor is inversely proportional to the length of the conductor and directly proportional to the area of cross section of the conductor. It means that with voltage constant, short and thick wire will produce more heat.

● **Example 1.1. Two resistors 6Ω and 24Ω are first connected in parallel and then in series. If a constant potential different is applied in both the combination, calculate the ratio of heat produced in these resistors in both the cases.**

○ **Solution :** Circuit diagrams in both the cases are shown in fig. 1.3.

(i) When the resistors are connected in parallel [fig. 1.3 (i)], if H be the heat produced

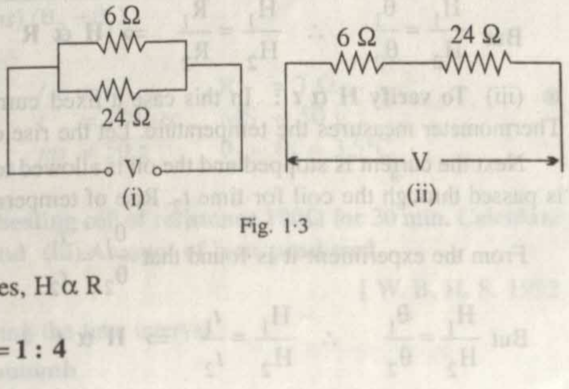


Fig. 1·3

$$H \propto \frac{1}{R} \quad \therefore \quad \frac{H_1}{H_2} = \frac{R_2}{R_1} = \frac{24}{6} = 4$$

$\therefore$ **$H_1 : H_2 = 4 : 1$**

(ii) When the resistances are connected in series, $H \propto R$

$$\therefore \quad \frac{H_1}{H_2} = \frac{R_1}{R_2} = \frac{6}{24} = \frac{1}{4} \quad \therefore \ H_1 : H_2 = 1 : 4$$

❂ 1.5. Experimental verification of Joule's law ❂

- **Description :** Joule's law of heating can be verified by applying the principle of calorimetric measurement. Here we require a calorimeter with a stirrer and is provided with an ebonite lid having several opening, through which is inserted a coil and a thermometer. About $\frac{2}{3}$ part of the calorimeter is filled with an oil. The heating coil is connected in series with a battery (B), an ammeter (A), rheostat (Rh) and a key (K). Voltmeter V measures the potential difference across the coil. Thermometer measures the temperature of oil [fig. 1·4].

- (i) **To verify H α i^2 :** The initial temperature of the oil is measured. The key (K) is closed, Rh is adjusted to send a suitable current. Let the ammeter reads i_1. The current is passed for a measured time interval. Rise of temperature is θ_1.

Now, the current is switched off, oil and the calorimeter comes back to its initial temperature. Key is closed again, Rh is changed so that the ammeter reads i_2. The current is passed for equal intervel of time. Let the rise of temperature is θ_2.

From the experimental values it is found that $\frac{\theta_1}{\theta_2} = \frac{i_1^2}{i_2^2}$

Let the heat produced in the two cases be H_1 and H_2.

Since the rise of temperature is proportional to produced heat, we write

$$\frac{H_1}{H_2} = \frac{i_1^2}{i_2^2} \qquad \therefore \ \mathbf{H \ \alpha \ i^2}$$

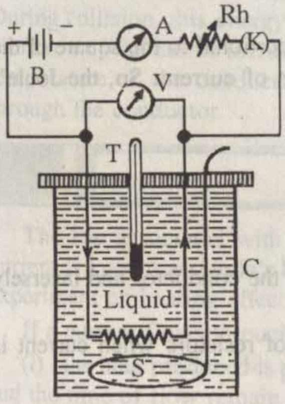


Fig. 1·4

- (ii) **To verify H α R :** Here we take two heating coils. Let the resistance of the first coil is R_1. It is put into the oil of the calorimeter and electrical connections are made as before. A fixed current is passed for a given time.

Rise in temperature is measured. Let it be θ_1.

Next current is stopped ; oil and the calorimeter comes back to initial temperature. Now the second is placed in position instead of first coil. Let the second coil resistance be R_2. Equal current is passed for equal time interval. Let now the rise of temperature of the oil be θ_2. From the results it is found that $\frac{\theta_1}{\theta_2} = \frac{R_1}{R_2}$.

$$\text{But } \frac{H_1}{H_2} = \frac{\theta_1}{\theta_2} \quad \therefore \ \frac{H_1}{H_2} = \frac{R_1}{R_2} \ \Rightarrow \mathbf{H \ \alpha \ R}$$

- (iii) **To verify H α t :** In this case a fixed current is passed through the heating coil for time t_1. Thermometer measures the temperature. Let the rise of temperature is θ_1.

Next the current is stopped and the oil is allowed to assume its initial temperature. Again same current is passed through the coil for time t_2. Rise of temperature θ_2 is measured by the thermometer.

From the experiment it is found that $\frac{\theta_1}{\theta_2} = \frac{t_1}{t_2}$

$$\text{But } \frac{H_1}{H_2} = \frac{\theta_1}{\theta_2} \quad \therefore \ \frac{H_1}{H_2} = \frac{t_1}{t_2} \ \Rightarrow \mathbf{H \ \alpha \ t}$$

✪ 1.6. Determination of J by Electrical Method ✪

- **Experiment :** The meahanical equivalent of heat is measured by Joule's calorimeter by utilising heating effect of electric current. At first an oil of known mass and specific heat is taken in the calorimeter. The herating coil should be kept completely immersed in the oil.

The experimental arrangement and circuit connection are shown in fig. 1·5. The heating coil C, battery B, ammeter, A rheostat Rh and a key K all put in a series combination. The voltmeter measures the voltage V across the coil. By adjusting the rheostat Rh, a constant current is passed through the heating coil for a definite time interval. Current is measured by ammeter, voltage by voltmeter and the rise of temperature is noted from the thermometer.

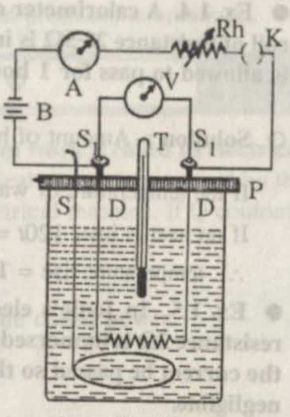


Fig. 1·5

- **Calculation :** Let V be potential difference between the two ends of the coil and a current i flows for a time t. Then the heat produced by the coil is

$$H = \frac{V\,it}{J}\ \text{cal}, \qquad [\text{ J = mechanical equivalent of heat }]$$

Suppose, m = mass of oil in the calorimeter
s = specific heat of oil
M = mass of calorimeter with the stirrer
S = specific heat calorimeter and stirrer
θ_1 and θ_2 = initial and final temperature of the oil

$\therefore$ The heat absorbed by calorimeter and oil is $H' = (MS + ms)(\theta_2 - \theta_1)$

If no heat is lost H = H′ $\quad \therefore \quad \frac{V\,it}{J} = (MS + ms)(\theta_2 - \theta_1)$

$$\therefore\ J = \frac{V\,it}{(MS + ms)(\theta_2 - \theta_1)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(1.4)}$$

All other quantities being known, J can be determined.

- **Example 1.2. A copper calorimeter of water equivalent 50g contains water of mass 50g. A heating coil of resistance 3Ω is put into the calorimeter and a current of 2A is passed for 2 min. If the temperature rise be 3·5°C, obtain the value of mechanical equivalent of heat.**

○ **Solution :** The required relation, $J = \frac{V\,it}{(MS + ms)(\theta_2 - \theta_1)}$

$$\therefore\ J = \frac{i^2 Rt}{(MS + ms)(\theta_2 - \theta_1)} = \frac{(2)^2 \times 3 \times 120}{(50 + 50)(3{\cdot}5)} = 4{\cdot}11\ \text{J-cal}^{-1}$$

i = 2A ; R = 3 Ω ;
t = 120 sec ; MS = 50 g ;
ms = 50 g ; $\theta_2 - \theta_1$ = 3·5°C

- **Ex. 1.3. A current of 2A is passed through a heating coil of resistance 100Ω for 30 min. Calculate : (i) Amount of charge passed, (ii) Work done and (iii) Amount of heat produced.**

[W. B. H. S. 1992]

○ **Solution :** (i) Amount of charge flowing during the time interval

$Q = it = 2 \times 30 \times 60 =$ **3600 Coulomb**

(ii) Work done, $W = i^2 Rt = (2)^2 \times 100 \times 30 \times 60 = \mathbf{7{\cdot}2 \times 10^5\ J.}$

(iii) Amount of heat produced, $H = \frac{W}{J} = \frac{7{\cdot}2\times10^5}{4{\cdot}2} = \mathbf{17{\cdot}28\times10^4\ cal}$

● **Ex. 1.4. A calorimeter of water equivalent 20g contains water of mass 100 gm at 30°C. A heating coil of resistance 28·5Ω is immersed into water and a potential difference of 2·0 V is applied. Current is allowed to pass for 1 hour. What is the rise of temperature of water ?**

○ **Solution :** Amount of heat produced, $H = \frac{V^2}{JR}t = \frac{4\times3600}{28{\cdot}5\times4{\cdot}2} = \mathbf{120\cdot3\ Cal}$

If the temperature of water rises by t°C, then heat required $H' = (20+100)\times t = 120t$

If no heat is lost, $120t = 120{\cdot}3$ $\quad\therefore\ t = 1°\text{C}$ (approx)

$\therefore$ temperature rise = **1°C**

● **Ex. 1.5. In Joule's electric calorimeter oil of mass 1·5 kg of sp. heat 0·6 cgs is taken. A coil of resistance 3Ω is immersed into the oil and a current of 3A is passed through the coil. For what time, the current be passed so that the temperature rises by 10°C ? Water equivalent of the calorimeter is negligible.** **[W. B. H. S. 1980]**

○ **Solution :** Heat absorbed by the oil, $H = ms\theta$ [θ = temperature rise]

Heat produced by the heating coil, $H' = \frac{i^2Rt}{J}$

$$\therefore\ \frac{i^2Rt}{J} = ms\theta \qquad \therefore\ t = \frac{ms\theta J}{i^2R}$$

$$\therefore\ t = \frac{1500\times0{\cdot}6\times10\times4{\cdot}2}{(3)^2\times3} = \mathbf{1400\ sec}$$

$\therefore$ The required time = **1400 sec.**

m = 1500 gm
s = 0·6 cgs
Q = 10°C
J = 4·2 J/Cal
I = 3A
R = 3 Ω

1.7. Principle of least heat

In an electric circuit, when the current is divided into a number of branches in such a manner that heating effect of current is minimum. So, total heat produced in different resistors of the circuit is also minimum.

It is known as principle of least heat.

● **Proof :** Here we establish the principle in the case of a simple circuit consisting of two resistors R_1 and R_2 joined in parallel [fig. 1·6].

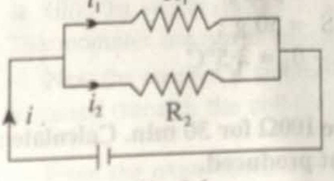


Fig. 1·6

Supply current is i. So, $i = i_1 + i_2$

So, the heat produced in the circuit per sec is

$$H = i_1{}^2R_1 + i_2{}^2R_2 = i_1{}^2R_1 + (i-i_1)^2R_2$$

Heat produced is minimum when $\frac{dH}{di_1} = 0$

Now, $\frac{dH}{di_1} = 2i_1R_1 - 2(i-i_1)R_2 = 2(i_1R_1 - i_2R_2)$

$\therefore$ for minimum heat, $i_1R_1 - i_2R_2 0 \Rightarrow \frac{i_1}{i_2} = \frac{R_2}{R_1}$

Hence, in the case of parallel combination resistances ratio of current in different branches is equal to inverse ratio of their resistances.

Now, from Ohm's law we know that in the case of parallel combination of resistors, i_1 and i_2 obey the same relation.

So, we conclude that current is so divided in different branches that heat produced is minimum.

✪ 1.8. Electrical energy and power ✪

➡ (a) **Electrical energy : The ability of an electric appliance to perform work is called its electrical energy. It is the maximum work that is available from a machine.** Electrical energy is measured by the work done to drive a charge under a potential difference applied to the electrical machine. If Q coulomb of charge flows due to a potential difference V volt is $W = VQ$

$\therefore$ Electrical energy, $E = VQ$ joule

➡ **(b) Power : Energy spent per unit time or work done per unit time called power.**

$$\therefore \text{ Power, } P = \frac{W}{t} = \frac{VQ}{t} = Vi$$

$$\therefore \quad P = i^2R = \frac{V^2}{R} \qquad [\because V = iR]$$

● **Unit of power :** (i) c. g. s. unit is erg. s^{-1}

(ii) SI unit is J. s^{-1} or watt.

● **Practical unit of energy :** (i) **Watt-hour**—The electrical energy supplied for 1 hour by a machine of power 1 watt is called one watt-hour energy.

(ii) **Kilowatt-hr or B. O. T. unit :** The commercial unit of electrical energy is kilowatt-hour (Kwh) or B. O. T. unit.

$$\therefore \quad 1 \text{ B. O. T.} = 1 \text{ Kwh} = 1000 \text{ watt} \times 3600 \text{ sec} = \mathbf{3{\cdot}6 \times 10^6 \text{ J}}$$

$$\therefore \quad \text{No. of B. O. T. units} = \frac{\text{No. of watt hr}}{1000} = \frac{\text{Volt} \times \text{Amp} \times \text{hour}}{1000}$$

➤ *1.8.1. Power of Electrical Appliances connected in parallel :*

Let $R_1, R_2, R_3, \ldots\ldots$ be the resistances of the electrical appliances. They operate at the same voltage V. Let $P_1, P_2, P_3, \ldots\ldots$ be their respective electrical powers. Then

$$P_1 = \frac{V^2}{R_1},\ P_2 = \frac{V^2}{R_2},\ P_3 = \frac{V^2}{R_3},\ \ldots\ldots$$

When the appliances are connected in parallel, their equivalent resistance R is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots\ldots$$

$$\therefore \quad \text{total power consumed is } P = \frac{V^2}{R} = V^2 \left[\frac{1}{R_1} + \frac{1}{R_2} + \ldots\ldots\right]$$

$$\therefore \quad P = \frac{V^2}{R_1} + \frac{V^2}{R_2} + \frac{V^2}{R_3} + \ldots\ldots$$

$$\therefore \quad P = P_1 + P_2 + P_3 + \ldots\ldots$$

➤ *1.8.2. Power of Electrical appliances connected in series :*

The equivalent resistance R when the appliances are connected in series is given by

$$R = R_1 + R_2 + R_3 + \cdots\cdots$$

$$\therefore \quad \text{Total power consumed is, } P = \frac{V^2}{R} = \frac{V^2}{R_1 + R_2 + R_3 + \cdots\cdots}$$

$$\therefore \quad P = \frac{V^2}{\frac{V^2}{P_1} + \frac{V^2}{P_2} + \frac{V^2}{P_3} + \cdots} = \frac{1}{\frac{1}{P_1} + \frac{1}{P_2} + \frac{1}{P_3} + \cdots}$$

$$\therefore \quad \frac{1}{P} = \frac{1}{P_1} + \frac{1}{P_2} + \frac{1}{P_3} + \cdots\cdots$$

⊙ 1.9. Efficiency of a machine ⊙

Efficiency of an electric machine is the ratio of the output or available energy to the input energy. An electric machine converts the supplied electric energy to other form of energy. But the available energy will always be less than energy supplied to it. So, the efficiency of a machine will always be less than 1 i.e. it a fraction. Percentage efficiency is always less than 100%.

- **A discussion :**

(i) The rating of an electric bulb is 220V—100W means that if a potential difference of 220V is applied, the bulb will burn with maximum brightness and it will spend energy at the rate of 100 J/sec.

$$\text{Maximum current through the bulb, } I = \frac{W}{V} = \frac{100}{220} = \mathbf{0 \cdot 45\ A}$$

$$\text{and its resistance, } R = \frac{V^2}{P} = \frac{220 \times 220}{100} = \mathbf{484\ ohm}$$

(ii) **Energy metre :** It is an instrument which measures electric energy consumed by a consumer. Electric supply company supplies energy to domestic or commercial establishments. Energy delivered is measured this energy metre.

⊙ 1.10. Power output of a cell : maximum power theorem ⊙

Any voltage source (say, a cell or a dynamo) may be treated as a series combination of an emf 'E' and an internal resistance r. If the external resistance connected to the cell be R, then current through R is

$$I = \frac{E}{R + r}$$

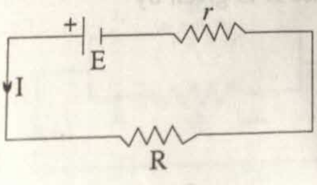


Fig. 1·7

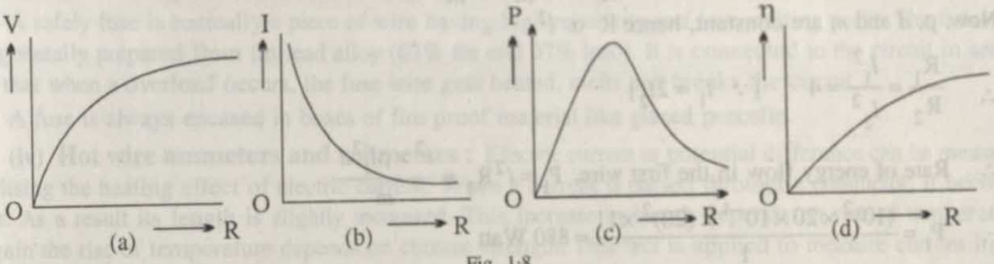
$$\therefore \quad \text{Potential difference across R is } V = IR = \frac{ER}{R + r}$$

$$\text{and power produced in R is } P = VI = \frac{E^2 R}{(R + r)^2} \quad \ldots\ldots\ldots\ldots \text{(i)}$$

$$\therefore \quad \text{The efficiency of the source is } \eta = \frac{\text{Power output across R}}{\text{Input power}}$$

$$\therefore \quad \eta = \frac{I^2 R}{I^2 (R + r)} = \frac{R}{R + r} = \frac{1}{1 + r/R}$$

It is evident that efficiency will be 100% when R is ∞. In fig. 1·8 we see different graphs showing the variation of V, I, P and η with resistance R.



(a) (b) (c) (d)

Fig. 1·8

The observations from the graphs are :

(i) The current in the circuit increases with the decrease of external resistance. When R = 0, the maximum circuit current is E/*r*.

$$\therefore \quad I_{max} = \frac{E}{r} \text{ when } R = 0$$

$$I_{min} = 0 \text{ when } R = \infty$$

(ii) With the increase of circuit current terminal potential difference decreases. The maximum and minimum value of V are

$$V_{max} = E \quad \text{when } I = 0,\ R = \infty$$

$$V_{min} = 0 \quad \text{when } I = \frac{E}{r},\ R = 0$$

(iii) Power produced in R is P = VI. As V increases and I decreases with increase of V, P will be maximum for a particular value of R. The condition for getting maximum power across the load can be obtained in the following way :

$$P = VI = \frac{E^2R}{(R+r)^2} = \frac{E^2R}{(R-r)^2+4rR} = \frac{E^2}{(R-r)^2/R+4r}$$

Hence, P will be maximum when the denominator is minimum. This occurs when **R = *r***. So, the **output power of a source of emf is maximum when the external resistance in the circuit is equal to the internal resistance of the source. It is known as maximum power theorem.**

$$\therefore \quad \text{Maximum power, } P_{max} = \frac{E^2}{4R} = \frac{E^2}{4r}$$

$$\therefore \quad \text{Efficiency under this condition, } \eta = \frac{R}{R+r} = \frac{R}{2R} = \frac{1}{2} = 50\%$$

● **Example 1.6. The mass of each of two uniform wires of same material is 1 gm. The length of one is double the other. They are joined in series and a current of 10A passes through it. If the length of longer wire be 20 cm, what is the rate of flow of energy through each wire ? Which one will be heated more ? Density of the wire is 11 g/c.c. and specific resistance 20 × 10^{-4} Ω-cm.**

[W. B. H. S. 1991]

○ **Solution :** Volume of wire, $V = \frac{m}{d}$ [m = mass, d = density]

$$\therefore \quad Al = \frac{m}{d} \quad [\ V = \text{length} \times \text{cross section} = Al\ ]$$

$$\therefore \quad A = \frac{m}{ld} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

∴ Resistance of the wire, $R = \rho \cdot \frac{l}{A} = \rho l \cdot \frac{ld}{m} = \frac{\rho l^2 d}{m}$

Now, ρ, d and m are constant, hence $R \propto l^2$

$$\therefore \quad \frac{R_1}{R_2} = \frac{l_1^2}{l_2^2} = 4 \qquad [\because l_1 = 2l_2]$$

∴ Rate of energy flow in the first wire, $P_1 = i^2 R_1 = \frac{i^2 \times \rho l_1^2 d}{m}$

$$\therefore \quad P_1 = \frac{(10)^2 \times 20 \times 10^{-4} \times (20)^2 \times 11}{1} = 880 \text{ Watt}$$

and rate of energy flow in the second wire, $P_2 = i^2 R_2 = i^2 \cdot \frac{R_1}{4} = 220$ watt

● **Example 1.7. The electric bulbs each of rating 200V—500W are connected in series and then joined in 110V line. Calculate the power produced in each bulb.** **[W. B. H. S. 2002]**

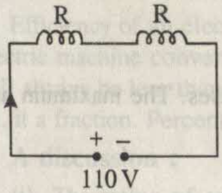


Fig. 1·8(a)

○ **Solution :** Resistance of each bulb $R = \frac{V^2}{P} = \frac{(220)^2}{500} = \frac{484}{5}$ ohm

Now, line current, $I = \frac{110 \times 5}{2 \times 484} = \frac{25}{44}$ A

∴ Power developed each bulb, $P = I^2 R$

$$\therefore \quad P = \left(\frac{25}{44}\right)^2 \times \frac{484}{5} = \mathbf{31{\cdot}25 \text{ Watt}}$$

✪ 1.11. Practical application of heating effect of current ✪

The heating effect of electric current has a wide range of applications in every daylife. Different electrical appliances have been designed by making use of heating effect of electric current. Here we discuss a few of them.

● **(i) Electric bulb :** Of all the electrical appliances designed by making use of heating effect of electric current. Electric incandescent lamp, ordinarily called an electric bulb is most widely used. It consists of a heating element called filament made of fine matallic wire enclosed in a glass tube filled with inert gas at a suitable low pressure. The material of the filament wire has a large value of resistivity and high melting point. Usually tungsten is used to make the filament of the bulb. When current is passed through the filament, it gets heated to incandescence. The temperature of the filament can be raised to about 2100°C in the vacuum lamp.

The electric power and voltage specification of the electric bulb is marked over it. Since $P = \frac{V^2}{R}$, it follows that the resistance of the filament of an electric bulb is inversely proportional to its power.

● **(ii) Electric appliance :** Electric heater, electric iron, electric stove, toaster, electric geyser etc., are some of the useful electric appliances which works on the principle of heating effect of electric current. In all these appliances resistance coil of high resistance and high melting point is used. The heating is usually made of michrome (an alloy of nickel and chromium). The coil is usually wound on fire clay, mica, porcelin or on any insulating material.

When current is passed through the coil, it becomes red hot and the appliance also becomes hot.

● **(iii) Electric fuse :** It is a safety device used to protect electric installations against overloading. By overload we mean a condition when large current flows through a circuit than it is designed to carry. This usually occurs due to short circuit. Fuse is also used in electric appliances to save them getting damaged due to flow of heavy current.

A safely fuse is basically a piece of wire having high resistivity and low melting point. The fuse wire is generally prepared from tin-lead alloy (63% tin end 37% lead). It is connected to the circuit in series, so that when a overload occurs, the fuse wire gets heated, melts and breaks the circuit.

A fuse is always encased in boxes of fire proof material like glazed porcelin.

● (iv) **Hot wire ammeters and voltmeters :** Electric current or potential difference can be measured utilising the heating effect of electric current. When a current is passed through a conductor, it becomes hot. As a result its length is slightly increased. This increase in length depends on rise of temperature. Again the rise of temperature depends on current strength. This fact is applied to measure current in hot wire instrument.

As the heating effect of electric current is proportional to the square of the current, such instruments can be used both on D. C. and A. C.

●● Short Answer Type Questions (with answers) ●●

● **Question 1. Two wires of equal length and area of cross-section but made of different materials are kept under same potential difference. Which one will produce more heat ?**

○ **Ans.** The wire having smaller resistivity will produce more heat.

Explanation : For a constant potential difference, $H \propto \frac{1}{R}$ $\therefore\ H \propto \frac{A}{\rho l}$

In this case A and l are constant. Hence $H \propto \frac{1}{\rho}$

$\therefore\ \frac{H_1}{H_2} = \frac{\rho_2}{\rho_1}$ Hence higher the resistivity less is the heat produced.

● **Q. 2. Define B. O. T. unit. Show that the unit of the product of potential difference and current and the unit of power are same.** **[W. B. H. S. 1998, 2001]**

○ **Ans. 1st part :** B. O. T. is the commercial unit electrical energy 1 B. O. T. = 1 Kwh.

So, 1 B. O. T. is the amount of electrical energy supplied for 1 hr by a machine of power 1 KW.

2nd part : Potential difference × current = Volt × Amp. = $Jc^{-1} \times cs^{-1} = JS^{-1}$

Now, $\text{Power} = \frac{\text{work}}{\text{time}}\ JS^{-1}$

● **Q. 3. Why is Joule's constant called mechanical equivalent of heat ?** **[W. B. H. S. 2000]**

○ **Ans.** When mechanical work is down, heat is produced. Heat and work are related to each other. The equivalence between heat and mechanical work is expressed by Joule's constant. For this reason Joule's constant is called mechanical equivalent of heat.

● **Q. 4. What is meant by $J = 4{\cdot}2\ J\ Cal^{-1}$** **[W. B. H. S. 1997]**

○ **Ans.** Mechanical equivalent of heat is $J = 4{\cdot}2\ J\ Cal^{-1}$ means that to produce 1 cal of heat, 4·2 J of work has to be performed.

● **Q. 5. What is the largest voltage you can safely put across a 98Ω-0·5 W resistor ?**

○ **Ans.** Here R = 18 ohm, P = 0·5 watt

Now, $P = \frac{V^2}{R}$ $\therefore\ V = \sqrt{PR} = \sqrt{0{\cdot}5 \times 98} = 7\,V$

● **Q. 6. The ratings of two electric bulb are 240V–1000W and 240V–100W. The filament of which bulb is thin ? If the two bulbs are connected in series and put across the main supply, which one will glow brighter ?** **[W. B. H. S. 1994]**

○ **Ans. 1st part :** For the same potential difference, $P \propto \frac{1}{R}$

$\therefore \quad P \propto \frac{A}{\rho . l} \quad \therefore \; P \propto A \; [\, \rho_1 l$ cm constant]

So, the bulb with higher power will have its filament of larger cross-section. Hence the filament of 100 W bulb is thin.

2nd part : When the bulbs are connected in series 100 W bulb will glow brighter than 1000W bulb. Reason is—as the bulbs are connected in series, equal current passes through them. So the bulb with higher resistance will produce more heat and glow brighter. The resistance of 100W bulb being larger will glow brighter.

● **Q. 7. 60V–120W bulb is required to be connected to 220 V d. c. line. What resistance should be connected in series so that it can glow with full brightness ?**

○ **Ans.** The resistance of the bulb is $R = \frac{V^2}{P} = \frac{60 \times 60}{120} = 30\Omega$

Its maximum or rated current, $I = \frac{V}{R} = \frac{60}{30} = 2A$

Let R be the required resistance to be connected in series so that it can glow full is 220V line. Then

$(R + 30) \times 2 = 220 \quad \therefore \; \mathbf{R = 80\ ohm}$

● **Q. 8. A 220V–60W carbon filament bulb and a 220V–60W metal filament bulb are connected in series with a 220V supply. Which bulb, if any, will glow brighter.**

○ **Ans.** When the two bulbs are connected in series and 220V is applied to the combination, carbon filament bulb will glow brighter.

Explanation : When the bulbs are connected in parallel to 220V supply both the bulbs will glow with full brightness and they have equal resistance and they have equal power.

Now, if the bulbs are connected in series and 220V is applied, they will carry current smaller than rated value. Their brightness will decrease and temperature will be smaller than in parallel combination. Carbon being a non-metal its resistance will be higher than the metal filament at lower temperature. As the bulb are now carrying equal current, carbon filament bulb will glow brighter than metal filament bulb.

● **Q. 9. The equation for power $P = i^2R$ shows that if the resistance decreases, the rate of heat produced due to Joule's heating also decreases. But the equation $P = \frac{V^2}{R}$ shows that the rate of heat produced increases if the resistance is smaller. How can you explain this apparent discrepency ?**

○ **Ans.** If the resistance in a circuit is constant, $P \propto R$. When several resistors are connected in series and a voltage is applied to them, equal currents flows through them. So, the rate of heat produced in each resistor is proportional to its resistance. It means that the resistor of higher resistance will produce heat at highter rate.

On the other hand, in the equation $P = \frac{V^2}{R}$, we get $P \propto \frac{1}{R}$ when the voltage V is constant. For example, when several resistors are connected in parallel and a potential difference is applied to the combination, rate of heat produced is inversely proportional to resistance. So in this case higher the resistance, heat is produced at smaller rate.

Hence in the first equation i is kept constant, but in the second equation is held constant. There is no contradiction between the two equations.

● **Q. 10. When an electric store is connected to a 220 V supply, it produces heat at the rate of 22000 cel / min. What is the resistance of the store ?**

○ **Ans.** The required relation : $\mathrm{H} = \frac{\mathrm{V}^2}{\mathrm{JR}}t \quad \Rightarrow \quad \mathrm{R} = \frac{\mathrm{V}^2 t}{\mathrm{JH}}$

$$\therefore \quad \mathrm{R} = \frac{(220)^2 \times 60}{4 \cdot 2 \times 22000} = \mathbf{31 \cdot 43\ ohm}$$

● **Q. 11. 40 lamps are first connected in series and 220V is applied to the combination. One lamp is fused. Then the rest 39 lamps are connected to the same source in series combination. In which case the bulbs glows brighter ? [I. I. T.]**

○ **Ans.** In the second case the bulbs glows brighter.

Explanation : Let the resistance of each bulb is R.

In the first case, line current, $i_1 = \frac{220}{40\mathrm{R}}$

$$\therefore \quad \frac{i_2}{i_1} = \frac{40}{39} \quad \therefore \ i_2 > i_1$$

Since $\mathrm{H} \propto i^2$, in the second case more heat is produced and so each bulb will glow brighter in the second case.

● **Q. 12. A 50W bulb is connected in series with a room heater and then to the main supply. If 100W bulb is used instead of 50W bulb, will the heater produce more or less or equal heat ? [I. I. T.]**

○ **Ans.** Room heater will produce more heat in the second case.

Reason : Resistance of 100W bulb is less than 50W bulb. Hence when 100W bulb is used the resistance in the circuit will decrease. Since the supply remains equal, current in the circuit will increase. Hence the rate of heat produced in the second case is larger.

● **Q. 13. The water in an electric kettle starts boiling 15 min after the switch is closed. If it is required to boil the same water in 10 min, is it required to increase or decrease the length of the heating coil ? [I. I. T.]**

○ **Ans.** To boil the water in 10 min instead of in 15 min, the length of the coil has to be decreased.

Explanation : Heat produced by the heating coil, $\mathrm{H} = \frac{\mathrm{V}^2}{\mathrm{R}}t$

Now, if V is constant, $\mathrm{H} \propto \frac{t}{\mathrm{R}}; \quad \mathrm{R} = \rho \frac{l}{\mathrm{A}}$

$$\therefore \quad \mathrm{H} \propto \frac{t\mathrm{A}}{\rho l} \quad \therefore \ \mathrm{H} \propto \frac{t}{l} \quad [\text{ if A and } \rho \text{ are constant }]$$

$$\therefore \quad \frac{\mathrm{H}_1}{\mathrm{H}_2} = \frac{t_1}{t_2} \times \frac{l_2}{l_1}.$$ Since equal amount of heat is required in both the cases,

$$\frac{l_2}{t_2} = \frac{l_1}{t_1} \qquad \therefore \ l_2 = \frac{l_1 t_2}{t_1} = l_1 \times \frac{10}{15} = \frac{2}{3} l_1$$

$\therefore$ Length of the heating coil in the second case should be $\frac{2}{3}$ of the initial length.

● **Q. 14. A battery of emf E and internal resistance r is put across an external resistance R. Show that when R = r, the rate of heat produced in external resistance will be maximum. What fraction produced in spent in external resistance under this condition ?**

○ **Ans.** Consider the fig. 1.9, current in the circuit, $i = \dfrac{E}{R+r}$

$\therefore$ Heat produced in external resistance, $H = \dfrac{E^2 R}{(R+r)^2}$

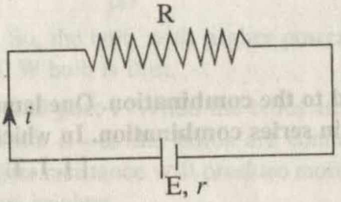


Fig. 1·9

Condition for maximum heat production is $\dfrac{dH}{dR} = 0$

$$\therefore \quad \frac{d}{dR}\left[\frac{R}{(R+r)^2}\right] = 0$$

$$\text{or,} \quad \frac{(R+r)^2 \cdot 1 - 2R(R+r)}{(R+r)^3} = 0. \quad \therefore \quad R = r$$

In can be shown that $\dfrac{d^2 H}{dR^2}$ is negative when R = r. So it is the condition for maximum heat production.

$\therefore$ Maximum power developed across R is $P_{max} = \dfrac{E^2 R}{(2R)^2} = \dfrac{E^2}{4R}$

$\therefore$ Now, maximum power developed by the cell, $P = EI = \dfrac{E^2}{R+r} = \dfrac{E^2}{2R} \quad \therefore \quad \dfrac{P_{max}}{P} = \dfrac{1}{2}$

$\therefore$ Half of the power produced by the battery is spent across the external resistance.

● **Q. 15. An accumulator is connected successively to two external resistors R and S. If both the resistors produce heat at equal rate. What is the internal resistance of the accumulator ?**

○ **Ans.** Let the internal resistance of the accumulator be r and its emf E.

In the first case, current through R is $i_1 = \dfrac{E}{R+r}$

$\therefore$ Rate of heat production, $Q_1 = \dfrac{E^2 R}{(R+r)^2}$

Similarly, the rate of heat produced in the resistor S is $Q_2 = \dfrac{E^2 S}{(r+S)^2}$

Now, by question, $Q_1 = Q_2 \quad \therefore \quad \dfrac{E^2 R}{(R+r)^2} = \dfrac{E^2 S}{(S+r)^2}$

$\therefore \quad \sqrt{R}\,(S+r) = \sqrt{S}\,(R+r)$

$\therefore \quad r(\sqrt{R} - \sqrt{S}) = R\sqrt{S} - S\sqrt{R} = \sqrt{RS}\,(\sqrt{R} - \sqrt{S}) \quad \therefore \quad r = \sqrt{RS}$

● **Q. 16. In an electric cell energy is produced at the rate 1 watt / Amp due to chemical reaction occurring inside the cell. When 10A current passes through the cell energy is spent at the rate 1 watt / sec. Find potential difference and internal resistance of the cell.**

○ **Ans.** When 10A current passes through the cell, its power = 10 watt

If the potential difference of the cell be V then,

10 = VI + loss of energy per sec inside the cell

$\therefore \quad 10 = V \times 10 + 1{\cdot}0 \qquad \therefore \quad V = 0{\cdot}9$ volt

Now, if the internal resistance of the cell be r, then loss of energy per sec within the cell is $I^2 r$

$$\therefore \quad I^2 r = 1 \quad \therefore \quad r = \frac{1}{I^2} = \frac{1}{100} = \mathbf{0{\cdot}01\,\Omega}$$

● **Q. 17. If the potential difference applied to a bulb decreases by 1%, what is the percentage change in power of the bulb ?**

○ **Ans.** Let the initial potential difference be V then final potential difference is 0·99 V.

Initial power, $P_1 = \frac{V^2}{R}$ and final power, $P_2 = \frac{(0 \cdot 99V)^2}{R}$

$$\therefore \frac{P_2}{P_1} = (0 \cdot 99)^2 = (1 - 0 \cdot 1)^2 = 1 - 0 \cdot 02$$

$$\therefore \frac{P_2 - P_1}{P_1} = 0 \cdot 02 \qquad \therefore \text{ loss of power} = 0 \cdot 02 \times 100 = \mathbf{2\%}$$

● **Q. 18. When a potential difference is applied to the parallel combination of two identical bulbs, power developed is P. What will be the power if the bulbs are connected in series under the same potential difference ?**

○ **Ans.** In the first case [fig. 1·10(i)], the bulbs are in parallel. If the applied potential difference be V then power produced by the combination is

$$P = \frac{V^2}{R/2} = \frac{2V^2}{R} \quad \ldots\ldots\ldots\ldots\ (i)$$

[R = resistance of each bulbs]

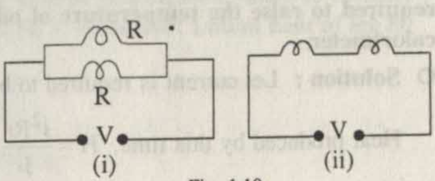


Fig. 1·10

In the second case [fig. 1·10(ii)], the bulbs are in parallel, power developed now is

$$P' = \frac{V^2}{2R} = \frac{P}{4} \qquad \therefore \mathbf{P' = \frac{P}{4}}$$

● **Q. 19. Which of the following quantities has the unit Amp-sec ? Emf, force, energy, power or charge. Explain your answer.** **[W. B. H. S. 1991]**

○ **Ans.** Amp-sec is the unit of charge. **Reason :** Amp-sec = current × time.

Now, current is the charge flowing per sec. So, $I = \frac{q}{t}$ or, $q = It$

$\therefore$ $\mathbf{q = Amp. \times sec}$

● **Q. 20. The rating of a resistor is 98Ω – 0·5W. What maximum voltage can be applied across the resistor ?**

○ **Ans.** The maximum voltage that can be applied to the resistor is obtained from

$$P = \frac{V^2}{R} \quad \therefore V = \sqrt{PR} \quad \therefore V = \sqrt{0 \cdot 5 \times 98} = 7 \text{ volt}$$

● **Q. 21. What should be the characteristics of a heating coil ?**

○ **Ans.** The heating coil material must have high resistivity and high melting point.

● **Q. 22. A heating coil is divided into two equal parts. One part is used as a heating coil under the same potential difference. Will more heat be produced in the present case ?**

○ **Ans.** In the second case rate of heat produced will be twice the rate of heat production of the first case.

Reason : The required relation for heat produced in time *t* in a coil of resistance R due to a potential difference V is

$$H = \frac{V^2}{R} \cdot t$$

Now, the resistance of one part of the coil, $R' = \frac{R}{2}$

$\therefore$ Heat produced now is $H' = \frac{V^2}{R'}t = \frac{2V^2}{R}t = 2H$

● **Q. 23. When current is passed through a steel wire it becomes red hot. If the lower half is immersed in water, in which part more heat is produced ? Explain your answer.**

○ **Ans.** The upper half which is above water will now produce more heat. So it will be more heated.

Reason : The part in water will be at lower temperature then the upper half. As resistance increases with rise temperature, so, the resistance of the upper half will be greater. Since same current passes through both the halves, the upper half will produced greater heat ($H \propto R$).

● ● Numerical Examples ● ●

● **Example 1.8. 1·5 Kg oil of specific heat 0·6 cgs is taken in a Joule calorimeter. A heating coil of resistance 3Ω is immersed in the oil and a current of 3A is passed through it. Calculate the time required to raise the temperature of oil by 10°C. J = 4·2 J / cal. Neglect water equivalent of the calorimeter.**

[W. B. H. S. 1980]

○ **Solution :** Let current is required to be passed for t sec.

Heat produced by this time, $H = \frac{i^2Rt}{J} = \frac{3^2 \times 3 \times t}{4 \cdot 2} = \frac{27t}{4 \cdot 2}$ cal

Heat required to raise the temperature of oil by 10°C is

$H = ms\theta = 1500 \times 0 \cdot 6 \times 10 = 9 \times 10^3$ cal

$\therefore \frac{27t}{4 \cdot 2} = 9 \times 10^3 \quad \therefore t = \frac{9 \times 10^3 \times 9 \cdot 2}{27} = \mathbf{1400\ sec}$

● **Ex. 1.9. An electric kettle of power 500 watt can raise the temperature of 1 litre water from 25° to 100°C in 15 min. What percentage of electric energy is spent in heating ?**

○ **Solution :** Heat produced by the heater in 15 min, $Q_1 = 500 \times 15 \times 60$ Joule

Heat absorbed to convert 1 litre water from 25°C to 100°C is

$Q_2 = m\,s\,\theta = 1000 \times 1 \times 75 = 75000 \text{ cal} = 7 \cdot 5 \times 4 \cdot 2 \times 10^4$ J

$\therefore$ Percentage heat spent $= \frac{Q_2}{Q_1} \times 100 = \frac{7 \cdot 5 \times 4 \cdot 2 \times 10^4}{500 \times 15 \times 60} \times 100 = \mathbf{70\%}$

● **Ex. 1.10. An electric furnace is made of three resistors of equal resistances. When the resistors are connected in parallel, the water in a kettle boils in 6 min. Now, if the resistors are connected in series after what time the water will boil ?**

[J. E. E. 1989]

○ **Solution :** Let the resistance of each resistor is R. So, in parallel combination equivalent resistance is $R_1 = \frac{R}{3}$.

$\therefore$ Heat produced the time t_1 is $Q_1 = \frac{V^2}{R_1}t_1 = \frac{3V^2}{R} \cdot t_1$ .... (i)

When the resistors are connected in series, equivalent resistance is $R_2 = 3R$

$\therefore$ Heat produced in time t_2, $Q_2 = \frac{V^2 t_2}{R_2} = \frac{V^2 t_2}{3R}$ .... (ii)

Now, $Q_1 = Q_2$. Hence $\frac{3V^2}{R} \cdot t_1 = \frac{V^2 t_2}{3R}$ $\therefore$ $t_2 = 9t_1 = 54$ **min**

● **Ex. 1.11. In a house there are three electric bulbs each of 60W, two bulbs each of 100W and three fans each of power 40W. Supply voltage 200V. If all these appliances are run simultaneously, what current flows in supply line ? If each bulb runs for 5 hrs per day and each fan runs for 15 hrs per day, what is the expenditure in one month of 30 days. 1 B. O. T. 50 P. [J. E. E. 1982]**

○ **Solution :** Total power of the bulbs and the fan = 3 × 60 + 2 × 100 + 3 × 40 = 500 W

and voltage is 200 V

$\therefore$ Supply line current, $I = \frac{500}{200} = 2 \cdot 5$ **A**

Now, energy spent in one day = (180 + 200) × 5 + 120 × 15 = 3700 watt-hr.

$\therefore$ Energy spent in one month of 30 days = 3700 × 30 = 111 Kwh = 111 B. O. T.

$\therefore$ Total expenditure = 111 × 50 P = **Rs. 55·05**

● **Ex. 1.12. When an electric kettle is connected to 220V line, 4A flows in the heating coil of the kettle. Calculate (a) Resistance of the heating coil, (b) if 40% of heat is lost, time taken to convert 1 kg ice at 0°C to steam at 100°C, (c) total expenditure if 1 B. O. T. costs 40P. Latent heat of ice 80 cal / gm and latent heat of steam = 540 cal / gm.**

○ **Solution :** (i) Resistance of the heating coil, $R = \frac{220}{4} = 55\Omega$

(ii) Heat required to convert 1 kg ice at 0°C to steam at 100°C is

$= 1000 \times 80 + 1000 \times 100 + 1000 \times 540 = 72 \times 10^4$ cal.

Heat produced by the heater $= 72 \times 10^4 \times \frac{100}{60} = 12 \times 10^5 \text{ cal} = 12 \times 4 \cdot 2 \times 10^5$ J

Now, $Vit = 12 \times 4 \cdot 2 \times 10^5$

$\therefore$ $t = \frac{12 \times 4 \cdot 2 \times 10^5}{220 \times 4} = \mathbf{5727 \cdot 3}$ **sec**

$\therefore$ time required = 1 hr 35 min 27·3 sec

(iii) Energy generated = $12 \times 4{\cdot}2 \times 10^5$ watt sec $= \frac{12 \times 4 \cdot 2 \times 10^5}{1000 \times 3600}$ Kwh = 1·4 Kwh = 1·4 B. O. T.

$\therefore$ Expenditure = 1·4 × 40 = **56 P**

● **Ex. 1.13. Three aluminium wires of equal cross-section, but of lengths 1m, 2m and 3m are connected in parallel to a battery. Next instead of these aluminium wires, a single wire of same material, but double the diameter is put across the battery. If current in both the cases is same, calculate the length of the wire.**

○ **Solution :** In the **first case** : Let area of cross-section of each wire in the first wire is A m^2 and resistivity = ρ Ω-cm

$\therefore$ Resistance of the first wire, $R_1 = \rho \cdot \frac{1}{A}$

and resistance of the second and third wire, $R_2 = \rho \cdot \frac{2}{A}$, $R_3 = \rho \cdot \frac{3}{A}$

If the applied voltage is V, current in the first wire, $i_1 = \frac{V}{R_1}$

Current in the second wire, $i_2 = \frac{V}{R_2}$ and the third wire, $i_3 = \frac{V}{R_3}$

$\therefore$ Current supplied by the battery, $i = i_1 + i_2 + i_3$

$$\therefore \quad i = V\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right] = V\cdot\frac{A}{\rho}\left(1+\frac{1}{2}+\frac{1}{3}\right) = \frac{11}{6}\frac{VA}{\rho}$$

In the **second case :** Let the required length of the wire be l metre and its area of cross-section $A_1 = 4A$ and the resistance of the wire, $R' = \rho\cdot\frac{l}{4A}$

$\therefore$ Current through the battery, $i' = \frac{V}{R'} = \frac{4VA}{\rho l}$

Since $i = i'$ then, $\frac{4VA}{l\rho} = \frac{11}{6}\frac{VA}{\rho} \Rightarrow l = \mathbf{2\cdot18\ m}$

● **Ex. 1.14. The rating of two electric bulbs are 25W–110V and 100W–110V. The bulbs are connected in series and the combination is put to a potential difference V. Will any bulb be fused ? If yes, which one ? If the parallel combination of the two bulbs be connected to the same potential difference V, what will happen ?**

○ **Solution : First case :** Resistance of the first bulb, $R_1 = \frac{V^2}{P} = \frac{110\times110}{25} = \mathbf{484\ \Omega}$

and the resistance of the second bulb, $R_2 = \frac{V^2}{P_1} = \frac{110\times100}{100} = \mathbf{121\ \Omega}$

Maximum current in the first bulb, $I_1 = \frac{P}{V} = \frac{25}{110} = 0\cdot23\ A$

and maximum current in the second bulb, $I_2 = \frac{P_1}{V} = \frac{100}{110} = 0\cdot92\ A$

When the bulbs are connected in series, total resistance,

$$R = R_1 + R_2 = 484 + 121 = \mathbf{605\ \Omega}$$

Now, if the combination is put to 220V supply line, then supply current,

$$I = \frac{220}{605} = 0\cdot36\ A \qquad \therefore\ I_1 < I \text{ and } I_2 > I$$

As the first bulb now carries current more than its rated current, so it will fuse.

Second case : If the bulbs are connected in parallel and if the supply voltage be 220 volt, then current supplied to the first bulb is

$$I' = \frac{220}{484} = \mathbf{0\cdot454\ A}$$

and current to the second bulb is $I'' = \frac{220}{121} = \mathbf{1\cdot82\ A}$

Now, $I' > I_1$ and $I'' > I_2$. Hence both the bulbs will fuse.

● **Ex. 1.15. (a) A battery made of six accumulators each of emf 2·0 V and internal resistance 0·5 ohm is being charged by connecting it to a 100V supply. (a) What resistance be connected in series with the battery so that current in the circuit does not exceed 8·0 A ? With the required resistance in the circuit (b) Calculate : (i) Energy supplied by the source and (ii) Loss of energy as heat and (iii) Energy stored in the battery after 15 min.**

○ **Solution :** (a) Let the required resistance be R [fig. 1·11]

$\therefore \quad V = nE + I\,(nr + R)$

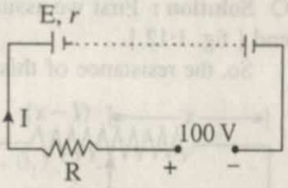


Fig. 1·11

$\therefore\ 100 = 6 \times 2{\cdot}0 + 8\,(0{\cdot}5 \times 6 + \text{R}) \quad \therefore\ \text{R} = 8\Omega$

$\therefore$ The required resistance is R = **8Ω**

(b) (i) Energy supplied by the source = 100 × 8 = 800 watt.

(ii) Loss of energy as heat

$= \text{I}^2\,(\text{R} + nr) = 8^2\,(8 + 3{\cdot}0) = 704$ watt

(iii) Energy stored in battery after 15 min

$= (800 - 704) \times 15 \times 60 = \mathbf{8{\cdot}64 \times 10^4\ J}$

● **Ex. 1.16. An electric kettle has two heating coils. When one coil is switched on, the water in the kettle boils in 6 min and when other coil is switched on it takes 8 min to boil the same water. Calculate the time required (i) when the coils are connected in series and (ii) when the coils connected in parallel.**

○ **Solution :** In both the cases equal amount of heat Q will be required to boil the water. Let the potential difference be V. First coil resistance is R_1 and that of the second coil is R_2.

When the first coil is switched on $\text{Q} = \dfrac{\text{V}^2}{\text{R}_1} t_1 \quad [t_1 = 6 \text{ min}]$ . . . . (i)

When the second coil is switched on $\text{Q} = \dfrac{\text{V}^2}{\text{R}_2} t_2 \quad [t_2 = 8 \text{ min}]$

From these two equations $\dfrac{t_1}{\text{R}_1} = \dfrac{t_2}{\text{R}_2} \quad \therefore\ \dfrac{t_1}{t_2} = \dfrac{\text{R}_1}{\text{R}_2}$ . . . . (ii)

(i) Let the water boils in time t_3 when the coils are connected in series. Then,

$$\text{Q} = \frac{\text{V}^2}{\text{R}_1 + \text{R}_2} \cdot t_3 \ \ldots\ldots\ldots\ldots \text{(iii)}$$

$\therefore$ From (i) and (iii) $\dfrac{t_3}{\text{R}_1 + \text{R}_2} = \dfrac{t_1}{\text{R}_1} \quad \Rightarrow t_3 = t_1 \cdot \dfrac{\text{R}_1 + \text{R}_2}{\text{R}_1}$

From the equation (ii), $\dfrac{\text{R}_2}{\text{R}_1} = \dfrac{t_2}{t_1} \quad \therefore\ \dfrac{\text{R}_1 + \text{R}_2}{\text{R}_1} = \dfrac{t_1 + t_2}{t_1}$

$$\therefore\ t_3 = t_1 \times \frac{t_1 + t_2}{t_1} = t_1 + t_2 = 6 + 8 = 14 \text{ min}$$

$\therefore$ Now the water will boil in 14 min.

(ii) Assume that water boils in time t_4 when the coils are connected in parallel for the same potential difference V. Then

$$\text{Q} = \frac{\text{V}^2\,(\text{R}_1 + \text{R}_2)}{\text{R}_1 \text{R}_2} \times t_4 \ \ldots\ldots\ldots\ldots \text{(iv)}$$

$$\therefore\ \frac{\text{R}_1 + \text{R}_2}{\text{R}_1 \text{R}_2} \cdot t_4 = \frac{t_2}{\text{R}_2} \quad \therefore\ t_4 = \frac{\text{R}_1}{\text{R}_1 + \text{R}_2} \times t_2 = \frac{t_1 t_2}{t_1 + t_2} = \frac{8 \times 6}{14} = 3{\cdot}43 \text{ min}$$

So, now the water will boil in **3·43 min**

● **Ex. 1.17. A D. C. source is connected to a rheostat. When the moving contact point is at a distance of *x* cm from any one side of rheostat, equal amount of heat is produced in both the cases. What is the internal resistance of the D. C. source. Given, the resistance of the rheostat is R per cm.**

[J. E. E. 1990]

○ **Solution :** First we assume that the contact point of the rheostat be at a distance of x cm from the left end [fig. 1·12].

So, the resistance of this part of the rheostat = xR ohm

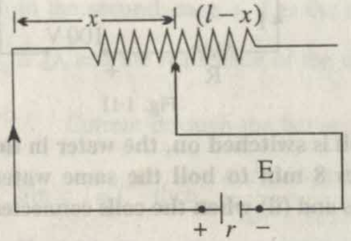


Fig. 1·12

Now, current in the circuit, $i_1 = \dfrac{E}{xR + r}$

[r = internal resistance of the cell]

∴ Rate of heat production, $Q_1 = \dfrac{E^2 xR}{(xR + r)^2}$ . . . . (i)

In the second case, the resistance of the portion of the rheostat included in the circuit is R ($l - x$), here l is the length of the rheostat.

∴ Circuit current, $i_2 = \dfrac{E}{R\,(l-x) + r}$

∴ Rate of heat production, $Q_2 = \dfrac{E^2\,(l-x)\,R}{[R\,(l-x)+r]^2}$ . . . . (ii)

Now, by question $Q_1 = Q_2$ ∴ $\dfrac{E^2\,x\,R}{(x\,R + r)^2} = \dfrac{E^2\,(l-x)\,R}{[R\,(l-x)+r]^2}$

∴ $(l-x)\,(xR + r)^2 = x\,[R(l-x) + r]^2$

or, $\sqrt{l-x}\;(xR + r) = \sqrt{x}\;[R\,(l-x) + r]$ or, $\sqrt{l-x}.r - \sqrt{x}.r = \sqrt{x}\,.R\,(l-x) - x\,R\,\sqrt{l-x}$

or, $r\,[\sqrt{l-x} - \sqrt{x}\,] = \sqrt{x}\,.R\,\sqrt{l-x}\,(\sqrt{l-x} - \sqrt{x})$ ∴ $\mathbf{r = R\,\sqrt{x\,(l-x)}}$

● **Ex. 1.18. In the circuit given in fig. 4·13, due to the flow of current heat at the rate of 10 cal/sec is produced in 5Ω resistor. At what rate heat is produced in 4Ω coil ?**

○ **Solution :** Let the difference of potential between A and B is V. Then

$\dfrac{V^2}{5} = 10 \times 4{\cdot}2$ or, $V = 14{\cdot}5$ volt

∴ Current in 4 ohm resistor, $i_1 = \dfrac{V}{10} = \dfrac{14{\cdot}5}{10} = 1{\cdot}45$ A

∴ Rate of heat production in 4Ω resistor is

$P = i_1{}^2 R = (1{\cdot}45)^2 \times 4 = 8{\cdot}41\ \mathrm{J} = \mathbf{2\ cal\ s^{-1}}$

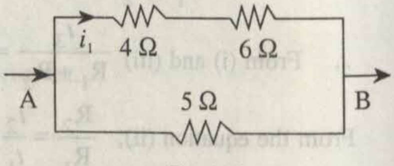


Fig. 1·13

● **Ex. 1.19. The circuit shown in fig 1·14 is kept inside an ice box. Temperature of ice is 0°C. How much ice has to be added to the box so that mass of ice in the box always remain unchanged. Latent heat of ice = 80 cal / g.**

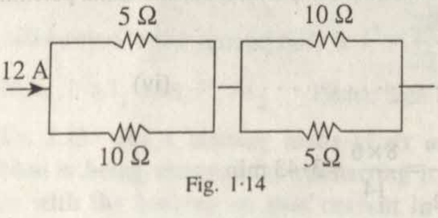


Fig. 1·14

○ **Solution :** Equivalent resistance, $R = \dfrac{20}{3}\ \Omega$

Heat produced in 10 min,

$H = i^2 Rt = (12)^2 \times \dfrac{20}{3} \times 600\ \mathrm{J} = 144 \times 4 \times 10^3 \times 0{\cdot}24$ cal

The mass of ice to be added to the box during this time is

$m = \dfrac{144 \times 4 \times 0{\cdot}24 \times 10^3}{80} = 1728\ \mathrm{gm} = \mathbf{1{\cdot}728\ kg}$

● **Ex. 1.20. The length of a copper wire is 0·1 m, area of cross-section 0·5 mm² and initial temperature 25°C. A current of 10A passes through it. (i) After what time, the wire will start melting ? (ii) If the length of the wire is made double, what is the required time ? Given, density of copper = 9 × 10³ kg/m³, Sp. heat 9 × 10⁻² K cal Kg⁻¹ °C⁻¹. Melting point of copper = 1075°C and its resistivity = 1·6 × 10⁻⁸ Ω-m.**

○ **Solution :** (i) Resistance of copper wire, $R = \rho \dfrac{l}{A}$

Heat produced, $H = \dfrac{I^2 R t}{J}$ .... (i)

Mass of copper wire $= l\,A\,d$ [d = density]

Heat required to raise its temperature from θ_1 or θ_2, $H' = l\,A\,d\,s\,(\theta_2 - \theta_1)$

$$\therefore \quad \frac{I^2 \rho\, l\, t}{AJ} = l\,A\,d\,s\,(\theta_2 - \theta_1)$$

$$\therefore \quad t = \frac{A^2\, d.s.\, J}{I^2 \,.\, \rho}\,(\theta_2 - \theta_1)$$

$A = 0{\cdot}5 \times 10^{-6}\ \text{m}^2$; $d = 9 \times 10^3\ \text{kg/m}^3$

$s = 90\ \text{cal / kg}^{-1}\ {}^\circ\text{C}^{-1}$

$\theta_2 - \theta_1 = 1075 - 25 = 1050\ {}^\circ\text{C}$

$I = 10\ \text{A}$; $\rho = 1{\cdot}6 \times 10^{-8}\ \Omega\text{-m}$

$$\therefore \quad t = \frac{(0{\cdot}5 \times 10^{-6})^2 \times 9 \times 10^3 \times 90 \times 4{\cdot}2 \times 1050}{(10)^2 \times 1{\cdot}6 \times 10^{-8}} = 558 \text{ sec } = \mathbf{9 \text{ min } 18 \text{ sec}}$$

(ii) If length of the copper wire is made double, its resistance will also be double. If the current is kept constant, heat produced H′ = 2H.

So, time required will be half of the above time.

● **Ex. 1.21. A heater spends 500 W under a potential difference of 220 V. (a) What is the resistance of the heating coil ? (b) If the potential difference be 200V, what will the power ? (c) If a bulb of rating 220V–100W is connected in series, calculate the power produced by the heater and the bulb. Assume the supply voltage to be 220 in this case.**

○ **Solution :** (a) Resistance of the heating coil, $R = \dfrac{V^2}{P} = \dfrac{(220)^2}{500} = \mathbf{96{\cdot}8\ \Omega}$

(b) If the applied voltage be reduced to 200V

Power is $P' = \dfrac{V^2}{R} = \dfrac{(220)^2}{96{\cdot}8} = \mathbf{413 \text{ watt}}$

(c) Resistance of the bulb, $R_1 = \dfrac{(220)^2}{100} = 484\ \Omega$

$\therefore$ Current through heater and the bulb, $i_1 = \dfrac{220}{484 + 96{\cdot}8} = 0{\cdot}379\ \text{A}$

$\therefore$ Power produced in heater, $P_1 = i_1{}^2 R = (0{\cdot}379)^2 \times 96{\cdot}8 = \mathbf{13{\cdot}9 \text{ watt}}$

and power produced in the bulb, $P_2 = i_1{}^2 R_1 = (0{\cdot}379)^2 \times 484 = \mathbf{69{\cdot}7 \text{ watt}}$

● **Ex. 1.22. A heating coil of resistance 10Ω is connected to a battery of internal resistance 40Ω. What resistance parallel to the coil be connected so that coil may produce heat $\frac{1}{4}$ th of the initial heat ?**

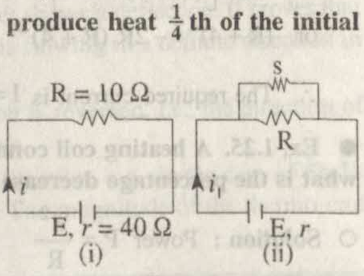


Fig. 1·15

○ **Solution :** Consider the fig. 1·15(i). The current through R is

$$i = \frac{E}{R + r} = \frac{E}{10 + 40} = \frac{E}{50}$$

$\therefore$ Heat produced, $H_1 \propto i^2$ .... (i)

In fig. 1.15 (ii) current supplied by the battery,

$$i_1 = \frac{E}{r + \frac{Rs}{R+s}} = \frac{E\,(R + s)}{r\,(R + s) + Rs}$$

$$\therefore \text{ Current through R, } i_2 = \frac{E\,(R + s)}{r\,(R + s) + Rs} \times \frac{s}{R + s} = \frac{E\,s}{r\,(R + s) + Rs}$$

$\therefore$ Heat produced in resistance R is $H_2 \propto i_2{}^2$

$$\therefore \quad \frac{H_1}{H_2}=\frac{i_1^{\,2}}{i_2^{\,2}} \qquad \therefore \quad \frac{i^2}{i_2^{\,2}}=4 \qquad \therefore \quad i=2i_2$$

$$\therefore \quad \frac{E}{50}=\frac{2\,E\,s}{r\,(R+s)+Rs} \quad \Rightarrow \quad \frac{1}{50}=\frac{2s}{40\,(10+s)+10s}$$

$$\therefore \quad 4\,(10+s)+s=10\,s \quad \Rightarrow \quad s=8\ \Omega \qquad \therefore \quad \text{Required shunt} = \mathbf{8\ \Omega}$$

- **Ex. 1.23. The temperature of a closed room are kept unchanged at 20°C by a heater of resistance 20Ω connected to 200V main. Temperature throughout the room is constant. Heat a conducted outside through a glass window of area 1 m^2 and thickness 0·2 cm. Calculate outside temperature. (Conductivity of glass is 0·2 cal $m^{-1}{}^{\circ}C^{-1}$ and mechanical equivalent of heat 4·2 J / cal.**

○ **Solution :** Heat produced per sec by the heater, $Q_1=\frac{V^2}{R}=\frac{(200)^2}{20}=2\times10^3\ \text{J / sec}$

Heat conducted by the window per sec, $Q_2=\frac{KA\,(\theta_2-\theta_1)}{x}$

$$\therefore \quad Q_2=\frac{0\cdot2\times10^{-2}\times10^4\ (20-\theta_1)}{0\cdot2}\ \text{cal} = 100\ (20-\theta_1)\ \text{cal}$$

By question, $100\ (20-\theta_1)=2\times10^3\times0\cdot24$

$\therefore \quad \theta_1=20-4\cdot8=\mathbf{15\cdot2^{\circ}C}$ $\quad$ [θ_1 = outside temperature ; θ_2 = temperature inside the room]

- **Ex. 1.24. A battery of emf 24V and internal resistance 4·0Ω is connected to the ends of a variable resistor. What should be the battery current so that rate of heat production in external resistance be maximum ?**

○ **Solution :** Let the external resistance be R.

Current in the circuit now is $I=\frac{E}{R+r}=\frac{24}{R+4}$

$\therefore$ Rate of heat production, $P=I^2R=\left(\frac{24}{R+4}\right)^2\times R$

The condition for maximum heat production is $\frac{dP}{dR}=0.$

$$\text{or,} \quad \frac{d}{dR}\left[\left(\frac{24}{R+4}\right)^2\times R\right]=0 \qquad \text{or,} \quad \frac{d}{dR}\ [576\,R\,(R+4)^{-2}\,]=0 \qquad \text{or,} \quad \frac{d}{dR}\ [R\,(R+4)^{-2}\,]=0$$

$$\text{or,} \quad (R+4)^{-2}-2R\,(R+4)^{-3}=0 \qquad \text{or,} \quad R=4\Omega$$

$\therefore$ The required current is $I=\frac{24}{R+4}=\frac{24}{4+4}=\mathbf{3\ A}$

- **Ex. 1.25. A heating coil connected to 115V supplied produces 500W. If the line voltage be 110V, what is the percentage decrease in power. Assume that the resistance of heater does not change.**

○ **Solution :** Power $P=\frac{V^2}{R}$ $\qquad \therefore \quad R=\frac{V^2}{P}=\frac{115\times115}{500}=26\cdot45\ \Omega$

$\therefore$ When the supply is 110V, the magnitude of power is $P'=\frac{V_1^{\,2}}{R}=\frac{110\times110}{26\cdot45}=\mathbf{457\ W}$

$\therefore$ Percentage decrease in power $=\frac{P-P'}{P}\times100=\frac{500-457}{500}\times100=\mathbf{8\cdot6\%}$

▲ THERMOELECTRICITY ▲

● **TOPICS :** ○ Introduction, ○ The Seebeck effect, ○ Thermo electric series, ○ Measurement of thermo-emf and emf-temperature graph, ○ Explanation of Seebeck effect from electric theory, ○ Laws of thermoelectric circuits, ○ Thermoelectric power, ○ Peltier effect, ○ Difference between Peltier effect and Joule heating effect, ○ Peltier coefficient, ○ Application of thermodynamics to a thermocouple : prediction of Thomson effect, ○ Thomson effect, ○ Total emf in a thermocouple, ○ Thermo-electric diagram and its uses, ○ Practical application of thermoelectric effect, ○ Exercise.

1.12. Introduction

The term 'thermoelectricity' implies direct conversion of heat into electricity. Discussions on thermoelectricity is based on three important thermoelectric phenomena—(a) Seebeck effect, (b) Peltier effect and (c) Thomson effect.

The device which converts heat into thermoelectricity is known as thermocouple. A thermocouple is an electric circuit made of two dissimilar metal wires joined at their ends [fig. 1.16]. Here, two metal wires A and B form a thermocouple. So, a thermocouple has two junctions.

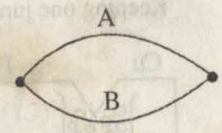
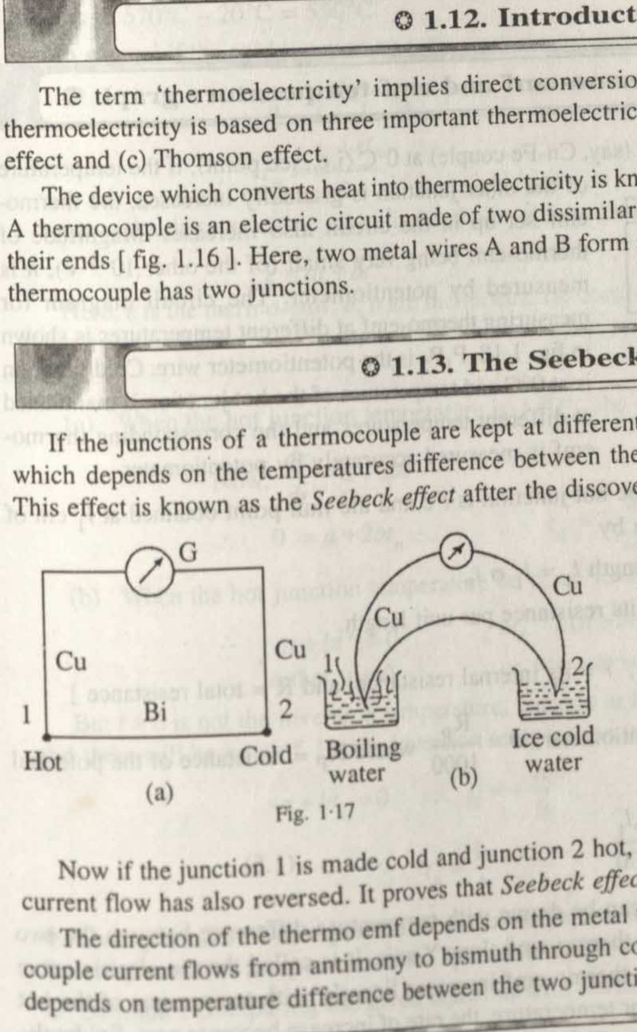


Fig. 1·16

1.13. The Seebeck effect

If the junctions of a thermocouple are kept at different temperatures, a current flows in the circuit which depends on the temperatures difference between the two junctions and the nature of the metals. This effect is known as the *Seebeck effect* aftter the discoverer, Thomas Johann Seebeck.



(a) (b)

Fig. 1·17

The electric current so obtained is called *thermoelectric* current which is the result of an emf in the circuit caused by the temperature difference between the junctions. This is termed as thermo emf. It is shown in fig. 1.17.

When the junction 1 of the Bi-Cu couple is kept in hot and junction 2 in cold, the galvanometer shows a deflection. If proves that the current is flowing in a definite direction in the couple.

Now if the junction 1 is made cold and junction 2 hot, the deflection is reversed, i.e., the direction of current flow has also reversed. It proves that *Seebeck effect* is *reversible*.

The direction of the thermo emf depends on the metal forming the junction, e.g., in the case of Sb-Bi couple current flows from antimony to bismuth through cold junction. The magnitude of the thermo emf depends on temperature difference between the two junctions.

1.14. Thermo electric series

It is found from experiment that the thermo emf set up in a thermocouple depends on two factors, viz., (a) the pair of metals forming the couple and (b) the temperature difference between the junctions.

Experimenting with different metals Seebeck arranged metals in a series known as Seebeck series. From this series (also called thermoelectric series) we can quickly get the direction of the current flowing in a couple made of any two metals in the series.

Metals are so arranged in the series that the greater the gap in between the given two metals, the greater is the thermo emf developed for a given temperature difference between the two junctions and the direction of current is always from the one which occurs earlier in the series to the one which occurs later in the series through the *cold junction*. Antimony occurs on the top of the series and bismuth at the bottom. Iron occurs earlier in the series than copper. Seebeck series is as follows :

Sb, As, Fe, Cd, W, Zn, Ag, Au, Cr, Sn, Pb, Hg, Cu, Pt, Pd, Co, Ni, Bi.

Thus the thermo emf for a Ni-Fe couple is greater than that for Cu-Fe couple for the same temperature difference.

❂ 1.15. Measurement of thermo-emf and emf-temperature graph ❂

Keeping one junction of a thermocouple (say, Cu-Fe couple) at 0°C (i.e., ice point), if the temperature of the other junction is gradually increased, the thermo-emf set up in the circuit also increases. Magnitude of thermo-emf being very small (of the other 10^{-3} V), it is measured by potentiometer. The circuit diagram for measuring thermo-emf at different temperatures is shown in fig. 1.18. P_1P_2 is the potentiometer wire. Cold junction is at 0°C and temperature of the hot junction is maintained at different temperatures and the corresponding thermo-emf is measured accurately by potentiometer.

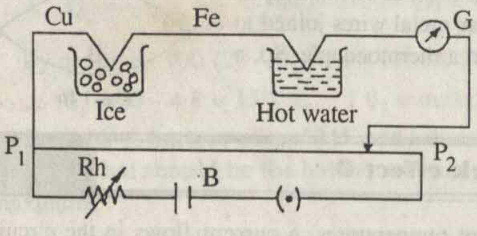


Fig. 1·18

- **Calculation :** Let the temperature of the hot junction is t°c and the null point obtained at l_1 cm of the wire P_1P_2. Then the thermo-emf is given by

$$e = \text{drop of potential over the length } l_1 = l_1 \, \sigma \, i_P$$

Here, i_P = Current in wire P_1P_2 and σ = its resistance per unit length.

Also, $i_P = \dfrac{E}{R+r}$ [E = emf of battery B, r = its internal resistance, and R = total resistance]

and resistance per unit length of the potentiometer, $\sigma = \dfrac{R_P}{1000}$ where R_P = resistance of the potential wire P_1P_2.

$$\therefore \quad e = \frac{E}{R+r} \cdot \frac{R_P l_1}{1000} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1.5)$$

- **Drawing of ($e - t$) graph :** A graph can be drawn with temperature difference between the two junctions along X-axis and the corresponding thermo-emf along Y-axis. It is called thermo-electric curve or ($e - t$) graph [fig. 1.19]. In the beginning thermo-emf increases linearly with temperature of the hot junction and then less rapidly and at a particular temperature, the rate of increase becomes zero. Evidently, it is the temperature for which emf is maximum. This temperature is called *neutral temperature* (t_n) of the couple. On further increasing the temperature of the hot junction the emf decreases continuously and falls to zero at some temperature called the *temperature of inversion* (t_i) as beyond this temperature the thermo-emf is inverted in its sign.

The nature of the graph is the same for any pair of metal in the Seebeck series. In most of the cases, the curve of fig. 1.19 represents a parabola to a first approximation.

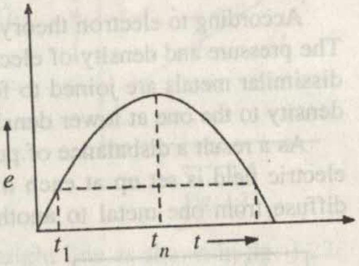


Fig. 1·19

It must be mentioned that for a couple, neutral temperature (t_n) is a constant and does not depend on cold junction temperature. For example, for Cu-Fe couple neutral temperature is 285°C. But the inversion temperature (t_i) at which emf is zero and then changes sign is not a constant for a couple. Its value depends on cold junction temperature. For example, for a Cu-Fe couple, if the cold junction be kept at 0°C then its inversion temperature is 570°C. But if the cold junction be at 20°C instead of 0°C, neutral temperature remains fixed at 285°C, but the inversion temperature lowers to 570°C − 20°C = 550°C.

In general if the cold junction be t_1°C, neutral temperature t_n°C and inversion temperature t_1'°C , then

$$t_n - t_i = t_i' - t_n$$

$$\therefore \quad t_n = \frac{t_i + t_i'}{2} \quad \ldots\ldots\ldots\ldots (1.6)$$

Since the thermoelectric curve is parabolic, it may be represented by the equation

$$e = at + bt^2 \quad \ldots\ldots\ldots\ldots (1.7)$$

Here, e is the thermo-emf; a, b are thermoelectric constants and t is the temperature difference between the hot and cold junction. We can calculate neutral temperature and inverson temperature from the equation (1.7).

(a) When the hot junction temperature is $t = t_n$, the slope of the thermoelectric curve $\frac{de}{dt} = 0$

$$\text{Now,} \quad \frac{de}{dt} = a + 2bt$$

$$\therefore \quad 0 = a + 2bt_n \qquad \therefore \quad t_n = -\frac{a}{2b} \quad \ldots\ldots\ldots (1.8)$$

(b) When the hot junction temperature be equal to inversion temperature, then $e = 0$.

$$\therefore \quad at + bt^2 = 0 \qquad \text{or,} \quad t\,(a + bt) = 0$$

$$\therefore \quad \text{either} \quad t = 0 \qquad \text{or,} \quad a + bt = 0$$

But $t = 0$ is not the inversion temperature, because at $t = 0$, both the junctions are at same temperature and there will be no emf. Hence inversion temperature is obtained from

$$a + bt_i = 0 \quad \therefore \quad t_i = -\frac{a}{b} \quad \ldots\ldots\ldots\ldots (1.9)$$

$$\therefore \quad t_n = \frac{t_i}{2}$$

- **Example 1.26. The equation of a thermoelectric curve of a thermocouple is $e = 16.7t - 0.0167t^2$. Calculate neutral temperature and inversion temperature.**

○ **Solution :** Comparing the given equation with the standard equation for thermoelectric curve we get

$$a = 16.7 \text{ and } b = -0.0167$$

$$\therefore \quad \text{Neutral temperature,} \quad t_n = -\frac{a}{2b} = \frac{7}{2 \times 0.0167} = 500°\text{C}$$

$$\text{and inversion temperature,} \quad t_i = 2t_n = 1000°\text{C}$$

1.16. Explanation of Seebeck effect from electric theory

According to electron theory, metals contain large number of free electrons behaving like perfect gas. The pressure and density of electron gas depend on the temperature and material of the metal. When two dissimilar metals are joined to form a thermocouple, electrons diffuse from the metal of higher electron density to the one at lower density.

As a result a disbalance of positive and negative charges is produced in the metals at the junction. An electric field is set up at each of the junctions, which grows in magnitude as more and more electrons diffuse from one metal to another. Ultimately further diffusion of electrons is stopped and a constant electric field is set up at the junctions. This state at the junction is represented by two tiny cells at the junctions as shown in fig. 1.20. If the temperature of the junctions are equal, the emfs of these two cells are equal and opposite and hence no current is produced in the circuit. On the other hand, if the junctions are at different temperatures, the emfs of the cells are unequal, hence a resultant emf acts in the thermocouple circuit which maintains the flow of electron in the circuit.

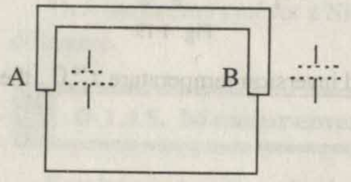


Fig. 1·20

1.17. Laws of thermoelectric circuits

There are two of thermoelectric circuits which are based on experimental facts :

- **Law of intermediate metal :** It states that the effective thermo-emf in a circuit is not changed by the introduction of additional metal (or, metals) at any point of a thermocouple, provided that the additional junctions are at the same temperature of the points of introduction.

- **Explanation :** A thermocouple is formed by metals A and B [fig. 1.21] with junction temperatures at t_1 and t_2 ($t_2 > t_1$). Let the emf acting in the couple is E_1^2 (A / s) .

Now, the junctions at t_1 is opened and a third metal C is introduced. It is assumed that the metal C is at the same temperature t_1. According to the law of intermediate metal we write

$$E_1^2 (A/B) = E_1^2 (A/C) + E_1^2 (C/B)$$

- **Law of intermediate temperature :** It states that for a given couple the thermo-emf for any given temperature difference is the sum of the thermo-emfs for any number of intermediate steps into which the given temperature difference may be subdivided.

The emf of a thermocouple between any two temperatures t_1 and t_3 is the sum of two emfs of the same thermocouple between t_1 and an intermediate temperature t_2 and between t_2 and t_3.

Thus, $E_{t_1}^{t_3} = E_{t_1}^{t_2} + E_{t_2}^{t_3} \quad (t_3 > t_2 > t_1)$

In general $E_{t_1}^{t_n} = E_{t_1}^{t_2} + E_{t_2}^{t_3} + \cdots\cdots + E_{t_{n-2}}^{t_{n-1}} + E_{t_{n-1}}^{t_n}$

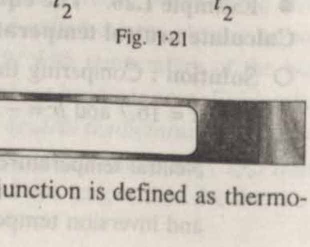


Fig. 1·21

1.18. Thermoelectric power

The rate of variation of thermo-emf with respect to temperature of hot junction is defined as thermo-electric power.

$\therefore$ Thermoelectric power, $P = \dfrac{dE}{dt}$

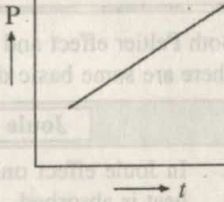


Fig. 1·22

Let two junctions of a thermcouple be at temperatures t and $(t + dt)$. Then the emf be dE, then the thermoelectric power at temperature t is

$$P = \frac{dE}{dt}$$

If the equation of thermo-emf for a couple be $E = at + bt^2$, then

$$P = \frac{dE}{dt} = a + 2bt$$

It is equation of a straight line. Hence, the plot of P against t is a straight line as shown in fig. 1.22. This is called *thermoelectric diagram* or *power diagram.*

- **N. B. :** It is not power in its true sense. Its unit is volt per °C or volt per Kelvin. The constants a and b are called the first and second thermoelectric constants.

1.19. Peltier effect

In the year 1834, J. C. A. Peltier discovered another phenomena related to thermoelectricity. It is known as Peltier effect and converse to Seebeck effect. According to Peltier effect, *when an electric current is sent in a thermocouple circuit by including a cell in the circuit, heat is either absorbed or evolved at the junctions and consequently, one junction is heated and the other is cooled. The rates of heat generation or absorption are proportional to the current.*

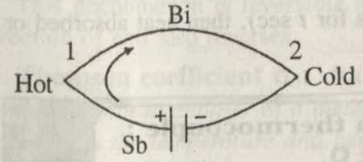


Fig. 1·23

Fig. 1.23 illustrates the effect. Here, we consider Antimony-Bismuth couple, the junctions being initially at the same temperature. A battery is included in the circuit which sends a current in the direction shown. The junction 1 gradually gets heated and the junction 2 gradually cooled. So, a difference of temperature is established between the junctions.

If the direction of current in the circuit is reversed, the effects at the two junctions get completely interchanged. Hence, *Peltier effect is also reversible.*

It is to be noted that the temperature changes associated with the Peltier effect are small. The detection or measurement of Peltier effect is rather difficult.

- **Demonstration of Peltier effect :** Two thick bars of antimony and bismuth are taken and a cross (×) is formed at their middle. A sensitive galvanometer is connected between two free ends of the cross and the remaining two ends are connected to a cell through a key. When the key is closed, an elctric current circulates in the cell circuit. As a result due to Peltier effect, the cross point is either heated or cooled relative to the other junctions. Immediately the galvanometer shows a deflection. It shows that a thermoelectric current is set up in the galvanometer circuit due to Seebeck effect. Thus both the Peltier and Seebeck effect is demonstrated by this experiment.

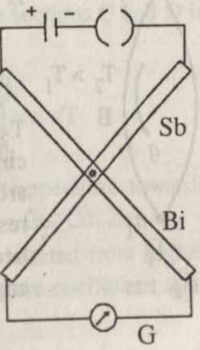


Fig. 1·24

If the current is reversed, the deflection of the galvanometer is also reversed. This shows the reversibility of Peltier and Seebeck effect.

✪ 1.20. Difference between Peltier effect and Joule heating effect ✪

Both Peltier effect and Joule heating effect are concerned with production of heat by electric current. Yet there are some basic differences between the two effects. The differences are summarised below :

Joule Effect	Peltier Effect
1. In Joule effect only heat is produced. No heat is absorbed.	1. In this effect heating and cooling takes place simultaneously.
2. Heating takes place throughout whole circuit.	2. Heating or cooling takes place only at the function of two dissimilar metals.
3. This effect is proportional to square strength of current and is irreversible.	3. This effect is proportional to current and is reversible.
4. Heat produced depends on resistance of the conductor.	4. Here heat produced does not depend a resistance of the conductor.

✪ 1.21. Peltier coefficient ✪

The amount of heat in joule gained or lost when a charge of one coulomb is passed across the junction of two dissimilar metals is defined as the Peltier coefficient of the junction. It is denoted by π. Its value depends on the temperature of the junction and material of the junction.

Its unit is joule/coulomb or volt. The unit of Peltier coefficient is similar to the unit of latent heat capacity which is 'joule per kg'. For this reason, Peltier coefficient is also called latent heat of electricity.

Hence, if at any junction of thermocouple Peltier coefficient be π J / coulomb, then heat is either absorbed or released. Now if a charge q coulomb (or, current I flows for t sec), then heat absorbed or released is, $W = \pm \pi q = \pm \pi I t$ joule

✪ 1.22. Application of thermodynamics to a thermocouple : Prediction of Thomson effect ✪

Peltier effect converts electrical energy to heat energy and it is reversible. For this reason, a thermocouple is considered to be equivalent to reversible Carnot engine. So, principle of thermodynamics can be applied to it. In Carnot's engine, the working substance heat Q_1 from the source kept at higher temperature T_2. A part of the heat is converted to useful work and the rest Q_2 is rejected to the sink kept at lower temperature T_1.

According to Carnot's theorem, $\frac{Q_1}{Q_2} = \frac{T_2}{T_1}$

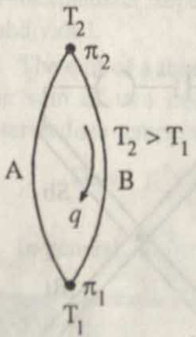


Fig. 1·25

Suppose, two metals A and B form a thermocouple [fig. 1.25]. Its hot junction is at T_2K and cold junction at T_1K. Due to Seebeck effect, a small current flows in the circuit. Let corresponding to a small current, an electric charge q is taken very slowly around the circuit. It π_2 and π_1 are the Peltier coefficient at the temperature T_2 and T_1 respectively, then heat absorbed (in joule) at the hot junction is $Q_2 = \pi_2 q$. While the heat rejected at the cold junction is $Q_1 = \pi_1 q$. From the analogy of reversible heat engine we write

$$\frac{\pi_2 q}{\pi_1 q} = \frac{T_2}{T_1} \quad \text{or,} \quad \frac{\pi_2 - \pi_1}{\pi_1} = \frac{T_2 - T_1}{T_1}$$

The absorbed heat minus the rejected heat, i.e., $(\pi_2 - \pi_1)\,q$ maintains the thermoelectric current. Hence, if E be the thermo-emf causing the Seebeck effect, the $Eq = (\pi_2 - \pi_1)\,q$.

Hence, $E = \pi_2 - \pi_1 = \dfrac{\pi_1}{T_1}(T_2 - T_1)$

Now, T_1 being constant, π_1 is also a constant and T_2 varies. Then

$$E \propto (T_2 - T_1)$$

It means that, Seebeck emf is proportional to the temperature difference between the two junctions. So, E vs. $(T_2 - T_1)$ graph will be a straight line as shown in fig. 1.26. It is called the theoretical thermo-electric curve. But from experiment it is found that the variation E with $(T_2 - T_1)$ is parabolic. The thermo-emf does not increase indefinitely with the increase of the difference of temperature between the hot and cold junction. Emf increases upto a certain maximum value and then decreases. So, there is some discrepancy between the two curves. This discrepancy was adequately explained by the third thermo-electric phenomena, called *Thomson effect.*

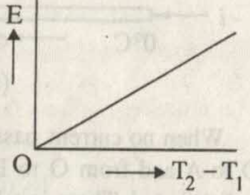


Fig. 1·26

✪ 1.23. Thomson effect ✪

According to Thomson effect, *if a metal rod is unequally heated, i.e., if a temperature gradient exists along a metal rod, an emf exists between any point of the rod.* So, there is a distributed emf along the length of an unequally heated rod.

So, when a current is passed through a thermocouple absorption or evaluation of heat takes place not only at the junction of dissimilar metals, but also along the length of the wire forming the couple.

This phenomenon is *reversible* in the sense that if the sign of temperature gradient is reversed, the direction of emf also reverses.

- **Thomson coefficient σ :** *It is defined as the work done in joule to transfer one coulomb charge from one point to another of a metal bar when temperature difference between the two points is 1°C. It depends on the temperature and material of the rod.* Its unt is joule/coulomb or volt. So, the Thomson coefficient is the emf that exists between two points of a rod when the two points differ in temperature of 1°C. This is similar to the unit of specific heat capacity which is 'joule per kg per kelvin'. Hence, Thomson coefficient may be called 'specific heat capacity of electricity'.

In fig. 1.27, the metal rod AB is so heated that its end A is at temperature T_1 and the other end at temperature T_2. C and D are two sections of the rod which are at temperatures T and T + dT respectively. If σ be the Thomson coefficient of the rod, then the emf between the sections C and D is

$$de = \sigma\, dT$$

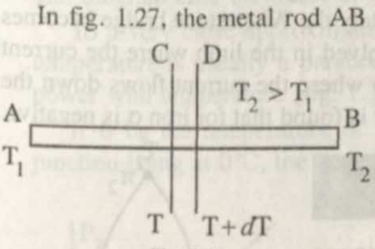


Fig. 1·27

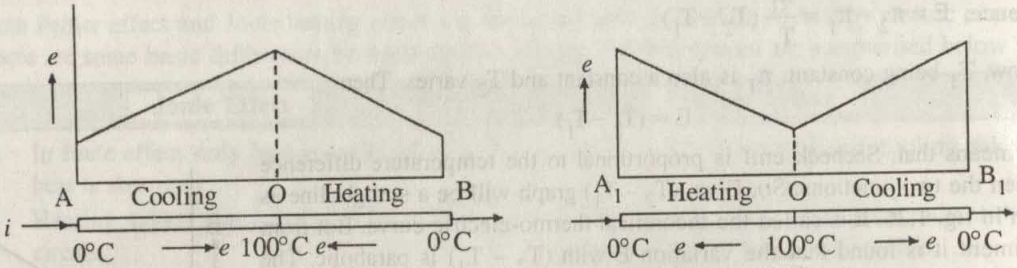
$\therefore$ Total emf between the two ends is $e = \int_{T_1}^{T_2} \sigma \,.\, dT$

Thomson coefficient may be positive or negative. If the emf acts from lower temperature towards higher temperature, then σ is taken to be positive. The metals having positive σ are : Cu, Cd, Zn, Ag.

On the other hands, the Thomson coefficient is considered negative when it is directed from higher temperature and towards the lower temperature end. Metals having negative temperature coefficient are : Fe, Pt, Pd.

Lead is a metal which does not show any Thomson effect.

- **Experiment :** The two ends of a copper rod are kept at 0°C and the middle point is at 100°C as shown in fig. 1.28(a).



(a) Copper (b) Iron

Fig. 1·28

When no current passes through the rod a uniform temperature gradient is set up along the rod form O to A and from O to B. Now a current is passed through the conductor. The temperature gradient is redistributed. There is a loss of heat along AO and gain of heat along OB. So, heat is absorbed along AO and evolution of heat occur along OB of the copper rod AB.

The opposite is the situation in the case of iron for which σ is negative. It is shown in fig. 1.28(b). The two ends of the iron rod are kept at 0°C and the mid point O_1 at 100°C. When current is passed through the rod along A_1B_1, heat is evolved along A_1O_1 and absorbed along O_1B_1. So, there will be heating along A_1O_1 and cooling over the length O_1B_1.

From the above consideration we define the Thomson coefficient as the energy absorbed or generated in joule due to a passage of one Coulomb of charge in the conductor through a temperature difference of 1K.

The Thomson coefficient can be demonstrated with the help of the arrangement shown in the fig. 1.29. A long iron rod is bent in the form of a U. The bent A of the U is heated by a bunsen burner till it is red hot. The two ends of the rod are kept cool by immersing them in mercury bath. Two identical insulated coils C_1 and C_2 are wound at the middle of the two limbs of the U and are connected to the two arms of Wheatstone bridge. When steady thermal state is attained the bridge is balanced. Now a current of about 10A is passed through the iron rod using the mercury baths as the terminals. Immediately the Wheatstone bridge becomes unbalanced. The deflection of the galvanometer shows that heat is evolved in the limb where the current flows up the temperature gradient. Heat is absorbed in the other limb where the current flows down the temperature gradient. Thus, the Thomson effect is demonstrated, and it is found that for iron σ is negative.

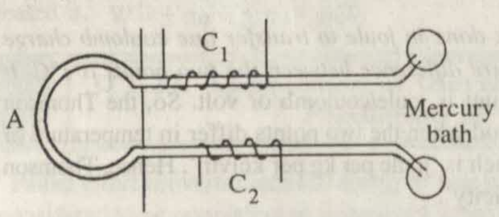


Fig. 1·29

1.24. Total emf in a thermocouple

When the two junctions of a thermocouple are at different temperatures, thermo-emfs act at the junction due to Peltier effect and also along the length of the metals due to Thomson effect. The resultant of all these emfs sends current in the thermocouple due to Seebeck effect. The complete expression for the thermo-emfs is deduced below :

A thermocouple is formed by two metals. Its one junction is at T_2 and the other junction at T_1 ($T_1 < T_2$). The Thomson coefficients for the metals

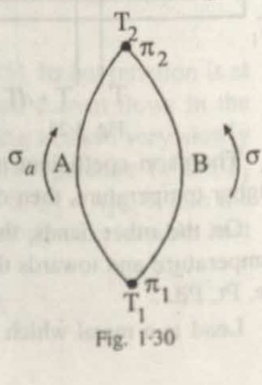


Fig. 1·30

A and B are σ_a and σ_b. We assume that both σ_a and σ_b are positive. Different emfs acting in the circuit are :

(a) Peltier coefficient at the junction at temperature $T_2K = \pi_2$

(b) Total Thomson emf for the metal $B = -\int_1^2 \sigma_b \, dT$

(– *ve* sign shows that it is directed from lower to higher temperature.)

(c) Peltier coefficient at the junction at temperature $T_1K = -\pi_1$

(d) Total Thomson emf for the metal $A = \int_1^2 \sigma_a \, dT$

∴ Total emf acting in the thermocouple circuit

$$e = \pi_2 - \pi_1 + \int_1^2 (\sigma_a - \sigma_b) \, dT \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

e is called Seebeck emf. So, the Seebeck emf is equal to algebraic sum of the Peltier and the Thomson emfs.

If the two junctions be at temperaturs T + *d*T and T and if the corresponding Peltier emfs be $(\pi + d\pi)$ and π, then

$$de + d\pi + (\sigma_a - \sigma_b) \, dT \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(1.10)}$$

1.25. Thermo-electric diagram and its uses

The plot of thermoelectric power $\left(\frac{dE}{dt}\right)$ against temperature difference or temperature of the hot junction (the cold junction being at 0°C) of a thermocouple consisting of the given metal and *lead* is called *thermoelectic diagram.*

This diagram represents diagrammatically the various thermoelectric effects. The diagram is also called power diagram, because here thermoelectric power is plotted against the temperature. It is also called Tait diagram after the name of the proposer.

To a very close approximation,the variation of Seebeck emf with temperature is usually a parabola, the variation of the thermoelectric power with temperature [fig. 1.31] will be straight line.

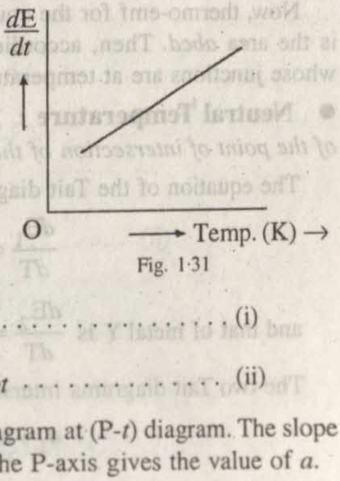


Fig. 1·31

If θ be the temperature of the hot junction in celsius, the cold junction being at 0°C, the equation of thermoelectric curve is

$$E = a\theta + b\theta^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

$$\therefore \frac{de}{d\theta} = a + 2bt \quad \text{or,} \quad P = a + 2bt \quad \ldots\ldots\ldots\ldots \text{(ii)}$$

It is the equation of thermoelectric diagram at (P-*t*) diagram. The slope of the curve is '*b*' and the intercept on the P-axis gives the value of *a*.

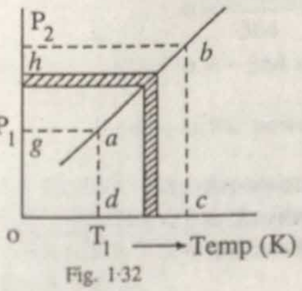


Fig. 1·32

- **(A)** Fig. 1.32 shows, the (*p-t*) diagram of a metal with respect to lead. Let the temperature of the cold junction be at T_1K (fixed) and t hot junction be at T_2K. Thermoelectric power at T_1K is $ab = P_1$ ar T_2K it is $bc = P_2$.

- **Peltier coefficient :** The Peltier coefficient π_1 at temperature T_1 is

$$\pi_1 = T_1 \left(\frac{dE}{dT}\right)_{T_1} = T_1 P_1$$

$$\therefore \quad \pi_1 = od \times ad = \text{area } odag$$

Again, the Peltier coefficient π_2 at temperature T_2 is

$$\pi_2 = T_2 \left(\frac{dE}{dT}\right)_{T_2} = T_2 P_2 = oc \times cb = \text{area } ocbh$$

- **Thomson coefficient :** $\sigma_a - \sigma_b = T\dfrac{dP}{dT} \quad \therefore \quad \int_1^2 (\sigma_a - \sigma_b)\, dT = \int_1^2 T.\, dP = \text{area } abhg$

because $(\sigma_a - \sigma_b)\, dT = T\, dP$ = small horizontal shaded area.

- **Total Thermo-emf :**

$$e = \pi_1 - \pi_2 - \int_1^2 (\sigma_a - \sigma_b)\, dT$$

$$= \text{area } acbh - \text{area } odcg - \text{area } abhg = \text{area } abcd$$

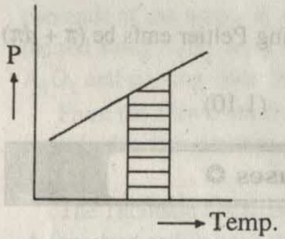


Fig. 1·33

Thus the total emf is given by the area and the Tait diagram. It is shown by the shaded area in the fig. 1.33.

- **(B)** In our discussion on thermo-electric power we have taken lead as reference metal. Now we consider the case of a thermocouple made of any two metals X and Y.

In fig. 1.34, the straight line A_1B_1 represents the thermoelectric power line for the metal X with respect to lead. Similarly, A_2B_2 is the thermoelectric power line for the metal Y with respect to lead. Two junctions of both the couples are kept at T_1 and T_2 $(T_2 > T_1)$.

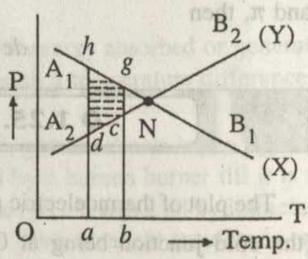


Fig. 1·34

Now, thermo-emf for the couple (X-Pb) is the area *abgh*. Also, the thermo-emf for the couple (Y-Pb) is the area *abcd*. Then, according the law of intermediate metal, the thermo-emf for the couple (X-Y) whose junctions are at temperature T_1 and T_2 is given by e = area *abgh* – area *abcd* = area *dcgh*.

- **Neutral Temperature :** *The neutral temperature of (X-Y) thermocouple is given by the temperature of the point of intersection of the Tait diagrams of the elements of the thermocouple.*

The equation of the Tait diagram of the metal X is

$$\frac{dE_1}{dT} = a_1 + b_1 T \quad [\text{T = temperature of the hot junction}]$$

and that of metal Y is $\dfrac{dE_2}{dT} = a_2 + b_2 T$.

The two Tait diagrams intersect at the point given by

$$a_1 + b_1 T = a_2 + b_2 T$$

$$\therefore \quad T = \frac{a_2 - a_1}{b_1 - b_2}$$

It is the neutral temperature for the couple.

❂ 1.26. Practical application of thermoelectric effect ❂

- **Thermoelectric pyrometer :** In a thermocouple the thermo-emf produced is proportional to the temperature difference between the hot and cold junction when the hot junction temperature is far from the neutral temperature. This property of a thermocouple is utilised for the construction of thermocouple thermometer or thermoelectric pyrometer.

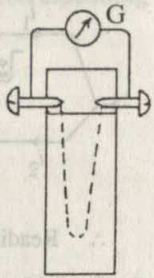


Fig. 1·35

It is cheap but very useful thermometer for measuring high temperature. By suitable selection of material, thermocouple thermometer can be used to measure wide range of temperature, viz, from – 240°C to 2000°C.

- **Thermopile :** A thermocouple consists of a number of bismuth-antimony couples in series so as to add the effects. One set of junction is protected and the radiation to be measured is allowed to fall on the other set [fig. 1.36]. The set of junctions exposed to the radiation is heated, a thermo-emf is produced and the galvanometer G shows a deflection connected to the instrument. The deflection is a measure of the radiation falling on the instrument.

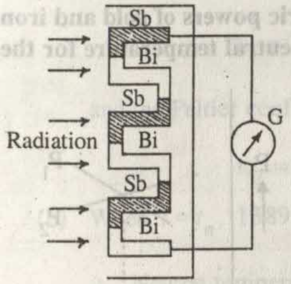


Fig. 1·36

Thermopile is very sensitive device to detect thermal radiations. It is used to measure very small temperature differences which arise in the measurement of radiation from stars, in the measurement of intensity of spectral lines, and in the measurement of feeble high frequency alternating current.

●● Numerical Examples ●●

- **Example 1.27. The thermoelectric power at 100°C for a iron-copper couple is 6.68 μV /°C neutral temperature is 282°C. Calculate thermo-electric power at 0°C.**

○ **Solution :** Let the equation for a Fe-Cu couple be given by $e = at + bt^2$, t = difference of temperature between hot and cold junction ; a and b are constants.

So, thermoelectric power $\mathrm{P} = \dfrac{de}{dt} = a + 2bt$

∴ From the given problem, $a + 2b \times 100 = 6.68$. . . . . (i)

Again, at $t = t_n$, $\dfrac{de}{dt} = 0$ ∴ $a + bt_n = 0$ ∴ $t_n = \dfrac{-a}{2b}$

$$\therefore \quad \frac{-a}{2b} = 282°\mathrm{C} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(ii)}$$

From equation (i) and (ii), $-564b + 200b = 6.68$

$$\therefore \quad b = \frac{-6.68}{364} = -1.835 \times 10^{-2}\,\mu\mathrm{V}.°\mathrm{C}^{-1}$$

$$\therefore \quad a = -564 \times (-1.835 \times 10^{-2}) = 10.35\,\mu\mathrm{V}.°\mathrm{C}^{-1}$$

$$\therefore \quad \text{Thermoelectric power at 0°C is } \left(\frac{de}{dt}\right)_{t=0} = a + 2b.0 = a = 10.35\,\mu\mathrm{V}.°\mathrm{C}^{-1}.$$

- **Ex. 1.28. The temperature difference between the two junctions of a thermocouple is kept constant. The resistance of the thermocouple is 5 ohm. Measurement by potentiometer shows that the thermo-emf is 3.9 mV. If a milli-voltmeter of resistance 60 ohm is connected to the thermocouple, what will be its reading ?**

○ **Solution :** The resistance of the thermocouple is 5 Ω and thermo-emf measured by potentiometer = 3.9×10^{-3}V.

$\therefore$ Current in the thermocouple, $i = \dfrac{3.9\times10^{-3}}{5} = \mathbf{0.78\ mA}$

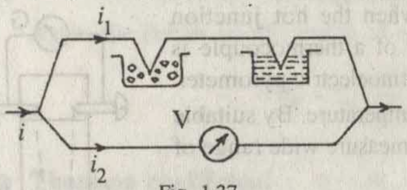


Fig. 1·37

When the voltmeter is connected, equivalent resistance of the circuit is $\mathrm{R} = \dfrac{60\times5}{65} = 4.62$ ohm

Again, $i_1\mathrm{R}_1 = i_2\mathrm{R}_2 \quad \therefore \quad 5i_1 = 60i_2 \Rightarrow i_1 = 12i_2$

or, $\quad 0.78 = 12i_2 \Rightarrow i_2 = \mathbf{0.06\ ma}$

$\therefore$ Reading of the voltmeter, $\mathrm{V} = i_2\mathrm{R}_2 = 0.06\times60 = \mathbf{3.6\ mV}$

● **Ex. 1.29. A thermocouple is made of gold and iron. The thermoelectric powers of gold and iron are given by $P_1 = 2.8 + 0.01t$ and $P_2 = 17.5 - 0.048t$ μV/°C. What is the neutral temperature for the gold-iron couple ?**

○ **Solution :** It is given that for gold $P_1 = 2.8 + 0.01t$ and for iron $P_2 = 17.5 - 0.048t$. Here, t = temperature of the hot junction. In fig. 1.38 (P_1-t) graphs are shown. The point of intersection of the two power lines is the neutral temperature for the couple. Hence, at the point of intersection.

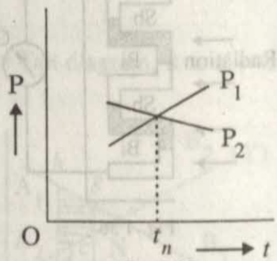


Fig. 1·38

$$2.8 + 0.01t_n = 17.5 - 0.048t_n$$

$$\therefore \quad t_n = \frac{14.7}{0.058} = 253.5°\mathrm{C}$$

$\therefore$ The required neutral temperature is **253.5°C.**

● **Ex. 1.30. The thermo-electric power for iron at t°C is $1734 - 4.87t$ centi μV / °C and that for copper is $136 + 0.95t$ centi μV / °C. Find the thermo-emf of iron-copper couple between the temperatures 20°C and 100°C.**

○ **Solution :** Here, $\dfrac{d\mathrm{E}_1}{dt} = 1734 - 4.87t$ and $\dfrac{d\mathrm{E}_2}{dt} = 136 + 0.95t$

Now, by the law of intermediate metals, $\mathrm{E} = \mathrm{E}_1 - \mathrm{E}_2$

$$\therefore \quad \frac{d\mathrm{E}}{dt} = \frac{d\mathrm{E}_1}{dt} - \frac{d\mathrm{E}_2}{dt} \qquad \therefore \quad \frac{d\mathrm{E}}{dt} = 1734 - 4.87t - 136 - 0.95t = 1598 - 5.82t$$

$$\therefore \quad \int_0^{100} d\mathrm{E} = \int_0^{100} 1598\,dt - \int_0^{100} 5.82t\,dt$$

$$\therefore \quad \mathrm{E}_0^{100} = 1598\times100 - \frac{1}{2}\times5.82\times(100)^2 = 130700 \text{ centi } \mu\mathrm{V}$$

$$\text{and} \int_0^{20} d\mathrm{E} = 1598\times20 - \frac{1}{2}\times5.82\times(20)^2$$

$$\mathrm{E}_0^{20} = 30796 \text{ centi } \mu\mathrm{V}$$

$\therefore$ By the law of intermediate temperatures

$$\mathrm{E}_{20}^{100} = \mathrm{E}_{20}^{0} + \mathrm{E}_0^{100} = \mathrm{E}_0^{100} - \mathrm{E}_0^{20} = 130700 - 30796 = 99905\times10^{-8} \text{ volt}$$

$$\therefore \quad \mathrm{E}_{20}^{100} = 0.99905 \text{ mV}$$

● **Ex. 1.31. The thermoelectric constant for copper are 2.76 μV / °C and 0.012 μV / $(°C)^2$. The corresponding quanties for iron are 16.65 μV / °C and – 0.03 μC / $(°C)^2$. Calculate the following quantities for iron-copper couple :**

(a) Peltier coefficients at 0°C and 100°C, (b) neutral temperature, (c) inversion temperature, (d) total thermo-emf for the temperature range 0°C and 100°C, (e) Thomson coefficient corresponding to 0°C and 100°C.

○ **Solution :** The thermoelectric power equation for copper is

$$P_{Cu} = 2.76 + 0.012t \quad \text{and for iron,} \quad P_{Fe} = 16.65 + 0.03t$$

Hence, power equation for iron-copper couple is

$$P_{Fe\text{-}Cu} = (16.65 - 0.03t) - (2.76 + 0.012t) = 13.89 - 0.042t$$

(a) Pelter coefficient at 0°C, $\pi_1 = T_1 \left(\frac{dE}{dT}\right)_{T_1}$

$$\therefore \quad \pi_1 = 273 \times (13.89 - 0.042 \times 0) = \mathbf{3792.0\ \mu V}$$

and the Peltier coefficient at 100°C, $\pi_2 = T_2 \left(\frac{dE}{dT}\right)_{T}$

$$\therefore \quad \pi_2 = 373\,(13.89 - 0.042 \times 100) = \mathbf{3614.4\ \mu V}$$

(b) When $t = t_n$, $13.89 - 0.042t_n = 0$

$\therefore$ Neutral temperature, $t_n = \frac{13.89}{0.042} = 330.7°C$

(c) Hence, inversion temperature, $t_i = 2t_n = 2 \times 330.7 = \mathbf{661.4°C}$

(d) For the couple, $\frac{dE}{dt} = 13.89 - 0.042t$

$$\therefore \quad E = \int_0^{100} 13.89\, dt - \int_0^{100} 0.042t\, dt = 13.89 \times 100 - 0.042 \times \frac{1}{2} \times (100)^2 = 1179\ \mu V$$

(e) Thomson coefficient, $\sigma = -T\frac{d^2E}{dt^2} \quad \therefore \quad \sigma_{Fe} = -T\frac{d^2E}{dt^2} = -(273 + t) \times (-0.03)$

$\therefore$ at 0°C, $\sigma_{Fe} = -(273 + 0)\,(-0.03) = \mathbf{8.19\ \mu V/°C}$

and at 100°C, $\sigma_{Fe} = -373 \times (-0.03) = \mathbf{11.19\ \mu V}$

also at 0°C, $\sigma_{Cu} = -(273 + 0)\,(0.012) = \mathbf{-3.28\ \mu V/°C}$

and at 100°C, $\sigma_{Cu} = -373 \times 0.012 = \mathbf{-4.47\ \mu V/°C}$

● E ◈ X ◈ E ◈ R ◈ C ◈ I ◈ S ◈ E ●

☐ HEATING EFFECT OF ELECTRIC CURRENT ☐

A. Objective type questions :

● **Fill in the blank spaces in the followings :**

1. Heating effect electric current depends on ______ of the conductor.
2. When a charge q is passed through a potential difference of V, the work done is W = ______ .
3. Heat produced due to a flow of current does not depend on ______ of current flow.

4. Heating effect electric current is a ______ process.
5. 1 B. O. T. unit is ______ Joule = ______ calorie.
6. Fuse wire is usually made of ______ and ______ .
7. Resistance of 100W – 220 bulb is ______ .

B. Very short answer type questions :

1. How does the heating effect depends on magnitude of current ?
2. If the resistance of a conductor increases, how does the heating effect change ?
3. Why is Joule's heating called irreversible ?
4. Distinguish between kilowatt and kilowatt hour.
5. What is the commercial unit of electrical energy ?
6. Why is fuse used in a circuit ?
7. What is the resistance of an electric bulb of rating 100 W–220 V ?
8. What should be the characteristics of heating coil ?

C. Short answer type questions :

1. Heat is continuously produced due to the current flowing through a conductor. But after some time, temperature reaches a steady maximum value. Explain. [W. B. H. S. 2002]
2. What is electric fuse ? How is it constructed and why ? [W. B. H. S. 2002]
3. What is B. O. T. unit ? Express it in calorie. [W. B. H. S. 1996]
4. A steady current is passing through coil. Will its temperature continuously go on increasing ? Explain with reason.
5. An accumulator is first connected to a resistor of resistance R_1 and then to another resistance R_2. For what value of internal resistance R_0 of the accumulator equal heat is produced in both the cases ?
6. Two bulbs of power 25 watt and 100W are connected in series and then to the supply main. Explain which bulb will glow brighter ?
7. What is meant by 'rating of an electric bulb is 60W–220V' ? What is its resistance when connected to 220V supply. [**Ans :** 806·7 Ω] [W. B. H. S. 1987]
8. What is B. O. T. unit ? Show that the unit of the product of potential difference and current is that of power ? [W. B. H. S. 2001]
9. Why is Joule's equivalent is called mechanical equivalent of heat ? [W. B. H. S. 2000]
10. The maximum power a 20Ω resistor can dissipate as heat without being damaged is 2 KW. Would you connect this resistor directly across 300 V d. c. source of negligible internal resistance ? [J. E. E. 2001]
11. When a current is passed through a conductor having some resistance, it is heated. Explain.
12. When a current is divided into a number of branches, the current is so divided that heat produced in minimum. Explain. [J. E. E. 1992]
13. A combination of several resistors are connected to a source of constant potential difference. Show that rate of heat production in any resistor (i) is proportional to the resistance in the case of series combination and (ii) is inversely proportional to the resistance in the case of parallel combination.
14. Three equal resistors are connected in series and a potential difference is applied across the combination. It produces 10W power. Now the resistor is connected in parallel and same potential difference is applied. What power will be produced now ? [**Ans :** 90 W]
15. Two resistors 6 ohm and 24 ohm are first connected in parallel and then in series. If same potential difference is applied to them, what is the ratio of heat produced in them ? [**Ans :** 25 : 4]
16. A 220V–60W carbon filament bulb is connected in series to a 220V–60W metal filament bulb. The combination is connected to 220V supply. Which one will glow brighter ? Explain your answer. [J. E. E. 1987]
17. The equation $P = i^2 R$ shows that the rate heat produced in proportional to resistance and the equation $P = \dfrac{V^2}{R}$ shows that the rate of heat produced is inversely proportional to the resistance. How can you explain this apparent discrepancy ?

18. Two identical coils are placed in two identical glass bulbs. One bulb is evacuated of air and the other contains hydrogen gas. The two coils are connected in series and a current is passed. Which bulb will glow brighter ?

19. Two heating coils are made of same material ; but the length and diameter of one coil are twice those of the other coil. The coils are connected in parallel and then to the supply line. Calculate the ratio of heat produced in them. **[Ans :** $H_1 : H_2 = 2 : 1$ **]**

20. Two heating elements A and B are kept in identical glass tubes as they are arranged to heat equal amount of water taken in two identical calorimeters. When the elements are connected in series, the element A produces more heat than the element B. But if they are is parallel B produces more heat than A. When measured by metre bridge, both the elements have equal resistance. What is the possible explanation of this findings ?

D. Essay type questions :

1. State Joule's law of heating from the point of view of energy. **[W. B. H. S. 2002]**

2. If Q charge flows through a conductor due to a potential difference V applied to it, how much work is done ? Does this work depend on resistance of the conductor ?

3. State the principle of least heat. Prove the principle in the case of two resistors connected in parallel.

4. Describe electrical method for determining mechanical equivalent of heat.

5. Establish Joule's law from the principle of conservation of energy.

6. When current is divided into parallel branches, the current is so divided that heating effect is minimum. Prove it.

E. Simple numerical problems :

1. A potential difference of 6V is applied to the series combination of 2Ω and 4Ω resistors. Compare the power produced in them. Also compare the power produced in the resistor when they are connected in parallel. **[Ans :** 1 : 2 ; 2 : 1 **]**

2. A 60W–120W bulb is required to be connected to 220V supply so that it may burn in full glow. What resistance are to be connected in series with it ? **[Ans :** 80Ω **] [J. E. E. 1990]**

3. Two electric bulb each of rating 220V–500W are connected in series to 110V supply line. Calculate the power generated by each bulb. **[Ans :** 31·24 Watt **]**

4. A cell of emf 1·5V and internal resistance $0{\cdot}1\ \Omega$ is connected to the series combination of resistance and an ammeter of negligible resistance. The ammeter reads 2·0A, calculate (i) rate of heat production in the cell and (ii) rate of heat production in the resistor. **[Ans :** 3W (ii) 2·6W **] [W. B. H. S. 2001]**

5. A heater of resistance $1{\cdot}40\Omega$ can carry maximum current of 1·2A is to be connected to 210V d. c. source. What minimum resistance be connected with it. **[Ans :** 36 ohm **] [W. B. H. S. 1996]**

6. How much work has to be done to transfer 30 coulomb of charge through a potential difference of 150V ? If this charge flows through a resistor, how much heat will be produced ? **[Ans :** 4500 J ; 1071·4 cal **]**

7. An electric motor of power $\frac{1}{8}$ HP is connected to a 220V supply line. What current flows through it ? What is the expenditure if it runs for 80 hrs ? Cost per unit 70 P. **[Ans :** 0·424 A ; Rs. 5·22 **] [W. B. H. S. 1996]**

8. How much heat will be produced if a current of 0·8 A passes through a resistance of 10Ω for 1 min. **[Ans :** 91·4 cal **] [W. B. H. S. 2003]**

9. How many electric bulbs each of power 60W can be run at a time if the line voltage be 220V and fuse current is 5A. **[Ans :** 18 **] [C. B. S. E. 1994]**

10. In a heater a nicrome wire is used. Its length is 10m, area of cross-section $2{\cdot}5 \times 10^{-6}\text{m}^2$ and resistivity $1{\cdot}1 \times 10^{-6}\ \Omega\text{-m}$. If 200V is applied to the heater, what will be its power ? **[Ans :** 9·091 KW **]**

11. A current of 2A is passed through a heating coil of resistance 50Ω for 5 min. Calculate (i) Charge passing through the coil (ii) work done by source of emf (ii) heat produced.
[Ans : (i) 600 coulomb (ii) 6×10^4 J (iii) $14{\cdot}2 \times 10^3$ cal **] [W. B. H. S. 1985]**

12. A calorimeter contains 1·5 Kg oil of sp. heat 0·6. A coil of resistance 3Ω is immersed into the oil and a current of 3A is passed through it. Through what time, current should be passed so that temperature of oil rises by 10°C ? Neglect water equivalent of the calorimeter. J = 4·2 J / cal.
[Ans : 1400 sec **] [W. B. H. S. 1980]**

13. The following appliances are connected in parallel to 100V main. (i) a 50W lamp (ii) a parallel combination of a resistor and an ammeter; the ammeter reads 0·5A (iii) resistance coil ; heat is produced in the coil at the rate of 50 cal / s (iv) a 100Ω resistor. What is (i) equivalent resistance of the circuit (ii) total power.
[**Ans :** 24·4Ω ; 410 W]

14. A heating coil of resistance 5Ω is connected to an electric cell of internal resistance 20Ω. What shunt be connected parallel to the coil so that it may produce heat equal to $\frac{1}{9}$ of the initial heat.
[**Ans :** 2 ohm] [**W. B. H. S. 1989**]

15. The resistance of four arms of a Wheatstone bridge is 100, 10, 500 and 50 ohm. What is the ratio of heat produced in different arms ? [**Ans :** 50 : 5 : 10 : 1] [**J. E. E. 2001; W. B. H. S. 1989**]

16. A circuit is made of three resistances in parallel. A potential difference of 50V volt is applied to the combination. The ratio of heat produced in the resistors is 1 : 2 : 3 and they produce 100 Wh in 4 hours. If the resistors are connected in series and then connected to 220V main, at what rate heat is produced in them ? [**Ans :** 44 Watt]

17. The water in a kettle starts boiling 15 minutes after the switch is closed. The length of the heating coil is 6 m. What change be effected in the heating coil so that water in the kettle will start boiling 10 min. after the switch is closed ? [**Ans :** the length of the coil be reduced to 4m] [**J. E. E. 1986**]

18. The mass of each of two uniform wires made of same material is 1g. The length of one wire is double the other. The wires are connected in series and a current of 10A is passed through them. If the length of the longer wire be 20 cm, calculate the rate of energy flow in each wire. Which one will be more hot ? Density and resistivity of the material of the wire are 11 g/c.c. and 20×10^{-4} Ω-cm.
[**Ans :** 880 W, 220W ; longer wire will be more hot]

19. The heating element of an electric furnace is made three coils of equal resistance. When the coils are connected in parallel, the water in a vessel starts boiling in 6 mins. If the coils are connected in series, after what time the same water will boil ? [**Ans :** 54 min] [**J. E. E. 1989**]

20. A cell of emf 1·5V and internal resistance 0·1Ω is put across the series combination of a resistor and an ammeter of negligible resistance. The ammeter reads 2·0 A. Calculate : (i) rate at which chemical energy is spent. (ii) rate of loss of energy in the cell. (iii) rate of loss of energy in the resistor. (iv) power produced by the source. [**Ans :** (i) 3W (ii) 0·4 W (iii) 2·6 W (iv) 2·6 W]

21. When an electric motor is connected to 50 V d. c., 12A current flows through it. If the efficiency of the motor be 30%, what is the resistance of the motor. [**Ans :** 2·9Ω]

22. The radius of cross-section of a fuse wire is 0·15 mm. It melts when a current of 15A passes through it. What should be the area of cross-section of a fuse wire made of same material so that it may melt when the current is 30A ? [**Hints :** $\frac{I^2}{r^3} = \text{constant}$] [**Ans :** 0·238 mm]

23. An electric bulb has rating of 100W–230V. If the supply voltage drops to 115V, what is the total heat produced by the bulb in 20 min ? [**Ans :** 30 KJ]

24. An electric kettle has two heating coils. When one of the coils is switched on, the water in the kettle begin to boil in 6 min. When the other coil is switched on, the boiling begins in 8 min. In what time will the boiling begin if both the coils are switched on simultaneously (a) in series and (b) in parallel ?
[**Ans :** (i) 14 min (ii) 3·53 min] [**I. I. T**]

25. A calorimeter of water equivalent 10g contains 500g water. A heating coil of resistance 10Ω is immersed into the water and a current of 2A is passed for 10 min when temperature rises by 11·8°C. Calculate mechanical equivalent of heat. [**Ans :** 4×10^7 erg / cal]

26. Two electric bulbs of rating 220V–40W and 220–60 W are connected in series and a supply voltage of 220V is applied to the combination. Calculate the rate of heat produced in them. J = 4·2 J / cal.
[**Ans :** 3·429 cal / s ; 2·286 cal / s]

27. A battery of emf 2V and internal resistance 0·1Ω are connected to the parallel combination of three resistors each of resistance of R. For what value of R, maximum heat is produced in the circuit ? [**Ans :** R = 0·3Ω]

28. Two resistors of resistances 3Ω and 5Ω are connected in parallel. What should be the value of third resistor and its mode of connection so that when a potential difference of 12V is applied to the combination, it produces 36W of power ? [**Ans :** 2·125Ω, in series]

29. An electric heater can raise the temperature of 5 Kg water from 29°C to 99°C in 10 min. If 60,000 J of energy is wasted, calculate the power of the heater. Water equivalent of the heater 150g. [**Ans :** 2·623 KW]

30. A 22 ohm resistor is connected in series with the parallel combination of 10Ω and 40Ω resistors. Now, a battery of emf 10V and negligible internal resistance is applied to the combination. Compare the rate of heat produced in them. [**Ans :** 16 : 4 : 55]

◢ F. Harder numerical problems :

1. 220 identical electric bulbs each of resistance 300 ohm are connected in parallel and a battery is connected across the combination. EMF of the battery is 100V and internal resistance 0·5 ohm. What power is produced by each bulb ?

If one bulb is fused, what is the percentage increase of power produced by one bulb now ? [**Ans :** 0·25%]

[**Hints :** Initially, $P = \dfrac{E^2R}{N^2\left[r+\frac{R}{N}\right]^2}$, finally $P' = \dfrac{E^2R}{(N-1)^2\left[r^2+\frac{R}{N-1}\right]^2}$

$\therefore$ Percentage rise $= \dfrac{P'-P}{P}\times 100 = \dfrac{2r}{rN+R}\times 100 = \mathbf{0\cdot 25\%}$]

2. A wire of length *l* and of certain diameter is heated by *x* number of cells. Now the length of the wire is made *kl* without changing the diameter. How many cells are required now to heat to wire ? [**Ans :** *kx*]

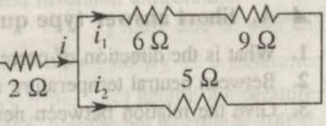


Fig. 1·39

3. In the circuit shown in fig. 1·39 heat at the rate of 10·24 cal / sec is produced in the 5Ω resistor. Calculate : (i) Rate at which heat is produced in 2 ohm resistor (ii) Potential difference across the 6 ohm resistor. [**Ans :** (i) 7·28 cal / sec, (ii) 5·88 V] **[I. I. T. 1974]**

4. A fuse of lead wire has an area of cross-section 0·2 mm^2. On short-circuit the current in the fuse wire reaches 30A. How long after the short circuit, will the fuse begin to melt ? For lead, specific heat 0·032 cal g^{-1}°C^{-1}, melting point 327°C, density 11·34 g / c.c. and resistivity 22 × 10^{-6} Ω-cm. Initial temperature of wire is 20°C. Neglect heat loss. [**Ans :** 0·095 sec] **[I. I. T. 1976]**

5. A factory requires 90 KW power. The energy is sent by cable of total resistance 2·5Ω. If 10% of power produced is lost during transmission, then calculate (a) line current (b) potential difference at the generating station, (c) potential drop due to line resistance. [**Ans :** (a) 63·25A (b) 1681 V (c) 158·13V] **[J. E. E. 1986]**

6. N electric bulbs each of resistance R are connected to a d. c. generators of internal resistance *r*. Energy radiated from a bulb is proportional to the square of heat produced. Show that, to get maximum light from the bulbs the bulbs are to be so connected that *x* bulbs are in series where $x = \sqrt{\frac{Nr}{R}}$.

7. The length of each side of a cubical box is 50 cm and thickness of the wall is 1 mm and its thermal conductivity 4 × 10^{-4} cal / (cm sec °C^{-1}). The temperature inside the box is kept at a temperature of 100°C above the surrounding temperature by a heater. If the heater is connected to 400 V d. c., what will be its resistance ? [**Ans :** 6·36Ω] **[I. I. T.]**

8. Two cells each of emf E and internal resistance *r* are connected in parallel and then to a resistance R. For what value of R, heat produced is maximum ? [**Ans :** $R = \frac{r}{2}$; $P_{max} = \frac{E^2}{2r}$]

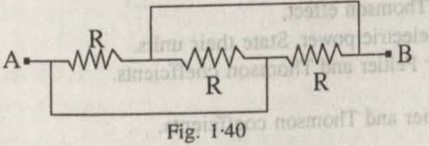


Fig. 1·40

9. In the fig 1·40, a battery of emf E and internal resistance *r* is conected between the points A and B. For what value of R, heat produced in the circuit be maximum. [**Ans :** R = 3*r*]

10. An electric motor is moving up an inclined plane of inclination 1 : 20 with a velocity of 15 miles / hr. The resistance to motion is 50 pound per ton. Efficiency of electric motor is 85% and the efficiency of the car is

80%. Calculate : (i) motor output (ii) if the supply line voltage is 220V, current through motor. Assume, mass of car = 3 ton. **[Ans : (i) 18·1 KW (ii) 0·7 A]**

11. The temperature of water in a tap is 22°C. It is required to draw a continuous supply of water at the temperature of the human body (37°C) flowing the rate of 1 litre per minute. How much power will be consumed by a heater coil placed in the path of water ? What will be the current in the coil if the supply voltage is 210V ? **[Ans : 1050 W ; 5A]**

11. The rating of a heater is 1000W–100V. A resistor R and another resistor 10Ω are connected to the heater as shown in fig. 1.41. The combination is connected to 100V supply. For what value of R, the heater will produce 62·5 watt ? **[Ans : 5Ω]**

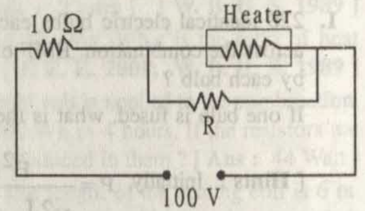


Fig. 1·41

12. The lengths, diameters and resistivities of two wires X and Y are in the ratio 3 : 1, 1 : 2 and 1 : 20 respectively. To the parallel combination of the wires a battery of emf 2V and negligible internal resistance is connected. What is the ratio of heat produced in the wires ? If the two wires produce heat at the rate of 0·5 cal /sec, find the resistance of the two wires.

[Ans : $H_1 : H_2 = 5 : 3$, $R_1 = 3{\cdot}05\Omega$, $R_2 = 5{\cdot}08\Omega$]

❑ THERMOELECTRICITY ❑

◢ A. Short answer type questions :

1. What is the direction of current through the hot junction of a Sb-Bi couple ?

2. Between neutral temperature and inversion temperature, which one is not constant for a couple ?

3. Give the relation between neutral temperature and inversion temperature for a thermocouple when the cold junction is at t_1°C.

4. Which one of the following is called 'Specific heat capacity of electricity' ?
(a) Peltier coefficient (b) Thomson coefficient (c) Thermoelectric emf (d) Thermoelectric power

5. Which one of the following is called 'Specific latent heat capacity' ?
(a) Peltier coefficient (b) Thomson coefficient (c) Thermoelectric emf (d) Thermoelectric power

6. What is the unit of 'a' in the equation : $e = at + bt^2$?

7. What is the unit of 'a' in the equation : $P = a + bt$?

8. Define Peltier coefficient. What is its unit ?

9. Define Thomson coefficient. What is its unit ?

10. What is a thermoelectric curve ? Write its equation.

11. Define thermoelectric power. What is its unit ?

12. When is Thomson coefficient positive or negative ? Give example.

13. State the law of intermediate metals of thermoelectricity.

◢ B. Essay type questions :

1. What is Seebeck effect and what is its origin ?

2. Explain the terms neutral temperature, inversion temperature, thermoelectric power. Describe an experiment to measure accurately the thermo-emf of a thermocouple.

3. State and explain Peltier effect. In what way is Peltier effect different from Joule heating effect ?

4. Describe an experiment to demonstrate Seebeck and Peltier effect. Apply laws of thermodynamics to Peltier phenomenon only and show how it led to the discovery of Thomson effect.

5. Define Peltier coefficient, Thomson coefficient and Thermoelectric power. State their units.

6. From thermodynamical consideration derive expressions for Peltier and Thomson coeffcients.

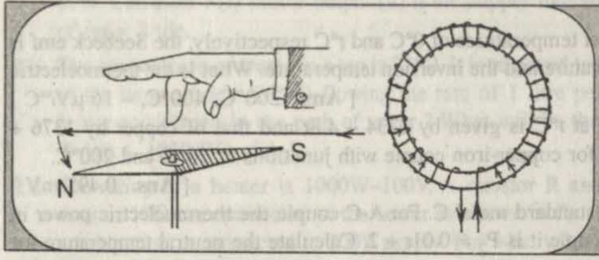
7. Derive the relations : $\pi = T\dfrac{dE}{dT}$ and $\sigma = -T\dfrac{d^2E}{dT^2}$ for Peltier and Thomson coeffcients.

8. What are Tait diagram ? Explain how the Peltier coefficient, Thomson coefficients, total thermo-emf and neutral temperature can be measured from these diagrams.

C. Numerical problems :

1. For a thermocouple, the junctions of which are at temperatures of 0°C and t°C respectively, the Seebeck emf is $E = 16t - 0.04t^2\ \mu V$. Calculate the neutral temperature and the inversion temperature. What is the thermoelectric power at inversion temperature ? [**Ans.** 200°C, 400°C, – 16 μV/°C]
2. The thermoelectric power for iron against lead at t°C is given by $1734 - 4.8t$ and that of copper by $1376 + 0.95t$ in centimicrovolt / °C. Calculate the emf for copper-iron couple with junctions at 20°C and 200°C. [**Ans.** 0.495 mV]
3. The metals A and B form a couple each with a standard metal C. For A-C couple the thermoelectric power in μV/°C is $P_1 = -0.04t + 16$ while for the B-C couple it is $P_2 = 0.01t + 2$. Calculate the neutral temperature for the A-B couple. [**Ans.** 280°C]
4. The thermo-electric power of iron is 17.5 μV/°C at 0°C and 5 μV/°C at 250°C. For cadmium the thermoelectric power is 3 μV/°C at 0°C and 15 μV/°C at 300°C. Calculate neutral temperature for iron-cadmium couple. [**Ans.** 161°C]
5. When the temperature difference between the hot and the cold junctions of a thermocouple is 100K, an emf of 1mV is generated. When the temperature of the cold junction is raised by 20 K, determine the percentage by which the emf is changed. Assume that the thermoelectric power remains constant over the whole temperature range. [**Ans.** 20%]
6. The equation of the thermoelectric curve for a couple is $e = 16.5t - 0.0165t^2\ \mu V$. Calculate its neutral temperature and inversion temperature. [**Ans.** 500°C, 1000°C]
7. One junction of a thermocouple is maintained at 0°C and the other at t°C. Its emf equation is $e = at + bt^2$. If $a = 10\ \mu V/°C$ and $b = -0.025\ \mu V/(°C)^2$, calculate its neutral temperature and inversion temperature. [**Ans.** 200°C, 400°C]
8. For a copper-iron thermocouple, the equation of its thermo-electric curve is $e = a\theta + \frac{1}{2}b\theta^2 \mu V$ (θ = difference of the junction temperatures). Calculate : (a) Peltier coefficient, π ; (b) $\sigma_{Ca} - \sigma_{Fe}$; (c) neutral temperature; (d) inversion temperature and (e) total thermo-emf. Assume, cold junction at 0°C and the hot junction at 100°C, $a = 13.89$ and $b = -0.042$.
[**Ans.** (a) $\pi_1 = -3792\ \mu V$, $\pi_2 = -3614\ \mu V$; (b) 1356 μV; (c) 330.7°C; (d) 661.4°C]

ELECTROMAGNETISM–I



CHAPTER 2

MAGNETIC FIELD DUE TO A CURRENT

• **TOPICS :** ○ Magnetic field; ○ Oersted's experiment; ○ Direction of magnetic field due to a current; ○ Nature of magnetic fields due to current carrying conductors of different shapes; ○ De La Rive's floating battery; ○ Biot-Savart's law; ○ Magnetic field due to a long straight conductor; ○ Magnetic field due to a moving charge; ○ Magnetic field due to a circular coil carrying current; ○ Field at a point on the axis of a solenoid; ○ Ampere's circuital law; ○ Application of Ampere's circuital law; ○ Current loop as a magnetic dipole; ○ Force on a charged particle in a magnetic field; ○ Force on a current carrying conductor in a magnetic field; ○ Force between two long parallel conductors carrying current; ○ Lorentz force; ○ Magnetic field due to a moving charge; ○ Magnetic force between point charges; ○ Barlow wheel; ○ Experiment on the mutual force between two parallel currents; ○ Torque on a current loop in a magnetic field; ○ Galvanometers; ○ Table galvanometer; ○ Ammeter and Voltmeter; ○ Short answer type questions (with answers); ○ Exercise.

❂ 2.1. Magnetic field ❂

We know that an electric field produced by a charged body around it, can exert a force on a charge placed in the field. It is immaterial whether the charge is at rest or in motion inside the electric field–in both the cases, a force acts on the charge.

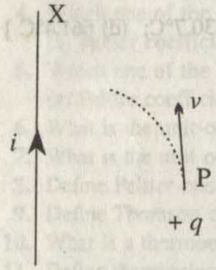


Fig. 2·1

Now, if a charge q is placed at rest at a point P near a conductor carrying a current i, it experiences no force. It proves that there is no electric field at P near the conductor. But, if the charge at P is given a velocity v in the direction of current [Fig 2.1], immediately the moving charge is attracted by the wire. Evidently a new type of field is produced at P which exerts a force on the moving charge and it does not exert any force when the charge is at rest. This field is different from the electric field which always exerts a force on a charge whether it is at rest or in motion.

Evidently this field is quite different from electric field. This new field is termed as **magnetic field.** The force exerted by a magnetic field is called magnetic force.

An electric field exists around a charged rod. Similarly, a magnetic field exerts around a current carrying conductor. A magnetic field exerts a force on a charged particle only when it is in motion. Like electric field, the magnetic field is a vector field—it has both magnitude and direction. **The magnetic field is denoted by $\vec{B}$ vector. $\vec{B}$ vector is also called magnetic induction or magnetic flux density.**

As electric field is expressed by electric lines of force, similarly the $\vec{B}$ vector field is expressed by lines of induction. The magnetic field is related to lines of induction by the following :

(i) The tangent drawn at any point on a line of induction gives the direction of $\vec{B}$ vector.

(ii) The number of lines of induction passing normally through a unit area is proportional to the magnitude of $\vec{B}$ vector. Hence $\vec{B}$ is stronger at a point where lines of inductor are very close.

(iii) Total magnetic flux φ_B flowing through an area is related to $\vec{B}$ vector as :

$$\varphi_B = \int_s \vec{B} \cdot \vec{ds} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2.1)$$

The science of electricity and the science of magnetism developed independently until 1820, when Hans christian Oersted first discovered a connection between them. He showed that current in a wire can produce magnetic effects, namely, it can deflect a magnetic needle placed near it. Oersted's simple experiment proves that electric current produces magnetic field because the magnetic needle is only deflected by magnetic field. The result of his experiment gave birth to a new science called **electromagnetism**.

Twelve years latter at 1832 Michael Faraday experimentally showed that when the current in a circuit is varied (*i.e.* made on or off) a transient current is induced in a second neighbouring circuit. Similarly, effect is noticed when a magnet is taken towards or away from a closed circuit.

So, **Oersted's experiment demonstrates that moving charges produce magnetic field and Faraday's experiment demonstrates that a changing magnetic flux or a moving magnet can produce electric current.**

Out of three effects of electric currents (heating effect, chemical effect and magnetic effect) heating and electric effects of electric current depends on external conditions ; e.g., current flowing through a resistor produces heat and chemical dissociation occurs only when current flows through an electrolyte. But magnetic effect of electric current does not depend on any external condition. Because moving charges always produce magnetic field around the current.

✪ 2.2. Oersted's experiment ✪

In 1820, H. C. Oersted first discovered the magnetic effect of electric current. The experimental arrangement is shown in fig 2.2. The wire AB is set parallel to the magnetic needle which is freely pivoted [Fig 2.2 (a)] when the key is not inserted, the wire and the needle both are pointing north-south direction. As soon as key is inserted, the magnetic needle swings to a new position as shown in fig 2.2 (b). When the current in the circuit is reversed the needle is deflected in opposite direction [fig 2.2(c)].

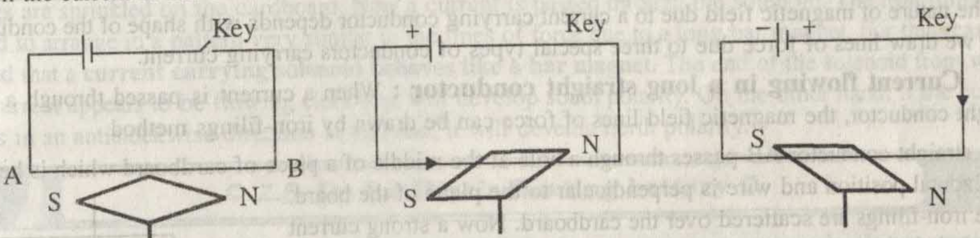


Fig. 2·2

Evidently, the deflection of the needle is due to the magnetic field. Thus a **moving charge** *i.e.* current produces a magnetic field. Also it is found that direction of deflection of the needle depends on (i) relative positions of needle and the wire and (ii) also on the direction of current.

✪ 2.3. Direction of magnetic field due to a current ✪

For a given direction of current in a wire the following rules may be applied to find the direction of deflection of the needle due to a current. Needless to ray that the direction of deflection of the north pole of the needle denotes the direction of lines of force of the magnetic field. Because, the direction of force on the north pole in the direction of magnetic field.

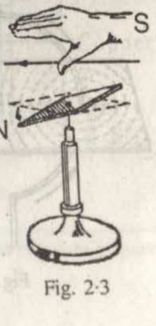


Fig. 2·3

- (i) **Oersted's rule :** Stretch the right hand and place it above the needle and the wire so that the wire remains in between the needle and the hand. If the four fingers (excepting the thumb) point the direction of current, then the thumb gives the direction of deflection of the north pole of the needle.

● (ii) **Ampere's swimming rule :** If a swimmer is imagined to be swimming along the wire in the direction of current with his arms outstretched and facing the needle. Then the N-pole of the needle with be deflected towards his left arm [Fig 2.4].

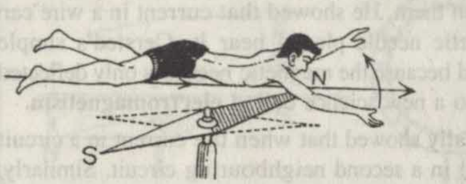


Fig. 2·4

● (iii) **Maxwell's Corkscrew rule :** If a right-handed cork-screw is assumed to be driven along the wire in the direction of the current, then the direction in which the thumb rotates gives the direction of deflection of the N-pole of the needle [Fig 2.5].

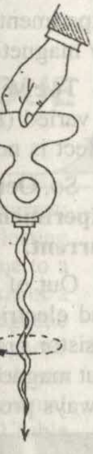


Fig. 2·5

● (iv) **Thumb rule :** According to this rule, stretch the first three fingers of your right hand so that they are at right angles to each other. If the fore-finger points to the direction of current and the middle finger towards the magnetic needle, then the thumb gives the direction of deflection of the north pole of the needle [Fig 2.6].

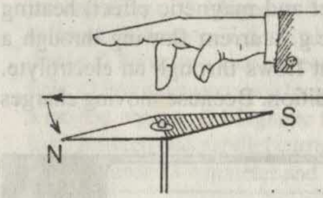


Fig. 2·6

✪ 2.4. Nature of magnetic fields due to current carrying conductors of different shapes ✪

The nature of magnetic field due to a current carrying conductor depends with shape of the conductor. Here we draw lines of force due to three special types of conductors carrying current.

● (i) **Current flowing in a long straight conductor :** When a current is passed through a long straight conductor, the magnetic field lines of force can be drawn by iron-filings method.

A straight conductor AB passes through a hole at the middle of a piece of cardboard which is held in a horizontal position and wire is perpendicular to the plane of the board. Some iron-filings are scattered over the cardboard. Now a strong current is passed through the wire, the filings will be found to arrange along concentric circles surrounding the wire. This arrangement of iron filings represents the lines of force. The direction of the lines of force may be obtained by applying any of the rules discussed in Art. 2.3.

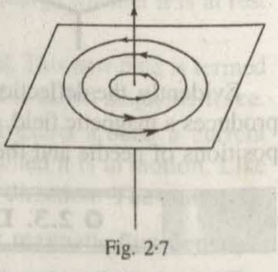


Fig. 2·7

● (ii) **Current flowing in a circular conductor :** To study the nature of magnetic field due to a circular coil carrying current, a short circular wire of several turns is fixed to a piece of horizontal cardboard so that the centre of the coil lies on the place of the board [Fig 2.8]. Here half of the coil remains above the board and the other half remains below the board. Some iron-filings are sprinkled on the board and a strong current is passed through the coil. The board is gently tapped. The iron filings are found to arrange in a definite pattern as shown in fig. This pattern represents the magnetic lines of force due to the circular coil. It is found that near the wire, the lines of force are closed loops. Near about the centre of the coil, the lines are straight and parallel and they are parallel

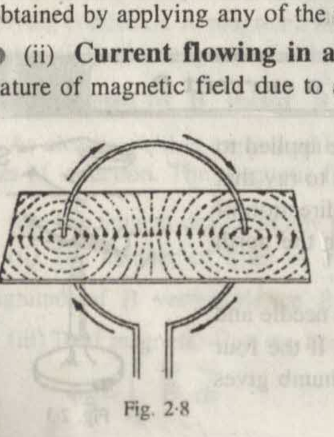


Fig. 2·8

to the plane of the coil. So, over a small area round the centre of the coil, the magnetic field is uniform and strictly parallel to the axis of the coil.

A circular conductor carrying current is equivalent to a magnetic shell.* The area of the coil is equal to the area of the magnetic shell with thickness equal to thickness of the wires. The circular coil develops north polarity on one face and other face develops both polarity like a magnetic shell. The face of the coil in which current appears clockwise, develops south pole and the face in which current appears anticlockwise, develops north polarity.

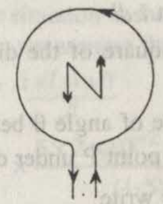


Fig. 2·9

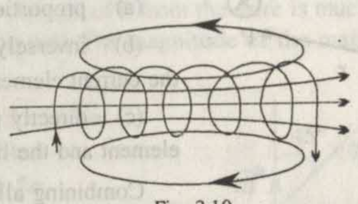


Fig. 2·10

(iii) **Current flowing in a solenoid :** If a long insulated wire is wound on an insulating uniform cylinder such that every turn of the coil is at right angle to the axis of the cylinder, then such a coil is called **solenoid**. So it is helix like structure and may be supposed to be made of a number of circular coils cell connected in series [Fig 2.10]

When a current is passed through the solenoid, the nature of lines of force is shown in the fig. Within the solenoid the lines of force are densely arranged and parallel to each other. The resultant magnetic field is very similar to that of a bar magnet. This can be demonstrated in the following way.

A long solenoid is fixed on a horizontal cardboard so that its axis lies on the cardboard some iron filings are sprinkled on the cardboard. Now a current is passed through the solenoid. The iron filings are found to arrange in a pattern very similar to the lines of force due to a long bar magnet. For this reason it is said that a **current carrying solenoid behaves like a bar magnet**. The end of the solenoid from which the current appears to be flowing clockwise will develop south polarity. On the other hand, if the current flows in an anticlockwise direction at that end, it will develop north polarity.

❂ 2.5. De La Rive's floating battery ❂

The fact that a long current carrying solenoid behaves as a bar magnet can be demonstrated an arrangement known as De la Rive's floating battery [Fig 2.11].

A zinc and copper plates are partly dipped in dil sulphuric acid taken in a wide and fairly long test tube. Two wires are soldered to the mouth of the test tube. A solenoid is connected to the two wires as shown in fig.

The whole arrangement is then allowed to float in water. To float it upright some mercury is put in the test tube. The solenoid is found to remain align in the north-south direction. If it is slightly deflected it comes back to the previous position after a few oscillations.

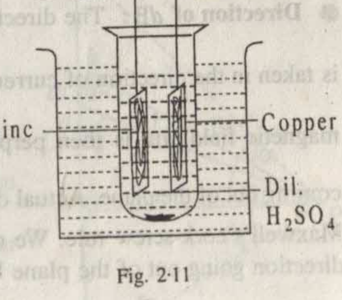


Fig. 2·11

Explanation : The two dissimilar plates of zinc and copper together with dil H_2SO_4 form a simple voltaic cell. A current flows through the solenoid. As a result, the solenoid behaves like a floating bar magnet and so it set itself in magnetic meridian just like a freely suspended magnet.

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* Magnetic shell is a thin sheet of magnetic shell which is magnetised perpendicular to the surface so that one face acquires north polarity and the other face acquire south polarity.

❂ 2.6. Biot-Savart's law ❂

The quantitative measurement of magnetic field produced by a current carrying conductor of any shape is obtained by applying **Biot-Savart's law**. It is a basic law in electricity and magnetism.

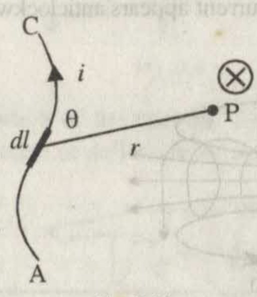


Fig. 2·12

According to the law, the magnetic field dB at any point P due to current i flowing through a small segment of length dl is

(a) proportional to current element $i\ dl$

(b) inversely proportional to the square of the distance r between the current element and the point p.

(c) directly proportional to the sine of angle θ between the current element and the line joining it with the point P under consideration

Combining all the above factors, we write

$$dB \propto \frac{idl\sin\theta}{r^2} \quad \text{or,} \quad dB = k\,.\,\frac{idl\sin\theta}{r^2} \; \ldots\ldots\ldots(2.1)$$

The value of the constant of proportionality k depends upon the nature of the medium between the observation point and the current carrying conductor and the system of units used to measure the quantities involved. If the medium be vacuum or air, then in SI

$$k = \frac{\mu_0}{4\pi} = 10^{-7}\ wb/(\mathrm{A}-m) \ [\ \mu_0 = \text{absolute permeability of free space}]$$

Therefore, in SI, Biot - Savart's law may be expressed as

$$dB = \frac{\mu_0}{4\pi}\cdot\frac{idl\sin\theta}{r^2} \ldots\ldots\ldots\ldots\ldots\ldots(2.2)$$

So, the total magnetic field at P due to the whole length of the conductar is

$$B = \frac{\mu_0}{4\pi}\int\frac{idl\sin\theta}{r^2} \ldots\ldots\ldots\ldots\ldots\ldots(2.3)$$

In the language of vector, the equation (2.2) can be written as

$$\vec{dB} = \frac{\mu_0}{4\pi}\,\frac{i\,\vec{dl}\times\hat{r}}{r^2} = \frac{\mu_0}{4\pi}\,\frac{i\,\vec{dl}\times\vec{r}}{r^3} \ldots\ldots\ldots\ldots(2.4)$$

- **Direction of $\vec{dB}$:** The direction of $\vec{dB}$ is same as that of the vector $i\,\vec{dl}\times\vec{r}$ when direction of $\vec{dl}$ is taken in the direction of current flow. Usually the plane of the diagram contains both $\vec{dl}$ and $\vec{r}$. The magnetic field $\vec{dB}$ is then perpendicular to the plane of the diagram, either going into the plane or coming out of the plane. Actual direction of $\vec{dB}$ is obtained according to the rules of cross product or by Maxwell's cork-screw rule. We denote the direction going into the plane by an **enclosed cross** and the direction going out of the plane by an encircled dot.

In SI, unit of $\vec{B}$ vector field is Wb/m^2 or Tesla (T) and c. g. s. system, its unit is Gauss (G). The relation between the two units is $1\,T = 10^4\,G$

So, the magnitude of $\vec{H}$ is $H = \frac{1}{4\pi} i \int \frac{dl \sin\theta}{r^2}$ $\therefore\ \vec{H} = \frac{1}{4\pi} i \int \frac{\vec{dl} \times \vec{r}}{r^3}$ ....(2.5)

Unit of H is $\mathbf{Am^{-1}}$

- **Example 2.1. A wire placed in North-South direction carries a current of 5A from south to north. Find the magnetic field due to a wire of length 2 cm at a point 150 cm north-east from the piece**

○ **Solution :** The situation is shown in fig 2.13. As the distance of P from the wire is much larger that the length of the wire, we assume the wire to be a small element. The magnitude of the magnetic field is

$$dB = 10^{-7}\ \frac{i\, dl \sin\theta}{r^2}$$

$$\therefore\quad dB = 10^{-7} \times \frac{5 \times 2 \times 10^{-2} \times \sin 45^\circ}{(1\cdot5)^2}$$

$$= \mathbf{3\cdot152 \times 10^{-9}\ T.}$$

$dl = 2 \times 10^{-2}\, m$
$i = 5A$
$r = 1\cdot5m$
$\theta = 45^\circ.$

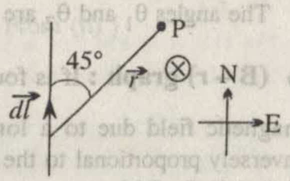


Fig. 2·13

The direction is perpendicular to the plane of the paper and inwards.

▲ Application of Biot-Savart's law

◎ 2.7. Magnetic field due to a long straight conductor ◎

The lines of force of the magnetic field produced by a current carrying long straight conductor are circular with the centres lying on the conductor. Also these lines of force are situated in a plane perpendicular to the plane of the conductor [Fig 2.14].

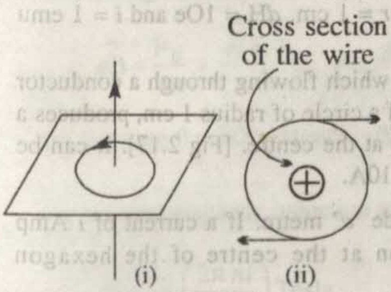


Fig. 2·14

XY is a long straight conductor which carries a steady current *i*. We require to calculate $\vec{B}$ vector field at P at a distance *r* from the conductor [Fig 2.15]

Now, the magnetic field at due to the element *ab* = *dl* at P is

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{i\, dl \sin(180^\circ - \theta)}{x^2} = \frac{\mu_0}{4\pi} \cdot \frac{i\, dl \sin\theta}{x^2}$$

From the right angle triangle *abc*, *ac* = *dl* sin θ and from the triangle *aPc*, *ac* = *x.dα*

$\therefore\quad dl \sin\theta = ac = x\,.\,d\alpha$

$$\therefore\quad dB = \frac{\mu_0}{4\pi} \cdot \frac{i\, x \cdot d\alpha}{x^2} = \frac{\mu_0}{4\pi} \cdot \frac{i}{x}\, d\alpha$$

Again, $\frac{r}{x} = \cos\alpha$ or $\frac{1}{x} = \frac{1}{r}\cos\alpha$ $\therefore\ dB = \frac{\mu_0}{4\pi} \cdot \frac{i}{r} \cos\alpha\,.\,d\alpha$

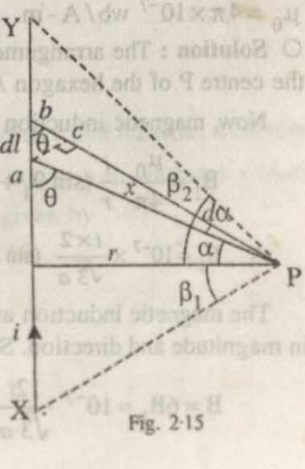


Fig. 2·15

The field at P due to all other small elements are in the same direction (perpendicular to the plane of the paper and directed in wards).

Hence the resultant field at P due to the whole length of the conductor XY is

$$B = \frac{\mu_0}{4\pi} \cdot \frac{1}{r} \int_{-\beta_1}^{\beta_2} \cos\alpha\,.\,d\alpha = \frac{\mu_0}{4\pi}\ \frac{i}{r}(\sin\beta_2 + \sin\beta_1) \ldots\ldots\ldots (i)$$

- **Special case :** (i) If the canductor is very long and the point P is close to the conductor

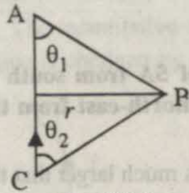


Fig. 2·15(a)

Then, $\beta_1=\beta_2=\pi/2$ then, $B=\frac{\mu_0}{4\pi}\cdot\frac{2i}{r}\ wb/m^2$ .... (2.6)

∴ magnitude of $\vec{H}$ vector, $|\vec{H}|=\frac{1}{4\pi}\cdot\frac{2i}{r}\ \text{A}m^{-1}$

(ii) If the conductor is short one then from the equation (i),

$$B=\frac{\mu_0}{4\pi}\frac{i}{r}(\cos\theta_1+\cos\theta_2)$$

The angles θ_1 and θ_2 are shown in fig 2.15 (a).

- **(B – *r*) graph :** If is found from the equation 2.6, $B\,\alpha\,\frac{1}{r}$. So, the magnetic field due to a long straight conductor carrying current is inversely proportional to the distance from the conductor. The graph is shown in fig 2.16.

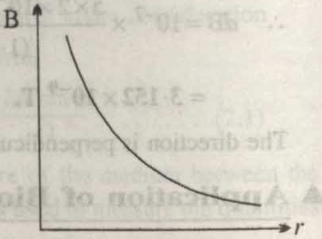


Fig. 2·16

- In c. g. s. system, $H=\frac{2i}{r}$ Oe ;

Here *r* is measured in cm and current *i* is measured in emu.

- **A discussion : ❖ Definition of 1 c. g. s. emu of current**—From Biot-Savart's law, magnetic field intensity at distance *r* from a current element *i dl* is

$$dH\,\alpha\,\frac{i\,dl\sin\theta}{r^2}\qquad\therefore\quad dH=k\frac{i\,dl\sin\theta}{r^2}$$

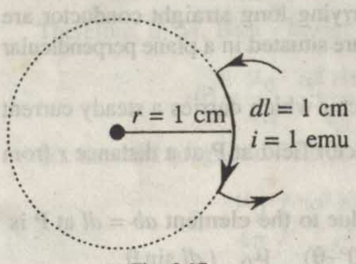


Fig. 2·17

Now, if dl = 1 cm, θ = 90°, r = 1 cm, dH = 1Oe and i = 1 emu then $k = 1$

Hence, 1 emu is the current which flowing through a conductor of length 1 cm bent in the arc of a circle of radius 1 cm, produces a magnetic field of intensity 1Oe at the centre. [Fig 2.17]. It can be shown that 1 emu of current = 10A.

- **Example 2.2.** A conductor is bent into a regular hexagon of side '*a*' metre. If a current of *i* Amp is passed through the conductor, calculate magnetic induction at the centre of the hexagon $\mu_0=4\pi\times10^{-7}$ wb / A - m.

○ **Solution :** The arrangement is shown in the fig. 2.18. We require to calculate magnetic induction at the centre P of the hexagon ACDEFG.

Now, magnetic induction at P due to AC arm of the hexagon is

$$B_1=\frac{\mu_0}{4\pi}\cdot\frac{i}{r}(\sin\theta_1+\sin\theta_2)\quad\Bigg|\quad r=\frac{\sqrt{3}}{2}a\,;\ \theta_1=\theta_2=30^\circ$$

$$\therefore\quad B_1=10^{-7}\times\frac{i\times2}{\sqrt{3}\,a}\ (\sin30^\circ+\sin30^\circ)=10^{-7}\times\frac{2i}{\sqrt{3}a}$$

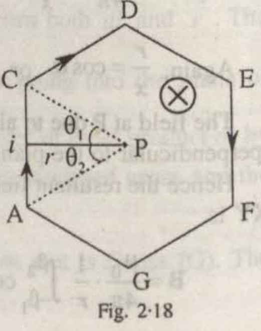


Fig. 2·18

The magnetic induction at P due to each of the other sides an equal both in magnitude and direction. So the total field at the point P of the hexagon is

$$B=6B_1=10^{-7}\ \frac{12i}{\sqrt{3}\,a}=10^{-7}\times\frac{4\sqrt{3}}{a}\ i\ wb/m^2$$

✪ 2.8. Magnetic field due to a moving charge ✪

From Biot-Savart's law, the magnetic induction at a distance *r* from the current element *i dl* is

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{i\,dl \sin\theta}{r^2} \quad \ldots\ldots\ldots\ldots \text{(i)}$$

This magnetic field is total sum of the magnetic fields produced by each of small moving charges inside the conductor. Let charge of each particle be $+q$, their average velocity v. If A be the area of cross section and n be the number of particles per unit volume then $i = nqAv$. . . . . (ii)

Then the free charge on the current element dl is

$dQ = qn\,A\,dl$. So, $i\,dl = (dQ)\,v$ . . . . (iii) [From (ii)]

$\therefore$ From the equations (i) and (iii), $dB = \dfrac{\mu_0}{4\pi} \cdot \dfrac{(dQ)\,v \sin\theta}{r^2}$

Now, if we consider finite length of the conductor

$$B = \frac{\mu_0}{4\pi} \cdot \frac{Q\,v \sin\theta}{r^2} \quad \ldots\ldots\ldots\ldots \text{(2.7)}$$

✪ 2.9. Magnetic field due to a circular coil carrying current ✪

● (a) **At the centre of the coil :** Let the radius of the circular coil is r and number of turns n. It carries a current i. [Fig 2.19].

We require to calculate magnetic induction at the centre O of the circular coil.

Now, magnetic induction at a distance r due to a current element $i\,dl$ is

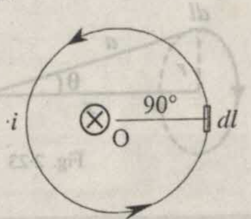


Fig. 2·19

$$\overrightarrow{dB} = \frac{\mu_0}{4\pi}\, i\, \frac{\overrightarrow{dl} \times \overrightarrow{r}}{r^2} \quad \text{or,} \quad dB = \frac{\mu_0}{4\pi}\, \frac{i\,dl \sin\theta}{r^2}$$

In this case $\theta = 90°$. Hence, $dB = \dfrac{\mu_0}{4\pi} \cdot \dfrac{i\,dl}{r^2}$

Hence total magnetic induction at 0 is $B = \dfrac{\mu_0}{4\pi}\, \dfrac{i}{r^2} \sum dl = \dfrac{\mu_0}{4\pi}\, \dfrac{i}{r^2} \times 2\pi rn$

$$\therefore \quad B = \frac{\mu_0}{4\pi}\left(\frac{2\pi ni}{r}\right) \text{Tesla} \quad \ldots\ldots\ldots\ldots \text{(2.8)}$$

$$\therefore \quad H = \frac{1}{4\pi}\, \frac{2\pi ni}{r^2}\ \text{A}m^{-1} \quad \ldots\ldots\ldots\ldots \text{(2.9)}$$

● (b) **On the axis of the circular coil :** Consider a circular loop of radius r and it carries a current i. We require to calculate magnetic induction $\overrightarrow{B}$ at a point P on the axis of the loop at a distance x from its centre 0 [Fig 2.20]. It can be shown that the magnetic induction at P is given by

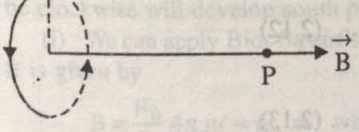


Fig. 2·20

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2\pi nr^2 i}{(r^2 + x^2)^{3/2}}\ \text{T}$$

$$\text{and } H = \frac{1}{4\pi}\, \frac{2\pi nr^2 i}{(r^2 + x^2)^{3/2}}\ \text{A}m^{-1}$$

Evidently, the magnitude of magnetic induction is maximum at the centre and gradually decreases along the axis of the coil. It is shown in fig 2.21.

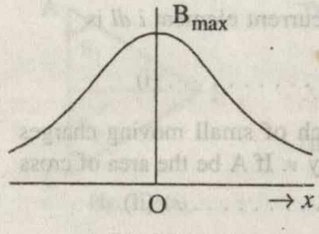


Fig. 2·21

Now, $A = \pi r^2$ is the area of the circular coil. If $x >> r$

then, $$B = \frac{\mu_0}{4\pi} \cdot \frac{2\,i\,n\,A}{x^3} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2.10)$$

From the equation 2.10, we see that a circular coil carrying current is equivalent to a magnetic dipole. Its magnetic dipole moment is $M = i\,n\,A$.

$$\therefore \quad B = \frac{\mu_0}{4\pi} \cdot \frac{2M}{x^3}\ \text{T} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2.11)$$

We can compare this with the expression for the electric field due to an electric dipole at a point on the dipole axis. It is

$E = \frac{1}{4\pi \in_0} \frac{2p}{x^3}$ where p is electric dipole moment.

The magnetic field at a point not on the axis is difficult to calculate. Here we show qualitatively in fig 2.22 the magnetic field lines due to a circular current.

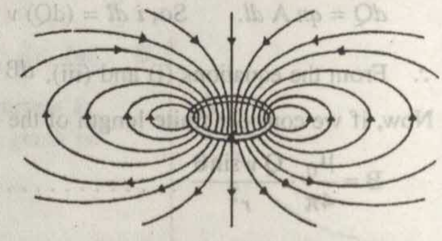


Fig. 2·22

● **Deduction for the equation 2.9 :** We calculate the magnetic induction at the point P situated on the axis of a circular coil of radius r, having n turns and it carries a current i as shown in fig 2.23. Magnetic induction dB at P due to the small segment dl of the coil is

$$dB = \frac{\mu_0}{4\pi} \cdot i \frac{\vec{dl} \times \vec{a}}{a^3}$$

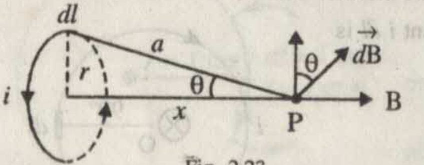


Fig. 2·23

In this case $\vec{a} \perp \vec{dl}$. $\vec{dl}$ is perpendicular to the plane of the paper. As $\vec{dB}$ is perpendicular to the plane in which $\vec{a}$ and $\vec{dl}$ lie, so $\vec{dB}$ will lie in the plane of the paper [Fig 2.23].

Next we resolve $\vec{dB}$ into two rectangular components : the component $dB \sin\theta$ is along the axis of the coil and $dB \cos\theta$ perpendicular to the axis. If we consider other dl elements of the coil, due to symmetry all ($dB \cos\theta$) components cancel each other, *i. e.* $\sum dB \cos\theta = 0$. But all the $dB \sin\theta$ components are added. Hence resultant magnetic induction at P is

$$B = \int dB \sin\theta = \frac{\mu_0}{4\pi} \int \frac{i\,dl}{a^2} \sin 90^\circ \cdot \sin\theta$$

$$= \frac{\mu_0}{4\pi} \frac{i}{a^2} \sin\theta \int dl = \frac{\mu_0}{4\pi} \frac{i}{a^2} \cdot \frac{r}{a} \cdot 2\pi nr \quad \left[a = \sqrt{r^2 + x^2}\right]$$

$$\therefore \quad B = \frac{\mu_0}{4\pi} \cdot \frac{2\pi n r^2 i}{(r^2 + x^2)^{3/2}}\ wb/m^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (2.12)$$

$$\therefore \quad H = \frac{1}{4\pi} \frac{2\pi n r^2 i}{(r^2 + x^2)^{3/2}}\ \text{Am}^{-1} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2.13)$$

Now, moment of the coil $M = \pi r^2 ni$

$$\therefore \quad B = \frac{\mu_0}{4\pi}\,\frac{2M}{(r^2+x^2)^{3/2}} \quad \text{and} \quad H = \frac{1}{4\pi}\,\frac{2M}{(r^2+x^2)^{3/2}}$$

- **Example 2.3. The radius of a circular coil is 5 cm and number of turns 100. If it carries a current of 5A, calculate (a) magnetic moment of the coil, (b) magnetic induction at the centre of the coil.**

○ **Solution :** (a) Magnetic moment, $M = inA$

$$\therefore \quad M = 5 \times 100 \times \pi\,(5 \times 10^{-2})^2 = 3{\cdot}925\ \text{A}m^2$$

(b) Magnetic induction at the centre of the coil is $B = \frac{\mu_0}{4\pi} \cdot \frac{2M}{r^3}$

$$\therefore \quad B = 10^{-7}\ \frac{2 \times 3{\cdot}925}{(5 \times 10^{-2})^3} = 6{\cdot}28 \times 10^{-3}\ \text{T}$$

- **Ex. 2.4. A circular coil of radius 5 cm and having 50 turns is made of a wire whose diameter is 0·2 mm and resistivity $2 \times 10^{-6}\,\Omega - \text{cm}$. What potential difference be applied at the two ends of the coil so that the magnetic field produced at its centre be just equal to horizontal intensity of earth's magnetism. H = 0·314 0e**

○ **Solution :** The resistance of the coil, $R = \rho\,\frac{l}{\pi d^2/4}$ and $l = 2\pi r\,N$

$$\therefore \quad R = \rho \cdot \frac{8\pi r\,N}{\pi d^2} = \rho \cdot \frac{8 r N}{d^2}$$

$$\therefore \quad R = \frac{2 \times 10^{-6} \times 8 \times 5 \times 50}{(0{\cdot}02)^2} \qquad [\rho = 2 \times 10^{-6}\,\Omega - \text{cm},\ v = 5\,\text{cm},\ N = 50,\ d = 0{\cdot}02\ \text{cm}]$$

$= \mathbf{10\ ohm.}$

$$\text{Now,} \quad H = \frac{2\pi N i}{r} \;\Rightarrow\; i = \frac{rH}{2\pi N} = \frac{5 \times 0{\cdot}314}{2 \times 3{\cdot}14 \times 50} = \frac{1}{200}\ \text{emu} = \frac{1}{20}\,\text{A}$$

$$\therefore \quad \text{Required potential difference,} \quad v = Ri = 10 \times \frac{1}{20} = \mathbf{0{\cdot}5\ V}$$

○ 2.10. Field at a point on the axis of a solenoid ○

A solenoid is an insulated wire wound closely in the form of a helix. Generally, the length of the solenoid is large compared to its radius of cross-section [Fig 2.24(a)]. When a current is passed through the solenoid a magnetic field is produced. The nature of the field is shown by lines of force [Fig 2.24 (b)]. The nature of the field is very similar to that due to a bar magnet. So, a current carrying solenoids equivalent to a bar magnet. The end of the solenoid from which current appears to flow anticlockwise develops north polarity and if the current appears to be clockwise will develop south polarity.

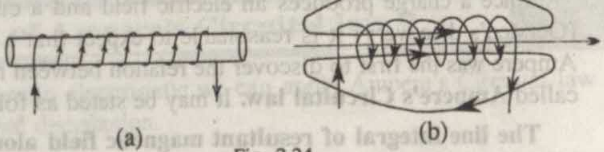


Fig. 2·24

(i) We can apply Biot-Savart's law to calculate the magnetic induction at any point inside the solenoid. It is given by

$$B = \frac{\mu_0}{4\pi}\,4\pi\,ni = \mu_0\,ni \quad \text{wb/cm}^2 \ \ldots\ldots\ldots\ (2.14) \quad \text{and} \quad H = ni \ \ \text{A}m^{-1} \ \ldots\ldots\ldots\ (2.15)$$

Here n = number of turns per unit length and i is the current.

In cgs system of units **$H = 4\pi ni$ 0e**

(ii) If the point P is near one end of the solenoid than

$$B = \frac{1}{2}\mu_0 ni \text{ wb/m}^2 \text{ ....(2.16)}$$

$$\text{and } H = \frac{1}{2} ni \text{ Am}^{-1} \text{ ....(2.17)}$$

So, magnetic induction at one end of a long solenoid is just half of its value at a point inside. The reason is the lines of force are very close inside the solenoid, but outside it is rare. [Fig 2.25]

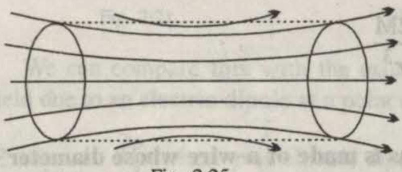


Fig. 2·25

In fig 2.26 we show a toroid of Anchor Ring. The magnetic induction inside the toroid can also be calculated by the equation applicable to a long solenoid.

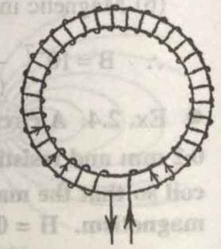


Fig. 2·26

$$\therefore \; B = \mu_0 ni = \mu_0 \frac{N}{2\pi R} i = \frac{\mu_0 Ni}{2\pi R} \text{ ....(2.18)}$$

Here N = total number of turns of the toroid and R = its radius.

- **Ex. 2.5. The number of turns of long solenoid is 20 cm^{-1}, what current be passed through it so that magnetic induction at any point inside is 2×10^{-2}T. $\mu_0 = 4\pi \times 10^{-7}$ wb/Am.**

○ **Solution :** The required relation : $B = \mu_0 ni$

$$\text{or, } i = \frac{B}{\mu_0 n} = \frac{2 \times 10^{-2}}{4\pi \times 10^{-7} \times 20 \times 100} = 7 \cdot 96\text{A}$$

$\therefore$ The required current is **7·96 A**

2.11. Ampere's Circuital law

Coulomb's law is a basic law in electrostatics. By applying this law we can calculate intensity of electric field at a point due any type of charge distribution. Another fundamental law in electrostatics is Gauss's law which gives the quantitative relationship between the surface integral of the electric field over a closed surface and the charges enclosed by the surface. Gauss's law provide an easy method to calculate electric field intensity in the case of symmetrical charge distribution. Now, the question arises : **Is there a similar relationship between magnetic field and current ?**

Since a charge produces an electric field and a current (moving charge) produces a magnetic field (Oersted's discovery) it is reasonable to expect that a magnetic analogue of Gauss's law can be found. Ampere was the first to discover the relation between magnetic field B and current i. This relationship is called **Ampere's Circuital law.** It may be stated as follows :

The line integral of resultant magnetic field along a closed curve is equal to μ_0 times the total current enclosed by the closed curve. μ_0 is absolute permability of free space. Thus

$$\int_c \vec{B} \cdot \vec{dl} = \mu_0 \; i \text{ .... 2.19}$$

Here i is the total current enclosed by the curve.

If θ be the angle between $\vec{B}$ and $\vec{dl}$, $\int_c \vec{B} \cdot \vec{dl} = \int_c B\, dl \cos\theta$

The closed path integral of $\vec{B}$ is shown in fig 2.27.

In fig 2.28, three current carrying conductors have been shown. The magnetic induction $\vec{B}$ is due to the current in all the conductors. In this case, to determine the integral only, the i_1 and i_2 are to be considered. Current i_3 will not be considered as it is not linked by the closed path. Also effective current is $i = i_1 - i_2$ as the currents are in opposite direction.

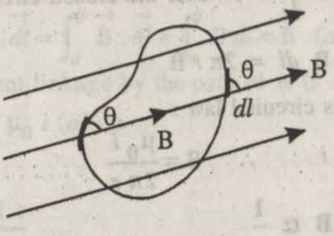


Fig. 2·27

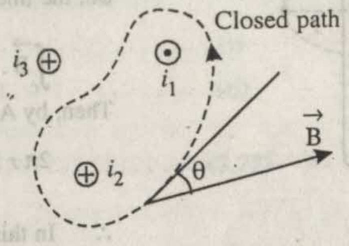


Fig. 2·28

● **Proof of the Circuital law :** Here we consider a circular path around a long straight wire carrying a current. Let XY be the long straight conductor and it carries a current I [Fig 2.29]. Circular lines of force are produced around the conductor. Let the radius of the circular path be r. dl is small segment of the path where $\vec{B}$ is the magnetic induction. Evidently $\vec{B}$ and $\vec{dl}$ acts in the same direction *i.e.* angle between then is zero. Hence the line integral for the circular path

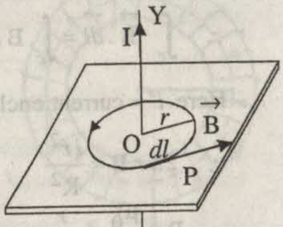


Fig. 2·29

$$\int_c \vec{B} \cdot \vec{dl} = \int_c B\, dl \quad \ldots\ldots\ldots\ldots\ldots \text{(i)}$$

Now, magnetic field at P is $B = \dfrac{\mu_0}{4\pi} \dfrac{2I}{r}$

$$\therefore \text{ From equ. (i) } \int_c \vec{B} \cdot \vec{dl} = \int_c \frac{\mu_0}{4\pi} \cdot \frac{2I}{r}\, dl = \frac{\mu_0}{4\pi} \frac{2I}{r} \int_c dl = \frac{\mu_0}{4\pi} \frac{2I}{r} 2\pi r$$

$\therefore \int_c \vec{B} \cdot \vec{dl} = \mu_0 I.$ It is Ampere's Circuital law. Here we has established the law for the special case of a field due to straight and long current carrying conductor, Ampere's law is true in general for any magnetic field for any assembly of current and for any path of integration.

● **Note :** The law has to be modified for the magnetic field arising from a time-varying electric field.

❂ 2.12. Application of Ampere's Circuital law ❂

Similar to the application of Gauss theorem in electrostatic we can apply Ampere's Circuital law to calculate magnetic field due to different current distribution.

▲ (A) Straight long conductor carrying current :

XY is a straight long conductor of radius R and a current i flows along its length. In this case the lines of magnetic induction are concentric circles with their centres lying on the axis of the wire. The planes of the circles are perpendicular to the axis of the wire.

(i) First we assume that the point P be at a distance r $(r > \mathrm{R})$ from the axis of the conductor where we like to calculate magnetic induction. A circle of radius r is drawn with its centre on the axis of the wire. Hence the lines of induction are parallel to the circular path. So, the direction of $\vec{\mathrm{B}}$ at every point is along the tangent to the circle at the point and its magnitude is same at all points on the circle.

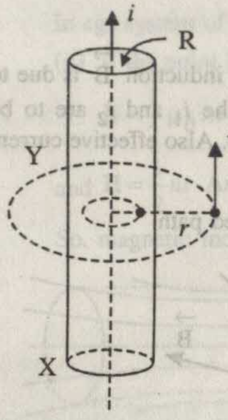


Fig. 2·30

So, the line integral $\int_c \vec{\mathrm{B}} \cdot \vec{dl}$ over the closed circular path is

$$\int_c \vec{\mathrm{B}} \cdot \vec{dl} = \int_c \mathrm{B} \; dl = 2\pi r \mathrm{B}$$

Then, by Ampere's circuital law :

$$2\pi r \mathrm{B} = \mu_0 i \qquad \therefore \qquad \mathrm{B} = \frac{\mu_0 i}{2\pi r} \quad \ldots\ldots\ldots\ldots\ldots \text{(2.20)}$$

$\therefore$ In this case $\mathbf{B} \propto \dfrac{\mathbf{1}}{r}$

(ii) Next we assume that the point P is inside the conductor. So $r < \mathrm{R}$ [Fig 2.31]. In this case also the closed circular path of radius r $(r < \mathrm{R})$ is parallel to the lines of induction and $\vec{\mathrm{B}}$ is tangent to the path.

$$\therefore \quad \int_c \vec{\mathrm{B}} \cdot \vec{dl} = \int_c \mathrm{B} \, dl = \mu_0 i' \quad \text{or,} \quad 2\pi r \mathrm{B} = \mu_0 i'$$

Here i' = current enclosed by the circular path $= i \dfrac{\pi r^2}{\pi \mathrm{R}^2} = i \dfrac{r^2}{\mathrm{R}^2}$

$$\therefore \quad 2\pi r \mathrm{B} = \frac{i r^2}{\mathrm{R}^2}$$

$$\therefore \quad \mathrm{B} = \frac{\mu_0}{2\pi} \, i \, \frac{r}{\mathrm{R}^2} \quad \ldots\ldots\ldots\ldots \text{(2.21)}$$

$\therefore$ In this case when $r < \mathrm{R}$, $\mathbf{B} \propto r$

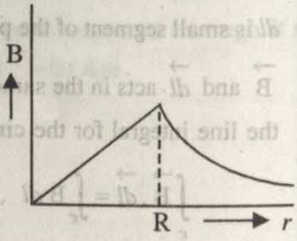


Fig. 2·31

(iii) Finally when the point P is on the surface of the conductor $r = \mathrm{R}$

$\therefore$ So, on the surface of the conductor, $\mathrm{B} = \dfrac{\mu_0}{2\pi} \cdot \dfrac{i}{\mathrm{R}}$.

The fig 2.31 shows the variation magnetic induction vector $\vec{\mathrm{B}}$ with distance from the axis of the conductor.

▲ (B) Long solenoid :

We consider a tightly wound solenoid of length L and radius R (L >> R). Its number of turns per unit length n. When current is passed through the solenoid, the lines of induction are parallel and closely packed inside the solenoid.

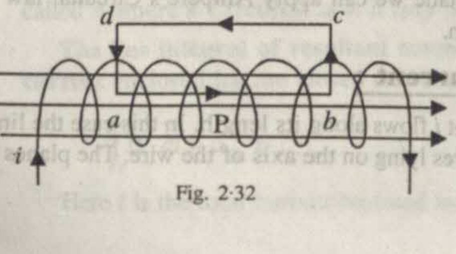


Fig. 2·32

To calculate the magnetic induction at a point P inside the solenoid by Ampere's circuital law, we consider a rectangular path *abcd* as shown in fig 2.32.

We has to calculate the line integral $\int_c \vec{\mathrm{B}} \cdot \vec{dl}$ over the closed *abcd*.

Here $\oint_c \vec{B}.\vec{dl} = \int_a^b \vec{B}.\vec{dl} + \int_b^c \vec{B}.\vec{dl} + \int_c^d \vec{B}.\vec{dl} + \int_d^a \vec{B}.\vec{dl}$

Now, magnetic induction outside the solenoid is negligible. Hence $\int_c^d \vec{B}.\vec{dl} = 0$

Again, the lines *bc* and *da* an perpendicular to the magnetic field.

$$\therefore \quad \int_b^c \vec{B}.\vec{dl} = \int_d^a \vec{B}.\vec{dl} = 0$$

Hence, $\oint_c \vec{B}.\vec{dl} = \int_a^b \vec{B}.\vec{dl} = \int_a^b B\,dl = B.(ab)$ .... (i)

Now, the current linkage by the path *ab* is $(i\,.\,ab\,.\,n)$ .... (ii)

$\therefore \quad B.(ab) = \mu_0\, i\,(ab)\, n$

$\therefore \quad B = \mu_0\, ni$ .... (2.22)

▲ (C) Toroid :

If a solenoid is bent into a circular shape and the ends are joined, we get a toroid. Alternatively, take a wooden ring and wind round it a large number of turns of an insulated copper wire. Now imagine that the wooden ring is not there. It is toroidal solenoid as shown in fig 2.33.

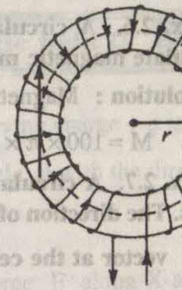


Fig. 2·33

The magnetic field in such a toroid can be obtained by using Ampere's circuital law.

Here we assume that the wire is wound tightly and turns are uniform. When a current is passed through the toroid, the lines of induction are wholly confined inside the toroid. Now, we consider a circular path of radius *r* inside the core of the toroid. By symmetry, the magnetic field will have equal magnitude at all points on this circle. Also, the field is everywhere tangential to the circle. Thus

$$\oint_c \vec{B}.\vec{dl} = \oint_c B\,dl = B\oint_c dl = 2\pi r B$$

If N be the total number of turns, this the current crossing the area bounded by the circle is N*i*, where i is the current in the toroid. Than by Ampere's circuital law :

$$2\pi r B = \mu_0 N i = \mu_0\,(2\pi r n)\, i \quad [n = \text{number of turns per unit length}]$$

$$\therefore \quad \mathbf{B = \mu_0\, ni}$$

❂ 2.13. Current loop as a magnetic dipole ❂

We have seen that a current carrying straight solenoid behaves like a bar magnet. If it is freely suspended it remains in the north-south direction like a bar magnet. A bar magnet is a magnetic dipole because it has two opposite poles at the two ends. To obtain the correlation, let us consider a circular current loop.

Now, the magnetic field on the axis at a distance *x* from the centre of a circular coil carrying current *i* is

$$B = \frac{\mu_0}{4\pi}\cdot\frac{2\pi n r^2 i}{(r^2 + x^2)^{3/2}} \quad [\ r = \text{radius and } n = \text{number of turns of the loop}\ ]$$

If $x >> r$, this expression reduces to

$$B = \frac{\mu_0}{4\pi}\cdot\frac{2\pi n r^2 i}{x^3} = \frac{\mu_0}{4\pi}\cdot\frac{2Ani}{x^3} \quad \text{......................... (i)} \quad [\pi r^2 = A, \text{ area of the coil}]$$

Let us compare it with the electric field due to an electric dipole on its axis at a large distance from it. This electric field is expressed by

$$E = \frac{1}{4\pi \in_0} \frac{2P}{x^3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(ii)}$$

Where p is the electric dipole moment. Evidently, B due to a current loop and E due to an electric dipole have the same $\left(\frac{1}{x^3}\right)$ dependence on distance. The equation (i), can be written as

$$B = \frac{\mu_0}{4\pi} \frac{2M}{x^3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(iii)}$$

Where $M = nAi$. By analogy with electric dipole moment P, the quantity M is called magnetic dipole moment. It is a vector.

$\therefore \quad \vec{M} = ni\vec{A} \quad [\vec{A}$ is the area vector]

Magnetic moment $\vec{M}$ is directed along the axis and perpendicular to the plane of the coil [Fig 2.34].

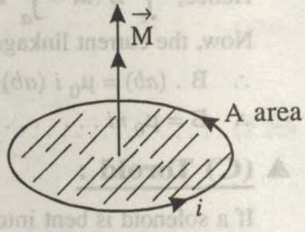


Fig. 2·34

- **Ex. 2.6. A circular coil of radius 10 cm and having 100 turns is carrying a current of 5A. Calculate magnetic moment of the coil.**

○ **Solution :** Magnetic moment. $M = nAi$

$$\therefore \quad M = 100 \times \pi \times (0 \cdot 1)^2 \times 5 = \mathbf{15 \cdot 7\ Am^2}$$

- **Ex. 2.7. A circular coil kept horizontally carries 2A current. Its radius is 8 cm and carries 10 turns. The direction of current is clockwise when seen from above. Calculate magnitude and direction of $\vec{B}$ vector at the centre of the coil.**

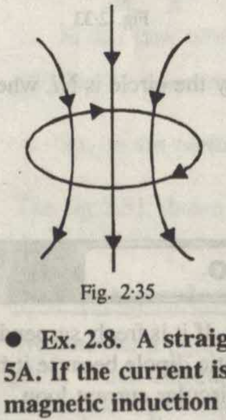


Fig. 2·35

○ **Solution :** The magnitude of $\vec{B}$ is obtained from the relation :

$$B = 10^{-7} \cdot \frac{2\pi ni}{r}$$

$$\therefore \quad B = 10^{-7} \times \frac{2 \times 3 \cdot 14 \times 10 \times 2}{0 \cdot 08} \quad [\ n = 10,\ i = 2\text{A and } r = 0 \cdot 08\text{m}\ ]$$

$$= \mathbf{1 \cdot 57 \times 10^{-4}\ T}$$

The direction of $\vec{B}$ vector is obtained by cork serew rule as shown in fig. 2.35.

- **Ex. 2.8. A straight long conductor of uniform radius of cross-section 5 cm carries a current of 5A. If the current is uniformly distributed over the whole cross-section of the conductor, calculate magnetic induction at a distance of 3 cm from the axis of the conductor.**

○ **Solution :** In fig. 2.36, P is a point at a distance of 3 cm from the axis of the conductor. Evidently the point is inside the conductor.

Here $r = 3$ cm, $R = 5$ cm.

The required relation to calculate magnetic induction at P is

$$B = \frac{\mu_0}{4\pi} \frac{2ir}{R^2} \quad \text{[vide Eq. 2.21]}$$

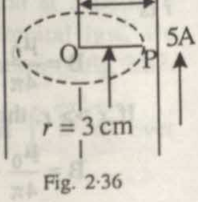


Fig. 2·36

$$\therefore\quad B = 10^{-7}\ \frac{2ir}{R^2}$$

$$= 10^{-7} \times \frac{2\times 5\times 0{\cdot}03}{(0{\cdot}05)^2} \quad [\, i = 5\text{A},\ r = 0{\cdot}03\ m,\ \text{R} = 0{\cdot}05\ \text{m}\,]$$

$$= \mathbf{1{\cdot}2\times 10^5\ T}$$

● **Ex 2.9. In fig 2·37, the conductor XY carries a current of 8A. What is the magnetic induction at P ?**

○ **Solution :** The magnetic induction at P is given by

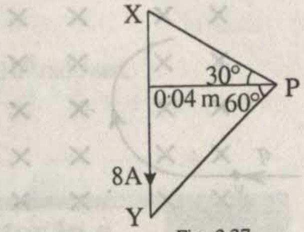


Fig. 2·37

$$B = \frac{\mu_0}{4\pi}\cdot\frac{i}{r}\ (\sin\theta_1 + \sin\theta_2)$$

$$= 10^{-7} \times \frac{8}{0{\cdot}04}\,(\sin 30^\circ + \sin 60^\circ)$$

$$= 2\times 10^{-5} \times (0{\cdot}5 + 0{\cdot}866) = \mathbf{2{\cdot}732\times 10^{-5}\ T}$$

2.14. Force on a charged particle in a magnetic field

We assume that the magnetic field vector $\vec{B}$ acts along Z-axis. A particle having charge + q is moving with velocity $\vec{v}$ in YZ plane making angle. θ with the direction of $\vec{B}$ as shown in fig 2.38.

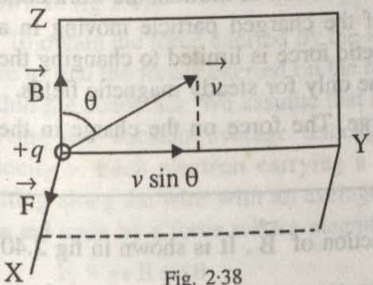


Fig. 2·38

The charged particle experiences a force $\vec{F}$ along X-axis. It is found that the magnitude of force is :

(i) Directly proportional to $|\vec{B}|$ i.e. $F \propto B$.

(ii) Directly proportional to the magnitude of charge i.e. $F \propto q$

(iii) Directly proportional to the component of velocity in a direction perpendicular to the direction of magnetic field *i.e.* $F \propto v \sin\theta$ combining the three factors,

$$F \propto Bq\,v\sin\theta \qquad \text{or,}\quad F = k.\,B\,q\,v\sin\theta.$$

In SI, the constant of proportionality is $k = 1$

$$\therefore\quad F = Bqv\sin\theta \quad \dots\dots (2.23)$$

$$\text{In vector form,}\quad \vec{F} = q\,(\vec{v}\times\vec{B}) \quad \dots\dots (2.24)$$

The force on the moving charge is perpendicular to both $\vec{v}$ and $\vec{B}$.

The equation 2.23 and 2.24 are consistent with experimental observations which are :

1. $F = 0$ if $v = 0$ *i.e.* a charge at rest does not experience any magnetic force.

2. $F = 0$, if $\theta = 0^\circ$ or 180° *i.e.* the magnetic force vanishes if v is either parallel or antiparallel to the direction of B, and

3. $F = F_{max}$, if $\theta = 90^\circ$ *i.e.* if v is perpendicular to B. The maximum magnetic force is $F_{max} = Bqv$

➤ 2.14.1 *Motion of charged particle in a magnetic field* :

The nature of the trajectory of moving charge in a magnetic field depends on direction of $\vec{v}$ with respect to $\vec{B}$. Here we consider the following cases :

(A) If the particle moves parallel to the field, $\theta = 0$ Hence $F = 0$. As no force acts on the particle, it will move along the field with uniform velocity along a straight line.

(B) If the direction of $\vec{v}$ is perpendicular to the magnetic field $\vec{B}$ *i.e.* if $\theta = 90°$. $F = F_{max} = Bqv$. As the force is perpendicular to both B and v, the charge will suffer a sideway deflection and so moves in a circle as shown in fig 2.39.

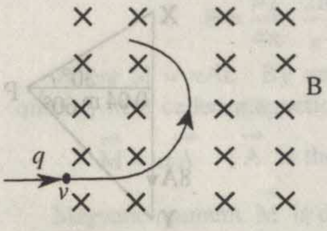


Fig. 2·39

If the radius of the circle be r, we write

$$\frac{mv^2}{r} = Bqv \qquad \therefore \qquad r = \frac{mv}{Bq}$$

The frequency of revolution in the magnetic field is

$$f = \frac{v}{2\pi r} = \frac{1}{2\pi}\frac{Bq}{m} = \frac{Bq}{2\pi m}.$$

It is to be noted that f does not depend on the velocity of the particle. The faster particle move in a bigger circle and the slower particle moving in smaller circle so that all of them complete one revolution in the same time. The frequency f is called by **cyclotron frequency**.

- **A discussion :** The magnetic force being perpendicular to the direction of motion, the work done by the field on the charge is zero. Hence the kinetic energy of the charged particle moving in a magnetic field will not change. Evidently, the effect of the magnetic force is limited to changing the direction of motion only. It should pointed out that this holds true only for steady magnetic fields.

  On the other hand, an electric field can acceleration of charge. The force on the charge in the electric field is in the direction of the field.

(C) If the direction of $\vec{v}$ make angle θ ($\theta < 90°$) with the direction of $\vec{B}$. It is shown in fig 2.40.

Here the magnetic field acts along X-axis. We resolve v into two components : (i) $v_x = v\cos\theta$ along the direction of the field B and (ii) $v_y = v\sin\theta$ along the perpendicular direction.

The component $v \cos \theta$ along the direction of the magnetic field remains unaffected. Hence the charge particle continues to move along the field with this velocity.

The other component of velocity $v \sin \theta$, being perpendicular to the direction of magnetic field, gives rise to a force (B q v sin θ). Due to this force the charged particle moves in a circular path. Its radius is

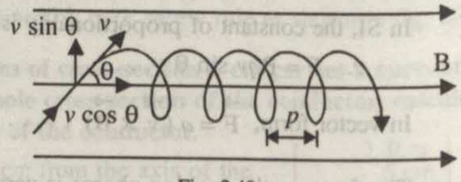


Fig. 2·40

$$R = \frac{m(v\sin\theta)}{Bq} \quad \text{and time period, } T = 2\pi\left(\frac{m}{qB}\right)$$

Hence if a charged particle enters a magnetic field at any angle θ, it possesses two concurrent motions— (i) a linear motion with a constant velocity $v \cos \theta$ and (ii) a circular motion of radius R. The resultant path is, therefore, a helix as shown in fig 2.40.

The pitch p of the helix i.e. the distance travelled by the charged particle along the direction of the field in one revolution is

$$p = (v \,.\cos\theta)\,T = \frac{2\pi mv\cos\theta}{T} = Bqv\cos\theta.$$

- **Example 2.10. A proton is moving at right angle to a magnetic field of flux density 0·4T in a circular path with a frequency of $6\cdot10\times10^6$ rev / sec. If charge of proton is $1\cdot6\times10^{-19}$ C, calculate its mass.**

○ **Solution :** Here, $\frac{mv^2}{r} = Bqv \quad \therefore \quad m = \frac{Bqr}{v} = \frac{Bq}{\omega}$

Here, $B = 0\cdot4T$, $q = 1\cdot6\times10^{-19}C$, $\omega = 2\pi n = 2\pi\times6\cdot1\times10^6 = 12\cdot2\pi\times10^6$ rad / sec,

$\therefore$ mass of proton, $m = \frac{0\cdot4\times1\cdot6\times10^{-19}}{12\cdot2\pi\times10^6} = \mathbf{1\cdot67\times10^{-27}\,kg}$

✪ 2.15. Force on a current carrying conductor in a magnetic field ✪

Flow of electric charges constitutes electric current. Also a moving charge experiences a force in a magnetic field. It, therefore, follows that when a current carrying conductor is put in a magnetic field, it also experiences a force of magnetic origin. The force on the electron in a currrent carrying conductor is shown in fig 2.41.

To obtain the force exerted by the magnetic field B on a conductor of length l carrying a current i, we have to find the forces exerted on the moving electrons within the conductor. We assume that the electrons are moving with constant average velocity called the drift velocity v. Each electron carrying a charge $-e$ and drifting along the wire with an average drift velocity v is acted upon by a force whose magnitude is given by

$$F_1 = evB\sin\theta$$

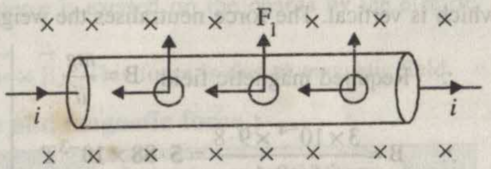


Fig. 2·41

Here θ is the angle between the wire and the direction of field. The direction of this force is normal to the plane containing v and B and the sense is shown in fig 2.41.

Now, if l be the length of the conductor, A its area of cross section and n be the number of free electrons per unit volume, then total number of electrons in l is $N = nAl$.

So, the total force on the wire of length l which carries a current i and makes angle θ with the field B is

$F = NF_1 = nAl\,e\,v\,B\sin\theta \quad \therefore \quad F = (n\,eAv)\,lB\sin\theta$

Now $neAv = i$. Hence, $F = B\,i\,l\sin\theta$ . . . . (2.25)

In vector form, $\vec{F} = i(\vec{l}\times\vec{B})$ . . . . (2.26)

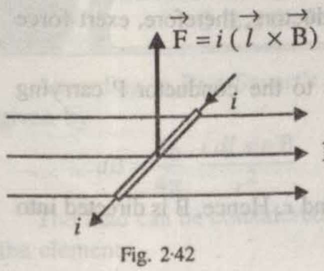


Fig. 2·42

The direction of the force is in the direction of $(\vec{l}\times\vec{B})$ where the vector l is taken along the direction of current. Evidently, the direction of $\vec{F}$ is perpendicular to both $\vec{l}$ and $\vec{B}$. The relation between $\vec{F}$, $\vec{l}$ and $\vec{B}$ can be described pictorially by **Fleming's left hand rule.**

- **Fleming's left hand rule :** The direction of magnetic force on a current carrying conductor put in a magnetic field is obtained by Fleming's left hand rule. The rule reads : If the forefinger, middle finger and the thumb of left hand are stretched mutually perpendicular to each other, such that the forefinger points the direction of magnetic field $\vec{B}$, the middle finger the direction of current I, then the thumb points the direction of the force $\vec{F}$ experienced by the conductor. The rule is illustrated in fig 2.43.

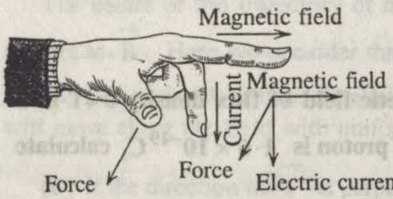


Fig. 2·43

- **Unit of B :** The unit of B can be obtained from the relation $F = Bqv\sin\theta$

$$\therefore\ B=\frac{F}{qv\sin\theta},\ \text{if } F=1\text{N},\ q=1\text{ C},\ v=1\text{ m/s and } \theta=90^\circ,$$

$$\text{then } B=\frac{1\text{N}}{1\text{C}\times 1\,ms^{-1}\times\sin 90^\circ}=\frac{1\text{N}}{1\,\text{A}\times 1m}\quad (\because\ \text{C}.s^{-1}=\text{A})$$

$$=\mathbf{1\,N\,A^{-1}m^{-1}}$$

In SI, the unit of B is Tesla (T). So magnetic field is said to be 1 tesla if a charge of 1C moving with speed of 1 ms^{-1} at right angle to the field experiences a force of 1N.

- **Ex. 2.11. A harizental wire of length 0·1 m carries a current of 5A. Find the magnetic field which can support the weight of the wire. The mass of the wire is 3×10^{-3} kg/m.**

○ **Solution :** The situation is shown in fig 2.44. The required F = Bil = mg. The wire is placed along X-axis and the magnetic field is applied along Y-axis. The wire will experience the force F = Bil along Z-axis which is vertical. The force neutralises the weight of the wire

$$\therefore\ \text{Required magnetic field, } B=\frac{mg}{il}$$

$$\therefore\ B=\frac{3\times10^{-4}\times9\cdot8}{5\times0\cdot1}=\mathbf{5\cdot88\times10^{-3}}\text{T}$$

$m = 3 \times 10^{-4}$ Kg
$g = 9\cdot8$ m/sec^2
$i = 5$A
$l = 0\cdot1$ m

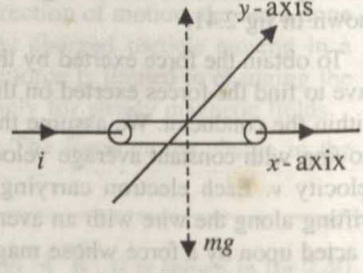


Fig. 2·44

✪ 2.16. Force between two long parallel conductors carrying current ✪

Like parallel currents attract each other and unlike parallel currents repel each other. We shall deduce an expression for the force. We consider two parallel conductors P and Q kept at a separation of r and they are carrying current i_1 and i_2 in the same direction. Each conductor carrying current produces magnetic field around it. So, the magnetic force of one exerts force on the other. The two current carrying conductors, therefore, exert force on one another.

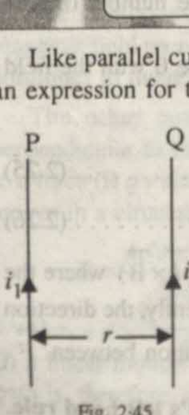


Fig. 2·45

Now, the magnetic field at the site of Q due to the conductor P carrying current i_1 is $B=\frac{\mu_0}{4\pi}\ \frac{2i_1}{r}$ T

The direction of B is perpendicular to both i_1 and r. Hence, B is directed into the page and is right angle to the wire Q.

Since B is perpendicular to i_2, the magnitude of force exerted by P by unit length of Q is

$$F = Bi_2 = \frac{\mu_0}{4\pi} \cdot \frac{2 i_1 i_2}{r} \text{ N}m^{-1} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.27)$$

Similar amount of force will also be acting on P due to Q.

❖ **Definition of 1A current :** From equation 2.27, we can define 1A current

$$\text{Here } F = 10^{-7} \; \frac{2\, i_1\, i_2}{r} \; \text{N}m^{-1}$$

$$\text{If } i_1 = i_2, \; F = 2 \times 10^{-7}\,\text{N} \text{ and } r = 1\,m \text{ then } i_1 = i_2 = 1\text{A}$$

In SI, unit current is of A. **One ampere is the steady current in two long straight and parallel conductors kept at a separation of 1m in air that causes each wire to experience a magnetic force of 2 × 10⁻⁷ N per metre length.**

❂ 2.17. Lorentz force ❂

If in a region both electric field $\vec{E}$ and magnetic field $\vec{B}$ are present, then a moving charge q experiences a force given by

$$\vec{F} = q\,\vec{E} + q\,(\vec{v} \times \vec{B})$$

This force is known as Lorentz force. So Lorentz force has two components

(i) One component is electric force, $\vec{F}_e = q\,\vec{E}$. This force is exerted on the charge by the electric field (ii) the other component is magnetic force $\vec{F}_m = q\,(\vec{v} \times \vec{B})$. This force is due to magnetic field.

Table : 2.1. Comparison between electric force and magnetic force :

	Lorentz electric force	**Lorentz magnetic force**
1. Magnitude	$F_e = q\,E$	$F_m = Bqv \sin\theta$
2. Direction	Along the direction of the field	in the direction perpenticular to the plane containing B and v and depends
3. Dependence on v	does not depend on velocity	on velocity of the charge
4. Work done	work is done ; velocity and kinetic energy increases	No work is done ; velocity and kinetic energy remains constant.

❂ 2.18. Magnetic field due to a moving charge ❂

According to Biot-Savart's law, the magnetic field at a point due to a small current element $i\,dl$ is given by

$$dB = \frac{\mu_0}{4\pi} \; \frac{i\,dl \sin\theta}{r^2}$$

The field can be considered as the resultant of the contribution made by all the charges dQ present in the element.

If $+q$ be the charge of each particle and n be the number of particles per unit volume, then,

$$dQ = qnA \cdot dl$$

Also, $i = nqAv$ $\quad\therefore\quad$ Current element, $i\,dl = v.\,dQ$

$$\therefore \quad dB = \frac{\mu_0}{4\pi}\ \frac{v\,.\,dQ\,.\sin\theta}{r^2}$$

Hence the total magnetic field due to a charge Q is

$$B = \frac{\mu_0}{4\pi}\ \frac{v\,Q\sin\theta}{r^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.28)$$

2.19. Magnetic force between point charges

Let two point charges q_1 and q_2 are moving with velocities v_1 and v_2 respectively. The magnetic field set up by the moving charge q_1 at the site of q_2 is

$$B = \frac{\mu_0}{4\pi}\cdot\frac{q_1\,v_1\sin\theta}{r^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (i)$$

Here r is the instantaneous distance between the two charges.

So, the force on charge q_2 is $F_m = Bq_2\,v_2\sin\varphi \ldots\ldots\ldots$ (ii)

Here φ is the angle between v_2 and B.

Hence from the equations (i) and (ii) we write $F_m = \dfrac{\mu_0}{4\pi}\cdot\dfrac{(q_1v_1\sin\theta)\,(q_2v_2\sin\varphi)}{r^2}$

- **Special case :** If $\theta = 90°$ and $\phi = 90°$, then

$$F_m = \frac{\mu_0}{4\pi}\cdot\frac{q_1q_2v_1v_2}{r^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (iii)$$

- **A discussion : Comparison between electric force and magnetic force :** The electrostatic force between q_1 and q_2 (by Coulomb's law)

$$F_e = \frac{1}{4\pi\epsilon_0}\ \frac{q_1\,q_2}{r^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (iv)$$

$\therefore$ by equations (iii) and (iv), $\dfrac{F_m}{F_e} = (\mu_0\,\epsilon_0)\,v_1v_2 = \dfrac{v_1v_2}{c^2}\left[\because\quad c = \dfrac{1}{\sqrt{\mu_0\,\epsilon_0}}\right]$

Now, $v_1 << c$ and $v_e << c$, so $v_1v_2 << c^2$ $\quad\therefore\quad$ $\mathbf{F_m << F_e}$

- **Ex. 2.12. In a hydrogen atom the electron is revolving in circular orbit of radius 0·52 Å around the proton with a velocity of $2{\cdot}2 \times 10^6$ m/s. Calculate (i) equivalent current (ii) equivalent magnetic moment of the orbit (iii) magnetic induction at the site of proton.**

○ **Solution :** (i) Due to motion of electron in a circular orbit, the equivalent current is $i = \dfrac{e}{T}$ where $T = \dfrac{2\pi r}{v}$

$$\therefore \quad i = \frac{ev}{2\pi r} = \frac{1{\cdot}6\times10^{-19}\times2{\cdot}2\times10^{6}}{2\pi\times0{\cdot}52\times10^{-10}} = \mathbf{1{\cdot}08\times10^{-3}}\ \text{A}$$

(ii) Equivalent magnetic moment of the orbit, $M = inA = i\,\pi\,r^2$ $[n = 1]$

$$\therefore \quad M = 1{\cdot}08\times10^{-3}\times3{\cdot}14\times(0{\cdot}52\times10^{-10})^2 = 9{\cdot}2\times10^{-24}\ Am^2$$

(iii) Magnetic induction at the centre, $B = \frac{\mu_0}{4\pi} \cdot \frac{2\pi i}{r}$

$$\therefore \quad B = 10^{-7} \times \frac{2\pi \times 1 \cdot 08 \times 10^{-3}}{0 \cdot 52 \times 10^{-10}} = 13\,\text{T}$$

● **Ex. 2.13. The area of a circular coil is $7 \cdot 5 \times 10^{-4}\, m^2$ and it carries a current of 12A. To an observer the current in the coil flows clockwise, what is the magnitude and direction of magnetic moment of the coil?**

○ **Solution :** Magnetic moment, $M = iA = 12 \times 7 \cdot 5 \times 10^{-4} = 9 \cdot 0 \times 10^{-3}\ \text{A}m^2$

Direction : It is perpendicular to the plane of the coil and along the axis and directed away from the observer.

$\therefore$ The required magnetic moment = $\mathbf{9 \cdot 0 \times 10^{-3}\ Am^2}$

● **Ex. 2.14. A current of 25 A is passed through a long vertical conductor An electron is moving parellel to the conductor with a velocity of 44×10^4 m/s at a distance of 7 cm from the conductor. How much force acts on it ?**

○ **Solution :** Force on the moving electron, $F = Bev \sin\theta$

Now, $\theta = 90°$, $F = Bev = \frac{\mu_0}{4\pi} \cdot \frac{2i}{r}\, ev$

$$= 10^{-7} \times \frac{2 \times 25}{7} \times 1 \cdot 6 \times 10^{-19} \times 44 \times 10^4$$

$$= \mathbf{0 \cdot 503 \times 10^{-19}\,N}$$

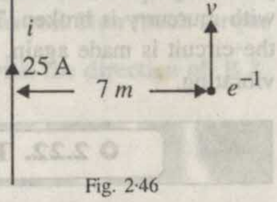


Fig. 2·46

✪ 2.20. Barlow wheel ✪

The action of magnet on current can be demonstrated by the experiment using Barlow's wheel.

● **Description :** It consists of a star-shaped wheel made of copper. It can rotate in a vertical plane about a horizontal axis passing through its centre [Fig 2.47]. Some mercury is kept in a slot cut on the wooden base. As the wheel rotates, its pointed teeth come in contact with mercury one after another. The poles of a strong horse shoe magnet are placed perpendicular to the length of the slot. The terminal of a cell are connected to the centre of the wheel and mercury so that a current flows along the vertical teeth.

When the current flows down the teeth dipped in mercury, a force is exerted on it—the direction of force is obtained from Fleming's left hand rule. Evidently, the tooth dipped in mercury experiences a force towards right. The wheel then rotates in the direction of the arrow. If the direction of current or the direction of magnetic field is reversed, the direction of rotation of the wheel is also reversed.

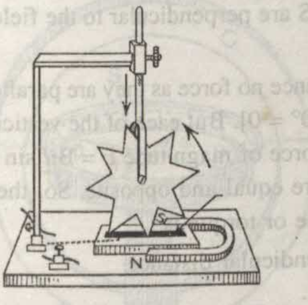
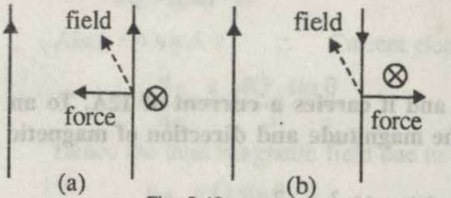


Fig. 2·47

✪ 2.21. Experiment on the mutual force between two parallel currents ✪

We have deduced an expression for the mutual force between two parallel currents. A current produces a magnetic field in its neighbourhood. Also, a current experiences a force when placed in a magnetic field. Hence, when a current is passed through two conductors placed near to each other, each will exert a force upon the other. It is found that



(a) (b)

Fig. 2·48

(i) Like parallel currents attract each other

(ii) Unlike parallel current repel each other. This is shown in fig 2.48.

- **Roget's vibrating spiral :** By this simple experiment the force of attraction between like parallel currents can be demonstrated.

It consists of a long spiral of copper wire and it is hung vertically from a fixed suppart [Fig 2.49]. The spring carries a small mutal weight W. A. metallic porper attached to w just dips into a pool murcury. If a strong current is passed through the spiral, it begins to vibrate up and down.

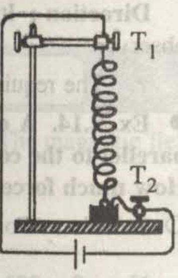


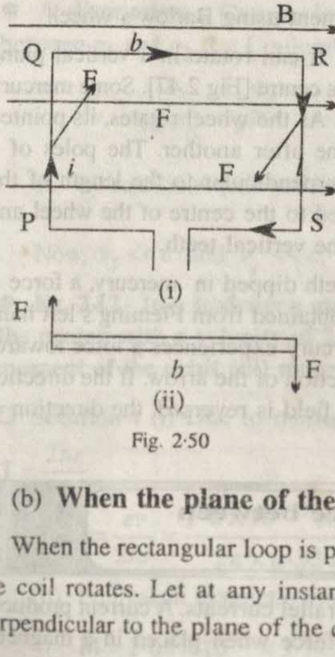
Fig. 2·49

- **Explanation :** Parallel currents in the same direction attract each other. Adjacent turns of the coil carry current in the same direction. So they attract each other producing a contraction of the spiral. The weight w is slightly raised and the contact with murcurry is broken. This breaks the circuit. So the weight W desconds and the circuit is made again. The above process is repeated and the spiral goes on vibrating.

2.22. Torque on a current loop in a magnetic field

A current loop placed in a magnetic field experiences a torque. We shall deduce expressions for the tarque in two cases :

(a) When the plane of the coil is parallel to the field.



(i)

(ii)

Fig. 2·50

In fig 2.50, PQRS is a rectangular loop of wire of sides b and l suspended in a uniform magnetic field $\vec{B}$. The plane of the coil is parallel to the field. The sides QR and PS are parallel to the lines of force. The other two sides PQ and RS are perpendicular to the field. Let the current in the coil is i

Evidently, QR and PS will experience no force as they are parallel to the field [$F = Bib \sin\theta = Bib \sin 0° = 0$]. But each of the vertical sides PQ and RS will experience a force of magnitude $F = Bil \sin\theta = Bil\ 90° = Bil$. These two forces are equal and opposite. So, they form a couple. Moment of the couple or torque is

$$\tau_1 = \text{one of the force} \times \text{perpendicular distance}$$

$$\therefore\ \tau_1 = Bil\ b = BiA$$

If the coil has n number of turns, total torque, $\tau = BiAn$ . . .(2.29)

(b) When the plane of the coil is at an angle with the magnetic field.

When the rectangular loop is parallel to the field, the maximum torque is $\tau = BiAn$. Due to the torque, the coil rotates. Let at any instant θ be the angle between the direction of $\vec{B}$ and that of the vector perpendicular to the plane of the coil [Fig 2.52].

Now, the magnitude of the torque acting on the coil is :

$$\tau' = \text{force} \times \text{perpendicular distance}$$

$$= Bil \times QR \sin\theta = Bil \times b \sin\theta$$

$$= BiA.\sin\theta$$

$\therefore$ If the coil has n turns, total torque,

$$\tau = BiAn.\sin\theta$$

$\therefore$ $\tau = BM\sin\theta$ [$M = i\,An$, the magnetic moment of the coil]

In vector notation the torque vector is given by

$$\vec{\tau} = \vec{M} \times \vec{B} \quad \ldots\ldots\ldots\ldots \text{(2.30)}$$

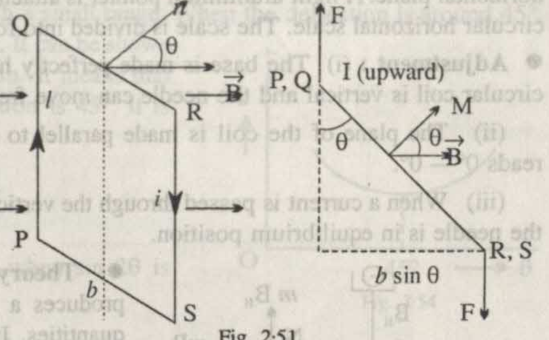


Fig. 2·51

This equation gives the magnitude as well as direction of τ. The torque on a current loop is the operating principle of the electric motor and galvanometers.

- **Ex 2.15. The radius of a circular coil is 8·0 cm and its number of turns 30. It is suspended in a magnetic field of flux density 1·0 T and a current of 1·0 A is passed through the coil. How much torque acts on the coil if the normal to the plane of the coil makes angle of 60° with the direction of $\vec{B}$?**

○ **Solution :** Torque acting on the coil is $\tau = BinA\sin\theta$.

Here, $n = 30$, $i = 1{\cdot}0\,A$, $A = \pi \times (0{\cdot}08)^2\ m^2$, $\theta = 60°$ and $B = 1{\cdot}0$ T

$\therefore$ $\tau = 1{\cdot}0 \times 1{\cdot}0 \times 30 \times 64\pi \times 10^{-4} = \mathbf{3{\cdot}1\ N-m}$

⚙ 2.23. Galvanometers ⚙

A galvanometer is an electromagnetic device which is used to detect the presence of current in a circuit. With certain modification in construction it is also used to measure steady current and voltage. We consider two types of galvanometer : (i) moving magnet galvanometer. *e.g.* tangent galvanometer and (ii) moving coil galvanometer, e.g. D'Arsonval type and ballistic type.

➤ *2.23.1. Tangent galvanometer :*

- **Principle :** The basic principle of this galvanometer is the effect of current on a magnet. When a current passes through a circular coil a magnetic field is produced along the axis of the coil. A magnetic needle placed in the field is deflected from its normal position.
- **Description :** It consists of a circular coil made of insulated copper wire wound on a circular wooden frame [Fig 2.52]. It is placed on a horizontal wooden base fitted with levelling screws. The ends of the coil are connected to two screw terminals (T_1, T_2) fixed on the base P. The circular wooden frame can be rotated about a vertical axis passing through

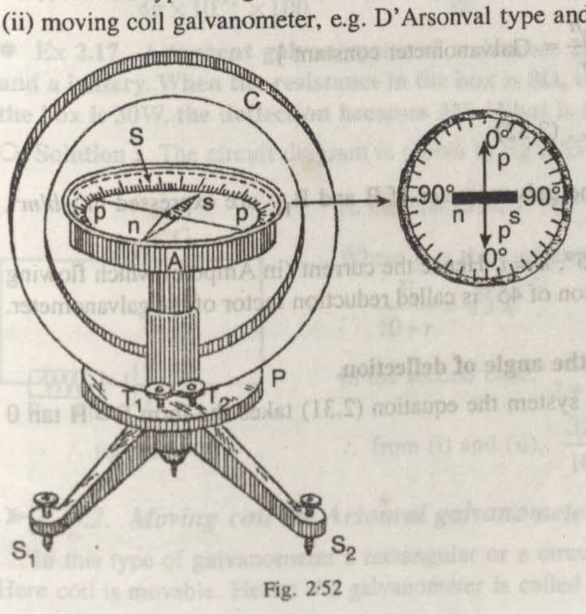


Fig. 2·52

to centre. A small magnetic needle is pivoted at the cnetre of the coil. The needle is free to rotate in a horizontal plane. A light aluminium pointer is attached at right angle to the needle which can move over a circular horizontal scale. The scale is divided into four quadrants, each having graduation from 0° to 90°.

- **Adjustment :** (i) The base is made perfectly horizontal by means of the levelling screws. Now the circular coil is vertical and the needle can move freely on the horizontal scale.

(ii) The plane of the coil is made parallel to the axis of the needle. Now, the aluminium pointer reads 0°— 0°.

(iii) When a current is passed through the vertical coil, the needle is delected. Reading is taken when the needle is in equilibrium position.

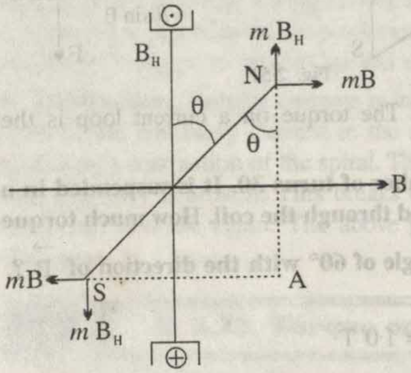


Fig. 2·53

- **Theory :** Here we deduce an expression for current which produces a deflection of the galvanometer in terms of known quantities. In fig 2.53, B_H the horizontal component of earth's magnetic intensity and B is the magnetic field intensity due to the vertical circular coil carrying current. Since the coil is in magnetic meridian, B and B_H are mutually perpendicular to each other. Due to join action of the two fields, let the deflection of the needle from the magnetic meridian be θ. The different forces acting on the needle at its equilibrium position, are shown in fig.

Here m is the pole strength of the needle. At equilibrium, torque due to (mB, mB) = torque due to (mB_H, mB_H)

$$\therefore \quad mB \times (AN) = mB_H\,(As) \Rightarrow B\,.(2l\cos\theta) = B_H\,.\,2l\,\sin\theta$$

$$\therefore \quad B = B_H\,\tan\theta \;\dotfill\; (2.31)$$

In SI, $B = \dfrac{\mu_0}{4\pi}\cdot\dfrac{2\pi ni}{r} = \dfrac{\mu_0}{2}\cdot\dfrac{ni}{r}$

n = turns of the coil
r = radius of the coil

$$\therefore \quad \frac{\mu_0}{2}\times\frac{ni}{r} = B_H\,\tan\theta$$

$$\therefore \quad i = \frac{B_H}{\mu_0 n/2r}\cdot\tan\theta = \frac{B_H}{G}\tan\theta \quad \left[\, G = \frac{\mu_0 n}{2r} = \text{Galvanometer constant}\,\right]$$

$$\therefore \quad i = k\tan\theta \left[\, k = \frac{B_H}{G}\,\right] \;\dotfill\; (2.32)$$

The quantity k is called reduction factor of the galvanometer. If B and B_H are expressed in wb/m^2, k is in ampere.

- **Definition of *k* :** In equation (2.32), if θ = 45°, $k = i$. Hence the current (in Ampere) which flowing through a tangent galvanometer produce a deflection of 45° is called reduction factor of the galvanometer.

From the equation (2.32), $i \propto \tan\theta$

The current is proportional to the **tangent of the angle of deflection.**

- **A discussion :** (A) When we consider c. g. s. system the equation (2.31) takes the form F = H tan θ [Here we put B = F and B_H = H]

$$\therefore \quad \frac{2\pi ni}{r} = H\tan\theta \quad \text{or,} \quad i = \frac{rH}{2\pi n}\cdot\tan\theta$$

$$\therefore \quad i = k\,\tan\theta$$

Now, i is in emu and k is also in emu.

(B) The value of tan θ varies from 0 to ∞. Theoretically, it can measure current of any magnitude. But in practice, it can measure current accurately with a small range. When the deflection is around 45°, the accuracy in measurement is within allowable limit. It can be shown that when the deflection is just 45°, error is minimum. It means that the galvanometer is most sensitive when the deflection is 45°. It is shown below :

$$i = k \tan\theta \quad \therefore \quad \frac{di}{i} = \frac{\sec^2\theta \cdot d\theta}{\tan\theta} = \frac{2 \cdot d\theta}{\sin 2\theta}$$

So, the proportional error $e = \frac{di}{i}$ is minimum, when $\sin 2\theta$ is maximum.

$\therefore$ for minimum error, $\sin 2\theta = 1 \quad \therefore \quad \theta = 45°$

The approximate variation of '*e*' with 'θ' is shown in fig 2.54.

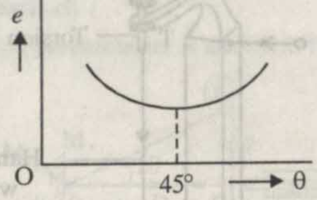


Fig. 2·54

(a) The magnetic needle used in the galvanometer should be as small as possible. Reason is—the magnetic field acting on the needle is not uniform. The magneting field produced by the coil is maximum $\left[B = 10^{-7}\frac{2\pi ni}{r}\right]$ only at the centre of the coil and gradually decreases in all directions. So, if the needle be larger in length, magnetic field at different points on the needle are different. So there will be some error in the measurement.

● **Ex. 2.16. In a tangent galvanometer, the radius of the coil is 5 cm and its number of turns 100. For what current, the deflection will be 45° ? Earth's horizontal magnetic intensity = 0·18 × 10⁻⁴ T.**

○ **Solution :** The required relation : $i = \frac{2r B_H}{\mu_0 n} \cdot \tan\theta$

$$\therefore \quad i = \frac{2 \times 5 \times 0{\cdot}18 \times 10^{-4}}{4\pi \times 10^{-7} \times 100} \tan\theta = \frac{0{\cdot}18}{4\pi} = \mathbf{0{\cdot}014A}$$

● **Ex 2.17. A tangent galvanometer of resistance 2Ω is connected in series with a resistance box and a battery. When the resistance in the box is 8Ω, the deflection is 60°. But when the resistance in the box is 30W, the deflection becomes 30°. What is the internal resistance of the battery ?**

○ **Solution :** The circuit diagram is shown in fig 2.55.

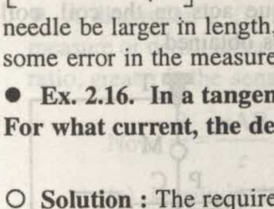


Fig. 2·55

In the first case, $i = \frac{E}{2+8+r} = k\tan 60°$

Where E is the emf and *r* is the internal resistance of the battery.

$$\therefore \quad \frac{E}{10+r} = \sqrt{3}\,k \quad \ldots\ldots\ldots\ldots\ldots \text{(i)}$$

In the second case, $\frac{E}{2+30+r} = k\tan 30°$ or, $\frac{E}{32+r} = \frac{k}{\sqrt{3}} \quad \ldots\ldots \text{(ii)}$

$\therefore$ from (i) and (ii), $\frac{32+r}{10+r} = 3 \quad \therefore \quad \boldsymbol{r = 1\Omega}$

➤2.23.2. *Moving coil De Arsonval galvanometer :*

In this type of galvanometer a rectangular or a circular is kept suspended is a strong magnetic field. Here coil is movable. Hence the galvanometer is called a suspended coil galvanometer.

- **Principle :** Action of magnet on current. When a current carrying coil is placed in a magnetic field, it experiences a force and it is deflected following Fleming's left hand rule.
- **Description :** Fig 2.56 illustrates the essential parts of a moving coil De Arsonval type galvanometer. It consists of a rectangular coil *c* of fine copper wire suspended between the poles of a strong permanent horse-shoe magnet NS by means of phosphor bronze strip. The strip serves (i) as a suspension for the coil and (ii) as are of the terminals for passing current. The other terminal is a flexible string at the base. The upper end of the suspension is attached to a torsion head T which can be rotated to set the coil in the zero position. A mirror is attached to the phospher bronze strip which measures deflection with a lamp and scale arrangement.

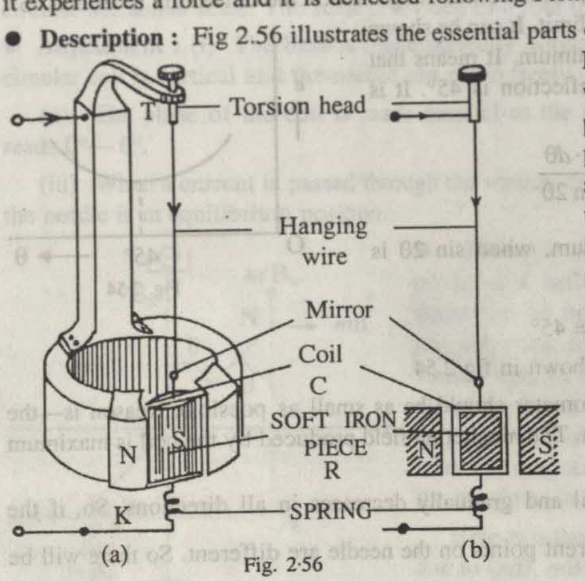


Fig. 2·56

- **Theory : 1st case :** The working of the galvanometer is based on the fact that when a current is passed through a coil placed in a magnetic field, a torque acts on the coil, coil rotates and deflection is obtained.

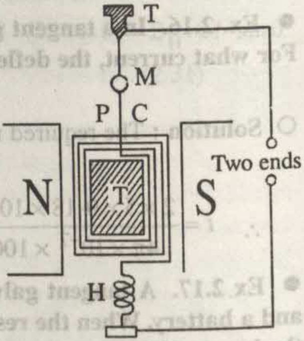


Fig. 2·57

In fig 2.57, the rectangular coil is suspended in a uniform magnetic field B. When no current flows in the coil, then the plane of the coil is parallel to the magnetic field and the suspension is not twisted. As soon as current passes through the coil, a torque acts on it. It is deflected and the suspension is twisted. The strip provides a counter torque and opposes the magnetic torque. At equilibrium, when the angle of deflection is θ, magnetic torque (τ_0) is equal to restoring torque (τ_R) on the strip due to twist.

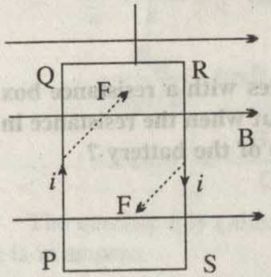


Fig. 2·58

Now, $\tau_D = in\,AB \sin\theta$, and torsional couple, $\tau_R = c\theta$

Where *c* = torsional constant *i.e.* restoring torque for unit twist.

$$\therefore \quad in\,AB \sin\theta = c\theta$$

$$\therefore \quad i = \frac{c}{nAB}\;\frac{\theta}{\sin\theta}$$

2nd case : In this case the field is made radial instead of horizontal field. This is done by making the faces of the magnet concave as in fig 2.59. Now, the field lines are radial. Now for any position of the rectangular coil, the vertical side of the coil will remain perpendicular to this field lines. So now

$$C_B = \text{in AB} \quad [\because \quad \sin\theta = 1]$$

$\therefore$ At equilibrium, in AB = $c\theta$

$$\therefore \quad i = \frac{c}{n\,AB}\,\theta \qquad \therefore \; i = G\theta \qquad \therefore \; i \,\alpha\, \theta$$

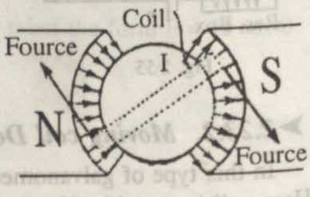


Fig. 2·59

The quantity $G = \dfrac{c}{nAB}$ is called galvanometer constant.

The current in the coil is proportional to the angle of deflection.

- **Theory of measurement of angle of deflection (Lamp and scale method) :**

The lamp and scale arrangement for measuring the steady deflection of galvanometer is shown in fig 2.60. M is the initial position of the magnet attached to the suspension strip. A lamp is placed in front of the mirror. Light from the lamp is focussed on the mirror by a small convex lens fitted to the galvanometer frame. The light after reflection produces a spot of light 'O' of the scale. Due to deflection of the galvanometer coil, two mirror also rotates and the spot of light shifts O_1. Let $OO_1 = d$. If θ be the angle of rotation of the suspension strip, the reflected ray rotates by 2θ. Hence

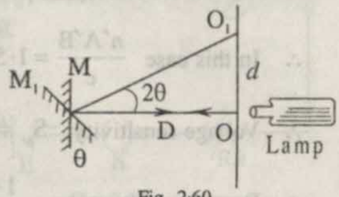


Fig. 2·60

$$\tan 2\theta = 2\theta = \frac{d}{D} \quad \text{[As } \theta \text{ is small]}$$

$$\therefore \quad i = \frac{c}{n\,AB} \cdot \frac{d}{2D} \qquad \frac{G}{2D}\, d = kd \quad \therefore \quad i \,\alpha\, d$$

Hence, **the current is proportional to the linear displacement of the spot of light on the scale :**

- **A discussion :** (i) **Sensitivity of the instrument :** A galvanometer is said to be sensitive, if it can measure or detect externals small current. Sensitivity is measured by the ratio θ/i. Higher the value of the ratio, greater is the sensitivity of the instrument.

Now, $\dfrac{\theta}{i} = \dfrac{nAB}{c} \cdot$ Hence

(a) sensitivity increases if the strength of the field B is greater. So, stronger the horse shoe magnet, more sensitive will be the galvanometer.

(b) by increasing the area A and number of turns n, sensitivity can be increases. But if A and n are increased, the moment of inertia I and resistance R of the coil increases. So it is not wise to increase A and n beyond a limit.

(c) smaller the torsional constant, larger is the sensitivity. For this reason phosphor bronze strip is used.

(ii) **Figure of merit :** It is the current which flowing through a moving coil galvanometer produces a deflection of 1 mm of the spot of light on the scale placed at a distance of 1 m from the galvanometer mirror. Hence. figure of merit, $\text{F.M.} = \dfrac{i}{d}\ \text{A}/mm.$

Where d mm is the deflection due to a current i Amp.

(iii) **Current sensitivity :** Current sensitivity (S_c) is defined as the deflection in mm of the spot of light on the scale placed at a distance of 1 m from the galvanometer mirror due to a current of 1 μA. So if due to a current of i μA, the deflection be d mm of the spot of light on the scale placed at a distance of 1 m from the galvanometer mirror, then $S_c = \dfrac{d}{i}\ mm/\mu\text{A}$

(iv) **Voltage sensitivity, S_V :** It is defined as the deflection in mm of the spot of light on the scale placed at a distance of 1 m from the galvanometer mirror due to a voltage of 1 μV.

$$\therefore \quad S_V = \frac{d}{V}\ mm/\mu\text{V}$$

where d mm is the deflection of the spot of light due to a voltage of V μV.

● **Ex. 2.18. To increase the current sensitivity by 50%, the resistance of a moving coil is doubled. What is the percentage change in voltage sensitivity ?**

○ **Solution :** For a moving coil galvanometer $i = \frac{c\theta}{n\text{AB}}$.

$\therefore$ Current sensitivity, $\text{S}_i = \frac{\theta}{i} = \frac{n\text{AB}}{c}$

$\therefore$ In this case $\frac{n'\text{A}'\text{B}}{c} = 1{\cdot}5\,\frac{n\text{A B}}{c} \quad \therefore\ n'\text{A}' = \frac{3}{2}n\text{A}$

$\therefore$ Voltage sensitivity, $\text{S}_v = \frac{\theta}{v} = \frac{n'\text{A}'\text{B}}{\text{CR}'} = \frac{3}{2}\,\frac{n\text{A}}{\text{C}}\,\frac{\text{B}}{2\text{R}} = \frac{3}{4}\,\frac{n\,\text{AB}}{\text{CR}}$

$\therefore$ Percentage decrease $= \frac{1-3/4}{1} \times 100 = 25\%.$

✪ 2.24. Table galvanometer ✪

Like suspended coil galvanometer double galvanometer is also a moving coil galvanometer. Both have the same working principle. It is actually a pivoted type galvanometer. In this galvanometer the frame of the coil rests vertically on two jewelled pivots working within two metal pillars. The deflection of the galvanometer is measured by a light aluminium pointer fixed with the axile of the coil. Pointer moves on a circular graduated scale, when a current is passed through the coil, it is deflected. The coil comes to an equilibrium position when deflecting torque is balanced by restoring torque exerted by the hair spring fixed with the coil.

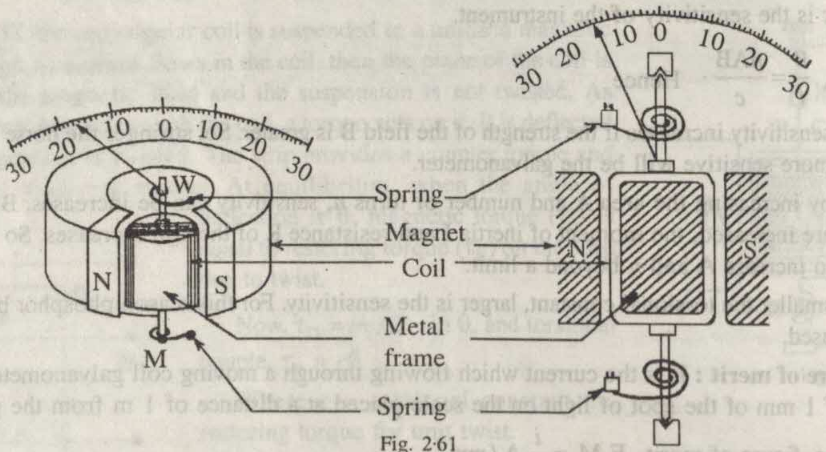


Fig. 2·61

✪ 2.25. Ammeter and Voltmeter ✪

With some modifications in construction, moving coil table galvanometer can be converted into ammeter and voltmeter. All these instruments are based on the same principle of electromagnetism.

Ammeter is used to measure current in a circuit and it is connected in series with the circuit so that main current may pass through it. Now current in the circuit must not change appreciately when the ammeter is inserted in the circuit, **an ammeter resistance must be low.**

An ammeter, therefore, has a low resistance shunt connected across the coil. An ideal ammeter would have zero resistance.

A voltmeter measures potential difference across a part of a circuit. It is connected parallel to the resistor across which potential difference is to be measured. Its resistance, therefore, must be very high so that after its connection main circuit current must not change appreciably. So, the resistance of the galvanometer should be very high. Hence, to convert a galvanometer into an ammeter high resistance is connected in series with the coil. The resistance of an ideal voltmeter is infinite.

The method of connection of ammeter and voltmeter in a circuit is shown in fig 2.62. Ammeter is always connected in series and voltmeter is always connected parallel to the circuit. Ammeter is denoted by A and voltmeter by V.

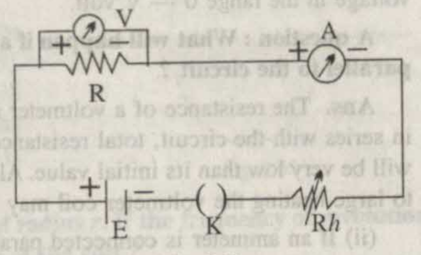


Fig. 2·62

➤2.25.1 Ammeter from galvanometer :

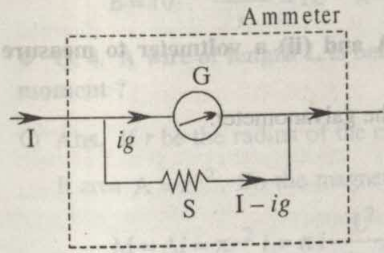


Fig. 2·63

Ammeter a low resistance moving coil table galvanometer. A galvanometer can be converted into an ammeter by connecting a low resistance shunt parallel to the coil. So, when an ammeter is inserted into the circuit, it does not affect appreciably circuit current to be measured so that measurement of current is almost error free.

The circuit 2.63 shows that conversion of galvanometer into an ammeter. G is the galvanometer resistance and S is the shunt. The value of 'S' depends on the range of the instrument. Range means maximum current to be measured.

- **Calculation for S :** Let i be the range of the ammeter, i_g is the galvanometer current for full scale deflection. The current distribution is shown in fig. As galvanometer and the shunt are in parallel, we get

$$S(i - ig) = G\, i_g$$

$$\therefore \quad S = \frac{i_g G}{i - i_g} = \frac{i_g\, G}{i} \quad [\text{ when } \; i >> i_g]$$

Evidently when 'S' is connected parallel to the galvanometer, the instrument will be able to measure current from 0 – iA.

➤2.25.2 Voltmeter from Galvanometer :

Voltmeter is a high resistance moving coil table galvanometer. So, a galvanometer is converted into a voltmeter by connecting a high resistance R in series with the galvanometer. The value of R depends on range of the instrument. Hence when a voltmeter is connected parallel to a resistor, it does not affect the resistor current appreciably. So the error in measurement is negligible.

The circuit 2.64 shows the method of conversion of galvanometer into a voltmeter. G is galvanometer resistance and i_g is maximum galvanometer current. V is the maximum voltage to be measured.

- **Calculation for R :** Evidently, the maximum value of R is to be so chosen that the galvanometer current must not exceed i_g.

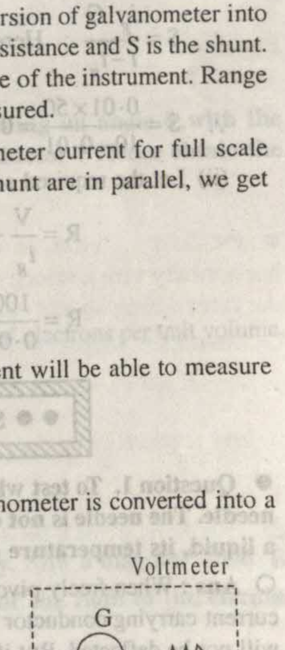


Fig. 2·64

$$\therefore \quad (G+R)\,i_g = V \qquad \therefore \quad R = \frac{V}{i_g} - G$$

So, when R is connected in series with the galvanometer, the instrument will be able to measure voltage in the range 0 — V volt.

A question : What will happen if a voltmeter is connected in series and an ammeter is connected parallel to the circuit ?

Ans. The resistance of a voltmeter is high and that of ammeter is low (i) If a voltmeter is connected in series with the circuit, total resistance of the circuit will increase greatly. So, the main circuit current will be very low than its initial value. Also if this current be greater than maximum voltmeter current, due to large heating the voltmeter coil may burn out.

(ii) If an ammeter is connected parallel to a resistor, the total resistance of the will decrease largely. Circuit current will increase appreciably. Large part of the main current will now pass through the ammeter. So it may burn out if the current exceeds its rated current.

● **Ex. 2.19. The resistance of a table galvanometer is 5Ω and it gives a full scale deflection when the current through it is 10 mA.**

How can you convert it to (i) an ammeter to measure 10A and (ii) a voltmeter to measure maximum 100V.

○ **Solution :** (i) A suitable shunt has to be connected parallel to the galvanometer.

Let the value of the shunt be 'S' ohm.

$$\therefore \quad S = \frac{i_g\,G}{i - i_g}\ ; \quad \text{Here } i_g = 0{\cdot}01\text{ A},\ i = 10\text{A},\ G = 50\text{ ohm}$$

$$\therefore \quad S = \frac{0{\cdot}01 \times 50}{10 - 0{\cdot}01} = \mathbf{0{\cdot}05}\ \text{(nearly)}$$

(ii) Let the required series resistance be 'R' ohm to convert the galvanometer into voltmeter : Then

$$R = \frac{V}{i_g} - G\ ; \quad \text{Here, } V = 100\text{ volt},\ G = 50\Omega\ ,\ i_g = 0{\cdot}01\text{A}$$

$$\therefore \qquad R = \frac{100}{0{\cdot}01} - 50 = 10000 - 50 = \mathbf{9950\ ohm}$$

● ● Short Answer Type Questions (with answers) ● ●

● **Question 1. To test whether a conductor is carrying a current or not, it is placed near a magnetic needle. The needle is not deflected, but when the conductor is immersed in a calorimeter containing a liquid, its temperature gradually rises. Explain the observation.**

○ **Ans :** When freely pivoted, a magnetic needle always align in the north south direction. If the steady current carrying conductor is kept along east-west direction, then no force will act on the needle and so it will not be deflected, But if the wire is immersed in water, heat will be produced due to Joule heating. So temperature of water will rise.

Second explanation : If the current through the conductor be high frequency A.C., then also there will be no deflection of the needle, because direction of current in the wire also changes at double the frequency of the current, so net deflection is zero. But here also heat will be produced as heat generated is proportional to square of the current and does not depend on direction of current.

- **Q. 2. A circular coil carries current i_1 A straight conductor carrying current i_2 is placed along the axis of the coil. What is the mutual force between the currents ?**

○ **Ans.** The mutual force between the currents is zero.

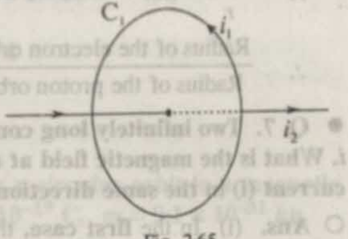


Fig. 2·65

Reason : The magnetic field produced by the circular coil will be perpendicular to the plane of the coil along its axis [Fig 2.65]. Hence the direction of the magnetic field produced by coil C_1 will be along the current i_2. The angle between the magnetic field and direction of current is zero. So the force is $F = Bi_2l \sin 0° = 0$. Hence no force acts on the coil or the conductor.

- **Q. 3. A particle having charge q is moving along a circle of radius r. If the frequency of revolution be n, what is the magnetic induction produced at the centre of the coil ?**

○ **Ans.** The equivalent current is $i = nq$. Hence the magnetic induction at the centre of the circle is

$$B = 10^{-7}\,\frac{2\pi ni}{r} = 10^{-7} \times \frac{2\pi \times ni}{r} = 10^{-7} \times \frac{2\pi ni}{r}\quad wb/m^2$$

- **Q. 4. A wire of length L is bent into a circle. If i be the current in the wire what is the magnetic moment ?**

○ **Ans.** If r be the radius of the circle L, then $2\pi = L \quad \therefore \quad r = \frac{1}{2\pi}$.

It area $A = \pi r^2$, So the magnetic moment of the circular wire is

$$M = Ai = \pi r^2\, i = \pi i \frac{L^2}{(2\pi)^2} = \frac{L^2 i}{4\pi}.$$

- **Q. 5. A conductor of length l and carrying current i is placed making an angle θ with the direction of magnetic field B. How much force will be acting on the conductor ? Hence obtain the expression for the force acting on a moving charge in the field.**

○ **Ans.** The magnetic force on the conductor $\vec{F} = i\,(\vec{l} \times \vec{B})$

$$\therefore \quad |\vec{F}| = Bil \sin\theta$$

2nd part : The current in the conductor is $F = n\,e\,A\,v$. Here n = number of electrons per unit volume.

$\therefore$ magnetic force, $F = B\,neA\,vl \sin\theta$

Now, total free charge on the conductor, $q = n\,e\,A\,l$

Hence the force on a charge θ moving in a magnetic field is

$$\therefore \quad F = B\,q\,v \sin\theta \qquad \therefore \quad \vec{F} = q\,(\vec{v} \times \vec{B})$$

- **Q. 6. An electron and a proton are moving with equal kinetic energy. Now a magnetic field $\vec{B}$ is applied perpendicular to the direction of motion. What is the ratio of the radii of the circular paths of the two particles in the field ?**

○ **Ans.** Let m, v be the mass and velocity of the electrons and M, V be the corresponding quantities for the proton. Then

$$\frac{1}{2}mv^2 = \frac{1}{2}MV^2 \qquad \therefore \quad \frac{m}{M} = \frac{V^2}{v} \quad \ldots\ldots\ldots (1)$$

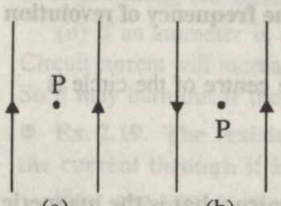
If r be the radius of the path of the electron and R that for proton.

$$\therefore \quad \frac{mv^2}{r} = Bev \qquad \therefore \quad r = \frac{mv}{Be} \text{ and } R = \frac{MV}{Bc} \therefore \frac{r}{R} = \frac{mV}{mv} = \frac{V}{v}.$$

$$\therefore \quad \frac{\text{Radius of the electron orbit}}{\text{Radius of the proton orbit}} = \frac{\text{velocity of proton}}{\text{velocity of electron}}$$

- **Q. 7. Two infinitely long conductors are placed at a separation *r* and each is carrying a current *i*. What is the magnetic field at a point midway between the conductors when the conductors carry current (i) in the same direction and (ii) in the opposite direction ?**

○ **Ans.** (i) In the **first case,** the infinitely long parallel conductors are carrying equal current in the same direction [Fig 2.66(a)]



(a) (b)

Fig. 2·66

The field at the point P midway between the conductors will be zero.

Reason : The magnetic field at P due to the conductor XY is $B_1 = \frac{\mu_0}{4\pi} \frac{2i}{r/2} = \frac{\mu_0}{4\pi} \frac{4i}{r}$ directed into the plane of the paper similarly, the field at P due to the conductor $X_1 Y_1$ is $B_2 = \frac{\mu_0}{4\pi} \frac{2i}{r/2} = \frac{\mu_0}{4\pi} \frac{4i}{r}$ directed out of the plane of the paper.

$\therefore$ Resultant field at P is $B = B_1 - B_2 = 0$

(ii) 2nd the **second case,** the conductors are carrying current in the opposite direction [Fig 1.67 (b)]. The total magnetic field at P due to the two conductors will be added. Here the magnitudes the field produced by the two conductors are equal and acts along the same direction.

$\therefore$ total field at P is $B = \frac{\mu_0}{4\pi} \cdot \frac{4i}{r} \times 2 = \frac{\mu_0}{4\pi} \cdot \frac{8i}{r}$.

• • Numerical Examples • •

- **Ex. 2.20. A electron of mass $9{\cdot}11 \times 10^{-31}$ kg and having charge $-1{\cdot}6 \times 10^{-19}$C is projected horizontally with a velocity $1{\cdot}5 \times 10^5$ m/s into a vertical magnetic field of flux density 0·01 T. Calculate the force acting on the electron. Also find its acceleration.**

○ **Solution :** Magnetic force on the electron, $F = B\, q\, v \sin\theta$

$$\therefore \quad F = 0{\cdot}01 \times 1{\cdot}6 \times 10^{-19} \times 1{\cdot}5 \times 10^5 \times \sin 90° = 2{\cdot}4 \times 10^{-16}\ \text{N}$$

and acceleration of the electron, $a = \frac{F}{m} = \frac{2{\cdot}4 \times 10^{-16}}{9{\cdot}11 \times 10^{-31}} = 2{\cdot}6 \times 10^{14}\, m/s^2$

- **Ex. 2.21. A proton is revolving in a circular orbit at right angle to a magnetic field B = 0·4 T. If the charge of proton be $1{\cdot}6 \times 10^{-19}$C calculate the mass of proton. Given frequency of rotation = $6{\cdot}10 \times 10^6$ rps**

○ **Solution :** The required relation : $\frac{mv^2}{R} = B\, q\, v$

$\therefore$ Angular velocity, $\omega = \frac{v}{R} = \frac{q}{m} B$

$\therefore \quad m = \frac{qB}{\omega} = \frac{1{\cdot}6 \times 10^{-19} \times 0{\cdot}4}{12 \cdot 2\pi \times 10^6}$

$= \mathbf{1{\cdot}67 \times 10^{-27}}$ **kg.** $\therefore$ It is the mass of proton.

$q = 1{\cdot}6 \times 10^{-19} c$

$B = 0{\cdot}4$ T

$\omega = 2\pi \times 6{\cdot}1 \times 10^6\ m/s.$

- **Ex. 2.22. A particle of mass $3{\cdot}3 \times 10^{-31}$ kg and charge $+\,1{\cdot}6 \times 10^{-19}c$ is projected at right angle to a magnetic field of B = 1·0 T. Calculate its frequency of rotational motion.**

○ **Solution :** Time-period, $T = \dfrac{2\pi m}{q\text{B}}$

$m = 3{\cdot}3 \times 10^{-31}$ kg

$q = 1{\cdot}6 \times 10^{-19}$C

$\text{B} = 1{\cdot}0$ T

$$\therefore \quad T = \frac{2\pi \times 33 \times 10^{-31}}{1{\cdot}6 \times 10^{-19} \times 1{\cdot}0} = \mathbf{1{\cdot}3 \times 10^{-7}\,s}$$

- **Ex. 2.23. An electron of kinetic energy 2×10^{3} eV is describing a circular orbit in a magnetic field of B = 0·2 T. What is the radius of the orbit. Given, $e = 1{\cdot}6 \times 10^{-19}$ C, $m = 9{\cdot}1 \times 10^{-31}$ kg.**

○ **Solution :** Kinetic energy, $\text{E} = \dfrac{1}{2}mv^2 = 2 \times 10^3 \times 1{\cdot}6 \times 10^{-19} = \mathbf{3{\cdot}2 \times 10^{-16}\,J}$

$$\therefore \quad \text{Velocity of electron } v = \sqrt{\frac{2 \times 3{\cdot}2 \times 10^{-16}}{9{\cdot}1 \times 10^{-31}}} = \sqrt{\frac{6{\cdot}4}{9{\cdot}1}} \times 10^{15} = \mathbf{2{\cdot}65 \times 10^{7}}\ \text{m/s}$$

$$\therefore \quad \text{Radius of the electron orbit is } \text{R} = \frac{mv}{e\text{B}} = \frac{9{\cdot}1 \times 10^{-31} \times 2{\cdot}65 \times 10^{7}}{1{\cdot}6 \times 10^{-19} \times 0{\cdot}2} = \mathbf{7{\cdot}54 \times 10^{-4}\,m}$$

- **Ex. 2.24. A α-particle is describing a circle of radius 45 cm in a magnetic field of intensity B = 1·2 T. Calculate (i) velocity of α-particle (ii) its kinetic energy (iii) the potential difference through which the particle must pass to acquire this amount of kinetic energy. Mass of α-particle = $6{\cdot}64 \times 10^{-27}$ kg, its charge, $q = 3{\cdot}2 \times 10^{-19}$ C.**

○ **Solution :** (i) $\dfrac{mv^2}{r} = \text{B}\,q\,v \quad \therefore \quad v = \dfrac{\text{B}\,q\,r}{m}$

$$\therefore \quad \text{Velocity of α-particle, } v = \frac{1{\cdot}2 \times 3{\cdot}2 \times 10^{-19} \times 0{\cdot}45}{6{\cdot}64 \times 10^{-27}}$$

$$\therefore \quad v = \mathbf{2{\cdot}6 \times 10^{7}\,m/s}$$

(ii) Now, $mv = \text{B}qr \quad \therefore$ momentum, $\text{P} = \text{B}qr$

$$\therefore \quad \text{Kinetic energy, } \text{E} = \frac{\text{P}^2}{2m} = \frac{\text{B}^2 q^2 r^2}{2m}$$

$$\therefore \quad \text{E} = \frac{(1{\cdot}2)^2 \times (3{\cdot}2 \times 10^{-19})^2 \times (0{\cdot}45)^2}{2 \times 6{\cdot}64 \times 10^{-27}} = 2{\cdot}25 \times 10^{-12}\ \text{J}$$

(iii) If v be the required potential difference, $\text{E} = vq$

$$\therefore \quad \text{V} = \frac{\text{E}}{q} = \frac{2{\cdot}25 \times 10^{-12}}{3{\cdot}2 \times 10^{-19}} = \mathbf{7{\cdot}03 \times 10^{6}\ Volt}$$

- **Ex. 2.25. An electron after passing through a potential difference of 1 Kv, enters into a magnetic field B = 29×10^{-3} T making an angle of 30° with the direction of B. Calculate the pitch of the helical path followed by the electron.**

○ **Solution :** Centripetal force on the electron = $\text{B}qv \sin\theta$

$$\therefore \quad \text{B}\,q\,v \sin\theta = m\omega^2 r \qquad \therefore \quad \omega = \frac{\text{B}q \sin\theta}{m}$$

$$\therefore \quad \text{The time period, } \text{T} = \frac{2\pi m}{\text{B}q \sin\theta}$$

$$\therefore \quad T=\frac{2\pi\times 9\cdot 1\times 10^{-31}}{29\times 10^{-3}\times 1\cdot 6\times 10^{-19}\times \sin 30^{\circ}}=2\cdot 46\times 10^{-9}\text{ sec.}$$

Now, velocity acquired by the electron while passing through a potential difference of 10^3 volt is given by

$$v=\sqrt{\frac{2\,V\,q}{m}}=\sqrt{\frac{2\times 10^{3}\times 1\cdot 6\times 10^{-19}}{9\cdot 1\times 10^{-31}}}=\sqrt{\frac{32}{9\cdot 1}}\times 10^{7}=\mathbf{1\cdot 875\times 10^{7}\ ms^{-1}}$$

$\therefore$ pitch of the path, $p = T.\, v\cos\theta = 2\cdot 46\times 10^{-9}\times 1\cdot 875\times 10^{7}\cos 30^{\circ}$

$$\therefore \quad p=\frac{2\cdot 46\times 1\cdot 875\times \sqrt{3}}{2}\times 10^{-2}m=4\times 10^{-2}m.$$

● **Ex. 2.26. The radius of a circular coil is 8·0 cm and number of turns 30. It is suspended in a magnetic field B = 1·0 T so that normal to the plane of the coil makes angle 60° with the direction of B. If a current of 1·0A passes through the coil, calculate :**

(i) magnetic moment of the coil and (ii) torque on the coil.

○ **Solution :** (i) Magnetic moment of the coil, $M = niA$

$$\therefore \quad M=30\times 6\times \pi\times (8\times 10^{-2})^{2}=3\cdot 617\ Am^{2}$$

(ii) Torque on the coil, $\tau = MB\sin\theta$

$$\therefore \quad \tau = 3\cdot 617\times 1\cdot 0\times \sin 60^{\circ} = \mathbf{3\cdot 13\ N\text{-}m}$$

● **Ex. 2.27. A potential difference of 300V is applied between two horizontal parallel plates and the separation between the plates is 2 cm. A magnetic field B = 1·5 T is applied in the space between the plates perpendicular to the electric field. An electron goes through the space between the plates without any diflection. What is the velocity of the electron ?**

○ **Solution :** The situation is shown in fig 2.67. As the electron goes undeviation, we write

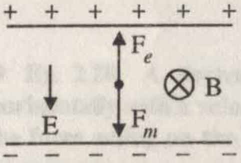


Fig. 2·67

magnetic force = electric force on the electron.

$$\therefore \quad Bqv = Eq = \frac{V}{d}q \quad \therefore \quad v=\frac{V}{d\,.\,B}$$

$$\therefore \quad v=\frac{300}{0\cdot 02\times 1\cdot 5}=10^{4}\,\text{m/s}$$

● **Ex. 2.28. A battery is connected in series with a tangent galvanometer of resistance 2Ω and resistance box. When 8Ω resistance is inserted in the resistance box, the deflection of the galvanometer is 60°. But when the resistance is 32 ohm, the deflection is 30°. What is the internal resistance of battery ?**

○ **Solution :** The circuit diagram is shown in fig 2.68. In the first case, circuit current, $i_1=\dfrac{E}{10+r}$ and in the second case circuit current,

$$i_2=\frac{E}{32+r}\quad [\ r = \text{internal resistance of the battary}\ ]$$

$$\therefore \quad \frac{E}{10+r}=10k\tan 60^{\circ}=10k\times\sqrt{3} \text{ and } \frac{E}{32+r}=10k\tan 30^{\circ}=10k/\sqrt{3}$$

$$\therefore \quad \frac{32+r}{10+r}=3 \quad \therefore \quad \mathbf{r=1\ ohm}$$

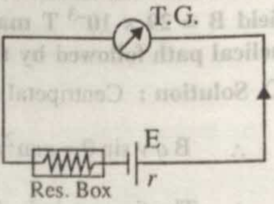


Fig. 2·68

● **Ex. 2.29. A table galvanometer of resistance 238 ohm gives a deflection of 5 division due to a current of 1 mA. What change be made in the galvanometer so that it may give a deflection of 10 division due to a current of 1A ?**

○ **Solution :** A suitable low resistance shunt is required to be connected across the galvanometer. Let the value of the shunt be S [Fig 2.69].

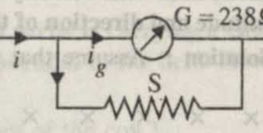


Fig. 2·69

Now, $S = \dfrac{i_g G}{i - i_g}$

Here $i_g = \dfrac{10^{-3}}{5}$ A / div $i = \dfrac{1}{10} = 0{\cdot}1$ A/div and G = 238 Ω

$$\therefore \quad S = \frac{10^{-3} \times 238}{5 \times 0{\cdot}1} = \mathbf{0{\cdot}476\,\Omega}$$

● **Ex. 2.30. The resistance of a millivoltmeter is 25 ohm and it can measure maximum 30 mV. How the instrument can be used in the following cases : (i) to measure 1 to 30V (ii) to measure 0·1 to 3A.**

○ **Solution :** (i) To convert a millivoltmeter into a voltmeter a high resistance has to be connected in series with the instrument. Let the required resistance be R ohm.

$$\therefore \quad R = \frac{V}{i_g} - G \text{ Here } i_g = \frac{30 \times 10^{-3}}{25} = 1{\cdot}2 \times 10^{-3}\,\text{A},\ G = 25\Omega$$

$$\therefore \quad R = \frac{30}{1{\cdot}2 \times 10^{-3}} - 25 = 25000 - 25 = \mathbf{24975\,\Omega}$$

(ii) The given millivoltmeter can be converted into an ammeter by connecting a low resistance shunt parallel to the instrument. If 'S' be the required shunt, then

$$S = \frac{i_g G}{i - i_g} = \frac{1{\cdot}2 \times 10^{-3} \times 25}{3 - 1{\cdot}2 \times 10^{-3}} = \mathbf{0{\cdot}01\,\Omega}$$

∴ Required shunt = **0·01 ohm**

● **Ex. 2.31. A tangent galvanometer of resistance 200 ohm when connected in series with a battery of negligible internal resistance and a resistance R, gives a deflection of 45°. Now a 200 ohm resistor is connected parallel to the galvanometer. The galvanometer now produce a deflection of 30°. Calculate the value of R.**

○ **Solution :** (i) In the first case [Fig 2.70 (i)] current in the circuit is $i_1 = \dfrac{E}{200 + R}$

$$\therefore \quad \frac{E}{200 + R} = 10k \tan 45^\circ = 10k \ldots\ldots\ldots\ldots \text{(i)}$$

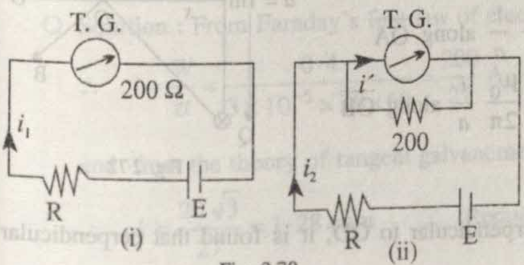


Fig. 2·70

In the second case [Fig 2.70 (ii)], current in the circuit $i_2 = \dfrac{E}{100 + R}$

∴ Galvanometer current,

$$i' = \frac{E}{100 + R} \times \frac{200}{400} = \frac{E}{2(100 + R)}$$

$$\therefore \quad \frac{E}{2(100 + R)} = 10\,k \tan 30^\circ = \frac{10k}{\sqrt{3}} \ldots\ldots \text{(ii)}$$

From equations (i) and (ii), $\frac{2\,(100+R)}{200+R}=\sqrt{3}.$

$\therefore\quad 200+2R=\sqrt{3}\,(200+R) \qquad \therefore\quad R=\mathbf{546\cdot 3}$ **ohm**

● **Ex. 2.32. A proton of mass $1{\cdot}67\times10^{-27}$ kg and charge $1{\cdot}6\times10^{-19}$C enters at right angle to a magnetic field with velocity of $1{\cdot}6\times10^{5}$ m/s. It describes a semi circle of radius 4 cm. Calculate the magnitude and direction of the magnetic field. Calculate also the time, it takes to traval the distance.**

○ **Solution :** Assume that magnetic induction is B tesla. The magnetic field provides the necessary centripetal force on the proton. The trajectory of the proton is shown fig 2.71. Hence the magnetic field is perpendicular to the plane of the fig and directed inwards as shown by ⊗ symbol.

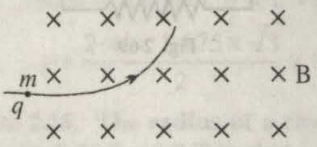


Fig. 2·71

$$\therefore\quad \frac{mv^2}{r}=Bqv \quad \therefore\quad B=\frac{mv}{qr}.$$

$$\therefore\quad B=\frac{1{\cdot}6\times10^{-27}\times1{\cdot}6\times10^{5}}{1{\cdot}6\times10^{-19}\times4{\cdot}2\times10^{-2}}$$

$m = 1{\cdot}67\times10^{-27}$ kg
$v = 1{\cdot}6\times10^{5}$ m/s
$q = 1{\cdot}6\times10^{-19}$ C
$r = 4{\cdot}0\times10^{-2}$ m

Next, the time taken by the particle to describe the semicircle is

$$t=\frac{T}{2}=\frac{\pi R}{v}=\frac{3{\cdot}14\times4{\cdot}0\times10^{-2}}{1{\cdot}6\times10^{5}}=\mathbf{1{\cdot}57\times10^{-6}}\ \text{sec}$$

● **Ex. 2.33. An electron initially at rest after passing through a potential difference of 1000V enters into a magnetic field. (i) Calculate the velocity of electron when it enters into the magnetic field (ii) If it describes a circle of radius 1·0 m, what is the intensity of magnetic field ? $\frac{e}{m}=1{\cdot}76\times10^{11}\,\mathrm{C\,.\,kg^{-1}}$**

○ **Solution :** (i) $\frac{1}{2}mv^2=eV \quad \therefore\ v=\sqrt{\frac{2\,eV}{m}}$

$$\therefore\quad v=\sqrt{2\times1{\cdot}76\times10^{11}\times1000}=1{\cdot}875\times10^{7}\,\mathrm{ms^{-1}}$$

(ii) $\frac{mv^2}{r}=Bev \quad \therefore\quad B=\frac{mv}{re}$

$$\therefore\quad B=\frac{1{\cdot}875\times10^{7}}{1{\cdot}76\times10^{11}\times1{\cdot}0}=\mathbf{1{\cdot}07\times10^{-4}\ T}$$

● **Ex. 2.34. Two long and straight parallel conductors are in air at a separation of 1·0 m. [Fig 2.72]. The current in both the conductors is 5A but in opposite direction. What is the magnetic field at a distance $x = 0{\cdot}2$ m ?**

○ **Solution :** The conductors P and Q carry current in the same direction and the point O is at equal distance from conductors. So the magnitudes of the magnetic fields at O produced by the conductors are equal.

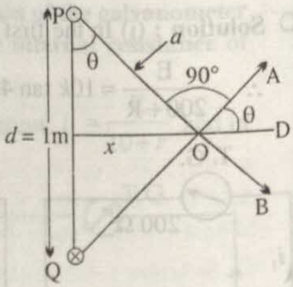


Fig. 2·72

$\therefore$ Field at O due to the conductor P is $B_1=\frac{\mu_0}{2\pi}\,\frac{i}{a}$ along $\overline{OA}$

and the field at O due to the conductor Q is $B_2=\frac{\mu_0}{2\pi}\,\frac{i}{a}$ along $\overline{OB}$

where $a^2=\left(\frac{d}{2}\right)^2+n^2$

Where B_1 and B_2 are resolved along OD and perpendicular to OD, it is found that perpendicular components cancel each other.

Hence the total intensity at O is $B = 2B_1 \cos\theta = 2 \times \frac{\mu_0}{2\pi} \cdot \frac{i}{a} \cdot \frac{d}{2a} = \frac{\mu_0}{2\pi} \cdot \frac{i\,d}{a^2}$

Here $\mu_0 = 4\pi \times 10^{-7}\,\text{SI}$, $i = 5\text{A}$, $d = 1 \cdot 0m$, $a^2 = (0 \cdot 5)^2 + (0 \cdot 2)^2 = \mathbf{0 \cdot 29}\,\boldsymbol{m^2}$

$\therefore \quad B = \frac{2 \times 10^{-7} \times 5 \times 1 \cdot 0}{0 \cdot 29} = 3 \cdot 45 \times 10^{-6}$ along $\overline{OD}$

● **Ex. 2.35. A rectangular coil of area 7·5 × 10^{-3} m^2 and having 100 turns is carrying a current of 0·1 A. It is placed in a magnetic field of B = 1·5 T so that its plane is parallel to the field. How much work is done to rotate it by 180° ?**

○ **Solution :** Work done, W = MB (1 – cos θ) [M = magnetic moment of the coil]

Here $M = nAi = 100 \times 7 \cdot 5 \times 10^{-3} \times 0 \cdot 1 = 7 \cdot 5 \times 10^{-2}\ Am^2$

$B = 1 \cdot 5\ wb/m^2$, $\cos\theta = \cos 180° = -1$

$\therefore \quad W = 7 \cdot 5 \times 10^{-2} \times 1 \cdot 5 \times (1+1) = \mathbf{0 \cdot 225\ J}$

● **Ex. 2.36. In a de Arsonsval galvanometer, the area of the coil is 50 cm^2 and number of turns 30. It is suspended in a magnetic field of intensity. B = 0·03 T. What current be sent through the coil so that it is deflected by 5° ? Torsional constant, τ = 10^{-8} N–m/deg.**

○ **Solution :** From the theory of moving coil galvanometer, $i = \frac{c}{nAB} \cdot \theta$

Here $c = 10^{-8}$ N–m / deg, $n = 30$, $A = 50 \times 10^{-4} m^2$, $B = 0 \cdot 03$ T and $\theta = 5 \times \frac{\pi}{180}$ rad.

$\therefore \quad i = \frac{10^{-8} \times 5\pi}{30 \times 50 \times 10^{-4} \times 0 \cdot 03 \times 180} = \mathbf{9 \cdot 7 \times 10^{-7}}\,\text{A}$

∴ The required current = **9·7 × 10^{-7}**A

● **Ex. 2.37. The intensity of earth's magnetic field is 5 × 10^{-5}T. A long wire is kept horizontally at a height of 0·1 m from the earth's surface. When a certain current is passed through the wire, then the magnetic field produced by the wire just balances the earth's magnetic field. Calculate the current in the wire.**

○ **Solution :** Let the required current in the wire be *i*A. If the wire be at a height *r* from the earth's surface, then the magnetic field at a point on the wire is

$$B = 10^{-7} \times \frac{2i}{r} = 2 \times 10^{-7} \times \frac{i}{0 \cdot 1}\ wb/m^2 = 2 \times 10^{-6}\ i\ wb/m^2$$

$\therefore$ By question, $2 \times 10^{-6} i = 5 \times 10^{-5} \quad \therefore\ \mathbf{i = 25A}$

● **Ex. 2.38. Through the series combination of a copper voltameter and a tangent galvanometer a current is passed and 0·4 g copper is deposited at the cathode of the voltameter. If the deflection of the galvanometer be 30°, calculate its reduction factor. ECE of copper = 3 × 10^{-5} g/C.**

○ **Solution :** From Faraday's first law of electrolysis, W = *z i t*

$\therefore \quad i = \frac{W}{zt} = \frac{0 \cdot 4}{3 \times 10^{-5} \times 30 \times 60} = \frac{200}{27}$ A.

and from the theory of tangent galvanometer, $\frac{200}{27} = 10\,k \tan 30°$

$\therefore \quad k = \frac{20\sqrt{3}}{27} = \mathbf{1 \cdot 28\ emu} \qquad \therefore$ Reduction factor of the galvanometer = **1·28 emu**

● **Ex 2.39. Two long and straight parallel wires are kept at a separation of 2 cm in air, current is one wire is 60A. What should be the current in second wire so that it may experience a force of 1200 dyne/m.**

○ **Solution :** The force per unit length, $F = 10^{-7} \times \frac{2i_1 \, i_2}{r}$

$$\therefore \quad 10^{-7} \times \frac{2i_1 \, i_2}{r} = 1200 \times 10^{-5}$$

$$\therefore \quad i_2 = \frac{12 \times 10^{-3} \times 2 \times 10^{-2}}{2 \times 60} \times 10^7 \quad [i_1 = 60\text{A}, \; r = 0{\cdot}02 \, m]$$

= **20A.** It is the required current.

● **Ex 2.40. Two parallel plates are kept horizontally at a separation of 3 cm and a potential difference of 600V is applied between the plates. An electron is projected horizontally between the plates with a velocity of 100 m/s. Now a bar magnet of magnetic moment 1000 cgs is so placed that its field is perpendicular to the electric field between the parallel plates. At what distance from the initial direction of motion, the magnet be held so that electron goes undeviated ?**

○ **Solution :** Let the bar magnet be placed at a distance r from the initial direction of motion of the electron. At the electron goes undeviation it implies that magnetic force on the electron is equal and opposite to electric field on it.

$$\therefore \quad \text{B}ev = \text{E}e, \; \text{B} = \frac{\text{E}}{v} = \frac{600}{0{\cdot}03 \times 100} = 200\text{T} \quad \left[\because \; \text{E} = \frac{\text{V}}{d}\right]$$

Now, intensity of the magnetic field due to the magnet is

$$\text{B} = 10^{-7} \, \frac{2\text{M}}{r^3} = 200. \qquad \therefore \quad r^3 = \text{M} \times 10^{-9}$$

$$\therefore \quad r^3 = \text{M} \times 10^{-9}$$

$$r^3 = 1 \times 10^{-9} \quad [\text{M} = 1000 \text{ cgs} = 1 \text{ A}m^2]$$

$$\therefore \quad r = 10^{-3} m = \mathbf{1 \, mm}$$

So, the magnet be placed 1 mm away.

● **Ex. 2.41. An electron of energy 10^4 eV describes a circular path in a plane at right angle to a uniform magnetic field of 0·01T.**

(i) What is the radius of the circular path ? (ii) What is cyclotron frequency ? (iii) What is the period of revolution ?

○ **Solution :** The radius of the electron orbit, $r = \frac{mv}{q\text{B}}$.

Now, velocity of electron is obtained from, $\frac{1}{2} mv^2 = k\text{E}$.

$$v = \sqrt{\frac{2 \, . \, k\text{E}}{m}} = \sqrt{\frac{2 \times 10^4 \times 1{\cdot}6 \times 10^{-19}}{9{\cdot}1 \times 10^{-31}}} \quad \left[k\text{E} = 10^4 \times 1{\cdot}6 \times 10^{-19} \text{ J}\right]$$

$$= 5{\cdot}9 \times 10^7 \text{ m/s}.$$

$$\therefore \quad r = \frac{9{\cdot}1 \times 10^{-31} \times 5{\cdot}9 \times 10^7}{1{\cdot}6 \times 10^{-19} \times 0{\cdot}01} = 3{\cdot}4 \times 10^{-2} m = 3{\cdot}4 \text{ cm}$$

(ii) Cyclotron frequency, $\upsilon = \frac{qB}{2\pi m}$.

$$\therefore \quad \upsilon = \frac{0 \cdot 01 \times 1 \cdot 76 \times 10^{11}}{2\pi} = 2 \cdot 8 \times 10^{8} \text{ Hz}$$

(iii) Time period of revolution, $T = \frac{1}{\upsilon} = \frac{10^{-8}}{2 \cdot 8} = \mathbf{3 \cdot 6 \times 10^{-9}}$ sec

• E ◇ X ◇ E ◇ R ◇ C ◇ I ◇ S ◇ E •

A. Objective type questions :

(i) Fill in the blanks :

1. SI unit of magnetic induction vector $\vec{B}$ is ——

2. The electric current in a long straight conductor is I, then magnetic induction $\vec{B}$ at a distance *r* from it is ——

3. A conductor of length L is bent into a circle and a current I passed through it. What is its magnetic moment ? ——.

4. PQRS circuit is formed by joining two semi-circular conductors of radius r_1 and r_2. If the current in the circuit be I, the magnetic induction at the centre O is ——.

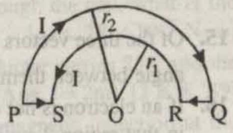


Fig. 2·73

5. A battery is connected between any two points of a circular coil. Magnetic induction B at its centre is ——.

6. The radius of a circular coil is 2π and its number of turns is 50. A current of 5A is passed through it. The magnetic induction at the centre is ——.

7. A conductor of length *l* and carrying current 5A is put at 30° to a magnetic field B. The force on the conductor is ——.

8. The magnetic force on a particle having charge *q* projected perpendicular to a magnetic field B with a velocity *v* is ——. The force is called —— ——. The radius of the circular path in the field is ——.

9. To convert a galvanometer into an ammeter a low resistance is to be connected —— to the galvanometer. But to convert it to a voltmeter a high resistance is to be connected —— to the galvanometer.

10. An electron is moving horizontally from east to west. A magnetic field is acting vertically downward. The force on the electron acts towards ——.

B. Very short answer type questions :

1. Define 1A current interms of the force between two parallel current carrying conductors.

2. What is meant by magnetic induction 1T ?

3. A circular coil of radius 10 cm, number of turns 10 carries current 10A. What is its magnetic moment ?

4. Between ammeter and voltmeter, which one has higher resistance ?

5. Write the expression for Lorentz force.

6. A coil of magnetic moment M is in equilibrium in a magnetic field B. How much work is done to deflect it by 180° ?

7. A current carrying conductor is placed in a magnetic field. When is the force is (a) maximum and (b) zero ?

8. What is the fundamental difference between ammeter and voltmeter ?

9. The magnetic moment of a magnetised steel wire of length *l* is M. What is its magnetic moment when it is but in a semi-circle ?

10. What should be the relation between the magnitude and direction of electric field and magnetic field so that a charged particle may go undeviated through these fields ?

11. What is the nature of the magnetic field in a moving coil galvanometer ?

12. Name the physical quantity whose S.I. unit is *wb*. m^{-2}. Is it a vector or scalar quantity ?

◢ C. Short answer type questions :

1. Define 1 e.m.u of current. How it is related to ampere ? **[W. B. H. S. 2002]**
2. What is the working principle of Barlow's wheel ? Is it a motor ? **[W. B. H. S. 2002]**
3. Why is the deflection of a tangent galvanometer is kept around 45° ? **[W. B. H. S. 2001]**
4. State the working poinciple of moving coil galvanometer.
5. State Biot-Savart's law of electromagnetism.
6. State Fleming's left hand rule.
7. Draw the lines of force due to a long solenoid carrying current.
8. A current *i* flows in a circular coil of radius *r* and having *n* turns what is its magnetic moment ? Give its SI unit.
9. What is the reduction factor of a tangent galvanometer ?
10. Define current sensitivity and voltage sensitivity of a moving coil galvanometer.
11. How is the deflection in a moving coil galvanometer is measured by lamp and scale method ?
12. The electron in a hydrogen atom is revolving round the nucleus in a circular orbit of radius *r*. If its time period be T, what is (i) equivalent current in the orbit (ii) magnetic induction at the nucleus (iii) magnetic moment of the orbit.
13. What is Lorentz force. Write its expression.
14. A stream of electrons travelling with speed *v*. m/s at right angle to a uniform magnetic field B, is deflected in a circular path of radius *r*. Prove that $\frac{e}{m} = \frac{v}{rB}$.
15. Of the three vectors in the equation $\vec{F} = q(\vec{v} \times \vec{B})$ which pairs are always at right angles ? Which may have any angle between them ?
16. If an electron is not deflected in passing through a certain space, can we be sure that is there is no magnetic field in that region ?

◢ D. Essay type questions :

1. State and explain Ampere's circuital law. Hence obtain an expression for the magnetic field at a point due to a long straight conductor carrying a current.
2. Using Biot-Savart's law derive an expression for the magnetic field at the centre of a circular coil of radius '*r*' and having N turns. Indicate direction of magnetic field.
3. Derive an expression for the torque on a rectangular coil of area A, carrying a current *i* in a magnetic field B. The angle between the direction of B and the vector perpendicular to the plane of the coil is θ.
4. Derive an expression for the force experiened by a current carrying straight conductor plaecd in a magnetic field. When is the force maximum ?
5. Describe the motion of a charged particle in a uniform magnetic field. Obtain an expression for the radius of the path of the charged particle moving perpendicular. to uniform magnetic field. Show that the time taken to complete one revolution by the particle is independent of its speed.
6. What is a Lorentz force ? Obtain an expression for it.
7. Deduce an expression for the force per unit length experience by each of two parallel current carrying conductor.
8. Explain how you can convert a galvanometer into an Ammeter and into a voltmeter.

◢ E. Simple numerical problems :

1. A conductor of length 5 cm carrying a current 2A is put perpendicular to a magnetic field of flux density 2000 G. What magnetic force acts on the conductor ? What will be the force on the conductor if it is placed parallel to the magnetic field. **[Ans. 0·02 N ; 0] [W. B. H. S. 2002]**
2. A single turn circular coil of radius 10 cm carries 10A current. What is the intensity of the magnetic field at the centre of the coil ? $\mu_0 = 4\pi \times 10^{-7}\ wb/\text{Am}$. **[Ans : $112{\cdot}8 \times 10^{-7}$ T]**
3. The radius of a circular coil is 20 cm. What a current of 5A is passed through the coil, the intensity of the magnetic field at the centre of the coil is $5{\cdot}024 \times 10^{-4}$ T. Calculate the number of turns of the coil. **[Ans. 40]**
4. A wire of length 9·42 cm is bent into a circle. If it carries a current of 2A, what is its magnetic moment ? **[Ans. $15{\cdot}48 \times 10^{-4}\ \text{A}m^2$]**

5. A long straight conductor carries a current of 4A. What is the magnetic induction at a distance of 10 cm from the conductor ? If a magnetic pole of strength 5 cgs is placed at the point, how much force will act on it ? [**Ans.** 8×10^{-6} T ; 0·4 dyne]

6. A beam of alpha particles and of protons of the same velocity v, enters a uniform magnetic field at right angle to the field. What is the ratio of the radii of the two circles described by the two types of particles. [**Ans.** 2 : 1]

7. An electron of kinetic energy 25 keV moves perpendicular to the direction of uniform magnetic field of $0{\cdot}2 \times 10^{-3}$ T. Calculate the time period of revolution of the electron in the magnetic field. [**Ans.** $1{\cdot}79 \times 10^{-7}$s]

8. An electron being accelerated through 100V enters a uniform magnetic field of 0·004 T perpendicular to the direction of motion. Calculate the radius of the path described by the electron. [**Ans.** $r = 8{\cdot}43 \times 10^{-3}$m]

9. A semi circular arc of radius 20 cm carries a current of 10A. Calculate the magnitude of the magnetic field at the centre of the arc. [**Ans.** $1{\cdot}57 \times 10^{-5}$ tesla]

10. A wire of length 2*m* is placed in a magnetic field of intensity 0·15 *Oe*. If a current of 10A is passed through the wire, what force will act on it in the following cases : (i) when the wire is perpendicular to the field, (ii) when the wire is at angle of 45° with field direction and (iii) when the wire is parallel to the field. [**Ans.** (i) 3×10^{-4}N (ii) $21{\cdot}12 \times 10^{-5}$N (iii) 0] **[W. B. H. S. '84]**

11. A circular coil of radius 20 cm and having 20 turns is placed in magnetic meridian. If a current is passed through the coil, a small magnetic needle placed at the centre is deflected by 45°. What is the current in the coil. H = 0·38 Oe. [**Ans.** 0·6 A]

12. The measurement of the coil of a moving coil galvanometer is 3 cm × 2 cm and its number of turns is 60. It is suspended in a magnetic field of strength 500 Oe. If a current of 1 mA is passed through the coil, what is the deflecting torque ? [**Ans.** 18×10^{-7} N–*m*]

13. Two straight long parallel conductors are in air and their separation is 4 cm. One conductor carries 25A and the other 5A current. What length of any conductor will experience a force of 125 dyne ? [**Ans.** l = 2m] **[J E E '89]**

14. A circular coil of radius 10 cm and having 200 turns carries 5A current. Calculate magnetic induction at the centre. [**Ans** $6{\cdot}28 \times 10^{-3}$T]

15. A circular coil of radius 10 cm and having 20 turns carries 1A current. Calculate magnetic induction at a point on the axis which at a distance of 20 cm from the centre. [**Ans.** $6{\cdot}28 \times 10^{-5}$ T]

16. In a hydrogen atom the electron rotates in a circular orbit of radius $5{\cdot}3 \times 10^{-11}$m at a frequency $1{\cdot}6 \times 10^{15}$. Calculate magnetic induction at the centre. $e = 1{\cdot}6 \times 10^{-19}$C [**Ans.** 12·51 T]

17. A tangent galvanometer of resistance 10Ω is connected in an electric circuit. Total circuit resistance including galvanometer is 100 ohm. The galvanometer shows a deflection of 60°. What shunt be connected across the galvanometer so that the deflected reduces to 30°. [**Ans.** 4·5 ohm]

18. A magnetic field of flux density 0·1 T is acting along Z-axis. An electron is projected into the field along X-axis. What is the magnitude and direction of the force on the electron. [**Ans.** F = $3{\cdot}2 \times 10^{-4}$N, Y-axis]

19. An electron is moving along a circular orbit in a magnetic field B = $2{\cdot}4 \times 10^{-2}$T with the speed $5{\cdot}0 \times 10^{6}$ m/s. Calculate (i) Effective force on electron (ii) Radius of the orbit (iii) Energy gained by electron after 1 revolution. [**Ans.** (i) $1{\cdot}92 \times 10^{-14}$N (ii) 1·18 mm (iii) 0]

20. A beam of electron is allowed to pass through a region when an electric field and a magnetic field are acting at right angle to each other and also perpendicular to the electron motion. If the electron beam passes through the region undeviated, what is the velocity of electrons ? [**Ans.** $1{\cdot}7 \times 10^{7}$ m/s]

21. Two long parallel wires are kept at a separation of 20 cm and each of then carries a current of 10A in opposite direction. What is the intensity of electric field at a point midway between the two wires ? [**Ans.** $4{\cdot}0 \times 10^{-5}$T]

22. A wire is bent into a circle of radius 0·5 m and it carries a current of 2A. Calculate (i) magnetic moment of the circle. (ii) If the axis of the coil is at right angle to a magnetic field B = 2·0 T, what is the torque acting on the coil ? [**Ans.** (i) 1·57 Am^2 (ii) 3·14 N-m]

23. The effective force between two long parallel wires kept at a separation of 0·01 m is 16×10^{-3} N/m. The current is one is double the other. What is the current is each wire ? [**Ans.** 20A, 40A]

24. A rectangular coil (12 cm × 10 cm) of 50 turns carrying a current of 3A is placed in a magnetic field 0·4T. Calculate (i) magnetic moment of the coil, (ii) maximum torque. [**Ans.** (i) 1·8 Am^2 (ii) 0·72 N-m]

25. The electron density in a copper wire of area of cross-section 0·6 mm^2. It carries a current of 5·0 A. A magnetic field 0·15 T is applied perpendicular to the conductor. Calculate the force on each electron.

[**Ans.** $1{\cdot}25 \times 10^{-24}$ N]

26. A galvanometer of resistance of 10Ω gives full scale deflection due to a current of 120 *m*A. How can you convert it into a voltmeter to measure upto 2·4 volt. [**Ans.** series resistance R = 19900 Ω]

26a. How can you convert the galvanometer of problem 26 into an ammeter to measure maximum 10 A ?

[**Ans.** shunt, S = 0·001 Ω]

27. A milliammeter of resistance 20 ohm can measure upto 5 mA. What change is to be made so that it measure a current of 5A. [**Ans.** S = 0·02 Ω]

28. A moving coil galvanometer gives full scale deflection due to a current of 10 mA. How it can be converted into a voltmeter to measure upto 100V. [**Ans.** Series resistance, R = 9950 Ω]

29. The current sensitivity of a milliammeter is 1 mA/div. It is converted to a voltmeter by connecting a resistance in series with the coil. If the total resistance of the instrument be 500 ohm, what is the value of each division of the voltmeter ? [**Ans.** 0·5 V]

30. A tangent galvanometer and silver voltameter are connected in series and a current is passed through it. The deflection of the galvanometer is 30° and 0·112 g silver is deposited at the cathode of the voltameter in 5 min. Calculate the reduction factor of the galvanometer. [**Ans.** 0·06 emu] [**H. S. '88**]

31. The measurement of the coil of a galvanometer is 2 cm × 8 cm and its number of turns 300. The magnetic field is radial and its intensity is 0·25 T. The torsional constant of the suspension $c = 10^{-6}$ N–m/deg. If the maximum deflection of the galvanometer be 60°, what is the maximum current for the galvanometer. [**Ans.** 4×10^{-4}A]

32. A uniform magnetic field of 1·0 T is acting horizontally along south to north. A proton of energy 5·0 Mev enters the field in the vertically downward. Calculate (i) total force on the proton and its direction. Mass of proton $= 1{\cdot}7 \times 10^{-27}$ kg [**Ans.** (i) $4{\cdot}96 \times 10^{-12}$N (ii) East]

F. Harder numerical problems :

1. The number of turns of a circular coil is 50 cm radius 5 cm. The diameter of the wire is 0·2 mm and resistivity 2×10^{-6} ohm–cm. What potential difference be applied to the coil so that the magnetic field produced at the centre of the coil will neutralise the horizontal component of earth's magnetism ? H = 0·314 Oe

[**Ans.** 0·5 V] [**JEE '94**]

2. The length of a solenoid is 40 cm and number of turns 500 and carries a current of 3A. A circular coil of radius 0·01 m and having 10 turns is kept inside the solenoid so that its axis perpendicular to the axis of the coil. If a current of 0·4A is passed through the coil, how much torque will act on it ? [**Ans.** $5{\cdot}92 \times 10^{-6}$N–*m*]

3. An electron accelerated by a potential difference V = 1KV moves in a uniform magnetic field at an angle $\alpha =$ 30° to the field B = 29 *m*T. Calculate the pitch of the helical trajectory. [**Ans.** $1{\cdot}7 \times 10^{-3}$ m]

4. A copper rod weighing 150g rests on two rails 30 cm apart. It carries the current of 50 A from one rail to the other. The coefficient of friction 0·6. What is the smallest magnetic field that would cause the rod to slide ?

[**Ans.** 0·588 T]

5. The length of a solenoid is 1m and average radius 3 cm. Wire is wound on it is 5 layers. The number of turns in each layer is 850. When 5A current is passed through the solenoid, calculate (i) magnetic induction inside the solenoid and (ii) magnetic flux inside it. [**Ans.** (i) $2{\cdot}7 \times 10^{-2}$ T ; (ii) $1{\cdot}9 \times 10^{-5}$ *wb*]

6. The mass of Li$^+$ ion is $1{\cdot}16 \times 10^{-26}$ kg. It is revolving in a circular orbit of radius 2·0 cm in a magnetic field B = 0·4 T. Calculate (i) velocity of the ion (ii) time to complete half revolution. (iii) potential difference through which the iron has to be passed to acquire this velocity. [**Ans.** (i) $1{\cdot}1 \times 10^5$ m/s (ii) $5{\cdot}7 \times 10^{-7}$s (iii) 439V]

7. The free electron density in copper wire 8×10^{28} m^{-3}. A copper wire of length 1m and area of cross-section $8{\cdot}0 \times 10^{-6}$ m^2 is placed at right angle to a magnetic field B = 5×10^{-3}T. If the wire experiences a force of $8{\cdot}0 \times 10^{-2}$ N, calculate drift velocity of electron in copper wire. [**Ans.** $1{\cdot}5625 \times 10^{-4}$ m/s]

8. A horizontal wire of length 0·1 m carries a current of 5A. Calculate the magnetic field so that the magnetic force on the wire just balances the weight of the wire. What is the direction of the field ? Mass of the wire 3×10^{-3} kg/m. [**Ans.** $5{\cdot}88 \times 10^{-3}$ T]

9. An electron is moving in a stable orbit. If the angular momentum of the orbit be $\frac{h}{2\pi}$, what is the magnetic moment of the orbit due to motion of electron ? $h = 6{\cdot}6 \times 10^{-34}$ J-s. [M = $9{\cdot}2 \times 10^{-24}$ Am^2]

10. Three long parallel wires are arranged as shown in fig 2.74. Calculate the force on 25 cm length of the wire C. [**Ans.** $F = 4 \times 10^{-4}$ N]

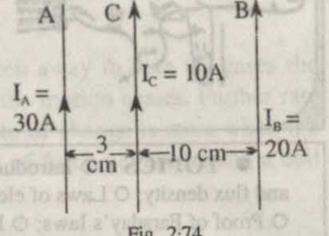


Fig. 2·74

11. A long straight conductor carries a current of 2A. An electron is moving along a parallel line at a distance of 0·1m with a velocity of 4×10^4 ms^{-1} in the direction opposite to the direction of current in the wire. What force will act on the moving electron due to the magnetic field produced by the wire ? [**Ans.** $7{\cdot}812 \times 10^{-4}$ N]

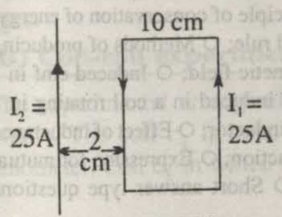


Fig. 2·75

12. A rectangular current loop (25 cm × 10 cm) carrying 25A current is placed near a long straight conductor as shown in fig 2.75. Calculate the force acting on the doop. [**Ans.** 7·812 × 10–4 N]

13. A closed circuit is formed of a wire ABC [Fig 2.76]. If current in the circuit be 5A what is the intensity of the magnetic field at the centre O of the circular wire. The radius of curvature of the curved part of the circuit is 12 cm and ∠AOC = 90°. [**Ans.** $27{\cdot}8 \times 10^{-6}$T]

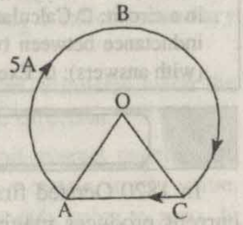


Fig. 2·76

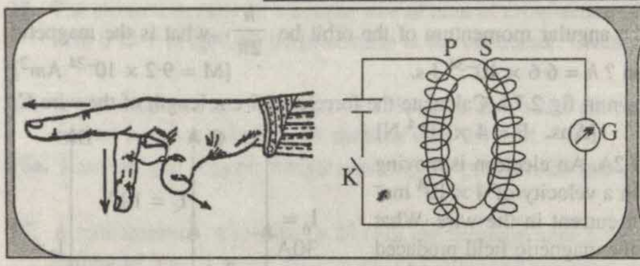
14. A small magnetic needle is pivoted at the centre of a vertical circular coil. When the coil is at angle of 30° with the magnetic meridian and a current of 1·25A is passed through the coil, the needle remains in east-west direction. If the radius of the coil be 20 cm and carries 10 turns, obtain the horizontal intensity of earth's magnetism. [**Ans.** H = 0·196 G]

15. A copper rod of length 50 cm and mass 20 g is placed across two parallel and horizontal metallic rails. The rails carry 40A current. A magnetic field $B = 5 \times 10^{-2}$ T is applied perpendicular to the rails. If the rod is about to slide over the rails, find the coefficient of friction between the rod and the rails. $g = 10$ m/s^2 [**Ans.** 0·5]

16. In question 15, if the rails are frictionless, what will be the velocity of the rod after 0·1 sec ? [**Ans.** 0·5 m/s]

17. The area of the rectangular coil of a de Arsonval type galvanometer is 2 cm^2 and its number of turns is 50. The intensity of the magnetic field is 3000 G. Due to a current of 10 μA, the galvanometer gives a deflection of 50. Find torsional constant of the suspension string of the galvanometer. [**Ans.** 21·6 dyne-cm]

18. The current sensitivity of a moving coil galvanometer is 0·5 div/μA. When it is connected in series with a battery of emf 2·2 V and a resistance of 4200 Ω, it gives a deflection of 250 div (full scale). What changes be made in the galvanometer so that, it may give full scale deflection due to a current of 3·0 mA. [**Ans.** shunt of 40 Ω]



CHAPTER 3

ELECTROMAGNETIC INDUCTION

• **TOPICS :** ○ Introduction; ○ Faraday's experiment on electromagnetic induction; ○ Magnetic flux and flux density; ○ Laws of electromagnetic induction; ○ Lenz's law and principle of conservation of energy; ○ Proof of Faraday's laws; ○ Direct of induced current : Fleming's right hand rule; ○ Methods of producing induced emf; ○ EMF induced in a conductor rotating perpendicular to a magnetic field; ○ Induced emf in a conductor moving perpendicular to a magnetic field : motional emf; ○ EMF induced in a coil rotating in a magnetic field; ○ Eddy current; ○ Self-induction; ○ Energy associated with an inductor; ○ Effect of inductance in a circuit; ○ Calculation of self-inductance of a long solenoid; ○ Mutual induction; ○ Expression for mutual inductance between two long solenoids; ○ Non-inductive resistance coil; ○ Short answer type questions (with answers); ○ Exercise.

✪ 3.1. Introduction ✪

In 1820 Oersted first demonstrated by a simple experiment (called Oersted experiment) that electric current produces magnetic field in the surrounding. Oersted's experiment shows that moving charges produce magnetic field. Attempts were made to observe the reverse effect, *i.e.*, generation of electric current by means of a magnetic field.

After several years (in the year 1831) Michael Faraday in U. K. showed by experiments that whenever the magnetic flux passing a through a coil changes, an emf is induced in the circuit and if the circuit is closed, a current flows in the circuit.

This phenomenon of generation of emf and production of current due to change of magnetic flux linking a circuit is known as electromagnetic induction.

This discovery of Faraday brings about a large change in the field of current electricity. Because, this basic phenomenon is the basis of working of a power generator, dynamo, transformer etc. because after its discovery it was possible to convert electrical energy into mechanical energy with high efficiency.

✪ 3.2. Faraday's experiment on electromagnetic induction ✪

Faraday and others conducted a few simple experiments to demonstrate the phenomenon of electromagnetic induction.

▲ (A) Coil-Magnet Experiment :

Fig 3.1 shows a circular coil of wire of one or more turns closed through a sensitive galvanometer G. A strong bar magnet NS is moved towards the coil along its axis. Immediately the galvanometer shows a deflection, showing presence of current in the circuit. Similarly, if the bar magnet is moved away from the coil, the deflection is new in the opposite direction. As soon as the movement of the magnet ceases, the deflection becomes zero. Further, the deflection is smaller

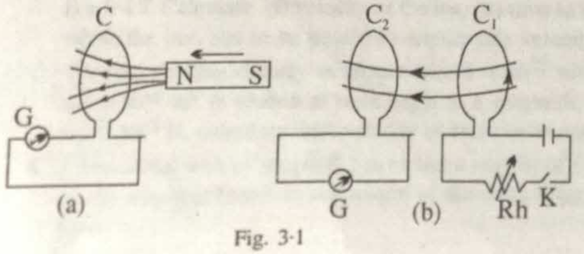


Fig. 3·1

if the magnet moves slowly compared to the case when the magnet moves faster. The conclusions form the experiment is that,

(i) An emf is included in the coil when there is relative motion between the magnet and the coil.

(ii) When the relation velocity between the magnet and the coil is large the induced emf is also large.

(iii) If relative velocity is zero induced emf is also zero.

- **Explanation :** When the bar magnet is taken towards the coil or taken away in both the cases the number of lines of force entering the coil changes and it is constant when the motion ceases. Further rate of change of lines of force is less when the magnet moves slowly. The rate of change is more when the motion of the magnet is faster. So, an emf is induced only when the lines of force or flux linking a coil changes.

▲ (B) Coil-coil experiment :

The experimental arrangement is shown in fig 3.1 (b). The coils c_1 and c_2 are placed side by side co-axially. The coil c_1 is connected to a battery through a rheostat and the coil c_2 is connected to a galvanometer. Coil c_1 in which source of emf is placed is called **primary** coil and c_2 is called **secondary** coil.

As soon as the key *k* is closed, the galvanometer gives a momentary deflection. Similarly when the key *k* is open, there is again a momentary deflection in G but now in the opposite direction.

The whole process can be understood on similar lines as in the previous coil-magnet experiment. When the primary circuit is switched on, the current in it increases from zero to a certain steady value. Similarly when the primary coil is switched off, the current decreases from the steady value to zero. In both the cases, current in the primary coil changes and so the magnetic lines of force entering the secondary change A momentary deflection of the galvanometer is observed implying that there is a current in the galvanometer and the secondary.

From this experiment also observation is : An emf is induced in a coil only when the magnetic lines of force linking the coil changes. If the lines of force is steady, no emf is induced.

Before discussing laws of electromagnetic induction, we shall define magnetic flux.

✪ 3.3. Magnetic flux and flux density ✪

The magnetic flux φ through any surface placed in a magnetic field $\vec{B}$ is measured by the number of lines of induction that passes normally through the surface. In fig 3.2, S is a surface placed in a magnetic field. ΔS is an infinitisimal element of area on this surface.

Such an element is treated as a vector $\vec{\Delta S}$ whose magnitude is ΔS and whose direction is outward drawn normal to the surface S. Then the magnetic flux Δφ passing through ΔS is the product of the magnitude of magnetic field B at a point on ΔS and the component of ΔS in the direction of the magnetic field. Hence

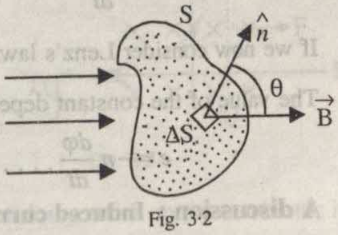


Fig. 3·2

$$\Delta\varphi = B\ \Delta S\cos\theta = \vec{B}\cdot\vec{\Delta S}$$

$\vec{B}$ is also called **magnetic flux density** vector. Evidently Δφ is not a vector.

∴ The total flux passing through S is

$$\varphi = \int_s \vec{B}\cdot\vec{\Delta S} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.1)$$

- **Special case :** (i) When the surface is a plane of area A and magnetic field is constant throughout the surface, then

$$\varphi = (\vec{B} \cdot \hat{n})\, A \quad [\hat{n} \text{ is a unit vector normal to the surface}]$$

(ii) If the magnetic field touches the surface tangentially, $\theta = 0°$ and $\varphi = 0$

- **Unit of φ :** The unit of magnetic flux in SI system is **weber** (Wb) and in cgs system the unit is Max well. 1 Wb is the amount of flux produced when a magnetic field of 1 T acts on a plane of area $1m^2$ held perpendicular to the direction of the magnetic field.

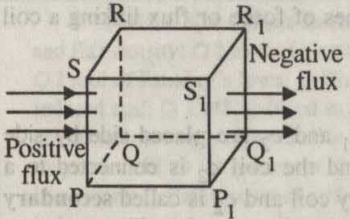


Fig. 3·3

- **Relation between wb and Maxwell :** $1\text{ Wb} = 1\text{T} \times 1m^2$

$\therefore \quad 1\text{ Wb} = 10^4\text{ G} \times 10^4\text{ cm}^2 = 10^8$ Maxwell.

- **A note :** The flux entering into the surface is treated as positive flux and the flux leaving the surface is negative. Because in the first case θ is acute and in the second case is obtuse. It is shown in fig 3.3. Flux through the surface PQRS is positive flux and the flux through $P_1Q_1\,R_1S_1$ is negative flux.

3.4. Laws of electromagnetic induction

On the basis of various experiments on electromagnetic induction, three laws have been put forward. First two laws were given by Faraday and are known as Faraday's law. The third law is due to Lenz and are known as Lenz's law. From Faraday's laws give the condition for electromagnetic induction and the magnitude of induced emf, while Lenz's law gives the direction of induced current.

Faraday's laws :

(i) Whenever there is a change in the magnetic flux linked with a coil an emf is set up in it and lasts as long as the magnetic flux linked with it is changing.

(ii) The magnitude of the induced emf is proportional to the rate of change of magnetic flux linked with the coil.

- **Mathematical form :** Let at any time flux passing through a coil is φ and its number of turns is *n*

So, the flux linking the coil = $n\varphi$

Then, according Faraday's second law emf *e* induced in the coil is given by

$$e \propto \frac{d}{dt}(n\varphi) \qquad \text{or,} \quad e \propto n\frac{d\varphi}{dt}.$$

If we now consider Lenz's law, $e \propto -n\,\dfrac{d\varphi}{dt} \quad \therefore \quad e = -k\,n\dfrac{d\varphi}{dt}$ [k = proportionality constant]

The value of the constant depends on the system of units. In SI units $k = 1$ and one can write

$$e = -n\frac{d\varphi}{dt} \quad \text{.......} \quad (3.2)$$

- **A discussion : Induced current and induced charge :** It is known that if *n* be the number of turns and $\dfrac{d\varphi}{dt}$ be the rate of change of flux passing through the coil, the induced emf in the coil is $e = -n\dfrac{d\varphi}{dt}$.

Now if the circuit is closed and total resistance of the circuit is R, then induced current is

$$i = \frac{|e|}{R} = \frac{n}{R}\,\frac{d\varphi}{dt}.$$

So, induced current in a coil depends on resistance of the coil, but the induced emf does not depend on resistance.

Now, $i\,dt = \frac{n}{R}\,d\varphi$

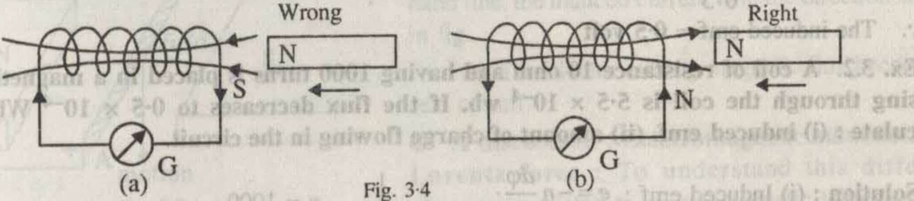
$$\therefore \int_0^t i\,dt = \frac{n}{R}\int_{\varphi_1}^{\varphi_2} d\varphi \quad \therefore \quad q = \frac{n}{R}(\varphi_2-\varphi_1) \qquad \ldots\ldots\ldots (3.3)$$

$$\therefore \text{ charge flowing in the circuit} = \frac{\text{no. of turns} \times \text{change of flux}}{\text{resistance of the circuit}}$$

Magnitude of the induced charge does not depend on time of change of flux.

❂ 3.5. Lenz's law and principle of conservation of energy ❂

Lenz's law complies with the principle of conservation of energy. Imagine for a moment that as the N-pole of a bar magnet is pushed towards a solenoid closed through a galvanometer, the induced current flows in clockwise direction so as to make the front face of the solenoid a S-pole [Fig 3.4(a)]. Then due to attraction between unlike poles, the magnet will be automatically accelerated towards the solenoid with no expenditure of energy. It means that it is possible to build current in the solenoid without any expenditure of energy. It is against the principle of conservation of energy. Hence, as the N-pole is brought close to the θ solenoid, the induced current should flow in the anti-clockwise direction [Fig 3.4(b)]



(a) (b)

Fig. 3·4

❂ 3.6. Proof of Faraday's laws ❂

Faraday's law of electromagnetic induction can be established from the principle of conservation of energy. In fig 3.5, the conductor PQ. can slide over the two long, straight and parallel conducting rails AC and DE. The rails are smooth and the conductor can more perpendicular to the rails. A current is passed through the rails and the conductor by a battery. A magnetic field of flux density B is applied perpendicular to the current as shown in fig. Now the magnetic force on the conductor PQ is

F = B*il* where *l* = length of the conductor PQ. Due to the force the conductor starts moving perpendicular to the rails.

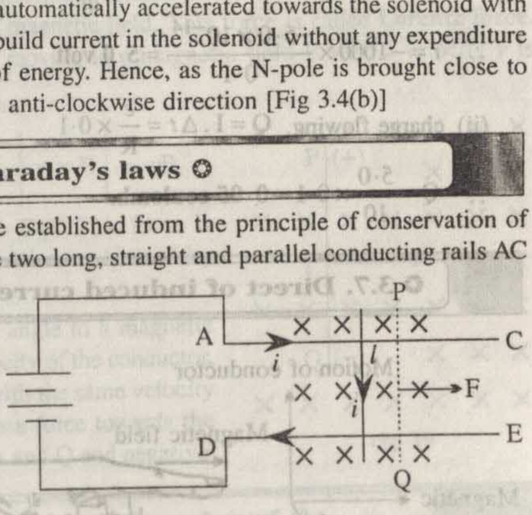


Fig. 3·5

Let at any instant, linear velocity of the conductor is *v*, then rate of doing work on the conductor is,

$$Fv = Bil\,v$$

Power produced by the battery = E*i*

Next, if R be the resistance of the circuit, rate of heat produced in the circuit = i^2R. Hence by the principle of conservation of energy

$$Ei = i^2R + Bil\,v \qquad \therefore \quad i = \frac{E - Blv}{R} = \frac{E+e}{R} \qquad \ldots\ldots\ldots (3.4)$$

Here $e = -Blv$ is the emf induced in the conductor due to its motion in the magnetic field.

$$\therefore \quad e = -Blv = -Bl\frac{dx}{dt} = -B\frac{dA}{dt}$$

$$\therefore \quad e = -\frac{d\varphi}{dt}$$: It is Faraday's laws of electromagnetic induction.

● **Example 3.1. The measurement of a rectangular coil is 0·1 m × 0·05 m and its number of turns is 100. It is placed in a magnetic field with its plane perpendicular to the magnetic field. The strength of the field is changed from 0·1T to 0·05T. Calculate the magnitude of the induced emf in the coil.**

○ **Solution :** The induced emf, $e = -n\frac{d\varphi}{dt} = -nA\frac{dB}{dt}$

Here, $n = 100$, $A = 0{\cdot}1 \times 0{\cdot}05 = 5 \times 10^{-3}\ m^2$

$dB = 0{\cdot}05 - 0{\cdot}1 = -0{\cdot}05$ T and $dt = 0{\cdot}05$ sec.

$$\therefore \quad e = \frac{100 \times 5 \times 10^{-3} \times 0{\cdot}05}{0{\cdot}5} = \mathbf{0{\cdot}5\ volt}$$

$\therefore$ The induced emf = **0·5 volt**

● **Ex. 3.2. A coil of resistance 10 ohm and having 1000 turns is placed in a magnetic field. Flux passing through the coil is 5·5 × 10⁻⁴ wb. If the flux decreases to 0·5 × 10⁻⁴ Wb in 0·1 sec. Calculate : (i) induced emf. (ii) amount of charge flowing in the circuit.**

○ **Solution :** (i) Induced emf : $e = -n\frac{d\varphi}{dt}$.

$$\therefore \quad i = -1000 \times \frac{-5{\cdot}0 \times 10^{-14}}{0{\cdot}1} = \mathbf{5{\cdot}0\ volt}$$

(ii) charge flowing, $Q = I\,.\,\Delta t = \frac{e}{R} \times 0{\cdot}1$

$$\therefore \quad Q = \frac{5{\cdot}0}{10} \times 0{\cdot}1 = \mathbf{0{\cdot}05\ coulomb.}$$

$n = 1000$

$d\varphi = \varphi_2 - \varphi_1$

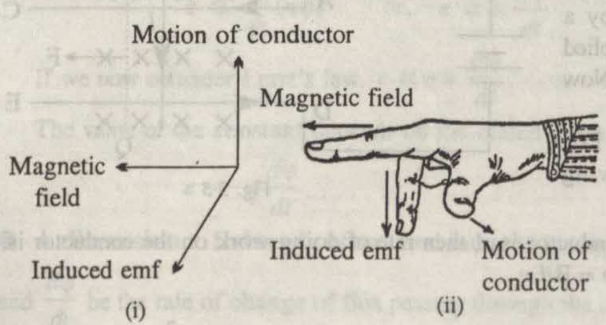
$= -5{\cdot}0 \times 10^{-4}$ wb

$dt = 0{\cdot}1$ sec.

$R = 10$ ohm

3.7. Direct of induced current : Fleming's right hand rule

The direction of the induced current in a conductor **moving** in a magnetic field is obtained from the **Fleming's right hand rule.** According to this rule : stretch the thumb, fore finger and middle finger of the right hand nutually perpendicular each other. If the fore-finger points in the direction of the magnetic field and the thumb in the direction of the motion of the conductor, then the middle finger will point in the direction of the induced current.



(i) (ii)

Fig. 3·6

The direction of the induced current in the dynamo or generator can be obtained from his right hand rule. For this reason Fleming's right hand rule is also called **dynamo rule.**

➤ 3.7.1. *Cause of induced emf in a conductor* :

The conductor PQ is placed at right angle to a uniform magnetic field. The field is perpendicular to the plane of the paper and directed inwards. The conductor PQ is in the plane of the paper [Fig 3.7]. Now it is moved towards right with a constant velocity *v*. Now, the electrons in the conductor will also more toward right. It can be assumed that the current is flowing towards left. So, according to Fleming's left hand rule, the free electrons in the conductor will more towards the end Q of the conductor. Because of excess electrons, the end Q becomes negative and the other hand P becomes positive due to deficit of electrons. Thus an emf is generated in the conductor PQ.

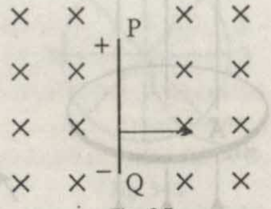


Fig. 3·7

➤ 3.7.2. *Motion of a linear conductor in a magnetic field* :

In fig 3.8, the linear conductor AB is moving downwards perpendicular to the magnetic field. The conductor cuts the magnetic field. So an emf is induced in the conductor. According to Fleming's right hand rule, the induced current is in the direction shown in fig.

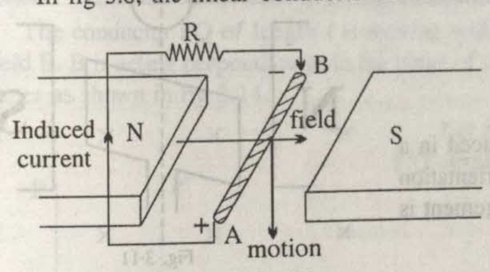


Fig. 3·8

So, a conductor cutting magnetic flux becomes a seat of emf.

- **A discussion : Electromagnetic induction and Lorentz force** : To understand this different phenomena related to electromagnetic induction, we require to consider the force on a moving charge in a magnetic field. This force is called **Lorentz force**. The force experienced by a particle having charge *q* moving in a magnetic field B with a velocity *v* is

$\vec{F} = q(\vec{v} \times \vec{B})$

If $\vec{F}$, $\vec{v}$ and $\vec{B}$ are mutually perpendicular then $\left|\vec{v} \times \vec{B}\right| = v\mathrm{B}$

∴ Effective magnetic force of the particle, $v = \mathrm{B}qv$. This force is perpendicular to the plane containing *v* and B. Its direction is obtained by Fleming's left hand rule.

The conductor PQ of length *l* is moving at right angle to a magnetic field of magnetic induction B [Fig 3.9]. If *v* be the velocity of the conductor, then an electron inside the conductor is also moving with the same velocity *v*. The electrons inside the conductor will experience a force towards the end Q. As a result, the end P develope positive charge and Q end negative.

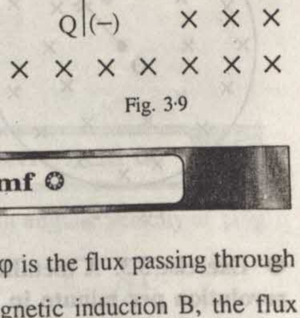


Fig. 3·9

✪ 3.8. Methods of producing induced emf ✪

The induced emf in a coil is given by the equation : $e = -n\dfrac{d\varphi}{dt}$ · where φ is the flux passing through the coil. If the plane of the coil be perpendicular to the flux lines of magnetic induction B, the flux passing through the coil is $\varphi = \mathrm{BA}$, A being the area of the coil [Fig 3.10 (i)]

However if the field lines are inclined at an angle θ with the axis of the coil, we resolve. B into two components B cos θ and B sin θ. No flux lines pass through the coil due to B sin θ component. Hence flux passing through the coil is $\varphi = \mathrm{BA}\cos\theta$. [Fig 3.10 (ii)]

Now, the expression for induced emf in the coil is $e = -n\dfrac{d}{dt}(\mathrm{BA}\cos\theta)$

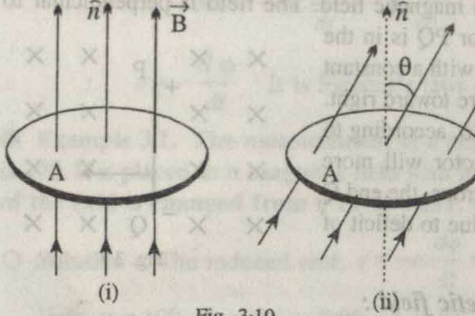


Fig. 3·10

Hence an emf will be induced if there is a change in the value of (BA cos θ). Evidently emf can be induced in a circuit (a coil or a conductor) by either changing B or by changing A or by changing θ alone.

- **(a) By changing B alone :** If the magnetic flux density B in a closed loop of conductor changes, then the flux linked with the loop will change thereby producing an induced emf in it.

$$\therefore \quad e = -n\frac{d}{dt}(\mathrm{BA}\cos\theta) = -n\mathrm{A}\cos\theta\,\frac{d\mathrm{B}}{dt}.$$

- **(b) By changing A alone :** The induced emf is

$$e = -n\,\mathrm{B}\cos\theta\cdot\frac{d\mathrm{A}}{dt}.$$

- **(c) By changing θ alone :** The emf can be induced in a coil placed in a constant magnetic field by changing the orientation of the coil with respect to the field direction. The arrangement is shown in fig 3.11.

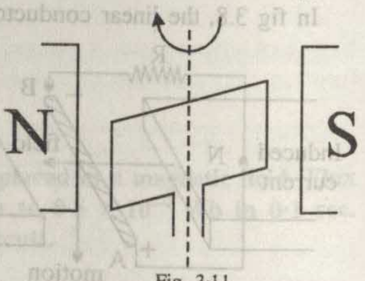


Fig. 3·11

3.9. EMF induced in a conductor rotating perpendicular to a magnetic field

The conductor OA of length l is rotating perpendicular to a magnetic field of flux density B about the point O with a constant angular velocity ω [Fig 3.12]. We require to calculate induced emf at the two ends of the conductor.

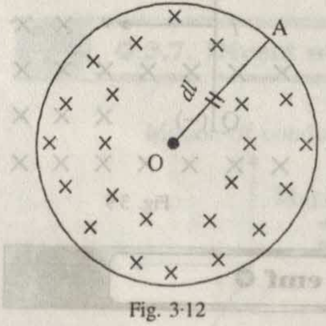


Fig. 3·12

Consider a small element dl of the conductor. If its linear velocity in the field be v, then the induced emf in the element is

$$d\mathrm{E} = \mathrm{B}v\,dl$$

Then the total emf induced in the conductor OA is

$$\mathrm{E} = \int_0^l \mathrm{B}\,v\,dl = \int_0^l \mathrm{B}\,l\omega\,dl = \frac{1}{2}\mathrm{B}\,\omega\,l^2 \quad [\because\ v = \omega l]$$

∴ EMF induced across a conductor rotating perpendicular to a magnetic field is

$$\mathbf{E} = \frac{1}{2}\mathbf{B}\,\omega\,l^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.5)$$

- **Exercise 3.3. A metallic wheel with 8 metallic spokes each 0·5 m long is rotating at speed of 120 revolution per minute in a plane perpendicular to a magnetic field of strength 0·4 G. Find the magnitude of the induced emf between the axle and rim of the wheel.**

○ **Solution :** The situation is shown in fig 3.13. Each spoke of length 0·5 m is rotating at right angle to the magnetic field $\mathrm{B} = 0{\cdot}4 \times 10^{-4}\mathrm{T}$ with a constant angular velocity $\omega = 2\pi \times \dfrac{120}{60} = 4\pi$ rad/s

So, the emf induced across spoke or the emf between the rim and axle of the wheel is

$$e = \frac{1}{2} \mathrm{B} l^2 \omega$$

$$\mathrm{B} = 0{\cdot}4 \times 10^{-4}\ \mathrm{T}$$

$$l = 0{\cdot}5\ m$$

$$\omega = 2\pi n = 2\pi \times 2 = 4\pi\ \mathrm{rad/s}$$

$$\therefore \quad e = \frac{1}{2} \times 0{\cdot}4 \times 10^{-4} \times (0{\cdot}5)^2 \times (4\pi)^2$$

$$= \mathbf{6 \cdot 28 \times 10^{-5}}\ \text{volt}$$

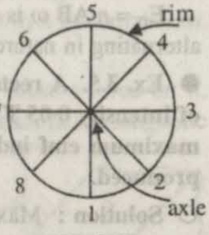


Fig. 3·13

3.10. Induced emf in a conductor moving perpendicular to a magnetic field : motional emf

The conductor PQ of length l is moving with constant linear velocity v at right angle to a magnetic field B. B is acting perpendicular to the plane of the paper and the conductor is moving in the plane of the paper as shown in fig 3.14.

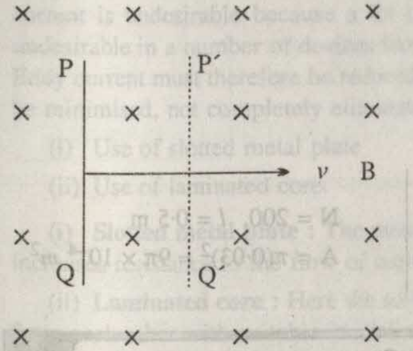


Fig. 3·14

Let in a small time dt the conductor moves to the position P′Q′ displacement of the conductor, $\mathrm{PP'} = v\,dt$

∴ change in area in time dt,

$$d\mathrm{A} = \mathrm{PP'\,Q'\,Q} = \mathrm{PP'} \times \mathrm{PQ} = v\,dt\,.\,l = v\,l\,dt$$

$$\therefore \quad \text{rate of change of area, } \frac{d\mathrm{A}}{dt} = v\,l$$

∴ Magnitude of the induced emf across the conductor,

$$e = n\frac{d\varphi}{dt} = n\,\mathrm{B}\frac{d\mathrm{A}}{dt}$$

$$\therefore \quad e = 1.\mathrm{B}\,.\,vl = \mathrm{B}lv$$

$$\therefore \quad e = \mathbf{B}\boldsymbol{lv} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.6)$$

This induced emf is called **motional emf.**

● **Exercise 3.4. A train is running in the north-south direction with a velocity of 108 km/hr. What emf will be induced across the axle of a pair of wheel due to its motion perpendicular to the vertical component of earth's magnetism ? Given, length of the axle 2m and vertical component = 0·8 G.**

○ **Solution :** The induced emf is $e = \mathrm{B}_v\,.\,l\,v$

Here $\mathrm{B}_v = 0{\cdot}8 \times 10^{-4}\,\mathrm{T}$, $l = 2m$, $v = 30\ \mathrm{ms}^{-1}$

$$\therefore \quad e = 0{\cdot}8 \times 10^{-4} \times 2 \times 30 = 4{\cdot}8 \times 10^{-3} v = \mathbf{4{\cdot}8\ mV}$$

3.11. EMF induced in a coil rotating in a magnetic field

A rectangular coil 'C' of area A is rotating in a magnetic field B with constant angular velocity ω [Fig 3.15]. At time $t = 0$, the axis of the coil was parallel to the direction of B. In time t the coil is rotated by $\theta = \omega t$.

∴ Magnetic flux passing through the coil at this instant is

$$\varphi = \mathrm{BA}\cos\theta = \mathrm{BA}\cos\omega t.$$

∴ Instantaneous emf induced in the conductor is

$$e = -n\frac{d\varphi}{dt} = -n\frac{d}{dt}(\mathrm{B\,A}\cos\omega t) = n\mathrm{AB}\omega\sin\omega t$$

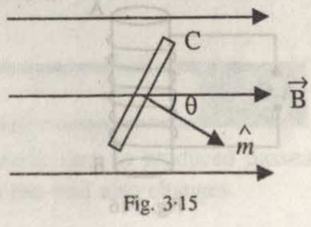


Fig. 3·15

$\therefore\ e = \mathrm{E_0} \sin \omega t$ .... (3.7)

$\mathrm{E_0} = n\,\mathrm{AB}\,\omega$ is called peak value of the induced emf. Evidently the emf. induced in the conductor is alternating in nature.

● **Ex. 3.5. A rectangular coil of area 0·01 m^2 and having 100 turns is rotating in a magnetic field of intensity 0·05 T with its axis perpendicular to the field. If the coil rotates at 50 rps, calculate the maximum emf induced in the coil. If the resistance of the coil be 30 ohm, find maximum power produced.**

○ **Solution :** Maximum value of induced emf in the rotating coil is

$$e_{\max} = n\,\mathrm{AB}\,\omega$$

$$\therefore\ e_{\max} = 100 \times 0{\cdot}01 \times 0{\cdot}05 \times 2\pi \times 50$$

$$= \mathbf{15{\cdot}7\ volt}$$

$n = 100,\ \mathrm{A} = 0{\cdot}01\ m^2$
$\mathrm{B} = 0{\cdot}05\ \mathrm{T}$
$\omega = 2\pi \times 50$ rad/s

$$\therefore\ \text{Maximum power, } \mathrm{P} = \frac{\mathrm{V}^2}{\mathrm{R}} = \frac{(15{\cdot}7)^2}{30} = \mathbf{8{\cdot}23\ Watt}$$

● **Ex. 3.6. A current of 5A is passed through a solenoid of length 50 cm, radius 3·0 cm and having 200 turns. When the switch is open, the current becomes zero within 10^{-3} sec. Calculate the emf induced across the switch. [$\mu_0 = 4\pi \times 10^{-7}$ *Wb*/A.m]**

○ **Solution :** Induced emf, $e = \mathrm{N}\dfrac{d\varphi}{dt}$, $\varphi = \mathrm{BA} = \mu_0 ni\mathrm{A}$

$$\therefore\ e = \mathrm{N}\frac{d}{dt}(\mu_0 ni\mathrm{A}) = \mu_0\,\mathrm{N}\,n\,\mathrm{A}\frac{di}{dt} = \mu_0\,\frac{\mathrm{N}^2}{l}\cdot\mathrm{A}\frac{di}{dt}\cdot$$

$$\therefore\ e = 4\pi \times 10^{-7} \times \frac{(200)^2}{0{\cdot}5} \times 9\pi \times 10^{-4} \times \frac{5}{10^{-3}}$$

$$= 4\pi \times 4 \times 18\pi \times 5 \times 10^{-4} = \mathbf{1{\cdot}42\ volt.}$$

$\mathrm{N} = 200,\ l = 0{\cdot}5$ m
$\mathrm{A} = \pi(0{\cdot}03)^2 = 9\pi \times 10^{-4}\ m^2$

✪ 3.12. Eddy current ✪

So for we have seen that currents are induced in a circuit made of conducting wires placed in a changing magnetic field. It was first observed by **Foucault** that when a solid conducting mass moves in a non-uniform magnetic field, induced currents are set up in the solid mass in the form of eddies. Such currents are known as **eddy currents**. So,

Eddy currents are the currents produced in solid metal mass when the flux threading through them changes.

Direction of the eddy currents can be found by applying Lenz's law or Fleming's right hand rule. Eddy currents are always produced in a plane perpendicular to the direction of magnetic field. Eddy current shows both heating and magnetic effects.

● **Demonstration :** The presence of eddy currents can be demonstrated by the followings experiments :

(i) Take an electromagnet fed by an a.c. source and place a light metal disc on its top face as shown in fig 3.16.

As the switch is closed to drive current through the coil of the electromagnet, the disc will be found to be thrown off.

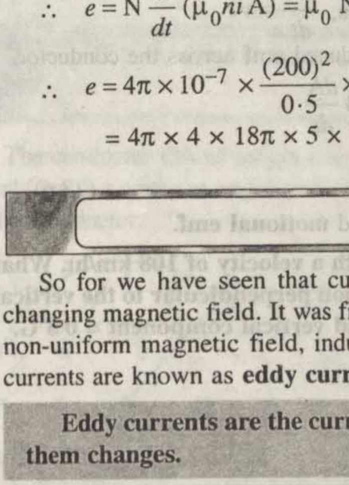


Fig. 3·16

- **Explanation :** As the current is switched on, the magnetic field at the disc rises from zero to finite value. Eddy currents are induced in the disc which is effectively converted into a small magnet. If the end A of the electromagnet starts acquiring N-polarity, then the disc will have *n*-pole next to it by Lenz's law. It results in a repulsive force. The light metallic disc is thus seen to be thrown off.

(ii) When a conductor moves in a magnetic field, eddy currents produced in it cause a force on it, which, by Lenz's law opposes the motion. Eddy currents damp the oscillating solid mass in the magnetic field.

Fig 3.17 shows a pendulum made of copper block set into oscillation in both the poles of a strong horse-shoe magnet. It will be found that on switching the current in the electromagnet, the motion of the pendulum will at once come to rest. Here eddy currents produced in the copper block oppose its motion in the magnetic field. In such a case, the solid body behaves as though it is embedded in a very viscous medium.

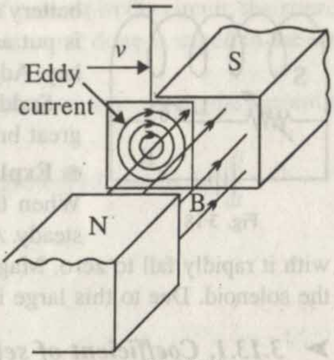


Fig. 3·17

- **Prevention of Eddy currents :** When a metal plate oscillates in a magnetic field or when the magnetic flux passing through a solid conducting mass changes with time, eddy currents are produced. The eddy currents heat up the metal plate or core. The heating due to eddy current is undesirable because a lot of useful energy is lost. This is undesirable in a number of devices like transformers, dynamo etc., where the coil is wound on iron core. Eddy current must therefore be reduced by some practical method. It is to be noted that these currents can be minimised, not completely eliminated. There are two methods which can weaken eddy currents :

(i) Use of slotted metal plate

(ii) Use of laminated core.

(i) **Slotted metal plate :** The metal plate is cut so as to increase the path for the eddy currents. This increases resistance to the flow of currents.

(ii) **Laminated core :** Here the solid metal mass is divided into thin sheets (called laminae) insulated from each other with suitable insulating materials. The planes of these laminations are arranged parallel to the magnetic flux. As a result the electrical resistance between the surfaces of laminations becomes very high. The currents are now confined to the individual laminae instead of the whole volume of solid. Thus eddy currents are reduced considerably, although the induced emf is unaltered. It implies low heating of the metal, which reduces the wastage of power. All A.C. machines like transformers and dynamos have laminated cores.

- **Application of eddy currents :** Eddy current are not always undesirable. Some applications of eddy currents are :

(i) Eddy current produce heat in a metal. Metals can be melted by placing them in a rapidly changing or high frequency magnetic field which produces large eddy current in the substance. Induction furnace works on the principle of eddy currents.

(ii) To stop electric trains, eddy current brakes are used. The principle of operation of these brakes is that when a magnetic field is suddenly applied to a rotating metal disc, eddy currents are set up in the disc which in turn exert a torque on the disc so as to stop it.

3.13. Self-Induction

A coil carrying current produces a magnetic field around it. The magnetic flux so produced passes through the coil itself. If the current strength changes, the flux linked with the coil also changes.

An induced emf is produced in the coil. It is the self induced emf. The phenomenon is known as **self-induction.** Hence,

Self induction is a phenomenon in which emf is induced in a coil due to its own current variation.

- **Demonstration :** To demonstrate the phenomenon of self-induction, we take a long solenoid. A battery and a key are connected to it. A series combination of a lamp and a rheostat is put across the solenoid. Now, a current is sent from the battery by closing the key. Adjust the rheostat so that the bulls just glows.

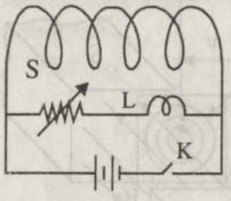


Fig. 3·18

Suddenly the switch 'K' is open. It is found that the lamp flashes out with great brilliance at the break.

- **Explanation :** The observation can be explained in terms of the induced emf. When the current was steady, the magnetic flux through the solenoid was also steady. As soon as the circuit is broken, the current and the magnetic field associated with it rapidly fall to zero. Magnetic flux linking the coil fall to zero rapidly. So large emf is induced in the solenoid. Due to this large induced emf, the lamp glows brilliantly at the break of the circuit.

➤ *3.13.1. Coefficient of self-induction*

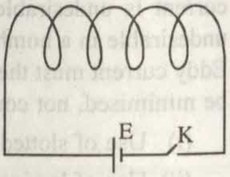


Fig. 3·19

Let after closing the key K at $t = 0$ of the circuit in fig 3.19, current through the coil at any instant be i. The magneting flux φ linked with the coil at any instant is proportional to the current in it at that instant.

Hence

$\varphi \propto i$ [φ = flux × number of turns]

$$\therefore \quad \varphi = Li \quad \ldots\ldots\ldots\ldots \quad (3.8)$$

The proportionality constant L is called the **coefficient of self induction** or simply **self inductance** of the coil. Its value depends on number of turns, area of cross-section of the coil and permeability of the material of the core on which the coil is wound.

If the current in the coil changes with time, the flux linked with the coil changes. The emf induced in the coil due to its own current variation is

$$e = -\frac{d\varphi}{dt} = -\frac{d}{dt}(Li) = -L\frac{di}{dt} \quad \ldots\ldots\ldots\ldots \quad (3.9)$$

So, the induced emf depends on self-inductance and on rate of current of its own current :

- **Definitions of L :** (i) From the equation $L = \frac{\varphi}{i}$; if i = 1, L = φ Thus **the self inductance of a coil is numerically equal to the magnetic flux linked with the coil due to unit current in it.**

Also from the equation (3.9) $L = \frac{|e|}{di/dt}$ $\quad \therefore$ If $\frac{di}{dt} = 1, \quad L = |e|$

So, the self inductance of a coil is numerically equal to the induced emf produced in the coil due to unit rate of change of current in the coil.

It is to be noted self inductance is a property of coil and is measured in terms of magnetic flux or induced emf.

- **Unit of self inductance, L :** The SI unit of self-inductance is **Henry** (H). In equation 3.8, if $i = 1$ Amp and $\varphi = 1$ Wb, then

$$L = \frac{1\,\text{Wb}}{1\text{A}} = 1\,\text{H}$$

Hence, the self-inductance of a coil is 1 H if a current of 1 A flowing in the coil produces magnetic flux of 1 weber.

3.14. Energy associated with an inductor

When the current flowing through a coil changes, an emf is induced in the coil due to its self inductance. This induced emf opposes applied emf. Hence to maintain the growth of current in the circuit, the source of current has to perform some work to overcome the opposing emf. This work done is stored in the coil in the form magnetic energy.

Again, when the current tends to fall, the stored magnetic energy is released to maintain the current in the coil.

Let during change of current, at any instant current is i and rate of growth of current is $\frac{di}{dt}$.

$\therefore$ The magnitude of the induced emf, $e = \mathrm{L}\frac{di}{dt}$.

$\therefore$ Rate of doing work, $= ei = \mathrm{L}\,i\frac{di}{dt}$.

$\therefore$ Total work done to grow current upto i_0 is $\mathrm{W} = \int_0^{i_0} e\,i\,dt$

$$\therefore \quad \mathrm{W} = \int_0^{i_0} \mathrm{L}\frac{di}{dt}\,i\,dt = \mathrm{L}\int_0^{i_0} i\,di = \frac{1}{2}\mathrm{L}\,i_0^{\,2}$$

Hence energy associated with the current carrying coil is

$$\mathbf{E} = \frac{1}{2}\mathbf{L}i_0^{\,2} \ \textbf{joule} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.10)$$

3.15. Effect of inductance in a circuit

The property of self-inductance of a coil always opposes any change of current in the coil. It opposes growth as well as decay of current in the coil. An ideal inductor has no influence on the steady current as it has no resistance. Fig 3.20 shows the growth and decay of current in an inductive circuit.

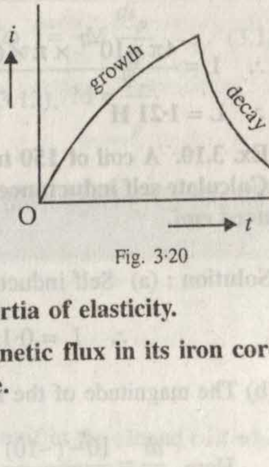


Fig. 3·20

The effect of inductance in a circuit is similar to the effect of mass in a mechanical system. When a source of emf tends to send current in an electric circuit containing an inductor, behaves the same way as the mass of a body does, when a force tends to cause a change in its state. Larger the mass of the body, more is the opposition it offers to the change in the state of the body. So, mass is taken as the measure of inertia. Similarly, higher the self inductance of a coil, more is the opposition it offers to the change in the current through the coil. Hence, self inductance is called **inertia of elasticity.**

● **Exercise 3.7. When the current in a solenoid is 2A, then the magnetic flux in its iron core is 3×10^{-4} Wb. If its number of turns is 400, calculate its self-inductance.**

○ **Solution :** Flux linkage, $\varphi = 400 \times 3 \times 10^{-4} = 12 \times 10^{-2}$ Wb – turn.

$\therefore$ Self inductance, $\mathrm{L} = \frac{\varphi}{i} = \frac{12 \times 10^{-2}}{2} = 6 \times 10^{-2} = \mathbf{0 \cdot 06\,H}$

● **Ex. 3.8. When the current in a coil grows from 0 to 2A in a time interval of 0·05 sec, 80V emf is induced in the coil. Calculate its self-inductance.**

○ **Solution :** Magnitude of induced emf due to self induction, $e = L\dfrac{di}{dt}$

$$\therefore \quad L = \frac{e}{di/dt} = \frac{80}{40} = 2\text{H} \quad \left[\frac{di}{dt} = \frac{2}{0\cdot 05} = 40\ \text{As}^{-1}\right]$$

∴ Self inductance of the coil is **2H**

✪ 3.16. Calculation of self-inductance of a long solenoid ✪

We consider a long and closely wound solenoid of radius r and length l ($l >> r$) as shown in fig 3.21. Its total number of turns N. Then the magnetic induction at any point inside the solenoid is

$$B = \mu_0\, ni = \mu_0\, \frac{Ni}{l} \quad \left[\, n = \frac{N}{l} = \text{number of turns per unit length}\right]$$

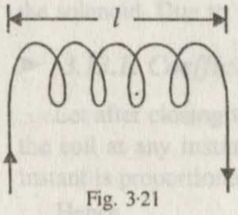


Fig. 3·21

The magnetic flux linkage with each turn is

$$\varphi_1 = \pi r^2 B = \frac{\mu_0 \pi r^2 N i}{l}.$$

∴ Total flux linkage with N turns of the solenoid

$$\varphi = N\varphi_1 = \frac{\mu_0 \pi r^2 N^2 i}{l}.$$

∴ Self inductance of the coil,

$$L = \frac{\varphi}{i} = \mu_0\, \frac{N^2 \pi r^2}{l} \quad \ldots\ldots\ldots\ldots\ldots (3.11)$$

● **Exercise 3.9. Calculate the self-inductance of an air-core solenoid 1m long, diameter 0·05 m and having 700 turns.**

○ **Solution :** Self-inductance, $L = \dfrac{\mu_0 \pi r^2 N^2}{l}$

$$\therefore \quad L = \frac{4\pi \times 10^{-7} \times \pi \times (0\cdot 025)^2 \times (700)^2}{1}\ \text{Henry}$$

∴ **L = 1·21 H**

$\mu_0 = 4\pi \times 10^{-7}\ \text{Hm}^{-1}$
$r = 0\cdot 025$ m
N = 700
$l = 1$ m

● **Ex. 3.10. A coil of 150 turns is linked with a flux of 0·01 wb when it carries a current of 10 A. (a) Calculate self inductance of the coil (b) If the current is uniformly reversed in 0·01 sec, find the induced emf.**

○ **Solution :** (a) Self inductance, $L = \dfrac{\text{flux linkage}}{\text{current}} = \dfrac{150 \times 0\cdot 01}{10} = \mathbf{0\cdot 15\ H.}$

$$\therefore \quad L = 0\cdot 15\ \text{H}$$

(b) The magnitude of the induced emf, $e = L\dfrac{di}{dt}$

$$\text{Here, } \frac{di}{dt} = \frac{10-(-10)}{0\cdot 01} = 2000\ \text{As}^{-1}$$

$$\therefore \quad e = 0\cdot 15 \times 2000 = \mathbf{300\ V}$$

❂ 3.17. Mutual Induction ❂

The phenomenon of inducing an of emf in a coil due to change of current or magnetic flux linked with a neighbouring coil is called mutual induction.

Let a pair of coil 1 and 2 be arranged such that they are co-axial and close to each other [Fig 3.22]. The coil-1 is closed through a battery and key. The coil - 2 is closed through a galvanometer. Magnetic lines of force due to the current in coil - 1 pass through the other coil (coil-2). So an emf will be set up in accordance with the laws of electromagnetic induction. The induced emf in the second coil lasts only during the change of current in the other coil. For this reason, as soon as the key K in coil - 1 is closed, the galvanometer in coil 2 gives a momentary deflection. Coil-1 is called primary coil or P-coil and coil 2 is secondary coil or S-coil.

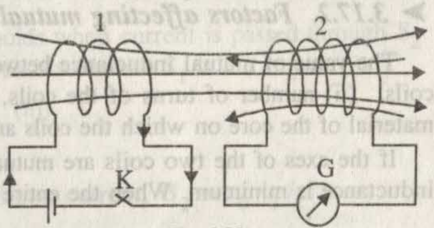


Fig. 3·22

❖ **Definition : The phenomenon of production of emf in one coil due to the change in current in the neighbouring coil and vice versa is called mutual induction.**

Several electrical devices like transformer, induction coil etc. are based on the phenomenon of mutual induction.

➤ 3.17.1. *Coefficient of mutual induction* :

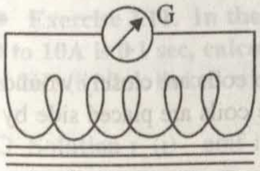


Fig. 3·22(a)

In fig 3.22(a), two coil P and S are placed close to each other. Let at any instant the current in P-coil is i_p, the magnetic flux φ_s linked with S-coil is found to be proportional to the current in P-coil. So,

$$\varphi_s \propto i_p \quad \text{or,} \quad \varphi_s = Mi_p \quad \ldots\ldots\ldots \quad (3.12)$$

The constant M is called the coefficient of mutual induction or simply mutual inductance between the two coils.

The induced emf in the S-coil is $e_s = -\frac{d}{dt}(\varphi_s) = -M\frac{di_p}{dt}$ .... (3.13)

❖ **Definitions of M :** (a) from the equation (3.12), $M = \frac{\varphi_s}{i_p}$.

So, if $i_p = 1$, $M = \varphi_s$.

Hence mutual inductance between two coil is numerically equal to the magnetic flux linking the second coil due to unit current flowing in the first coil.

(b) Again, from the equation (3.13), $M = \frac{|e_s|}{di_p/dt}$

So, if $\frac{di_p}{dt} = 1$, $M = |e_s|$

Mutual inductance between two coils is numerically equal to the induced emf in the second coil when the rate of change of current in first coil is unity.

● **Unit of M :** (a) SI unit of M is Henry (H)

From the equation (3.12), $1\,\text{H} = \frac{1\,\text{Wb}}{1\text{A}}$.

Hence, **mutual inductance of two coil is 1H when due to a current of 1A in one coil, 1 Weber of magnetic flux links the second coil :**

(b) Again, from the equation (3.13) $1\text{ H} = \dfrac{1\text{ volt}}{1\text{ Amp . sec}^{-1}}$

So, mutual inductance of two coils is 1H if a rate of change of current of 1 Ampere per sec in one coil induces an emf of 1 volt in the neighbouring coil.

➤ *3.17.2. Factors affecting mutual inductance between two coils :*

The value of mutual inductance between two coils depends on (i) geometrical shape and size of the coils, (ii) number of turns of the coils, (iii) separation between the coil, (iv) the permeability of the material of the core on which the coils are wound and (v) angular orientation between the coils.

If the axes of the two coils are mutually perpendicular as shown in fig 3.22(b) (i), then the mutual inductance is minimum. When the entire flux produced by the P-coil is linked with the secondary, then the mutual inductance is maximum. This can be increased μ_r times when the two coils are closely wound on an iron core of relative permeability μ_r as shown in fig 3.22(b)(ii). When the coils are placed side by side as in fig 3.22(b)(iii), M has intermediate value.

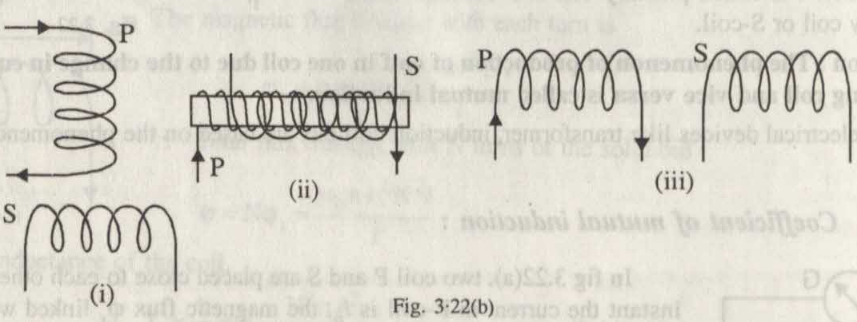


Fig. 3·22(b)

- **Coefficient of coupling :** The value of mutual inductance between two coils depends upon the manner in which the two coils are oriented relative to each other. The relative orientation of two coils is expressed by the quantity **coefficient of coupling** between them. It is a measure of the coupling between the coils. If L_1 and L_2 are the self inductances of the two coils and M is the mutual inductance of the two coils, then the Coefficient of coupling is given by

$$k = \frac{M}{\sqrt{L_1 L_2}} \quad \text{or,} \quad M = k\sqrt{L_1 L_2}$$

It is found that $k \leq 1$. $k = 1$ when all the flux produced by the P-coil links the S-coil.

✪ 3.18. Expression for mutual inductance between two long solenoids ✪

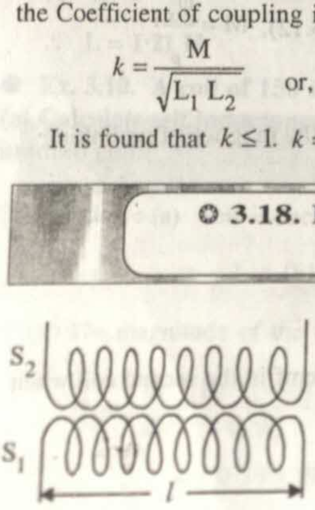


Fig. 3·23

S_1 and S_2 are two solenoids each of length l and one is situated inside the other [Fig 2.23]. Let a current i_1 flows in S_1. Then the magnetic flux linked with S_2 is

$$\varphi_{21} = M_{21} i_1 \cdots\cdots\cdots\cdots (i)$$

M_{21} is the mutual inductance of two solenoids when the flux links S_2 due to the current in S_1.

Magnetic field in side S_1, $B_1 = \mu_0 n_i i_1$ [n_1 = number of turns per unit length of S_1]

$\therefore$ flux linked with each turn of $S_2 = B_1 A$ [A = area of the coil]

$\therefore$ total flux linkage with S_2 ; $\varphi_{21} = B_1 A \times n_2 l$ [n_2 = number of turns per unit length of S_2]

$$\therefore \quad \varphi_{21} = \mu_0 n_1 n_2 A l\, i_1 \ldots\ldots\ldots\ldots \text{(ii)}$$

$\therefore$ From the relations (i) and (ii),

$$\mathbf{M_{21} = \mu_0 n_1 n_2 Al} \ldots\ldots\ldots\ldots\ldots\ldots \text{(3.14)}$$

Now, we calculate the mutual inductance between two solenoids when current is passed through S_2 and flux links S_1. If i_2 be the current in S_2, then the magnetic flux linked with S_1 is

$$\varphi_{12} = M_{12}\, i_2 \ldots\ldots\ldots\ldots \text{(iii)}$$

Now, M_{12} is the mutual flux between the two coils.

$\therefore$ Magnetic field produced inside the solenoid S_2 due to its own current, $B_2 = \mu_0 n_2 i_2$

$\therefore$ flux linked with each turn of $S_1 = B_2 A$

$\therefore$ total flux linkage with S_1, $\phi_{12} = B_2 A \times n_1 l$

$$\therefore \quad \varphi_{12} = \mu_0 n_1 n_2 A l\, i_2 \ldots\ldots\ldots\ldots \text{(iv)}$$

$\therefore$ From the relations (iii) and (iv), $M_{12} = \mu_0 n_1 n_2 Al \ldots\ldots$ (3.15)

So, in case of two solenoids mutual inductance remains same as long as no flux is wasted. Henry

$$M_{21} = M_{12} = M = \mu_0 n_1 n_2 Al$$

● **Exercise 3.11. In the case of mutual induction, if the current in the primary coil increases from 0 to 10A is 0·1 sec, calculate (i) emf induced in the second coil it the coefficient of mutual induction is 2H, (ii) flux linked with each turn if the number of turns in secondary is 500.**

○ **Solution :** (i) emf induced in secondary, $e_s = M\dfrac{di_p}{dt}$.

$$\therefore \; e_s = 2 \times \frac{10}{0{\cdot}1} = 200\text{ V} \quad \left[\because \; \frac{di_p}{dt} = \frac{10}{0{\cdot}1}\right]$$

(ii) Again, $e_s = -n_s \cdot \dfrac{d\varphi_s}{dt}$

$$\therefore \; d\varphi_s = \frac{e_s}{n_s} \times dt = \frac{200}{500} \times 0{\cdot}1 = 0{\cdot}04\text{ Wb / turn}$$

❂ 3.19. Non-inductive resistance coil ❂

The coil which is free from the effect of self inductance is called a non-inductive coil. In this case coil is so wound that its coefficient of self inductance is zero, Here the wire forming the coil is doubly wound on itself before being coiled. In such a coil, each turn is in close contact with a similar turn carrying current in opposite direction [Fig 3.24]. As a result, the magnetic effect of one turn is nutralised by the magnetic effect of the other turn. Now, the resultant magnetic flux and net self-inductance are very small. In resistance boxes, the resistance coils are wound non-inductively.

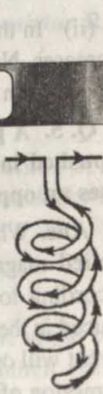


Fig. 3·24

● ● Short Answer Type Questions (with Answers) ● ●

● **Question 1. What is magnetic flux ? Is it a vector or a scalar ? Give its SI unit.**

○ **Ans.** The magnetic flux linked with a surface is the number of magnetic lines of force passing normally through that surface. If the direction of magnetic induction B is inclined at angle θ with the normal to the surface area A then, magnetic flux is $\varphi = BA\cos\theta = \vec{B}\cdot\vec{A}$

Magnetic flux is a scalar. Its SI unit is **weber** (Wb).

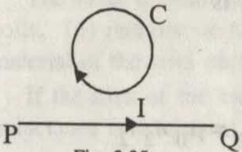


Fig. 3·25

● **Q. 2. A circular coil C is held near a straight conductor PQ which carries a study current I. What is the induced emf in the coil ?**

○ **Ans.** Induced emf will not be produced in the circular coil C.

Reason : The current I is steady. So, the magnetic flux linked with the loop also remain steady. No variation of flux linkage.

● **Q. 3. The electric current in a wire in the direction from B to A is decreasing what is the direction of induced current in the metallic loop above the wire as shown in fig 3.26.**

○ **Ans.** In the wire decreasing current flows from B to A. The decreasing magnetic field is produced, which is directed perpendicular to the plane of the looped in outward direction. So, the induced current in the loop should flow in a direction so that it produces magnetic field perpendicular to the plane of the loop and inward direction. So the induced in the loop will be in clockwise direction according to Maxwell's cork screw rule.

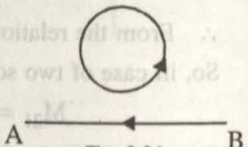


Fig. 3·26

● **Q. 4. In fig 3.27 (a), a rectangular coil 'C' is moving into the magnetic field which is acting in a direction perpendicular to the plane of the paper inwards. In fig 3.27 (b), the circular coil is moving out of the field.**

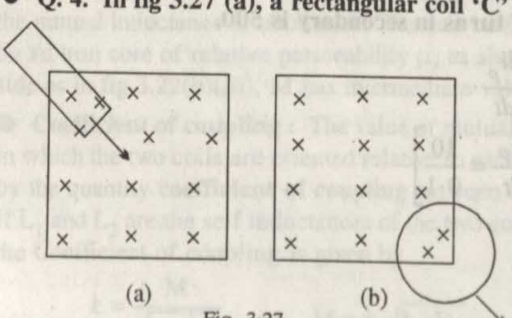


Fig. 3·27

○ **Ans.** (i) In fig 3.27 (a), the coil C is moving into the field. So, the flux linking the coil increases. According to Lenz's law, the direction of the induced current in the coil will be such so that increase of flux linkage is opposed. So, the magnetic field due to induced current will be opposite to the applied magnetic field. So the induced current in the coil C will be anticlockwise.

(ii) In this case the coil C_1 is going out of the magnetic field. So the flux linking the coil gradually decreases. Now, the direction of the induced current will be such that flux linking the coil increases. So, the direction of the induced current will be clockwise in the coil.

● **Q. 5. A part of a copper plate is partly inside the magnetic field. It is found that when the plate is pushed into or pulled out of the magnetic field, in both the cases an opposing force acts. Explain.**

○ **Ans.** When the copper plate is taken in or taken out of the applied magnetic field, magnetic flux linking the coil changes. According to the law of electromagnetic induction, a current is induced in the plate. This current is called **eddy** current. The induced current will oppose the cause of induction. Hence in both the cases the motion of the plate is opposed.

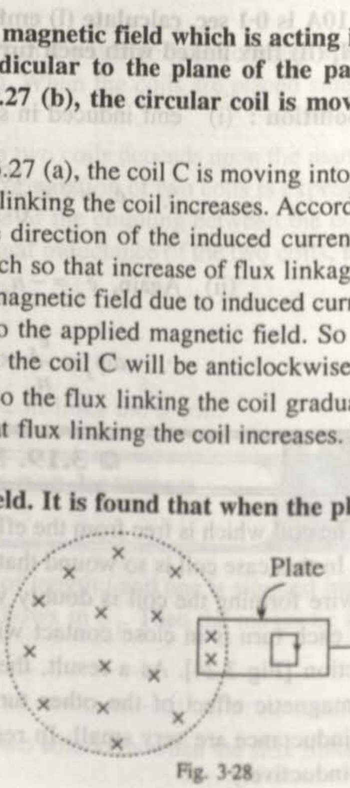


Fig. 3·28

● **Q. 6. Explain whether an induced current will be developed in a conductor, if it is moved in a direction parallel to a magnetic field.**

○ **Ans.** No current is induced in the conductor if it is moved parallel to the magnetic field.

Reason : In such a case, Lorentz force on free electrons in the conductor is zero [$\vec{v} \times \vec{B} = 0$]. Consequently no potential difference is created across the two ends of the conductor.

● **Q. 7. A bar magnet is dropped vertically along the axis of a horizontal copper ring. Will the acceleration of the falling magnet be equal to, greater than or less than due to gravity ?**

○ **Ans.** The acceleration of the falling magnet will be less than due to gravity.

Explanation : When the magnet falls, the magnetic flux through copper ring increases and it results in an induced emf in the ring. The direction of induced current in the ring is such that a similar S-pole is induced on the upper surface and it repels the incoming south pole. Hence its downward acceleration is less than the acceleration due to gravity.

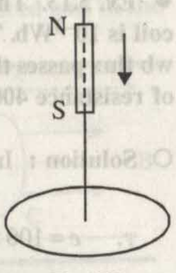


Fig. 3·29

● **Q. 8. Magnetic flux of 5 microweber is linked with a coil, when a current of 1 mA flows through it. What is the self inductance of the coil ?**

○ **Ans.** Self inductance, $L = \frac{\varphi}{i}$

$$\therefore \quad L = \frac{5\times10^{-6}}{1\times10^{-3}} = \mathbf{5\times10^{-3}\,H} \quad [\varphi = 5\times10^{-6}\,\text{Wb},\ i = 10^{-3}\,\text{A}]$$

● **Q. 9. A rate of change of current of 4 As^{-1} induces an emf of 20 mV in a coil. What is the self-inductance of the coil ?**

○ **Ans.** Induced emf, $e = L\,\frac{di}{dt}$.

$$\therefore \quad L = \frac{e}{di/dt} = \frac{20\times10^{-3}}{4}\left[e = 20\times10^{-3}\,\text{V},\ \frac{di}{dt} = 4\,\text{As}^{-1}\right]$$

$$\therefore \quad L = 5\times10^{-3}\,\text{H}$$

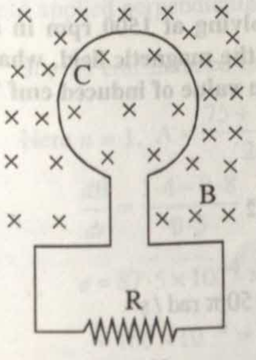


Fig. 3·30

● **Q. 10. A magentic field is applied at right angle to the plane of a circular coil as in fig 3.30. The magnetic flux changes following the relation $\varphi = 6t^2 + 7t + 1$; φ is in m.Wb and t is in sec. Find : (a) Induced emf in the coil at time t = 2 sec. (b) direction of current in resistor R.**

○ **Ans.** (a) The emf induced in the coil is

$$e = -\frac{d\varphi}{dt} = -(12t + 7) = \mathbf{-31\,mV}$$

The applied field is perpendicular to the plane of the paper and directed inwards. The direction of the induced current will be such so as to oppose the change of flux. So the induced current will be anticlockwise. Hence the current in R will be from left to right.

● ● Numerical Examples ● ●

● **Exercise 3.12. A conductor of length 30 cm is revolving about its one end at right angle to a magnetic field of intensity 5000 Oe and its makes 1000 rotation in a minute. How much emf is induced at its two ends.** **[J. E. E. '84]**

○ **Solution :** The induced emf is $e=\frac{1}{2}\mathrm{B}\,l^2\omega$

$$\therefore\ e=\frac{1}{2}\times 0\cdot 5\times(0\cdot 3)^2\times\frac{100\pi}{3}$$

$$= \mathbf{2\cdot 355\ volt}$$

$\mathrm{B}=0\cdot 5\ \mathrm{T},\ l=0\cdot 30\ \mathrm{m}.$

$$\omega=2\pi n=2\pi\times\frac{1000}{60}=\frac{100}{3}\pi\ \textbf{rad/s}$$

● **Ex. 3.13. The number of turns of a coil is 100 and its resistance 100Ω. Flux passing through the coil is 10^{-3}Wb. The intensity of the magnetic field is changed in time 0·1 sec so that now 0·2 × 10^{-3} wb flux passes through the coil. Calculate the induced emf. If the coil is connected to a galvanometer of resistance 400Ω, calculate the current through the galvanometer.**

○ **Solution :** Induced emf, $e=n\frac{d\phi}{dt}$

$$\therefore\quad e=100\times\frac{0\cdot 8\times 10^{-3}}{0\cdot 1}=\mathbf{0\cdot 8\ Volt}$$

∴ Current through the galvanometer

$$i=\frac{e}{\mathrm{R}}=\frac{0\cdot 8}{400+100}=\mathbf{1\cdot 6\times 10^{-3}\ A}$$

$x=100$

$d\varphi=(1-0\cdot 2)\times 10^{-3}=0\cdot 8\times 10^{-3}\ \mathrm{Wb}$

$dt=0\cdot 1\ \mathrm{sec}$

● **Ex 3.14. The mutual inductance between two neighbouring coils is 1·5 H. If the current in the primary coil is increased from 0 to 20A in 0·05 sec, what is the induced emf in the secondary coil. If the number of secondary turns be 800, what is the change of flux ? [J. E. E. '95]**

○ **Solution :** EMF induced in the secondary coil, $e_s=\mathrm{M}\frac{di}{dt}$

$$\therefore\quad e_s=1\cdot 5\times\frac{20}{0\cdot 5}=\mathbf{600\ volt}$$

∴ And change of flux, $\varphi=\mathrm{M}i=1\cdot 5\times 20=\mathbf{30\ wb}$

● **Ex 3.15. A coil of area 15 cm × 10 cm contains 400 turns. It is revolving at 1500 rpm in a magnetic field of intensity 20 Wb/m². When the coil is inclined at 30° with the magnetic field, what is the magnitude of the induced emf ? What is the maximum and minimum value of induced emf ?**

○ **Solution :** (i) EMF induced in the coil, $e=n\mathrm{AB}\,\omega\sin\omega t$

$$\therefore\ e=n\,\mathrm{A\,B}\,\omega\sin\theta$$

$$=400\times 15\times 10^{-3}\times 20\times 50\pi\times\sin 60^\circ$$

$$=6\times 10^3\pi\times 0\cdot 866=16{,}315\times 10^3\ \mathrm{V}$$

(ii) $\mathrm{E}_{\max}=n\mathrm{AB}\omega=6\pi\times 10^3=\mathbf{18\cdot 84\times 10^3\ V}$

$\mathrm{E}_{\min}=n\mathrm{AB}\,\omega\sin 0^\circ=0$

$n=400$

$\mathrm{A}=15\times 10^{-3}\ m^2$

$\mathrm{B}=20\ \mathrm{Wb/m^2}$

$$\omega=2\pi\times\frac{1500}{60}=50\,\pi\ \mathrm{rad/s}$$

$\theta=60^\circ$

● **Ex. 3.16. A search coil is placed at right angle to a magnetic field. The radius of the coil is 0·5 cm and number of turns 200. If the coil is quickly removed from the field, a charge of 3·14 μC flows through the coil. If the resistance of the coil be 300 Ω, what is the intensity of the magnetic field.**

○ **Solution :** Induced charge in the coil, $q=\frac{\mathrm{N}\,(\varphi_2-\varphi_1)}{\mathrm{R}}=\frac{\mathrm{NB}\pi r^2}{\mathrm{R}}$

Here $\varphi_2-\varphi_1=\mathrm{BA}=\mathrm{B}\pi r^2\qquad\therefore\ \mathrm{B}=\frac{q\mathrm{R}}{\mathrm{N}\,\pi r^2}$

$$\therefore \quad B = \frac{3\cdot14\times10^{-6}\times300}{200\times\pi\times(0\cdot5\times10^{-2})^2}$$

$$= \mathbf{6\times10^{-2}T}$$

$q = 3\cdot14\times10^{-6}$ C

R = 300 Ω ; N = 200

$r = 0\cdot5\times10^{-2}$ *m*.

● **Ex. 3.17. The length of a solenoid is 1m and number of turns 1000. A small coil of area 1 cm² and having 50 turns is placed inside the solenoid so that its axis is parallel to the axis of the solenoid. If the current in the solenoid increases from 1A to 2A in 01 sec, then what is the induced emf in the coil ?**

○ **Solution :** The arrangement is shown in fig 3.31. The solenoid 'S' is shown as a hollow cylinder. The coil C is placed inside the solenoid. If the current in the solenoid at any instant be *i*, flux density inside the solenoid is $B = \mu_0 ni$, here *n* = number of turns per unit length of the solenoid.

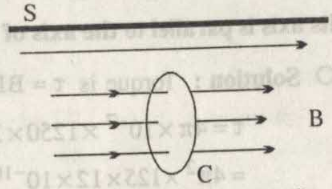


Fig. 3·31

The magnetic flux associated with the coil is

$$\varphi = N B A \cos0^\circ = N\mu_0\, ni\, A \quad [\text{ N = number of turns of C }]$$

∴ Induced emf in the coil is

$$e = \frac{d\varphi}{dt} = \mu_0 ANn\frac{di}{dt}$$

$$\therefore \quad e = 4\pi\times10^{-7}\times10^{-4}\times1000\times50\times10$$

$$= \mathbf{6\cdot28\times10^{-5}}\ \text{volt}$$

$\mu_0 = 4\pi\times10^{-7}$ H / m

$A = 1cm^2 = 10^{-4}\,m^2$

$N = 50\ ;\ n = 1000\,m^{-1}$

$$\frac{di}{dt} = \frac{2-1}{0\cdot1} = 10\,\text{As}^{-1}$$

● **Ex. 3.18. A wire of length 40 cm is bent into a rectangular coil of area 15 cm × 5 cm. A magnetic field at intensity 0·8 Wb/m² is applied perpendicular to the plane of the coil. In 0·5 sec, the coil is changed into a square coil of side 10 cm and the intensity of the magnetic field is increased to 1·4 Wb/m². What is the induced emf ?**

○ **Solution :** We know that if the area of the coil is A, number of turns N and intensity of the magnetic field applied perpendicular to the coil, then flux linked with the coil, N = *n*AB

So, the emf induced in the coil $e = n\,A\dfrac{dB}{dt} + nB\dfrac{dA}{dt}$

Here $n = 1$, $A = \dfrac{75+100}{2}\,cm^2 = 87\cdot5\times10^{-4}m^2$, $B = \dfrac{0\cdot8+1\cdot4}{2} = 1\cdot1\,\text{Wb}/m^2$

$$\frac{dB}{dt} = \frac{1\cdot4-0\cdot8}{0\cdot5} = \frac{6}{5} \quad \text{and} \quad \frac{dA}{dt} = \frac{100-75}{0\cdot5}\times10^{-4} = 50\times10^{-4}$$

$$\therefore \quad e = 87\cdot5\times10^{-4}\times\frac{6}{5} + 1\cdot1\times50\times10^{-4} = 10^{-4}\,[87\cdot5\times1\cdot2 + 1\cdot1\times50]$$

$$= 160\times10^{-4} = \mathbf{0\cdot016\ V}$$

● **Ex. 3.19. A copper wire of length 0·2 m is lying along X-axis. It starts moving along Y-axis with a velocity 4 m/s. A magnetic field of intensity 0·05 T is applied parallel to Z-axis. Calculate : (i) EMF induced in the wire, (ii) If a resistor of 2Ω is connected across the wire, that is the current through it, (iii) Force required to keep the coil moving, (iv) rate of work done by the applied force and (v) rate of production of heat.**

○ **Solution :** (i) Magnitude of the induced emf is $e = Blv$

∴ $e = 0\cdot05\times0\cdot2\times4 = \mathbf{0\cdot04V}$ [$B = 0\cdot05T$, $l = 0\cdot2$ m, $v = 4$ m/s]

(ii) Current in the wire, $i = \dfrac{e}{R} = \dfrac{0\cdot04}{2} = \mathbf{0\cdot02\ A}$

(iii) Force required to keep the coil moving, $F = Bil$

$\therefore \quad F = 0{\cdot}05 \times 0{\cdot}02 \times 0{\cdot}2 = 2 \times 10^{-4}\,\text{N}$

(iv) Rate of doing work, $P = Fv = 2 \times 10^{-4} \times 4 = \mathbf{8 \times 10^{-4}\ J.s^{-1}}$

(v) Rate of production of heat, $H = i^2R = (0{\cdot}02)^2 \times 2 = 8 \times 10^{-4}$ watt

- **Ex. 3.20. The length of a solenoid 0·4 m, number of turns 500 and current 3A. A small coil of radius 1 cm and having 10 turns carries a current of 0·4A. The coil is put inside the solenoid so that its axis is parallel to the axis of the solenoid. How much torque acts on the coil ? $\mu_0 = 4\pi \times 10^{-7}\,\text{H}m^{-1}$**

○ **Solution :** Torque is $\tau = \text{BINA}\sin\theta = \mu_0 ni\, I\, \text{NA}\sin\theta$

$$\tau = 4\pi \times 10^{-7} \times 1250 \times 3 \times 0{\cdot}4 \times 10 \times 10^{-4}\pi$$

$$= 4\pi^2 \times 125 \times 12 \times 10^{-10}$$

$$= \mathbf{5{\cdot}922 \times 10^{-10}\ N\text{-}m}$$

$$n = \frac{500}{0{\cdot}4} = 1250\,m^{-1}$$

$i = 3\text{A},\ I = 0{\cdot}4\text{A}$

$\text{A} = \pi \times (0{\cdot}01)^2 = \pi \times 10^{-4}\,m^2$

$\sin\theta = 1$

- **Ex. 3.21. A straight wire of length 1m is placed horizontally along east-west direction and it strarts falling freely. What will be its induced emf after 2 sec of fall. Horizontal intensity of earth's magnetism = 2 × 10⁻⁵ T.**

○ **Solution :** Velocity of the wire after 2 sec, $v = gt = 9{\cdot}8 \times 2 = 19{\cdot}6$ m/s.

$\therefore$ Induced emf, $e = Blv = 2 \times 10^{-5} \times 1 \times 19{\cdot}6$

$\therefore \quad \mathbf{e = 39{\cdot}2 \times 10^{-5}\ V}$

- **Ex. 3.22. A circular coil lying on a horizontal table is rotated through 180° in 0·5 sec. Its radius is 10 cm and has 500 turns. What is the average induced emf with coil ? Vertical component of earth's magnetism = 0·43 × 10⁻⁴T.**

○ **Solution :** Change of flux linking the coil when it is turned through 180° is $= n\text{AB} - (-n\text{AB}) = 2\,n\text{AB}$.

Here $n = 500,\ \text{A} = \pi \times (0{\cdot}1)^2 = 10^{-2}\pi\ m^2$

$$\therefore \text{ Induced emf, } e = \frac{2n\text{AB}}{t} = \frac{2 \times 500 \times 10^{-2}\pi \times 0{\cdot}43 \times 10^{-4}}{0{\cdot}5}$$

$\therefore \quad \mathbf{e = 2{\cdot}70 \times 10^{-3}\ volt}$

- **Ex. 3.23. The resistance of a coil is 10Ω and number of turns 1000. At any instant flux associatied with the coil is 5·5 × 10⁻⁴ Wb. In 0·1 sec, the flux changes to 0·5 × 10⁻⁴ Wb. Calculate (i) induced emf and (ii) charge flowing through the coil.**

○ **Solution :** (i) Induced emf, $e = n\dfrac{d\varphi}{dt}$.

$$\therefore \quad e = 1000 \times \frac{(5{\cdot}5 - 0{\cdot}5) \times 10^{-4}}{0{\cdot}1} = 5\text{V}.$$

(ii) Charge flowing through the circuit is

$$q = it = \frac{e}{\text{R}}t = \frac{5 \times 0{\cdot}1}{10} = \mathbf{0{\cdot}05\ coulomb}$$

- **Ex. 3.24. The diameter of a solenoid is 0·1m and number of turns 2 × 10⁴m^{-1}. A circular coil of radius 0·01 m and having 100 turns is placed co-axially inside the solenoid. The current in the**

solenoid is changed from + 2A to – 2A in 0·05 sec. What is the induced emf in the circular coil ? How much charge flows through the coil ?

The resistance of the coil = $10\pi^2$ ohm

○ **Solution :** Flux density inside the solenoid, $B = \mu_0 ni$

$\therefore$ Flux passing through the coil, $\varphi = \mu_0 n i A$

$\therefore$ Induced emf in the coil is $e = N\dfrac{d\varphi}{dt}\ \varphi = \mu\, N\, n\, A\dfrac{di}{dt}$.

$\therefore$ $e = 4\pi\times10^{-7}\times2\times10^{4}\times100\,\pi\times10^{-4}\times80$

$= 64\,\pi^2\times10^{-4}$ volt

$\therefore$ charge flow : $q = \dfrac{e}{R}\,t - \dfrac{64\pi^2\times10^{-4}}{10\pi^2}\times0\cdot05 = \mathbf{3\cdot2\times10^{-5}\ c}$

● **Ex. 3.25. A coil of area 8 cm × 2 cm is situated inside a magnetic field of intensity 0·3T. It is moving with a velocity of 1 cm/s, inside the field. Calculate the induced emf in the following cases :**

(i) When the coil moves perpendicular to the larger side

(ii) When the coil moves perpendicular to the shorter side.

○ **Solution :** (i) Here $B = 0\cdot3$ T, $v = 0\cdot01$ m/s, $l = 0\cdot08$ m

$\therefore$ Induced emf, $e = Blv = 0\cdot3\times0\cdot08\times0\cdot01$

$\therefore$ $e = \mathbf{2\cdot4\times10^{-4}}$ volt

(ii) In this case $l = 0\cdot02$ m

$\therefore$ $e = 0\cdot3\times0\cdot02\times0\cdot01 = \mathbf{0\cdot6\times10^{-4}}$ volt.

● **Ex. 3.26. To measure the intensity of the magnetic field produced by a permanent horse-shoe magnet, a search coil is placed perpendicular to the magnetic field. The area of the coil 2·0 cm² and number of turns 25. It is quickly removed from the magnetic field and a charge of 7·5 × 10⁻³ C charge flows through the coil. What is the intensity of the magnetic field ? Given, resistance of the coil is 0·5Ω.**

○ **Solution :** If Q be the charge flowing through the coil, then

$$Q = \frac{N(\varphi_2-\varphi_1)}{R} = \frac{N}{R}BA$$

$$\therefore\quad B = \frac{QR}{NA} = \frac{7\cdot5\times10^{-3}\times0\cdot5}{25\times2\times10^{-4}}$$

$\therefore$ $\mathbf{B = 0\cdot75\ T}$

$Q = 7\cdot5\times10^{-3}$C

$R = 0\cdot5\Omega$

$N = 25$

$A = 2\cdot0\times10^{-4}\ m^2$

$\therefore$ Intensity of the magnetic field of the magnet = **0·75 T**

● **Ex. 3.27. Calculate the self inductance of a air core solenoid. Given, length of the solenoid 1m, diameter 0·05 m and number of turns 700.**

○ **Solution :** Magnetic flux density inside a solenoid having n turns per unit length and carrying a current i is $B = \mu_0 ni$.

$\therefore$ Flux produced by the solenoid, $\varphi = BA = \mu_0 ni\,\pi r^2$

$\therefore$ Flux linking the solenoid, $N\varphi = \mu_0 n^2 l\,\pi r^2 i$ [Here $N = nl$, l = length of the solenoid]

$$\therefore \quad \text{Self inductance } L = \frac{N\varphi}{i} = \mu_0 n^2 l\, \pi r^2$$

$$= 4\pi \times 10^{-7} \times (700)^2 \times 1 \times \pi \times (2\cdot5 \times 10^{-2})^2$$

$$= \mathbf{1\cdot21 \times 10^{-3} H}$$

● **Ex. 3.28. A long and straight conductor carrying a current of 10A is located near a rectangular circuit. The length of its side parallel to the conductor is 0·2 m. The distance of one side from the conductor is 0·05 m and the distance of the other side is 0·1 m. The conductor lies in the plane of the circuit. How much magnetic flux passes through the circuit ? If the current in the conductor decreases uniformly to zero in time 0·02 sec, what is the induced emf in the circuit ?**

○ **Solution :** In fig 3.32, the rectangular coil PQRS is located near the straight conductor XY which carries a current i. The coil and the conductor are in the same plane. Now, the magnetic field at a distance x from the conductor is $B = \frac{\mu_0}{4\pi}\frac{2i}{x} = \frac{\mu_0 i}{2\pi x}$.

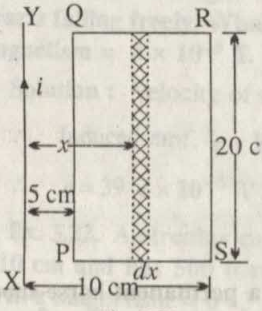


Fig. 3·32

So, the magnetic flux passing through the strip of thickness dx is

$$d\varphi = B\,.\,dA = B\, l dx \quad [l = 0\cdot2\ m]$$

$\therefore$ Total flux passing through the rectangular coil is

$$\varphi = \int_{0\cdot05}^{0\cdot1} \frac{\mu_0\, i}{2\pi x}\, l\, dx = \frac{\mu_0\, il}{2\pi}\left[\log_e x\right]_{0\cdot05}^{0\cdot1} = \frac{\mu_0\, il}{2\pi}\log_e 2$$

$$= 2\times10^{-7}\times10\times0\cdot2\times0\cdot693 \quad [i = 10A]$$

$$= \mathbf{2\cdot772\times10^{-7}\ Wb}$$

$$\therefore \quad \text{Induced emf, } e = \frac{d\varphi}{dt} = \frac{2\cdot772\times10^{-7}}{0\cdot02} = \mathbf{1\cdot386\times10^{-5}\ V}$$

The current in the coil will flow in the anticlockwise direction.

• E ◊ X ◊ E ◊ R ◊ C ◊ I ◊ S ◊ E •

◢ A. Objective type questions :

(i) Fill in the blanks :

1. The induced emf opposes the ——— of induction
2. Lenz's law is alternative form of ——— ———
3. Fleming's right hand rule is also called ——— rule.
4. A conductor of length l is moving with a velocity v at right angle to a magnetic field B. Its induced emf ——.
5. A conductor of length l is revolving about its one end at right angle to a magnetic field B with a constant angular velocity w. Its induced emf ———.
6. A coil having n turns carrying current i produces a flux φ. Its self inductance is L ———.
7. 1 Wb A^{-1}= ———.

◢ B. Very short answer type questions :

1. What is the relation between flux density and flux ?
2. What we get from Lenz's law ?
3. What is the principle of dynamo ?

4. What is eddy current ?

5. What material is chosen for transformer core ?

6. Define self inductance.

7. The current through the wire PQ is increasing [fig. 2.33]. In which direction does the induced current flow in the closed loop ?

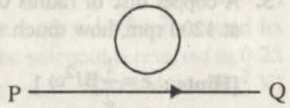


Fig. 3·33

8. Give the direction in which the induced current flows in the wire loop when the magnet moves towards it as shown in fig. 2.34.

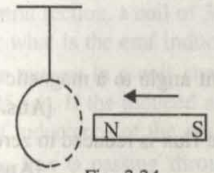


Fig. 3·34

9. A bar magnet falls through a metal ring from a height. Will its acceleration be equal to 'g' ?

◢ C. Short answer type questions :

1. What is meant by electromagnetic induction ?

2. What we mean by magnetic flux density and magnetic flux ? How are they related ? Which one is a vector ?

3. Define the SI unit of magnetic flux.

4. What is meant by 'magnetic flux density is 1 T' ? How is it related to c. g. s. unit ?

5. What is the unit of rate of change of magnetic flux with time ?

6. What is the difference between magnetic flux and magnetic flux linkage ?

7. State the condition for electromagnetic induction.

8. A train is moving along north-south direction. Will any emf will be induced across the axle.

9. A train is moving along east-west direction. Will the electric bulb burn connected across an axle.

10. A bar magnet is placed across the axis a coil. If the magnet is rotated about its axis, will any emf be induced in the coil ?

11. State Fleming's right hand rule.

12. An aeroplane is flying horizontally. An emf is found to be induced across its wing. Explain the reason. On which factor the magnitude of the induced emf depends ?

13. What is meant by 'self-inductance of a coil is 1 H' ?

◢ D. Essay type questions :

1. What is electromagnetic induction ? State Faraday's laws of electromagnetic induction.

2. State Lenz's law. Show that it follows from the principle of conservation of energy. Explain.

3. (a) State Lenz's law. Explain how this law follows from the principle of conservation of energy.

(b) A conductor is rotated about its one end perpendicular to a uniform magnetic field. Deduce an expression for the emf induced in the conductor.

4. A conductor of length l is moving with a uniform velocity v at right angle to a magnetic field B. Obtain an expression for the emf induced in the conductor.

5. A rectangular coil is rotated inside a magnetic field with a constant angular velocity. How much emf is induced in the coil ?

◢ E. Simple numerical problems :

1, The flux density of a magnetic field is 1·5 T. A circular coil of radius 10 cm and having 100 turns is placed in the magnetic field so that its axis makes angle of 60° with the direction of the field. Calculate :

(i) magnetic flux passing through the coil and (ii) magnetic flux linking the coil. [**Ans :** (i) $58{\cdot}85 \times 10^{-4}$ Wb (ii) $58{\cdot}85 \times 10^{-2}$ Wb-turn]

2. A coil of area 100 cm^2 is put inside a magnetic field of magnetic induction 2T so that its axits is parallel to the magnetic field. If the coil is removed from the field in 0·1 sec, calculate the induced emf. Assume that the coil contain 500 turns. **[Ans.** 100 V]

3. A coil of radius 10 cm with 100 turns is placed in a magnetic field, the axis of the coil being parallel to the field direction. If the intensity of the magnetic field changes from 0·1T to 0·3T in 0·05 sec, how much emf is induced in the coil ? **[Ans.** 12·6 volt]

4. A wire of length 0·3 m is rotated about its one end at right angle to a magnetic field at a speed of 1000 rpm. How much emf will in induced across the conductor ? **[Ans.** 7·065V]

5. A copper disc of radius 0·1 m is rotated in a magnetic field with its axis parallel to the field. If the disc rotates at 1200 rpm, how much emf will be induced between its centre and a point on the rim ? **[Ans.** 31·4 mV]

[Hints : $e=\frac{1}{2}Bl^2\omega$]

6. A straight conductor of length 0·4 m is moving with a velocity of 7·0 m/s at right angle to a magnetic field of intensity 0·9 T. How much emf is induced at its two ends ? **[Ans.** 2·52V]

7. The amount of magnetic flux passing with a coil of 50 turns is 0·4 Wb. In 1 sec the flux is reduced to zero ; what is the induced emf in the coil ? **[Ans.** 20 V]

8. A rectangular coil is placed at right angle to a magnetic field of intensity 3×10^{-2} T. If the area is decreased at the rate of 0·4 m^2/sec, what is the induced emf in the coil ? **[Hint :** $e=-B\frac{dA}{dt}$] **[Ans.** 12·0 mV]

9. A coil of area 200 cm^2 is placed in a magnetic field such that the plane of the coil is at 60° with the field direction. If 10^{-3} Wb flux passes through the coil, what is the intensity of the magnetic field ? **[Ans.** 0·058 T]

10. A millivoltmeter is connected between two horizontal railway lines which are at a separation of 1m. If a train moves with the velocity of 180 km/hr, what will be the reading of the voltmeter. The vertical component of earth's magnetism is $0{\cdot}2 \times 10^{-4}$ Wb/m^2. **[Ans.** 1mV]

11. A coil of area 100 cm^2 and having 100 turns is in a magnetic field of intensity 0·4 G. The coil is rotated through 180° in 0·1 sec. What is the induced emf ? **[Hint :** $d\varphi = nAB-(-nAB)$] **[Ans.** 0·04V]

12. A coil of area 10^{-3} m^2 and having 100 turns can rotate about an axis. It is placed in a magnetic field of intensity 1·0T. Now it is quickly rotated through 180°. If the resistance of the coil be 10Ω, how much charge flows through the coil ? **[Hints :** $q=\frac{2nAB}{R}$] **[Ans.** 0·02 Coulamb]

13. A closed coil of area 1·0 m × 1·0 m is capable of ratating about an axis. It is placed at right angle to a magnetic field of intensity 0·10 Wb/m^2. It is rotated through 180° in 0·01 sec. What is the induced emf ? If its resistance be 2·0Ω, how much current flows through it ? **[Ans.** 20V, 10A]

14. The self-inductance of an induction coil is 5H. If the current in the coil increases at the rate of $2As^{-1}$, what is the induced emf ? **[Ans.** 10 V]

15. The wing span of an aeroplane is 5m. It is flying horizontally with a velocity of 360 km/hr. Intensity of earth's magnetism at the place is $0{\cdot}4 \times 10^{-4}$ T and angle of dip 30°. How much emf is induced across the wing ? **[Ans.** 0·01 V]

16. A copper disc of radius 10 cm is rotating in a plane perpendicular to a magnetic field at a speed 20 rps. If the emf between the axis and a point on the rim be 3·14 mV, calculate the intensity of the magnetic field. **[Ans.** 5×10^{-3} T]

17. A train is moving due south with a velocity of 10 m/s. Vertical component of earth's field is $5{\cdot}4 \times 10^{-5}$ T. If the length of the axle is 1·2 m, how much emf will be induced in it. **[Ans.** $6{\cdot}48 \times 10^{-4}$V]

18. A coil of 100 turns and having area 0·05 m^2 is held perpendicular to a magnetic field of flux density 0·005T. Calculate :

(i) Magnetic flux passing through the coil.

(ii) Flux linkage with the coil.

(iii) The coil is rotated. If the plane of the coil makes angle of 60° with the magnetic field in time 100 μsec, what is the induced emf.

[Ans. (i) $2{\cdot}5 \times 10^{-4}$ Wb (ii) $2{\cdot}5 \times 10^{-2}$ Wb, (iii) 33·5 V]

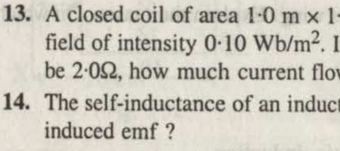


Fig. 3·35

19. In fig 3.35 ACDEHGFJ is a cube of side 'a'. It is placed in a magnetic field of flux density B. The magnetic field is perpendicular to the surface CHGD. Calculate magnetic flux passing through the surfaces: (i) ACDE, (ii) DEFG, (iii) AJFE and (iv) CHGD.

[Ans. (i) $\varphi = 0$, (ii) $\varphi = 0$, (iii) $\varphi = -Ba^2$ (iv) $\varphi = Ba^2$]

20. If the magnetic flux linked with a coil varies at the rate of 1 Wb/min what is the induced emf ? **[Ans.** $\frac{1}{60}$ volt]

21. A coil of area 400 cm^2 having 30 turns is making 1800 rev/min in a magnetic field of 1T. What is the peak value of the induced emf ? **[Ans. 226 V]**

22. A solenoid has 2000 turns wound over a length of 0·3 m. Its cross sectional area is ·2 × 10^{-3} m^2. Around its central section, a coil of 300 turns is wound. If an initial current of 2A flowing in the solenoid is reversed in 0·25 sec what is the emf induced in the coil ? **[Ans.** 4·8 × 10^{-2} V]

23. The current in a coil changes from 0 to 2A in 0·05 sec. If the induced emf is 80V, what is the self inductance of the coil ? **[Ans. 2H]**

24. The flux φ passing through a coil having 100 turns varies as shown in fig 3.36. What is the induced emf at

(i) $t = 1$ sec, (ii) $t = 5$ sec.

[Ans. (i) –20V (ii) 20V]

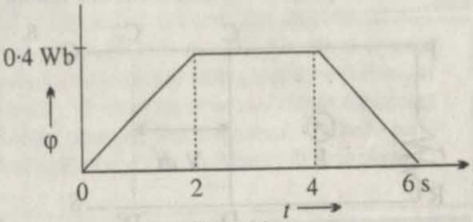


Fig. 3·36

25. A coil of self inductance 2H carries current i which varies with time as shown in fig 3.37. Calculate the induced valtage in the coil at $t = 0{\cdot}2s$ and $t = 0{\cdot}7s$. **[Ans.** (i) 10V (ii) 0]

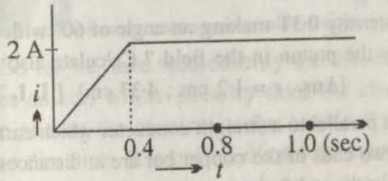


Fig. 3·37

26. How much emf will be induced in a coil of self inductance 1 mH when current in the coil changes from 5A to 3A in 10^{-3} sec ? **[Ans. 2V]**

27. Calculate mutual inductance between two neighbouring coils. Given, if a current of 3A is passed through one coil, flux linking each turn of other coil (of 1000 turns) is 10^{-4} Wb.

[Hints : $\varphi = Mi_p$] **[Ans.** 3·33 × 10^{-2} H]

28. When the current in the primary coil changes from 5·0A to 2·0A in 300 millisec, 0·5 V is induced in the secondary coil. Calculate mutual inductance between two coils. **[Ans. 0·05 H]**

F. Harder numerical problems :

1. A coil of resistance 100Ω, and having 100 turns is placed in a magnetic field of intensity 1 mWb. It is connected in series with a galvanometer of resistance 400Ω. The intensity of the magnetic field is changed to 0·2 mWb. in 10^{-3} sec. How much emf is induced in the coil ? What is the current in the circuit ?
[Ans. 0·8 V, 1·6 mA] [J. E. E. '87]

2. Calculate the self-inductance of a solenoid of length 1m and having 1000 turns. Area of cross-section of the solenoid 7 cm^2 and relative permeability = 1000. **[Ans. 0·88 A]**

3. The diameter of a solenoid is 0·1m and number of turns 2 × 10^4 m^{-1}. A small circular coil of radius 0·01 m having 100 turns is placed inside the solenoid co-axially. The current in the solenoid is changed from + 2A to – 2A in 0·05 sec. What is the induced emf in the coil ? How much charge is displaced during this time ? Resistance of the coil is 10π^2 ohm. **[Ans.** 63·17 mV ; 3·2 × 10^{-5} Coulomb]

4. A coil has resistance 20Ω and self-inductance 1·5 H. The emf induced in the coil changes with time as shown in fig 3·38 : What is the charge induced during the time interval from $t = 0$ to $t = 8$ μ sec? **[Ans.** 3μC]

[Hints : Magnitude of the induced emf,

$$[e = \frac{d\varphi}{dt} \, |\varphi_2 - \varphi_1| = \int e\,dt = \text{area OPQR and } q = \frac{\varphi_2 - \varphi_1}{R}]$$

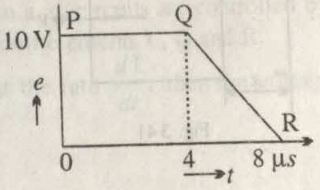


Fig. 3·38

5. Mutual inductance between two coils 5 × 10^{-3}H. The current in the coil changes according to the relation $I = I_0 \sin \omega t$. Here I_0 = 10A and $\omega = \frac{2\pi}{T}$ [T = 0·02 sec]. Find : (i) equation of the induced emf in the second coil and (ii) Maximum value of induced emf. **[Ans.** $e = -M\frac{di}{dt} = -15{\cdot}7 \cos 100\pi t$ - volt (ii) $E_{max} = 1{\cdot}57$ V]

6. The number of turns of a coil is 1000 and self inductance 1H. (i) If the current in the coil is 4A, what is the stored energy ? (ii) If a coil of 50 turns is wound at the middle of the solenoid, what is the mutual inductance between the coil and the solenoid. **[Ans.** (i) 8 J (ii) 0·05 H]

7. A search coil of area 1 cm^2 is of resistance 4Ω and 30 turns. It is connected to a charge measuring instrument of resistance 20 ohm. The coil is suddenly removed from a magnetic field of intensity 0·5T. How much charge flows through the instrument ? Assume that the coil was perpendicular to the magnetic field. **[Ans.** 62·5 μc]

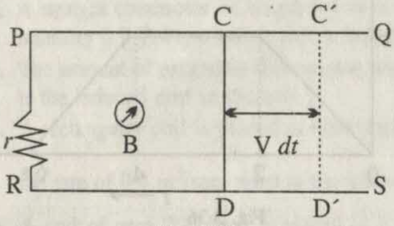


Fig. 3·39

8. PQ and RS are two parallel metal bars kept at a separation of l. A resistance r is connected across their ends. A wire CD is placed across the two bars [Fig 3.39]. The wire can move freely over the bars. Now a magnetic field B is applied perpendicular to the system. If the wire CD moves with a velocity v, what is the rate of doing work for the motion of the wire. **[Ans.** $W = \frac{1}{2}\frac{Bl^2v^2}{r}$]

9. A proton of velocity 4×10^5 m/s enters into a magnetic field of intensity 0·3T making an angle of 60° with the field direction. What is the radius of the helical path followed by the proton in the field ? Calculate also the distance travelled by the proton after one time period. **[Ans.** $r = 1{\cdot}2$ cm ; 4·37 cm] **[I. I. T.]**

10. A copper rod of length 0·19 m is moving with a velocity $v = 10$ m/s parallel to a straight conductor which carries a current of 5·0A. The rad is perpendicular to the conductor. The two ends of the copper bar are at distances of 0·01 *m* and 0·2 *m* from the conductor. How much emf is induced in the rod ? **[Ans.** 3×10^{-5}V]

11. The current in a coil of self-inductance 2·0 H changes according to the relation $i = 2 \sin t^2$ Amp. The time during which the current increases from 0 to 2A, how much work is done ? **[Ans.** 4J]

12. Two parallel metal rails AB and CD are at a separation of 1m and are held vertically. Two resistors r_1 and r_2 are connected at the two ends of the rails [Fig 3.40]. A horizontal metal bar of length L slides down the rails without any friction. A magnetic field of intensity 0·6 T is applied perpendicular to the plane of the rails. When a rod L, slides with a constant speed power loss in r_1 and r_2 are 0·75 W and 1·2W. Calculate constant terminal velocity of the rod L and the value of r_1 and r_2.

[Ans. $v = 1{\cdot}0$ m/s, $r_1 = 0{\cdot}474\Omega$; $r_2 = 0{\cdot}3\Omega$]

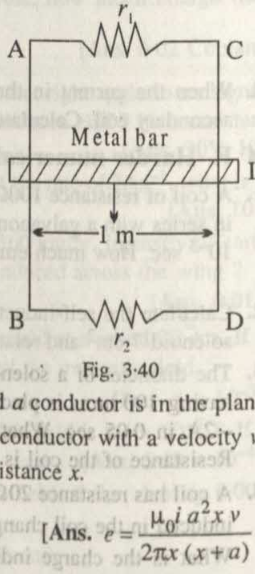


Fig. 3·40

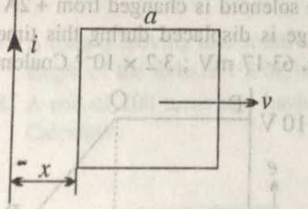


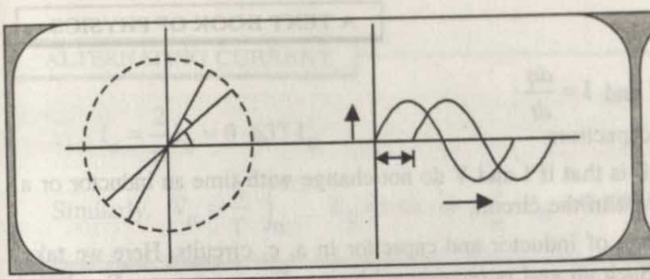
Fig. 3·41

13. A square coil of side '*a*' carrying current *i* and *a* conductor is in the plane [Fig 3.41]. The coil is moving away from the conductor with a velocity *v*. Obtain an expression for the emf interms of distance *x*.

[Ans. $e = \frac{\mu_0 i a^2 x v}{2\pi x (x+a)}$]

14. The conductor XY are carrying a current *i*. The rod PQ of length L is moving with velocity *v* parallel to the conductor. The distance of end P from the conductor is *r*. Obtain the induced emf is the rod PQ.

[Ans. $e = \frac{\mu_0 i v}{2\pi} \log_e\left(\frac{r+L}{r}\right)$]



CHAPTER 4

ALTERNATING CURRENT

● **TOPICS :** ○ Introduction; ○ The equation of alternating voltage and current; ○ Mean or average value of Alternating current and voltage; ○ Root mean square (r.m.s.) value of alternating quantity; ○ Phase of alternating current; ○ Three simple circuits in alternating electricity; ○ Analysis of an AC circuit containing resistance and inductance; ○ Analysis of an AC circuit containing capacitor and resistance; ○ Choke coil ; ○ Dynamo; ○ A. C. dynamo or generator; ○ D. C. dynamo or generator; ○ D. C. Motor; ○ Transformer ; ○ Short answer type questions (with answers); ○ Exercise.

4.1. Introduction

So far we have studied only d.c. circuits i.e. circuits in which the source of emf is a battery. A battery gives an emf whose polarity does not change with time and so the current in the circuit flows in the same direction. It is called **direct current or D. C.** The fig 4.1 shows some waveform of d. c. Evidently, the magnitude of direct current may increase, decrease or remains constant, but its direction does not reverse ; d. c. circuits contain resistor R as a basic element. The resistor controls the current and voltage in the circuit and the relationship between them is V = IR (Ohm's law)

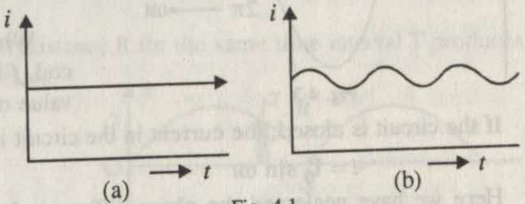


Fig. 4·1

In the present chapter we shall consider electric circuits which is supplied with an alternating voltage. Alternating voltage is a voltage whose polarity reverses with time. Alternating voltage applied to a circuit produces an alternating current (a. c.).

Alternating current is the current which continuously varies in magnitude and periodically reverses its direction. The same is true for alternating current. The fig 4.2 shows some waveforms for a. c.

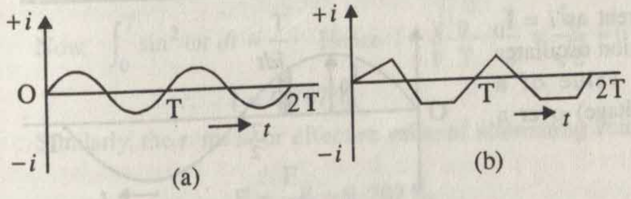


Fig. 4·2

In a. c. circuits, in addition to resistors R, two new basic elements are inductance L and capacitance C. The current and voltage in a. c. circuits are controlled by three circuit elements L, C and R.

If a circuit contains only art inductor L and if the current changes at the rate $\frac{dI}{dt}$, then the voltage across the inductor is

$$V = L\,\frac{dI}{dt} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

This is the voltage-current relationship for an inductor. Again if we have a circuit consisting only of a capacitor of capacitance C across which the voltage is changing at the rate $\frac{dV}{dt}$, then the current through the capacitor is

$$I = C\,\frac{dV}{dt} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(ii)}$$

which follows from the relation $q = \text{CV}$ and $\text{I} = \frac{dq}{dt}$.

It is the current-voltage relation for the capacitors.

It is evident from the relations (i) and (ii) is that if I and V do not change with time an inductor or a capacitor does not affect the voltage or current in the circuit.

In this chapter we shall consider the effect of inductor and capacitor in a. c. circuits. Here we take only sinusoidally varying emf and current. Such an emf is represented by $e = \text{E}_0 \sin\omega t$ or $e = \text{E}_0 \cos\omega t$. But in practical cases the dependence of voltage and current on time may be much more complicated.

4.2. The equation of alternating voltage and current

The magnitude of alternating current or voltage changes with time and direction reverses periodically.

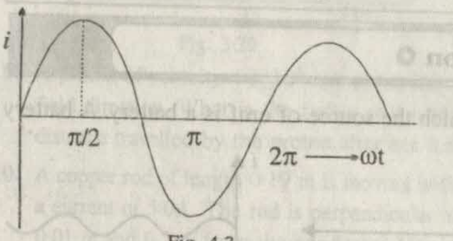


Fig. 4·3

Sinusoidally varying current or voltage is usually represented by sine or cosine function of time. A sinusoidal emf is produced when a coil rotates in a magnetic field (vide article 4.9) ; the emf so produced is given as

$$\text{E} = n\text{AB}\omega \sin\omega t = \text{E}_0 \sin \omega t$$

Where $\omega\ (= 2\pi f)$ is the angular velocity of the rotating coil, f is the frequency of rotation and E_0 is the maximum value of emf. It is also called **Peak value.**

If the circuit is closed, the current in the circuit is given by

$$i = \text{I}_0 \sin \omega t$$

Here we have neglected the phase difference between voltage and current. The fig 4.3 shows the variation of current with time.

Since alternating current varies with time continuously, to measure its effect we define (i) mean value of a. c. and (ii) root mean square value (or virtual value) of a. c.

4.3. Mean or average value of Alternating current and voltage

We usually represent the alternating current as $i = \text{I}_0 \sin \omega t$ (or $\text{I}_0 \cos \omega t$). A sine (or cosine) function oscillates symmetrically about zero, the average value of a sinusoidally varying quantity (current or voltage) over a complete cycle is zero. It is shown as—

$$i = \frac{1}{\text{T}}\int_0^{\text{T}} i\,dt = \frac{1}{\text{T}}\int_0^{\text{T}} \text{I}_0 \sin\omega t\,dt = 0$$

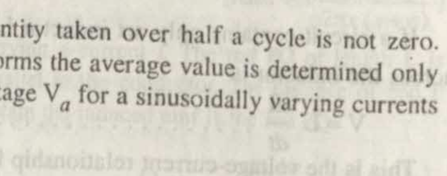


Fig. 4·4

However, the average value of a sinusoidally varying quantity taken over half a cycle is not zero. Hence it is customary, that for a symmetrically varying waveforms the average value is determined only over half a cycle. Hence the average current I_a and average voltage V_a for a sinusoidally varying currents and voltages are given by

$$\text{I}_a = \frac{1}{\text{T}/2}\int_0^{\text{T}/2} \text{I}_0 \sin \omega t \,.\, dt$$

$$= \frac{2\text{I}_0}{\text{T}\omega}\,[-\cos \omega t]_0^{\text{T}/2} = \frac{2}{\pi}\,\text{I}_0$$

$$\therefore \quad I_a = \frac{2}{\pi} I_0 = 0 \cdot 637\, I_0$$

$$\text{Similarly,} \quad V_a = \frac{2}{T}\int_0^{T/2} E_0 \sin \omega t \cdot dt = \frac{2}{\pi}\, E_0 = 0 \cdot 637\, E_0$$

✪ 4.4. Root mean square (r.m.s.) value of alternating quantity ✪

We define an effective value or r.m.s. value of an alternating current as—

It is that steady current which will produce the same heat in the same interval of time in a resistance as the given alternating current does.

Let the alternating current is $i = I_0 \sin\omega t$. During a small interval of time dt, the heat produced by the current i flowing through the resistance R is $i^2 R\, dt$. So the total heat produced in one cycle *i.e.* in time T

is $\int_0^T i^2 R\, dt$

Now, suppose a steady current I through the equal resistance R for the same time interval T produces equal amount of heat, then by definition I is the effective value of the alternating current i

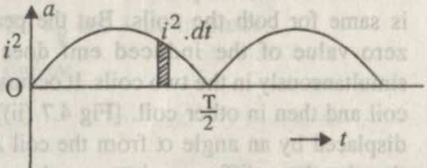


Fig. 4·5

$$\therefore \quad I^2RT = \int_0^T i^2 R\, dt$$

$$\therefore \quad I = \sqrt{\frac{1}{T}\int_0^T i^2\, dt}$$

Hence.I can be defined as the r. m. s. value of i

$$\therefore \quad \text{r. m. s. value of alternating current is } I = \sqrt{\frac{1}{T}\int_0^T I_0{}^2 \sin^2 \omega t\, dt}$$

$$\text{Now,} \quad \int_0^T \sin^2 \omega t\, dt = \frac{T}{2}. \quad \text{Hence } I = \sqrt{\frac{I_0{}^2}{T} \cdot \frac{T}{2}} = \frac{I_0}{\sqrt{2}} = 0 \cdot 707\, I_0$$

$$\therefore \quad I = 0 \cdot 707\, I_0$$

Similarly, the r. m. s. or effective value of alternating voltage $e = E_0 \sin \omega t$ is

$$E = \frac{E_0}{\sqrt{2}} = 0 \cdot 707\, E_0$$

● **Example 4.1. The emf. equation of an alternating current is $i = 40 \sin 400\, \pi t$ Amp. Find (i) peak value (ii) average value and (iii) r. m. s. value of the alternating current.**

○ **Solution :** (i) Peak value, $I_0 = 40$ A

(ii) Average value, $I_a = 0 \cdot 637\, I_0 = 0 \cdot 637 \times 40 = 25 \cdot 48$ A

(iii) r. m. s. value, $I = 0 \cdot 707 \times 40 = 28 \cdot 28$ A

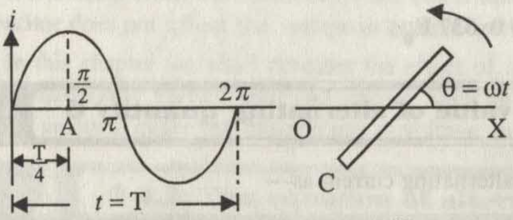
● **Ex. 4.2. An electric lamp is connected to 220V, 50 Hz supply. What is (i) effective voltage and (ii) peak voltage ?**

○ **Solution :** (i) Effective voltage, E = 220V

(ii) Peak voltage, $E_0 = \sqrt{2}\, E = \sqrt{2} \times 220 = \mathbf{311\ volt}$

4.5. Phase of alternating current

By the terms of **'phase'** of an alternating current we mean fraction of time period spent after the current crosses the zero reference line. In the fig 4.6 (i), the phase of the current at A is $\frac{T}{4}$ sec or $\frac{\pi}{2}$ rad, here T is time period. Similarly, the phase of the rotating coil is ωt. [Fig 4.6 (ii)]. This angle is also called phase angle.



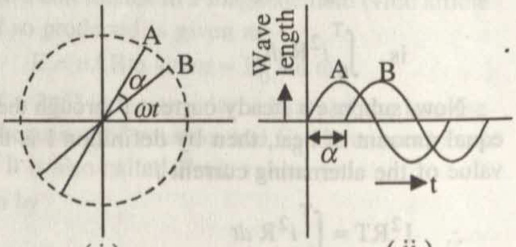
(i) (ii)

Fig. 4·6

➤ 4.5.1. Phase difference :

We consider two identical coils A and B which are inclined at an angle α and they are revolving in a magnetic field with constant angular velocity [Fig 4.7 (i)].

In this case, the peak value of the induced emf is same for both the coils. But the peak value or zero value of the induced emf does not occur simultaneously in the two coils. It occurs first in one coil and then in other coil. [Fig 4.7 (ii)] Coil B is displaced by an angle α from the coil A. It means that the phase difference between the coils is α.



(i) (ii)

Fig. 4·7

The current in coil A leads by angle α with respect to the current in coil B.

4.6. Three simple circuits in alternating electricity

The basic elements that control the current and voltage in electric circuits are resistance, inductance and capacitance. All the elements may not be present in a particular circuit. Also there must be a source of emf in the circuit. In direct current circuit *i.e.* the circuit connected to a battery, the element involved is a resistor.

But in alternating current circuits all the three elements may be involved. Depending on the complexity of connection of the elements, different types of complicated circuits are produced. Before solving for the behaviour of complicated circuits, we deduce the relationship between the voltage and current when a sinnsoidal voltage is applied to the three circuit elements (resistance inductance and capacitance) separately.

- **(A) Alternating circuit containing pure resistor**

Here we consider an a.c. circuit in which a resistor R is connected across an a. c. source of voltage [Fig 4.8(i)]. Let the applied a.c. voltage across the resistor is

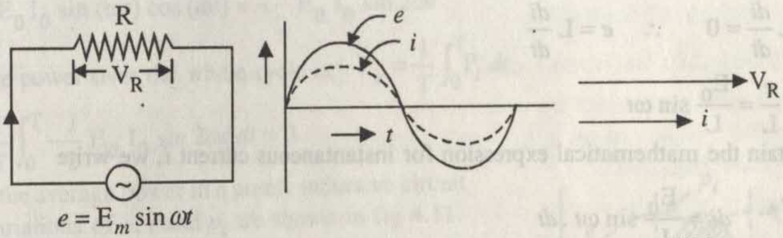
$$e = E_0 \sin \omega t \quad [\ \omega = 2\pi n] \ldots\ldots\ldots\ldots (4.1)$$

The instantaneous value of current is given by

$$i = \frac{e}{R} = \frac{E_0}{R} \sin \omega t = I_0 \sin \omega t \ldots\ldots\ldots\ldots (4.2)$$

Where $I_0 = \frac{E_0}{R}$ is the maximum value of current.

From the equations (4.1) and (4.2) it is found that both e and i are proportional to $\sin\omega t$. So, the current through the resistor is in phase with the voltage across it. In this case, emf and current increases or decreases simultaneously ; Because : (i) when $t = 0$, e and i both equal to zero and (ii) when $\omega t = \pi/2$



(i) (ii) (iii)

Fig. 4·8

or $t = \dfrac{\pi}{2\omega}$, e and i both are maximum. Thus we sec that e and i are always in phase. The variation of voltage and current with time in a purely resistive circuit is shown in fig 4.8 (ii).

The current and voltage vector are therefore collinear as indicated in the phasor diagram 4.8 (iii).

(a) **Potential difference across R :** $V_R = iR = I_0 R \sin\omega t$. So V_R and i are in phase.

(b) **Instantaneous power produced in R :** $P = ei = E_0 I_0 \sin^2 \omega t$

(c) **The average power,** $P_a = \dfrac{1}{T}\displaystyle\int_0^T ei\,dt$

$$\therefore \quad P_a = \frac{1}{T} E_0 I_0 \int_0^T \sin^2 \omega t\,dt = \frac{E_0 I_0}{2} = EI$$

= r. m. s. voltage × r. m. s. current.

The source of emf supply energy at this rate and is dissipated across, the resistor as heat. The variation of power over the whole cycle is shown in fig 4.9.

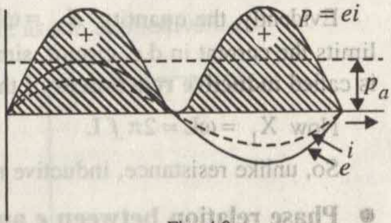
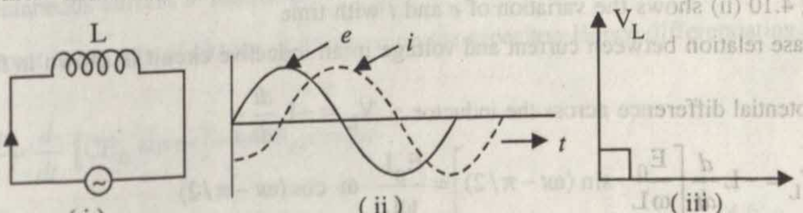


Fig. 4·9

• (B) Alternating circuit containing pure inductance

Now we consider an a. c. circuit consisting of a pure inductor L [Fig 4.10 (i)]. The voltage applied across the inductor is

$$e = E_0 \sin\omega t \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4.3)$$



(i) (ii) (iii)

Fig. 4·10

Due to the presence of inductance as well as due to alternating nature of the voltage applied, the current in the circuit varies with time. If i be the current at any instant of time.

The induced emf set up across the inductor is $V_L = -L\dfrac{di}{dt}\cdot$ This emf opposes the applied emf. The net voltage across the circuit is $\left(e - L\dfrac{di}{dt}\right)$. As the circuit does not contain any resistance, we write

$$e - L\frac{di}{dt} = 0 \qquad \therefore \quad e = L\frac{di}{dt}$$

$$\therefore \quad \frac{di}{dt} = \frac{e}{L} = \frac{E_0}{L}\sin \omega t$$

∴ to obtain the mathematical expression for instantaneous current i, we write

$$di = \frac{E_0}{L}\sin \omega t \, . \, dt$$

$$\therefore \quad i = \frac{E_0}{L}\int \sin \omega t \, dt = \frac{-E_0}{L}\frac{\cos \omega t}{\omega} + k \quad [k \text{ is a constant}]$$

The constant k represents a steady component of current. As the applied voltage is sinusoidal there cannot be any direct current superimposed on the alternating circuit. So, $k = 0$.

$$\therefore \quad i = -\frac{E_0}{\omega L}\cos \omega t = \frac{E_0}{X_L}\sin(\omega t - \pi/2)$$

$$\therefore \quad i = I_0 \sin(\omega t - \pi/2) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4.4)$$

∴ Peak value of the circuit current, $I_0 = \dfrac{E_0}{\omega L}$

Evidently, the quantity $X_L = \omega L$ plays the role similar to that of resistance R in d. c. circuit. As R limits the current in d.c. circuit, similarly X_L limits the current in inductive a.c. circuit. The quantity X_L is called **inductive reactances** of the coil.

Now $X_L = \omega L = 2\pi f L$

So, unlike resistance, inductive reactance increases with the frequency of the a.c. source.

- **Phase relation between *e* and *i* :**

The voltage $e = E_0 \sin \omega t$. Its phase is ωt. Also from equation (4.4) it is clear that the phose of the current i is $(\omega t - \pi/2)$. Thus the current lags behind the voltage by a phase angle of $\pi/2$ or 90°. Hence in an a.c. circuit containing a pure inductance, the current lags behind the voltage by $\pi/2$. It means : (i) When $t = 0$, $e = 0$, but $i = -I_0$ (maximum in opposite direction) and (ii) when $t = \dfrac{\pi}{2\omega}$, $e = E_0$, but $i = 0$

The fig 4.10 (ii) shows the variation of e and i with time

The phase relation between current and voltage in an inductive circuit is shown in fig 4.10 (iii).

(a) Potential difference across the inductor : $V_L = -L\dfrac{di}{dt}\cdot$

$$\therefore \quad V_L = -L\frac{d}{dt}\left[\frac{E_0}{\omega L}\cdot \sin(\omega t - \pi/2)\right] = \frac{E_0 L}{\omega L}\cdot \omega \ \cos(\omega t - \pi/2)$$

$$\therefore \quad V_L = E_0 \ \cos\left(\frac{\pi}{2} - \omega t\right) = E_0 \sin \omega t$$

So, the voltage across the inductor leads the current by 90°.

(b) Instantaneous power in the circuit, $P_i = ei$.

$$\therefore \quad P_i = E_0 \sin \omega t \cdot \frac{E_0}{\omega L} \sin(\omega t - \pi/2)$$

$$= -E_0 I_0 \sin(\omega t)\cos(\omega t) = -\frac{1}{2} E_0 I_0 \sin 2\omega t$$

(c) Average power over the whole cycle is, $P_a = \frac{1}{T}\int_0^T P_i \, dt$

$$\therefore \quad P_a = \frac{1}{T}\int_0^T -\frac{1}{2} E_0 I_0 \sin 2\omega t \, dt = 0.$$

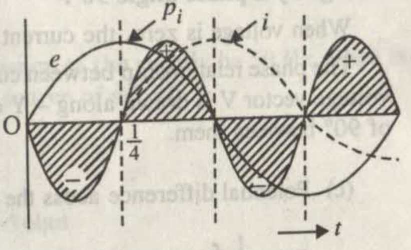


Fig. 4·11

Therefore, the average power in a purely inductive circuit is zero. The variations of e, i and p_i are shown in fig 4.11.

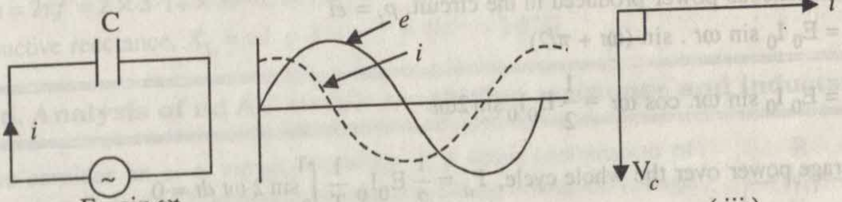
During one-fourth of the cycle ($t = T/4$) power is negative. It implies that energy stored in the coil returns to the source. During the next one-fourth of the cycle $\left(\frac{T}{4} \text{ to } \frac{T}{2}\right)$, energy is stored in the magnetic field of the coil and energy is positive. This process is repeated. Hence over the whole cycle average power is zero.

- **(c) Alternating circuit containing pure capacitor**

Here we consider a pure capacitor C, connected to an a. c. source of voltage $e = E_0 \sin \omega t$ as shown in fig 4.12 (i). At any instant t, the charge q across the capacitor is given by

$$q = ce = CE_0 \sin \omega t \quad \ldots\ldots (4.5)$$

As C is a constant, the charge across the capacitor will oscillate as the applied voltage.



$e = E_m \sin \omega t$

(i) (ii) (iii)

Fig. 4·12

Now, instantaneous current in the circuit $i = \frac{dq}{dt}$.

So, the current is the rate of change of charge q on the capacitor. Hence differentiating equ. (4.5), we get

$$i = \frac{dq}{dt} = \frac{d}{dt}\left[CE_0 \sin \omega t\right] = \omega C E_0 \cos \omega t$$

$$\text{or,} \quad i = \frac{E_0}{1/\omega C} \cdot \sin(\omega t + \pi/2) = I_0 \sin(\omega t + \pi/2) \quad \ldots\ldots (4.6)$$

$$\therefore \text{ Maximum current,} \quad I_0 = \frac{E_0}{X_c}\left[X_c = \frac{1}{\omega C}\right] \quad \ldots\ldots (4.7)$$

From the equation (4.7), it can easily be said that X_c behaves as the resistance of the circuit and its unit is also **ohm**. Hence X_c is called **capacitive reactance**.

As $X_c = \frac{1}{2\pi f C}$, capacitive reactance varies inversely as the frequency of a. c. supply voltage. X_c decreases with increase of frequency.

- **Phase relationship between *e* and *i* :**

Comparing $e = E_0 \sin \omega t$ with the equation (4.6), it is evident that current leads the voltage by a phase angle of $\pi/2$ or 90°. Hence **in an a. c. circuit consisting of an ideal capacitor, the current leads the voltage by a phase angle 90°.**

When voltage is zero, the current is maximum and vice versa as shown in fig 4.12 (ii).

The phase relationship between current and voltage is shown in the phasor diagram 4.12 (iii). Here the voltage vector V is drawn along – Y direction and the current vector I along + X direction with an angle of 90° between them.

(c) Potential difference acoss the capacitor, $V_c = \frac{q}{C} = \frac{1}{C}\int i\, dt$

$$\therefore \quad V_c = \frac{1}{C}\int I_0 \sin(\omega t + \pi/2)\, dt = \frac{1}{C} I_0 \int \cos \omega t\, dt$$

$$= \frac{I_0}{C}\ \frac{\sin \omega t}{\omega} = E_0 \sin \omega t$$

So, the voltage across the capacitor is $\frac{\pi}{2}$ rad or 90° behind the current. The phase difference has been shown in fig 4.12 (iii).

(b) Instantaneous power produced in the circuit, $p_i = ei$

$$\therefore \quad p_i = E_0 I_0 \sin \omega t \cdot \sin(\omega t + \pi/2)$$

$$= E_0 I_0 \sin \omega t . \cos \omega t = \frac{1}{2} E_0 I_0 \sin 2\omega t$$

(c) Average power over the whole cycle, $P_a = \frac{1}{2} E_0 I_0 \frac{1}{T}\int_0^T \sin 2\,\omega t\, dt = 0$

The average power is zero in a purely capacitive circuit similar to purely inductive circuit. The variations of *e*. *i* and P_i over a time period T is shown in fig 4.13. During one-fourth of a cycle, the source supplies energy to the capacitor and electric field is produced. During next one-fourth of the cycle, capacitor returns whole energy to the source. This proess continues. So, during each half cycle (0 to T/2), the power curve has two loop one positive and the other negative. To total power is zero. It is to be noted that the power varies with double the frequency as compared to the frequency of the voltage or current.

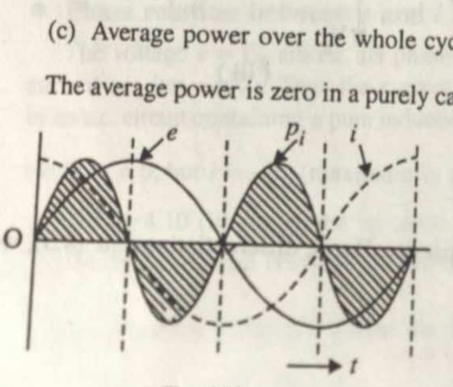


Fig. 4·13

- **Ex. 4.3. A circuit of pure inductance of 0·04 H is supplied with alternating current whose peak value of 15A and frequency 50 Hz. Determine : (i) Equation for current, (ii) equation for induced emf (iii) equation for applied emf and (iv) Maximum value of induced current.**

○ **Solution :** (i) Peak value of the alternating current, $I_0 = 15$ A and frequency, $f = 50$ Hz

∴ The equation for current, $i = I_0 \sin \omega t$

∴ $i = I_0 \sin 2\pi ft$

$= 15 \sin 314\,t$ Amp. [$\omega = 2\pi f = 2 \times 3{\cdot}14 \times 50 = 314$ rad/s]

(ii) The equation for induced emf : $e_i = -L\dfrac{di}{dt} = -L\dfrac{d}{dt}(15 \sin 314t)$

$$\therefore \quad e_i = -0{\cdot}04 \times 15 \times 314 \cos 314t$$

$$= \mathbf{-188{\cdot}4 \cos 314t \text{ volt}}$$

(iii) The equation for applied emf, $e = -e_i =$ **$188{\cdot}4 \cos 314t$ volt**

(iv) Peak value of the induced emf, $E_0 =$ **$-188{\cdot}4$ volt**

- **Ex. 4.4. The capacitance of a capacitor is 10μF. If the frequency of the supply be 50 Hz, what is its reactance ? What current it will draw from a 110V–50 Hz source of supply ?**

○ **Solution :** Capacitive reactance, $X_c = \dfrac{1}{2\pi fC}$

$$\therefore \quad X_c = \frac{1}{2 \times 3{\cdot}14 \times 50 \times 10^{-5}} \; [C = 10^{-5} F] \quad \text{or, } X_c = 318{\cdot}5 \text{ ohm}$$

r. m. s. voltage of the supply = 110 V

∴ r. m. s. current from the supply $= \dfrac{110}{318{\cdot}5} = \mathbf{0{\cdot}345\ A}$

- **Ex 4.5. A coil of self-indutance 5mH has a negligible resistance. Find the inductive reactance of the coil for current of frequency 50 Hz.**

○ **Solution :** $L = 5 \times 10^{-3}$ H, $X_L = \omega L = 2\pi fL$

Here $\omega = 2\pi f = 2 \times 3{\cdot}14 \times 50 = 314$ rad / s

∴ Inductive reactance, $X_L = \omega L = 314 \times 5 \times 10^{-3} =$ **1·57Ω**

✪ 4.6. Analysis of an AC circuit containing resistance and inductance ✪

Here we consider an a. c. circuit consisting of a series combination of resistor R and an inductor L connected to an alternating source of voltage $E = E_0 \sin \omega t$ as shown in fig 4.14.

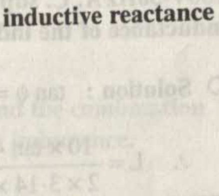


Fig. 4·14

Let I and E be the instantaneous current and emf in the circuit. V_R and V_L are the voltage at that instant across resistor R and inductor L respectively.

∴ $V_L = I\omega L = I X_L$ and $V_R = IR$, here X_L is inductive reaction To obtain mutual relation between E, V_R and V_L, we draw phasor diagram for current and voltage.

We have learned that V_L and V_R cannot be added arithmetically because of a phase difference between them. They have to be added vectorially. When an alternating current flows through a resistor, the voltage and current are in phase *i. e.* V_R and I are in phase. So in the phasor diagram V_R (= IR) and I are drawn as vectors OA and OD along X-axis. Here OA = V_R and OD = I.

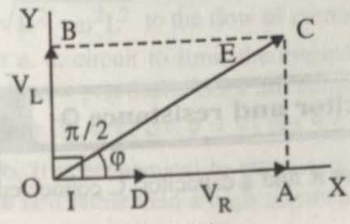


Fig. 4·14(a)

Again, when an alternating current flows through an inductor, the voltage leads the current by 90°. As the current vector I has been drawn along the X-axis, the valtage vector V_L is drawn along Y-axis [Fig 4.14(a)].

Here OB = V_L. Evidently OC represents instantaneous emf E. Now, from the triangle OAC,

$$OC = \sqrt{OA^2 + AC^2} = \sqrt{OA^2 + OB^2} = \sqrt{{V_R}^2 + {V_L}^2} = \sqrt{(IR)^2 + (IX_L)^2}$$

$$\therefore \quad E = I\sqrt{R^2 \times {X_L}^2}$$

$$\therefore \quad I = \frac{E}{\sqrt{R^2 + XL^2}} = \frac{E}{\sqrt{R^2 + \omega^2 L^2}} = \frac{E}{Z} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.8)$$

The quantity $Z = \sqrt{R^2 + \omega^2 L^2}$ is called the **impedance** of the circuit because it impedes the flow of alternating current in the circuit.

- **Phase relation between E and I :**

From the phason diagram 4.14, we sec that the voltage E leads the current I by a phase angle φ.

$$\tan\varphi = \frac{AC}{OA} = \frac{V_L}{V_R} = \frac{I\omega L}{IR} = \frac{\omega L}{R}$$

Hence, for an L-R circuit, the emf equation and current equation are represented by

$$E = E_0 \sin \omega t \text{ and } I = I_0 \sin(\omega t - \varphi)$$

Here E_0 and I_0 are the peak value alternating voltage and current.

- **Ex. 4.6. A series combination of a resistor of 100Ω and an inductor are conneted across a 100V-50Hz A. C. supply. If the phase angle between the voltage and current be π/4 rad, what is the inductance of the inductor.**

○ **Solution :** $\tan\phi = \frac{\omega L}{R} = \frac{2\pi f L}{R} \quad \therefore \quad L = \frac{R\tan\phi}{2\pi f}$

$$\therefore \quad L = \frac{10 \times \tan 45°}{2 \times 3{\cdot}14 \times 50} = 0{\cdot}032 \text{ H}$$

∴ Self inductane is L = **0·032 H**

- **Ex. 4.7. To the series combination of 25Ω resistor and 100 *m*H inductor an a.c. voltage of 100V, 50 Hz is applied. Calculate :**

(i) inductive reactance (ii) impedance (iii) circuit current and (iv) phase angle.

○ **Solution :** (i) Inductive reactance, $X_L = \omega L = 2\pi f L$

$$\therefore \quad X_L = 2 \times 3{\cdot}14 \times 50 \times 100 \times 10^{-3} = \mathbf{31{\cdot}4\Omega}$$

(ii) Impedance, $z = \sqrt{R^2 + {X_L}^2} = \sqrt{(25)^2 + (31{\cdot}4)^2} = \sqrt{1610{\cdot}96} = \mathbf{40{\cdot}14\Omega}$

(iii) r. m. s. current, $I = \frac{100}{40{\cdot}14} = 2{\cdot}49\Omega$

(iv) $\tan\varphi = \frac{X_L}{R} = \frac{31{\cdot}4}{25} = 1{\cdot}256 \quad \therefore \quad \varphi = \mathbf{51{\cdot}47°}$

○ 4.7. Analysis of an AC circuit containing Capacitor and resistance ○

We consider a circuit consisting of a series combination of a resistor R and a capacitor C connected to a source of voltage $E_0 = E_0 \sin \omega t$ as shown in fig 4.15.

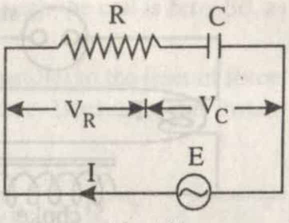


Fig. 4·15

Instantaneous voltage is E and corresponding current I. V_R and V_C are the voltages at that instant across the resistor R and capacitor respectively, where $V_R = IR$ and $V_C = I X_C$. Here X_C is the capacitive reactance.

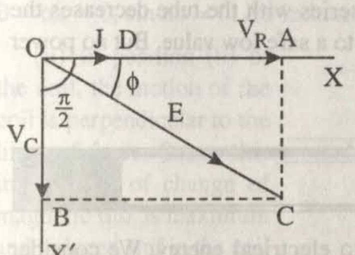


Fig. 4·16

To obtain a relation between E, V_R and V_C , phasor diagram for current and voltage is draw in fig 4.16. Since V_R and I are in phase, they are drawn along the X-axis with the vector OA representing V_R and the vector OD representing I. Again, when alternating current flows through a capacitor, the voltage across it lags behind the current by a phase angle of 90°. As I is along the X-axis, V_C is along the negative Y-axis, with OB representing the magnitude of V_C. The diagonal OC represents the instantaneous voltage. It is clear from the diagram.

$$OC = \sqrt{OA^2 + AC^2} = \sqrt{OA^2 + OB^2} = \sqrt{{V_R}^2 + {V_C}^2}$$

$$= \sqrt{(IR)^2 + (IX_C)^2} = I\sqrt{R^2\ {X_C}^2} = IZ$$

$Z = \sqrt{R^2 + {X_C}^2} = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$ is known as the impedance of the C-R circuit.

- **Phase relation between E and I :**

From the phasor diagram we see that the emf lags the current by the angle $\phi = \angle COA$

$$\therefore \quad \tan\phi = \frac{AC}{OA} = \frac{I\,X_C}{IR} = \frac{I/\omega C}{IR} = \frac{1}{\omega CR}$$

- **Ex. 4.8. A 10Ω resistor, a capacitor and an ammeter all connected in series and the combination is connected to 220V-50 Hz line. If the ammeter reads 2·0A, calculate capacitive inductance.**

○ **Solution :** Impedance of the circuit, $Z = \frac{E_0}{I_0} = \frac{220}{2} = 110\Omega$

Now, $Z = \sqrt{R^2 + {X_C}^2}$

$$\therefore \quad XC = \sqrt{Z^2 - R^2} = \sqrt{(110)^2 - 10^2} \quad [R = 10\Omega]$$

$$= \sqrt{(110)^2 - 10^2} = \sqrt{220 \times 100} = \mathbf{109 \cdot 54\ Ohm}$$

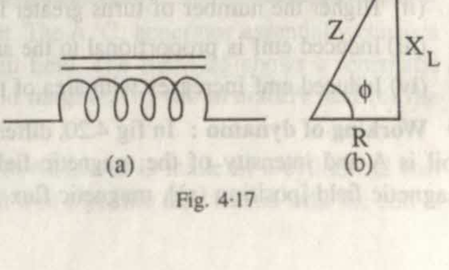
❂ 4.8. Choke coil ❂

A choke is an inductor (coil) having a large self-inductance, but its resistance is very low. It is made of large number of turns of thick copper wire wound on a laminated iron core. Because of its high inductance the choke coil offers a high impedance

$Z = \sqrt{R^2 + \omega^2 L^2}$ to the flow of current. It is therefore, used in an a. c. circuit to limit the current. As its resistance is very low, it practically does not consume any power,

Here $P_{av} = VI\cos\phi = VI\cos 90° = 0 \ (\phi \equiv 90°)$

So, it is economical to cut a.c. current by the use of a choke coil rather than a high resistance.



(a) (b)

Fig. 4·17

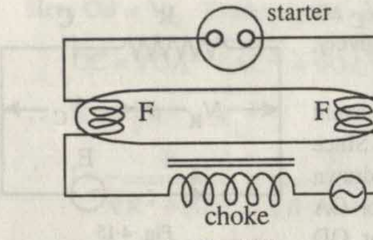


Fig. 4·18

● **Application :** The choke is used in series with flourescent tube working on a.c. mains [Fig 4.18]. If the tube is directly connected to 220V, a large current would flow through it and the tube may be damaged. The use of a choke in series with the tube decreases the voltage across the tube to a safe low value. But no power is wasted.

❂ 4.9. Dynamo ❂

A dynamo is a contrivance which converts the mechanical energy to electrical energy. We consider two types of dynamo (i) A. C. dynamo (ii) D. C. dynamo.

● **Principle of dynamo :** The principle dynamo is based on electromagnetic induction. A closed coil is rotated at constant angular velocity in a magnetic field. The magnetic flux linking the coil changes at constant rate. So, an emf is induced in the coil according to Faraday's law of electro magnetic induction. This induced emf is alternating one. If the two ends of the coil is connected to an external circuit, alternating current flows through it.

This alternating current can be converted to direct current by a device known as commutator.

In fig 4.19, the rectangular armature coil rotates about a horizontal axis with a constant angular velocity. The coil is so placed between the N and S poles that lines of induction are perpendicular to the axis of rotation of the coil. The emf induced in the coil is alternating in nature.

● **Expression for the induced emf :** Fig 4.19 (ii) shows the section of the coil and the magnetic field. Let the angular velocity of the coil be ω.

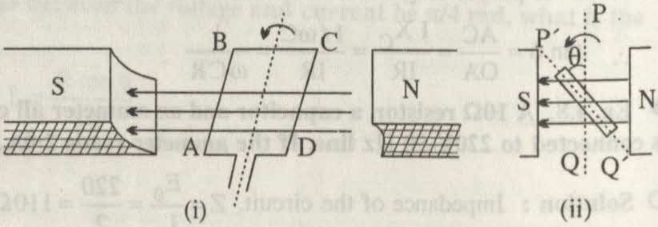


Fig. 4·19

At the position PQ, the plane of the coil is perpendicular to the lines of induction. In time t, the coil rotates by an angle θ. The amount of flux linking the coil at this instant is, $\varphi = n\text{AB}\cos\theta = n\text{AB}\cos\omega t$.

$$\therefore \quad \text{The induced emf : } e = -\frac{d\varphi}{dt} = n\text{AB}\omega\sin\omega t$$

$$\therefore \quad e = \text{E}_0 \sin\omega t \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.9)$$

Here E_0 is the peak value of the induced emf.

From the equation 4.9, we see that. —

(i) The induced emf is proportional to intensity of magnetic-field.

(ii) Higher the number of turns greater is the induced emf.

(iii) Induced emf is proportional to the angular velocity of the coil.

(iv) Induced emf increases with area of the coil.

● **Working of dynamo :** In fig 4.20, different positions of the armature coil are shown. The area of the coil is A and intensity of the magnetic field is B, when the plane of the coil is perpendicular to the magnetic field [position (a)], magnetic flux passing through the coil is BA. After the coil rotates by 90°

[position (b)], the plane of the coil is parallel to the field, the flux passing through the coil is zero. So, as the coil rotates the flux passing through the coil changes.

(i) When the coil is position (a), its instantaneous direction of motion is parallel to the lines of force, the rate of change of flux is zero. At this moment induced emf in the coil is zero. Because at this instant $\theta = \omega t = 0$, $\sin \omega t = 0$. So induced emf $e = 0$.

(ii) In position (b) of the coil, the motion of the coil is perpendicular to the lines of force. So, at this instant rate of change of magnetic flux is maximum and induced emf in the coil is also maximum. But from the position (a) to (b), amount of flux passing through the coil gradually decreases.

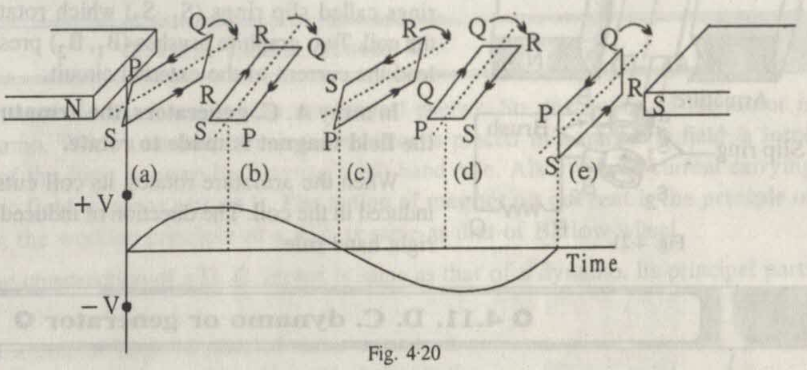


Fig. 4·20

In position (b), ω = 90°, plane of the coil is parallel to the field lines. So, $\sin \omega t = 1$ and induced emf, $e = E_0$. Evidently, as ωt changes from 0° to 90°, the induced emf gradually increases and becomes maximum when $\omega t = 90°$.

(iii) After further rotation by 90°, when the coil comes to the position (c) from the position (b), magnetic flux passing through the coil becomes maximum. At this instant, the motion of the coil is again parallel to the field. So in this position also no emf is induced in the coil.

In position (c), **$\omega t = 180°$, $\sin \omega t = 0$ and induced emf $e = 0$.**

(iv) Next the coil comes to the position (d) after rotating further by 90°. The plane of the coil is again perpendicular to the field and rate of change of lines of force is maximum. So, the induced emf is again maximum [similar to position (b)]. But in this position, the direction of emf is opposite.

In position (d) **$\omega t = 270°$, $\sin \theta = -1$ and $e = -E_0$**

(v) After rotating again by 90°, the coil comes to the position (e) from the position (d). This positions of the coil is similar to the position (a) of the coil. At this instant $\omega t = 360°$, $\sin \omega t = 0$ and $e = 0$.

Evidently from position (a) to (e), one full rotation of the coil is completed and time spent is equal to one time period. The variation of induced emf in the coil during one full rotation is shown in the lower portion of the fig 4.20. During the time interval from 0 to T/2 [from position (a) to (c)] the direction of the induced emf is opposite to the direction of the emf induced in the time interval from T/2 to T [position (c) to (e)].

So, the induced emf in the coil is alternating one.

✪ 4.10. A. C. dynamo or generator ✪

A. C. dynamo supplies alternating current to external circuit. The A. C. generator essentially consists of a coil of wire called armature rotating in a uniform magneti field. The fig. 4.21 shows a schematic diagram of an A. C. generator. Its basic elements are **(a) a field magnet, (b) the armature and (c) the slip rings and brushes.**

The rotating coil in which emf is induced is called armature ABCD. It is made of a cylindrical soft iron core over which a coil of copper is wound. Due to this soft iron core, the flux linked with the coil is increased by increasing the number of turns, the peak emf is increased. The armature is rotated by a steam engine, a petrol engine or a steam turbine.

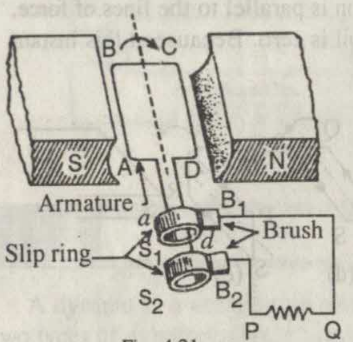


Fig. 4·21

The armature rotates at constant speed between the poles of a powerful field magnet which is generally an electromagnet (N, S).

The terminals of the armature coil are connected to two metallic rings called slip rings (S_1, S_2) which rotate with the same speed as the coil. Two graphite brushes (B_1, B_2) pressing against the two rings, lead the current to the external circuit.

In many **A. C. generators, the armature is kept stationary and the field magnet is made to rotate.**

When the armature rotates, its coil cuts the field lines, an emf is induced in the coil. The direction of induced emf is given by Fleming's right hand rule.

✪ 4.11. D. C. dynamo or generator ✪

A D. C. dynamo or generator consists of following parts

- (i) **Armature :** It is a coil made of copper wire wound on laminated iron core.
- (ii) **Field magnet :** The armature rotates in the field of a strong horse shoe magnet which is usually an electromagnet.
- (iii) **Commutator :** The commutator consists of a metal ring split into two halves which are insulated from one another. One end of the armature is connected to one-half of the split ring and the other end to the other half of the split ring. The commutator is rigidly attached to the armature and so it rotates with the same angular speed and about the same axis.
- (iv) **Brushes :** Two fixed graphite brushes pressing against the two halves of the commutator, lead the current to the external circuit.

❖ **Working of the generator :** When the armature is rotated in the magnetic field, an alternating voltage is induced in it, however, because of the split ring commutator, the connection to the external circuit is reversed twice in each revolution. So the current in the external circuit flows only in one direction.

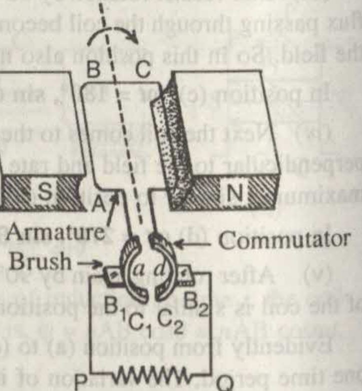
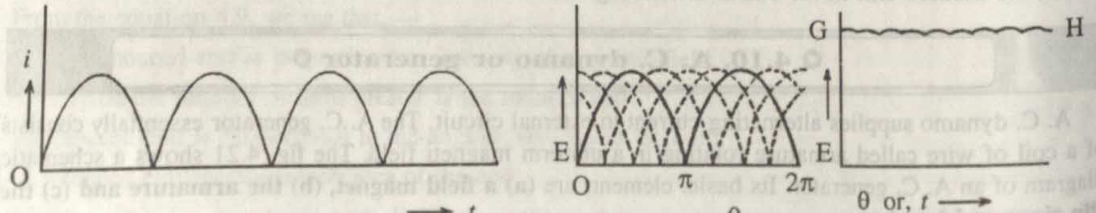


Fig. 4·22

It should, however, be noted that though the current in the external circuit flows in one direction, it is still not steedy. fig. 4.23 shows the emf-time relation of a single coil armature.



(i) (ii) (iii)

Fig. 4·23

In order to get a more steady and unidirectional emf. Several armature coils are used instead of a single coil. The coils are wound in slots spaced round a laminated soft iron cylinder. The coils are placed at equal angular intervals. The peak emf for each coil occurs at different moments during the rotation of the armature. The emfs due to all the coils are added. The resultant emf-time graph are shown in fig. 4.23.

Higher the number of coils, more steady will be the output emf and current.

✪ 4.12. D. C. Motor ✪

An electric motor converts electrical energy into mechanical energy. So, the action of the motor is opposite to that of a dynamo. When a current carrying conductor is placed in a magnetic field, a force acts on it. The direction of the force is given by Fleming's left hand rule. Also when a current carrying coil is placed in a magnetic field, a torque acts on it. **The action of magnet on current is the priniple of electric motor.** Evidently, the working principle of a d. c. is same as that of Barlow wheel.

● **(A) Description :** The construction of a D. C. motor is same as that of a dynamo. Its principal parts are :

● (i) **Armature :** It is a coil of large number of turns wound on a laminated cylindrical soft iron-core. It is capable of rotating between the poles of the field magnet about an axis which is perpendicular to the field direction.

● (iii) **Commutator :** It is a brass ring split into two each half being in contact with one side of the coil. The gap between the two halves of the ring is filled with some insuating material.

● (iv) The current is led to the coil through brushes made of graphite.

❖ **Working principle :** Let initially the plane of the coil is parallel to the magnetic field. When a current from a battery flows through the coil, a torque acts on the coil. Due to the torque direction of rotation of the armature is obtained from Fleming's left hand rule. When the plane of the coil becomes parallel to the field, the torque on the coil becomes zero. But due to inertia, the rotation is continued and the coil overshoots the vertical position. Immediately by the help of the commutator, the direction of current in the coil get reversed. The direction of torque remains unchanged and the coil rotates in the same direction.

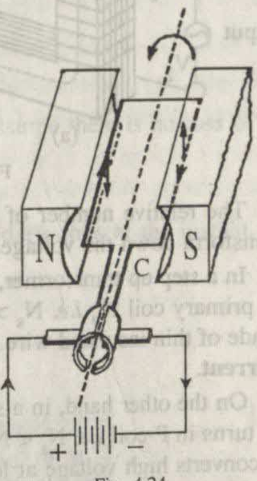


Fig. 4·24

Thus the electrical energy supplied to the armature is converted to mechanical energy of rotation of the coil. By increasing the current strength and using powerful magnet, the armature may be made to rotate at a large speed.

➤ *4.12.1. Back emf. in a motor*

When current is supplied to the armature coil of an electric motor from a battery, the coil begins to rotate in the magnetic field due to the field magnet. The flux linkage with coil changes and an emf is induced in the coil. By Lenz's law this emf opposes the voltage supplied to the motor. For this reason, this induced emf is called **back emf.**

If E be the emf of the battery connected to the armature coil and V be the back emf, then the armature current is

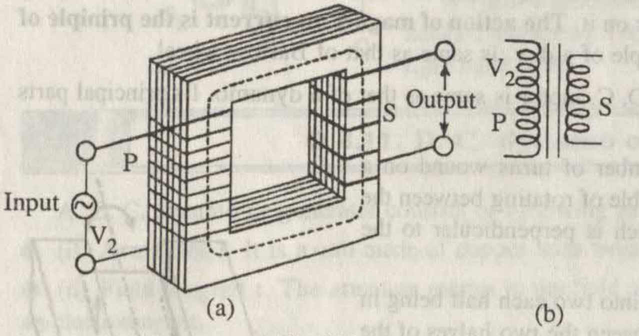
$$i = \frac{E - V}{r} \quad [\, r = \text{resistance of the armature coil}]$$

The back emf is proportional to (i) the strength of the magnetic field and (ii) the speed of rotation of armature.

When the motor is first switched on, the back emf is zero. The starting current is dangerously high ; to limit it, a variable resistance is inserted in series with the armature, which is gradually reduced to zero, as the rotor speed increases.

❂ 4.13. Transformer ❂

A transformar is an electrical device which is used for converting low alternating voltage at high current into high voltage at low current and vice versa.



(a) (b)

Fig. 4·25

- **Principle :** Its working is based on the principle of mutual induction. If two coils are inductively coupled and when current or magnetic flux changes in one coil then an emf is produced in the other coil.
- **Construction :** A transformer consists of two separate coils P and S wound on a soft iron laminated core. P is the primary coil to which a. c. input is applied and 'S' is the secondary coil from which transformed output is obtained. To reduce eddy current, the core is laminated [Fig 4.25]

The relative number of turns in the two coils depends on whether it is required to transform up or transform down the voltage.

In a step up transformer, the number of turns in secondary coil N_s is greater than the number of turns, in primary coil N_p *i.e.* $N_s > N_p$. The P-coil is made of thick insulated copper wire, while the S-coil is made of thin insulated wire. It is used to convert **low voltage at high current into a high voltage at low current**.

On the other hand, in a step down transformer, the number of turns in S-coil is less than the number of turns in P-coil *i.e.* $N_s < N_p$. The P-coils made of a thin wire, while the S-coil is made of a thick wire. It converts high voltage at low current into a low voltage at high current.

- **Theory :** Let an alternating voltage $e = E_0 \sin \omega t$ is applied to the primary and the secondary is open. A small current flows through the primary. The primary current magnetises the core and produces an alternating flux φ to flow in the core. If there is no leak of flux, the same flux passes through the secondary also. Hence flux linkage with P-coil, $\varphi_P = N_P \varphi$ and flux linkage with S-coil, $\varphi_s = N_s \varphi$

So, the back emf in the P-coil, $e_p = \frac{d\varphi_p}{dt} = -N_p \frac{d\varphi}{dt}$ and the induced back emf in the S-coil, $e_s = -\frac{d\varphi_s}{dt} = -N_s \frac{d\varphi}{dt}$ and the induced back emf in the S-coil,

$$e_s = -\frac{d\varphi_s}{dt} = -N_s \frac{d\varphi}{dt}$$

$$\therefore \quad \frac{e_s}{e_p} = \frac{N_s}{N_p}. \qquad \therefore \quad e_s = k\, e_p$$

The ratio $\frac{N_s}{N_p} = k$ is called **transformation ratio.**

- **EMF Equation :**

Consider the variation of flux with time graph as shown in fig 4.26. Flux equation is $\varphi = \varphi_m \sin \omega t$,

where $\omega = \dfrac{2\pi}{T} = 2\pi f$, f is the frequency of the a. c. supply, Magnetic flux increases from zero to maximum value in time

$$t = \frac{T}{4} = \frac{1}{4f}.$$

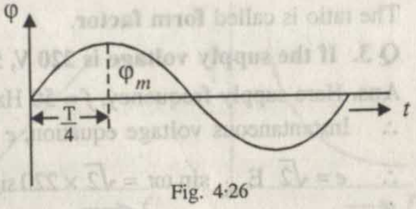


Fig. 4·26

Hence the average rate of change of flux $= \dfrac{\varphi_m}{1/4f} = 4f\,\varphi_m$

$\therefore$ Induced emf per turn $= \varphi f\,\varphi_m$ volt.

As flux φ is sine function, r. m. s. value of the induced emf is obtained by multiplying with form factor 1·11.

$\therefore$ r. m. s. value of emf induced per turn $= 1{\cdot}11 \times 4f\varphi_m = 4{\cdot}44\, f\varphi_m$

$\therefore$ total r. m. s. emf induced in the primary coil is $E_1 = 4{\cdot}44 f\, N_p \varphi_m$.

Similarly, the total induced emf in the S-coil $E_2 = 4{\cdot}44\, f N_3\, \varphi_m$.

$$\therefore \frac{E_2}{E_1} = \frac{N_s}{N_p} = k$$

For a step-up transformer, $k > 1$ and step down transformer $k < 1$. If we assume there is no loss of energy, then

Instantaneous power output = instantaneous power input $E_2 I_2 = E_1 I_1$

Here I_1 and I_2 are respectively the values of current in primary and secondary coils at the instant, when the respective values of the voltage across the two coils are E_1 and I_2

$$\therefore \frac{E_2}{E_1} = \frac{I_1}{I_2}$$

● ● Short Answer Type Questions (with answers) ● ●

- **Question 1. The equation of an alternating current is** $i = 10 \sin\left(200\,\pi t - \dfrac{\pi}{15}\right)$ **A. What is the frequency and peak value of the current ?** **[W. B. H. S. '98]**

○ **Ans.** The general equation of an alternating current is $i = I_0 \sin(\omega t - \phi)$

Here I_0 = peak value and $\omega = 2\pi f$, f = frequency comparing the given equation with this equation, we get peak value of the current I_0 = **10A** and frequency, $f = \dfrac{\omega}{2\pi} = \dfrac{200\,\pi}{2\pi} = \mathbf{100Hz}$

- **Q 2. What is the ratio of the r. m. s. and average value of a sinusoidally varying alternating emf ? What is the ratio called ?**

○ **Ans.** If E_0 be the peak value of the sinusoidally varying alternating emf, then its r. m. s. value is $E = \dfrac{E_0}{\sqrt{2}}$ and its average value, $E_a = \dfrac{2E_0}{\pi}$

$$\therefore \quad \frac{E}{E_a} = \frac{E_0}{\sqrt{2}} \times \frac{\pi}{2E_0} = \frac{\pi}{2\sqrt{2}} = 1{\cdot}11$$

The ratio is called **form factor**.

● **Q 3. If the supply voltage is 220 V, 50 Hz, what is the equation of the instantaneous voltage ?**

○ **Ans.** Here supply frequency, $f = 50$ Hz and $E_{rms} = 220V$.

$\therefore$ Instantaneous voltage equation, $e = E_0 \sin \omega t$

$$\therefore \quad e = \sqrt{2}\ E_{rms} \sin \omega t = \sqrt{2} \times 220 \sin (2\pi \times 50 f)$$

$$\therefore \quad \boldsymbol{e = 310{\cdot}2 \sin 100\, \pi f} \ \textbf{ volt}$$

● **Q 4. What value of current does an A. C. Ammeter measure ?**

○ **Ans.** An A. C. ammeter measures r. m. s. value of alternating current.

● **Q 5. What is the value of inductive reactance in the case of direct current ?**

○ **Ans.** In the case of direct current, the value of inductive reactance is zero. Because inductive reactance $X_L = 2\pi f L$. Now for direct current $f = 0$. Hence $X_L = 0$.

● **Q 6. In a pure inductor the current lags behind the voltage by π/2. What does it mean ?**

○ **Ans.** The statement means that when the voltage in the inductor is maximum (or minimum), the current will react its maximum (or minimum) value after at time $t = T/4$ where T is the time period.

● **Q 7. In an A. C. circuit, the equations for voltage and current are $e = 300 \sin (\omega t + \pi/2)$ and $i = 5 \sin \omega t$. What is the average power produced in the circuit ?**

○ **Ans.** In general instantaneous power, $P_i = ei = E_0 I_0 \sin (\omega t + \varphi) \,.\, \sin \omega t.$

$$\therefore \quad \text{Average power, } P_a = \frac{1}{T}\int_0^T E_0 I_0 \sin (\omega t + \varphi) \,.\, \sin \omega t \,.\, dt$$

$$P_a = \frac{E_0\, I_0}{T} \int_0^T (\sin \omega t \,.\, \cos \varphi + \cos \omega t \,.\, \sin \varphi) \,.\, \sin \omega t \ \ dt$$

$$= \frac{E_0 I_0}{T}\left[\cos\varphi \int_0^T \sin^2 \omega t\, dt + \sin\varphi \int_0^T \sin \omega t \cos \omega t\, dt\right]$$

$$= \frac{E_0 I_0}{T} \cdot \cos\varphi \cdot \frac{T}{2} \quad \left[\because \int_0^T \sin \omega t \cos \omega t\, dt = 0\right]$$

$$= \frac{E_0\, I_0}{2} \cdot \cos\varphi$$

In this case, $\varphi = \pi/2 \quad \therefore P_a = 0.$

● **Q 8. What is power factor ?**

○ **Ans.** In an a. c. circuit, the cosine of the angle between the voltage and current is called power factor.

$\therefore$ power factor = $\cos\varphi$

$$\therefore \quad \cos\varphi \frac{P_a}{E_{rms} \cdot I_{rms}}$$

● **Q 9. Show by graphs how does the inductive reactance and capacitive reactance vary with frequency of the supply.**

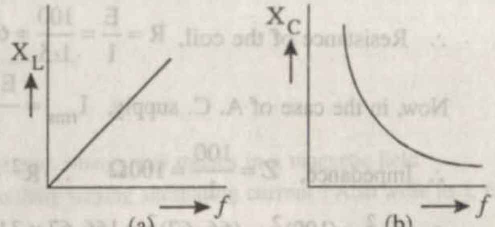
○ **Ans.** (i) Inductive reactance, $X_L = 2\pi f L$

$\therefore$ $X_L \propto f$ variation of X_L with f is shown in fig 4.27 (i).

(ii) Capacitive reactance,

$$X_C = \frac{1}{2\pi fc} \quad \therefore \; X_C \propto \frac{1}{f}$$

variation of X_C with f ($X_C - f$ graph) is shown in fig 4.27 (ii).



(a) (b)

Fig. 4·27

● **Q 10. For the same voltage why a. c. is more dangerous than d. c. ?**

○ **Ans.** Mamiximum value of a. c. voltage is higher than the d. c. voltage. For this reason a. c. is more dangerous than d. c.

• • Numerical Examples • •

● **Ex. 4.9. A voltage given by the equation *e* = 200 sin 314*t* volt is applied to a resistor of resistance 10 ohm. Calculate : (i) r. m. s. value of voltage, (ii) r. m. s. value of current and (iii) Power in the circuit.**

○ **Solution :** (i) Peak value of the voltage, $E_0 = 200$ volt.

$\therefore$ it r. m. s. value, $E_{rms} = \frac{200}{\sqrt{2}} = \mathbf{141\cdot4\ volt}$

(ii) r. m. s. value of current, $I_{r.m.s} = \frac{E_{r.m.s}}{R} = \frac{141\cdot4}{10} = \mathbf{14\cdot14\ A}$

(iii) Power generated, $P_a = E_{rms} \times I_{rms} = 141\cdot4 \times 14\cdot14 = \mathbf{2000\ V}$

● **Ex. 4.10. A capacitor of capacity 1μF is connected to 200 V, 50 Hz A. C. source. What the r. m. s. value of current through the capacitor ?**

○ **Solution :** Capacity, $C = 10^{-6}$ F, frequency, $f = 50$ Hz, $E_{rms} = 200$ V

Capacitive reactance, $X_C = \frac{1}{2\pi fC} = \frac{10^6}{2\pi \times 50} = \frac{10^4}{\pi}$ ohm

$\therefore$ R. M. S. value of current, $I_{rms} = \frac{E_{rms}}{X_C} = \frac{200\,\pi}{10^4} = \mathbf{0\cdot063A}$

● **Ex. 4.11. The inductance of coil is 44 mH. It is connected to 220 V - 50 Hz a. c. supply. Calculate reactance and current through the coil.**

○ **Solution :** Inductive reactance, $X_L = 2\pi fL$

$\therefore$ $X_L = 2\pi \times 50 \times 44 \times 10^{-3} = \mathbf{13\cdot82\ ohm}$

and the current strength, $I_L = \frac{220}{13\cdot82} = \mathbf{15\cdot92A}$

● **Ex. 4.12. When a potential difference of 10V D. C. is applied across an air core coil, the current is 1·5A. But an A. C. voltage of 100 V, 50 Hz produces a current of 1·0A in the same coil. What is the reactance of the coil ?**

○ **Solution :** In the case D. C. supply, current $I = \frac{E}{R}$

$\therefore$ Resistance of the coil, $R = \frac{E}{I} = \frac{100}{1 \cdot 5} = 66 \cdot 67\ \Omega$

Now, in the case of A. C. supply, $I_{rms} = \frac{E_{rms}}{Z}$

$\therefore$ Impedance, $Z = \frac{100}{1} = 100\Omega \quad \therefore R^2 + X_L{}^2 = (100)^2$

$\therefore X_L{}^2 = (100)^2 - (66 \cdot 67)^2 = 166 \cdot 67 \times 31 \cdot 33.$

$\therefore$ Reactance, $X_L = \mathbf{74 \cdot 63\ \Omega}$

● **Ex. 4.13. To the L-R circuit consisting of series combination of R = 10Ω ohm resistor and an inductor of L = 800 *m*H, an alternating voltage E = 200 sin 300*t* is applied. Find (i) impedance of the circuit. (ii) maximum circuit current and (iii) Power factor of the circuit. [C. B. S. E. 1994]**

○ **Solution :** Peak value of the voltage, $E_0 = 200V$, $\omega = 300$ rad/s

(ii) $X_L = \omega L = 300 \times 800 \times 10^{-3} = 240\Omega$

$\therefore$ Impedance of the circuit, $Z_L = \sqrt{R^2 + X_L{}^2} = \sqrt{(100)^2 + (240)^2} = 240 \cdot 2\ \Omega$

(ii) Maximum current, $I_0 = \frac{E_0}{Z_L} = \frac{200}{240 \cdot 2} = 0 \cdot 83A$

(iii) Power factor, $\cos\varphi = \frac{R}{Z_L} = \frac{10}{240 \cdot 2} = \mathbf{0 \cdot 0416}$

● **Ex. 4.14. A motor has an armature resistance of 5Ω and takes a current of 2A due to D. C. voltage of 100 V is applied to it, when operating at the rated speed. Calculate : (i) back emf (ii) Starting current (iii) Power input (iv) Output power (v) Efficiency**

○ **Solution :** (i) $I = \frac{E - e}{R} \quad \therefore$ back emf, $e = E - IR = 100 - 2 \times 5 = \mathbf{90\ V}$

(ii) Starting current, $I_s = \frac{E}{R} = \frac{100}{5} = 20A$

(iii) Input power = $Ei = 200 \times 2 = 200$ W

(iv) Output power = $ei = 90 \times 2 = 180$ W

(v) Efficiency, $\eta = \frac{\text{output}}{\text{input}} \times 100 = \frac{180}{200} \times 100 = \mathbf{90\%}$

• E ◇ X ◇ E ◇ R ◇ C ◇ I ◇ S ◇ E •

A. Objective type questions :

● Fill in the blanks :

1. General equation of an alternating emf is ———.
2. If peak value of the alternating current is E_0, it average value is ——— and its r. m. s value is ———.
3. In dynamo ——— energy is converted to ——— energy
4. The working principle of transformer is ———
5. Inductive reactance is ——— and capacitive reactance is ———.

6. In pure inductance voltage ——— the current by 90°.
7. In pure capacitance voltage ——— the current by 90°.
8. Impedance of a series L-R circuit is ———.
9. Inpedance of series C-R circuit is ———.
10. Power factor of pure inductive or capacitive circuit is ———.

◢ B. Very short answer type questions :

1. What is the source of alternating current ?
2. Write the expression for the peak value of the induced emf, when a coil rotates in a magnetic field.
3. What is the average value over a half cycle of a sinusoidally varying alternating current ? Also write its r. m. s. value.
4. What is the form factor ? What is its value when the sinusoidally varying alternating current is concerned ?
5. Write the expression for inductive reactance. Draw a graph showing its variation with frequency.
6. Write the expression for capacitive reactance. Draw a graph showing its variation with frequency.
7. In pure inductance or capacitance what is the phase difference between voltage and current ?

◢ C. Short answer type questions :

1. Define average value of alternating quantity. Why it is calculated over half cycle in the case of sinusoidally varying voltage and current ?
2. How one can calculate r. m. s. value of alternating quantity ?
3. What is farm factor ? Calculate its value for sinusoidal voltages.
4. What is inductive reactance ? Give its expression. On which factor its value depend ?
5. What is capacitive reactance ? Give its expression. On which factor its value depend ?
6. Why a d. c. voltmeter cannot measure A. C. voltage ?
7. The A. C. main of a house is 220V, 50 Hz. Write the equation of emf.

◢ D. Essay type questions :

1. (a) A rectangular coil is rotated in a magnetic field with a constant angular velocity. Obtain an expression for the induced emf.
   (b) On which factor the value of induced emf depend ?
2. What are meant by average value and r. m. s value of alternating voltage ? Obtain the expression for these quantities.
3. Obtain an expression for alternating voltage and current for the circuit containing. (i) resistance, (ii) inductance and (iii) capacitance. Show graphically the variation of voltage and current with time. Draw phasor diagram in all cases.
4. Obtain an expression for impedance of L-R circuit by phasor diagram.
5. Obtain an expression for impedance of C-R circuit by phasor diagram.

◢ E. Simple numerical problems :

1. The alternating current in circuit is $i = 6 \sin 314t$ Amp. What r. m. s. value of current ? **[Ans.** 4·24A]
2. An alternating current in a circuit is given by $i = 50 \sin 100\,\pi t$. Calculate : (i) frequency, (ii) average value of the current in the positive half cycle, (iii) effective value of current (iv) current $\frac{1}{300}$ sec after the current was zero.
   [Ans. (i) 50 Hz (ii) 31·85A (iii) 35·35A (iv) 43·3A]
3. An inductor of inductance 1·0 H is connected to A. C. source 110V, 70 Hz. Calculate (i) reactance (ii) effective value of current and (iii) maximum value of current. **[Ans.** (i) 440Ω (ii) 0·25A (iii) 0·354A]
4. An alternating current $i = 0{\cdot}2 \sin 200\,t$ is applied to a coil of inductance 2 mH. Calculate (i) induced emf in the coil and (ii) maximum induced current. **[Ans.** (i) $e = 0{\cdot}08 \cos 200t$ volt (ii) 0·2A]

   [Hints : $e = L\dfrac{di}{dt}$, $I_0 = \dfrac{E_0}{X_L}$]

5. An a. c. voltage 220V, 50Hz is applied to a 100 Ω resistor. Calculate : (i) Peak voltage (ii) Average voltage and (iii) r. m. s. value of current. **[Ans.** (i) 311·1V (ii) 198·1V (iii) 2·2A]
6. The frequency of alternating current is 50Hz. After what time, the current reaches its r. m. s. value ? **[Ans.** $2{\cdot}5 \times 10^{-3}$ sec]
7. The inductance of a coil is 14 mH. If the frequency of a. c. applied to the coil be 100 Hz, what is its reactance ? **[Ans.** 8·8Ω]
8. The reactance of a coil at 200 Hz frequency is 22 ohm. It is connected to 220V, 50 Hz supply. Calculate (i) inductance of the coil (ii) reactance. **[Ans.** (i) 17·5 mH (ii) 5·5Ω]
9. When a resistance is connected to 220V, 50 Hz supply, the current in the circuit is 2A. What capacitance be connected in series with the resistor so that current decreases to 1A ? **[Ans.** R = 110Ω, C = 16·7 μF]
10. When a 200V D. C. is applied to a coil, the current is 1·0A. But when a 200V, 50Hz A. C. is applied to the coil the current is 0·8 A. Calculate (i) resistance of the coil (ii) inductance of the coil. **[Ans.** (i) 200Ω and (ii) 0·48 H]
11. When an electric lamp is connected to 100 V D. C., the current is 10A. If it is required to be conneted to 200V, 50Hz, what inductance be connected in series with the lamp. **[Ans.** L = 0, 055 H]
12. An LR circuit of impedance 400 Ω and a resistance 300 Ω is connected to a 220V, 50Hz supply. Calculate (a) the inductance of the circuit and (b) the phase angle. **[Ans.**(a) 0·84 H (b) 41·4°]
13. An alternating voltage V applied to the series combination of resistor R and inductance L produces an a. c. of 1·0A. Potential difference across the resistor is 120V and that across the inductor is 160 V. Calculate : (i) Supply voltage V, (ii) impedance of the circuit and (iii) phase difference between voltage and current. **[Ans.** (i) V = 200V (ii) 200Ω (iii) $\cos\varphi = 0{\cdot}6$]
14. A bulb of resistance 10Ω and an inductance L are connected in series and an alternating voltage 100 V, 50 Hz is applied to the combination. If the phase difference between voltage and current by $\pi/4$ rad, calculate the value of L. **[Hints :** $\cos\varphi = R/z$] **[Ans.** 0·032 H]
15. A series combination of a 20Ω resistor and a 50 *m*H inductor is connected to a 100V, 50Hz supply. Calculate, (i) circuit resistance (ii) r. m. s. current in the circuit. **[Ans.** (i) 25·43Ω, (ii) 3·93A]
16. When 110V D. C. is applied to a coil the current is 11A. But when 110 V, 50 Hz A. C. is applied the current reduces to 0·5A. Calculate (i) resistance of the coil (ii) circuit impedance and (iii) inductance. **[Ans.** (i) 10Ω (ii) 220Ω (iii) 0·7 H]
17. To the series combination of a 20 ohm resistor and a capacitor an alternating voltage 220 V - 50Hz is applied. If the circuit current is 2·5A, calculate reactance of the capacitor. **[Ans.** 85·7 Ω]
18. An alternating current of 1·5 mA and angular frequency 300 rad s^{-1} flows through a 10kΩ resistor and a 0·5 μF capacitor in series. Find the r. m. s. voltage across the capacitor and impedance of the circuit. **[C. B. S. E. 93]** **[Ans.** (i) 10V (ii) 12 kΩ]
19. An emf of $E = 100 \sin 2000\,\pi t$ is applied to a circuit containing a capacitance of 0·1 μF and a resistance of 500Ω in series obtain the equation for instantaneous current in the circuit. **[Ans.** $i = 0{\cdot}06 \sin(2000t + 1{\cdot}27)$]

◢ F. Harder numerical problems :

1. The current equation in a circuit is $I = 20 \sin \omega t + 10 \sin 2\omega t$. What is the rms value of current ? **[Ans.** 15·8 A]
2. In an ideal transformer, the number of turns of the primary coil is 200 and that in the secondary coil is 1000. If the supply voltage 200V and input power 10KW, calculate (i) output voltage and (ii) current in primary coil. **[Ans.** (i) 1000V (ii) 50A.] **[C. B. S. E. 2002]**
3. A transformar has 200 windings in the primary and 400 windings in secondary. The primary is connected to an a. c. supply of 110 V and a current of 10A flows in it. Calculate voltage and current in secondary winding. **[Ans.** 220V, 5A]

MODERN PHYSICS – II

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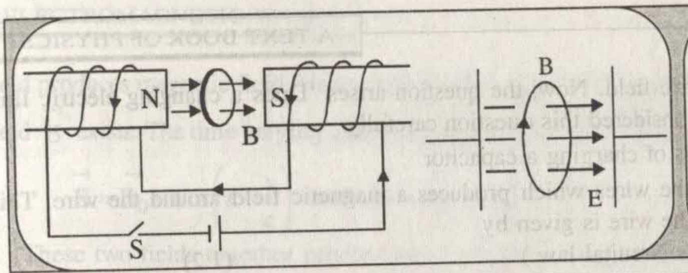


CHAPTER 1

ELECTROMAGNETIC WAVES

● **TOPICS :** ○ Concept of displacement current and origin of electromagnetic waves; ○ Electromagnetic waves; ○ Characteristics of electromagnetic waves; ○ Transverse nature of electromagnetic waves; ○ Maxwell's equations; ○ Electromagnetic spectrum; ○ Energy associated with electric and magnetic field of electromagnetic waves; ○ Transmission of EM Radiation through atmosphere; ○ Short answer type questions (with answers); ○ Exercise.

❂ 1.1. Concept of displacement current and origin of electromagnetic waves ❂

The origin of electromagnetic waves is related to the concept of displacement current. This concept was introduced by **Maxwell.**

▲ A. Faraday's experiment and Maxwell's hypothesis :

Maxwell's hypothesis was based on the results of Faraday's experiment on electromagnetic induction. Suppose a conducting loop is placed between the poles of an electromagnet [Fig 1.1]. As soon as the switch 'S' is closed, the current in the electromagnet starts rising. As a result magnetic flux linking the loop placed between the poles of the electromagnet grows with time. If at any instant the magnetic field is B, then the magnetic flux φ_B passing through the loop of area A is $\varphi_B = BA$. Consequently, the emf induced in the coil is $e = -\dfrac{d\varphi_B}{dt}$

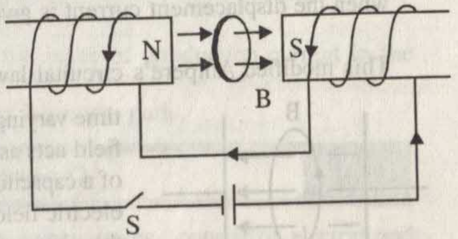


Fig. 1·1

This induced emf can be attributed to the electric field E induced in the coil whose value is given by

$$e = \oint \vec{E}.\,d\vec{t} = -\frac{d\varphi_B}{dt} \cdots\cdots\cdots\cdots (1.1)$$

The induced electric field survives so long as magnetic flux φ_B is changing. As soon as current in the electromagnet is fully established, the magnetic field B (hence φ_B) becomes steady. Immediately induced electric field disappears.

▲ B. Maxwell's thought experiment :

Maxwell thought a similar situation as above in the space between the two plates of a parallel plate capacitor [Fig 1.2]. As soon as the switch's is closed, a temporary changing current flows and charge q starts accumulating on the capacitor plates. At the same time the electric field which is related to growth of charge with time, starts growing. Now, if E be the electric field, the electric flux φ_E passing through the loop of area A (not shown) placed between the plates is $\varphi_E = EA$. Evidently electric flux φ_E also grows with time.

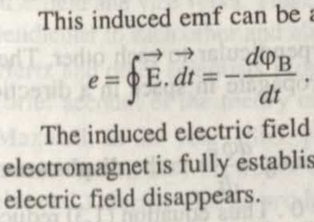


Fig. 1·2

Thus, during the process of charging, the electric flux φ_E passing through the loop varies with time. From Faraday's experiment, we have seen that a changing magnetic field induces electric field. Now, the question arises 'Does a changing electric flux induces a magnetic field ?' Maxwell considered this question carefully.

He observed that during the process of charging a capacitor

(i) a charging current I flows in the wires which produces a magnetic field around the wire. This magnetic field B at a distance r from the wire is given by

$\mathbf{B.\,2\,\pi\, r = \mu_0\, I}$ (It is Ampere's circuital law) .... (1.2)

(ii) The conduction current that flows through the connecting wires cannot pass across the gap between the plates of the capacitor. But a compass needle placed in the gap between the two plates of the capacitor shows a deflection indicating the existence of magnetic field in the gap between the plates. Maxwell assumed a flow of current across the gap so as to explain the existence of magnetic field between the plates. He named it as **displacement current** and attributed its origin to the varrying electric field between the plates of the capacitor. Maxwell proved that the time varying electric field E across the plates of the capacitor give rise to magnetic field B, just as time varying magnetic field B induces electric field E. If the induced magnetic field B is represented by circular lines of force, then for a circular path of radius r,

$$B.2\pi r = \mu_0 \in_0 \frac{d\varphi_E}{dt} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1.2a)$$

Thus, through the connecting wires there is a flow of coduction current I and through the gap across the plates of the capacitor, there is a flow of displacement current I_D. In this light, Ampere's circuital law is modified to $\oint \vec{B}.\vec{dl} = \mu_0(I + I_D)$ .... (1.3)

when the displacement current is given by $I_D = \in_0 \dfrac{d\varphi_E}{dt}$ .... (1.4)

This modified Ampere's circuital law together with Faraday's law $\left[\oint \vec{E}\,.\,\vec{dl} = -\dfrac{d\varphi_B}{dt}\right]$ show that a time varying electric field is a source of magnetic field and a changing electric field acts as a source of magnetic field. Thus in the space between the plates of a capacitor, both the electric field and magnetic field exist. The time varying electric field between the plates of the capacitor is directed perpendicular to the plane of the plates and the time varying magnetic field produced by the electric field is along the perimeter of a circle in a plane parallel to the plates of the capacitor [Fig 1.3].

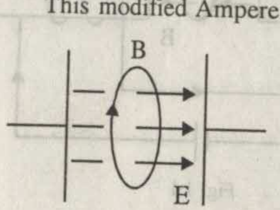


Fig. 1·3

So, the time varying electric field E and the magnetic field B are perpendicular to each other. These two fields combine to constitute electromagnetic waves. These waves propagate in space in a direction perpendicular to direction of propagation of both the fields.

- **A discussion :** In equation (1.3), I is the conduction current and $I_d = \in_0 \dfrac{d\varphi_E}{dt}$ is the **displacement current** between the plates of the capacitor. Now, outside the plates $\varphi_E = 0$. Thus equation (1.3) reduces to equation (1.2). Also between the plates of the capacitor I = 0. Then equation (1.3) reduces to equation (1.2a) so, we conclude that—

(i) When a capacitor is connected to a battery by conducting wires, conduction current flows through the wire and displacement current flows between the plates of the capacitor.

(ii) There two currents are separately discontinuous. But in a closed circuit, sum of these currents $(I + I_d)$ is continuous.

(iii) These two currents are equal in magnitude. But they are of different nature.

(iv) Variation of electric flux between the plates of the capacitor produces displacement current.

Both the currents can produce magnetic field. Conduction current which is due to a motion of electrons produces magnetic field around it. On the other hand, displacement current due to time varying electric field produces magnetic field. Hence between the plates of the capaciter both electric field $\vec{E}$ and magnetic field $\vec{B}$ exists. The time varying electric and magnetic fields can be represented by the following equations.

$$\vec{E} = \vec{E}_0 \sin \omega \left(t - \frac{x}{c} \right) \quad \text{and} \quad \vec{B} = \vec{B}_0 \sin \omega \left(t - \frac{x}{c} \right)$$

These two fields together produce electromagnetic wave which is perpendicular to both electric and magnetic field.

To show **$(I + I_d)$ is continuous :** Ampere's circuital law gives a relation between magnetic field and current. The law is $\int_c \vec{B} \cdot \vec{dl} = \mu_0 I$. During charging of capacitor, a current I enters one plate and leaves the other plate. I is not continuous between the plates, as no charge passes through the space between the plates. Hence $(I + I_d)$ will be continuous if the displacement current between the plates be equal to conduction current through the wire. It can be shown in following way :

These electric field E in the gap of a parallel plate capacitor of plate area A is $E = \frac{q}{\varepsilon_0 A}$ where q is the charge on the capacitor plates at any instant of time during the charging process.

Differentiating, $\frac{dE}{dt} = \frac{1}{\varepsilon_0 A} \frac{dq}{dt} = \frac{I}{\varepsilon_0 A}$ .... (i)

Now displacement current, $I_d = \varepsilon_0 \frac{d\varphi_E}{dt} = \varepsilon_0 \frac{d}{dt}(EA) = \varepsilon_0 A \frac{dE}{dt}$ .... (ii)

Combining (i) and (ii), we get, $I_d = \varepsilon_0 A \frac{I}{\varepsilon_0 A} = I$ which is the value of conduction current in the connecting wires. Thus, the sum $(I + I_d)$ has the same value along the entire path.

❂ 1.2. Electromagnetic waves ❂

Maxwell gave the idea of electromagneting waves. Electromagnetic waves consist of electric and magnetic fields, both varying with space and time. A time varying magnetic field acts as a source of electric field and vice versa. Thus, one provides the source of the other. The electric and magnetic fields are perpendicular to each other and also perpendicular to the direction of propagation of electromagnetic waves.

Hertz and other scientists produced and studies electromagnetic waves experimentally. The following is a brief account of the theory of electromagnetic waves.

Maxwell, in the year 1865, predicted the existence of electromagnetic waves on the basis of some equations known as Maxwell's equation [vide next article 1.5]. According to him oscillating (i.e. accelerated) electric charge produces time varying magnetic field, which in turn produces a time varying electric field. These two fields are mutually perpendicular and one produces the other.

Thus Electromagnetic waves consist of time varying electric and magnetic fields acting at right angles to each other as well as at right angles to the direction of propagation of waves.

❂ 1.3. Characteristics of electromagnetic waves ❂

A detailed study shows that a plane progressive electromagnetic wave has the following characteristics.

(i) Electromagnetic waves are produced by accelerated charges.

(ii) Electromagnetic waves propagate in the form of varying electric and magnetic fields. The fields are not only perpendicular to each other but also perpendicular to the direction of propagation of the wave. In other words, electromagnetic waves are transverse in nature. So, if the wave propagates in X- direction, the electric field is in Y-direction, there magnetic field is in the Z-direction [Fig 1.4]

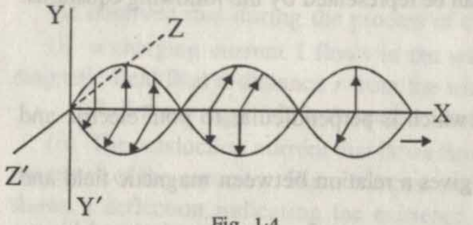


Fig. 1·4

The direction of propagation is along the vector $\vec{S} = \frac{1}{\mu_0}\left(\vec{E} \times \vec{B}\right)$, $\vec{S}$ is called Poynting vector. Its SI unit W / m^2

(iii) Electromagnetic wave do not require any material medium for their propagation.

(iv) In free space, electromagnetic waves travel with a velocity $c = \frac{1}{\sqrt{\mu_0 \in_0}}$ where μ_0 and $\in_0$ are the permeability and susceptibility of free space. It is also equal to the velocity of light in free space.

Also, in a material medium, the velocity of electromagnetic wave is $v = \frac{1}{\sqrt{\mu \in}}$

Where μ and ∈ are absolute permeability and absolute permittivity of that medium.

(v) Electromagnetic waves obey principle of superposition.

(vi) The ratio of the amplitudes of electric and magnetic fields is always constant and is equal to the velocity of the electromagnetic wave. So, $c = \frac{E}{B}$

(vii) Electromagnetic waves carry energy as they propagate through space. This energy is equally divided between $\vec{E}$ and $\vec{B}$.

(viii) Electromagnetic waves can transfer energy as well as momentum to objects placed in their path.

(ix) The electric vector is responsible for optical effects due to an electromagnetic wave. For this reason, electric vector is also called **light vector**.

✪ 1.4. Transverse nature of electromagnetic waves ✪

Electromagnetic waves are transverse waves. Here the electric vector $\vec{E}$ and the magnetic vector $\vec{B}$ and the direction of propagation are mutually perpendicular to each other. The two mutually perpendicular time varying electric and magnetic fields may be represented by

$$E = E_0 \sin\omega\left(t - \frac{x}{c}\right) = E_0 \sin(\omega t - kx) \quad \text{and} \quad B = B_0 \sin\omega(t - x/c) = E_0 \sin(\omega t - kx)$$

where $\omega = 2\pi\upsilon = \frac{2\pi}{T}$ and $k = \frac{2\pi}{\lambda}$

This shows that both the electric and magnetic field oscillate with the same frequency υ and there is no phase difference between them.

The two fields combine to constitute electromagnetic waves which propagate in space in a direction perpendicular to the direction of both the fields.

As said earlier, the velocity of electromagnetic waves in free space is $c = \frac{1}{\sqrt{\mu_0 \in_0}}$.

It is also given by the ratio of the amplitudes of the time varying electric magnetic fields i.e. $c = \frac{E}{B}$

1.5. Maxwell's equations

Maxwell's equations represents the four basic laws of electricity and magnetism. These four laws are :

(i) Gauss's law in electrostatics
(ii) Gauss's law in magnetism
(iii) Faraday's law of electromagnetic induction and
(iv) Maxwell Ampere's Circuital law.

The behaviour of electric and magnetic fields is best described by these laws and hence by Maxwell's equations. These equations predict that the time and space dependent electric and magnetic fields propagate as transverse waves, called electromagnetic waves, which possess velocity equal to that of light. Maxwell's four equations along with their salient features are being discussed here.

(i) First equation :

This represents Gauss's law in electrostatics. The law states that **the total normal electric flux through any closed surface S is always equal to $\frac{1}{\varepsilon_0}$ times the net charge enclosed by the surface.**

Mathematically, $\int_s \vec{E}.\,\vec{ds} = \frac{q}{\varepsilon_0}$ [in SI system]

This equation is known as Maxwell's first equation. It is time independent steady state equation. It is true both for stationary and moving charges. From this equation we can infer that the electric lines of force do not constitute continuous closed path.

(ii) Second equation :

Maxwell's Second equation represents Gauss's law in magnetism. It states that **the net magnetic flux crossing any closed surface is zero**.

Mathematically, $\int_s \vec{B}.\,\vec{ds} = 0$

It is a time independent equation. A direct consequent of Gauss's law in magnetism is that an isolated magnetic monopole does not exist, we can also infer from this equation that magnetic lines of force constitute continuous closed path.

(iii) Third equation :

Maxwell's third equation represents Faraday's law of electromagnetic induction. It states that **emf induced in a circuit is numerically equal to rate of change of magnetic flux linking the circuit.**

Mathematically, $e = -\frac{d\varphi_B}{dt}$ [φ_B = flux linkage]

The negative sign shows that induced emf opposes the rate of change of flux.

Now, emf can be defined as the line integral of electric field, the above relation may be expressed as

$$\oint \vec{E}\,.\,\vec{dt} = -\frac{d\varphi_B}{dt} = -\frac{d}{dt}\oint_s \vec{B}\,.\,\vec{ds}$$

In this particular form the law states that **the line integral of the electric field along a closed path is equal to the rate of change of magnetic flux through the surface bounded by that closed path.**

Evidently, it is a time dependent equation, we can infer from this equation that the time variation of the magnetic field generates an electric field.

(iv) Fourth equation :

This equation represents Maxwell-Ampere's circuital law. It states that **the line integral of the magnetic field over a closed path is equal to μ_0 times the sum of conduction and displacement currents :**

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 \left[I_c + \epsilon_0 \frac{d\varphi_E}{dt} \right]$$

It is Maxwell's fourth equation. It is a time dependent equation. We can infer from this equation that the time variation of electric field produces magnetic field.

1.6. Electromagnetic spectrum

After the discovery of electromagnetic waves by Maxwell in 1865, the other types of electromagnetic radiations like X-rays, gamma rays, ultraviolet, infrared, microwave and radiowaves were discovered. These radiations have different ranges of frequencies. So electromagnetic waves cover wide range of frequencies or wavelengths. The classification of electromagnetic waves does not have sharp boundaries and they overlap. This is due to the fact that the classification of electromagnetic waves is done according to their main sources. Different sources produce waves in overlapping range of frequencies.

Electromagnetic spectrum is orderly distribution of electromagnetic radiations in accordance with their wavelengths or frequencies.

- **Classification of electromagnetic waves :**

Name	**Frequency H_z**	**Wavelength (m)**	**How generated**
γ-rays	$5 \times 10^{10} - 3 \times 10^{19}$	$6 \times 10^{-13} - 1 \times 10^{-10}$	Radio active nuclei
X-rays	$3 \times 10^{19} - 1 \times 10^{+16}$	$1 \times 10^{-10} - 3 \times 10^{-8}$	(a) Bombardment of high energy electrons on a high atomic number target. (b) Energy changes of innermost orbital electrons.
Ultraviolet	$1 \times 10^{16} - 8 \times 10^{14}$	$3 \times 10^{-8} - 4 \times 10^{-7}$	Excitation of atoms
Visible	$8 \times 10^{14} - 4 \times 10^{14}$	$4 \times 10^{-7} - 8 \times 10^{-7}$	Excitation of atoms
Infra-red	$4 \times 10^{14} - 3 \times 10^{11}$	$8 \times 10^{-7} - 1 \times 10^{-3}$	Hot bodies
Microwaves	$3 \times 10^{11} - 1 \times 10^{9}$	$1 \times 10^{-3} - 3 \times 10^{-1}$	Oscillating currents
Radio waves	$3 \times 10^{9} - 3 \times 10^{4}$	$0{\cdot}1 - 10^{4}$ m	Oscillating currents

- **Visible radiation :** It forms a very narrow part of the electromagnetic spectrum having wave length roughly from 8×10^{-7} m which corresponds to red colour to about 4×10^{-7} m which corresponds to violet colour. Human eye is sensative to only visible part of the electromagnetic spectrum. Visible light is emitted by atomic excitation.

- **Ultraviolet rays :** The region of electromagnetic spectrum covers from $3{\cdot}8 \times 10^{-7}$ m to nearly 6×10^{-10} m with frequency lying in the range 8×10^{14} Hz to 1×10^{16} Hz. The ultraviolet rays are part of solar spectrum. These waves are produced by atoms and molecules in electrical discharge. Ultraviolet radiations are used in medical sciences.

- **X-rays :** X-rays are electromagnetic waves of very short wavelengths. This part of the electromagnetic spectrum extends from wavelengths of nearly 10^{-9} *m* down to wavelengths of nearly 6×10^{-12} *m*. The range of frequencies of X-rays is approximately lie between 3×10^{17} Hz and 5×10^{19} Hz. There rays are of atomic origin and are produced when a target of an element of atomic number is bombarded by high speed electrons. There rays find several applications in medical science and industry.

- **Gamma-rays :** Electromagnetic waves of extremely high frequencies are known as γ-rays. There waves are of nuclear origin. Their wavelength ranges from nearly 10^{-10} *m* to well below 10^{-14} *m* with the frequency range from 3×10^{18} Hz to more than 3×10^{22} Hz. γ -rays are highly energetic radiations and are mainly emitted by radio active substance.

- **Infra red spectrum :** If the wavelength of the electromagnetic radiations exceeds the visible limit, that are known as infra red radiations. They cover wavelengths from 10^{-3} m to $7{\cdot}8 \times 10^{-7}$ *m*. The frequency range is from 3×10^{11} Hz upto 4×10^{14} Hz. Infra red radiations are produced by any hot body.

- **Microwaves :** Electromagnetic waves having wavelengths of a few millimetres to about a metre are called microwaves. The frequency range is from 10^{9} Hz upto 3×10^{11} Hz. There are produced by oscillating electric circuits. Microwaves are used in communication for transmission of rader and TV signals.

- **Radiowaves :** Radiowaves are electromagnetic wave of long wavelengths. There have wavelengths ranging from a few kilometre down to $0{\cdot}3$ *m*. The frequency range is from a few Hz to 10^{9} Hz. Radio waves are generated by electronic oscillating circuits. These waves are used in television and radio broad casting systems.

1.7. Energy associated with electric and magnetic field of electromagnetic waves

Energy density of electric field is $K_E = \frac{1}{2} \epsilon_0 E^2$ and energy density of magnetic field is

$$K_B = \frac{1}{2}\left(\frac{B^2}{\mu_0}\right).$$

So, an electromagnetic wave, which consists of coupled electric and magnetic fields contains energy. Evidently, the propagation of the wave must be accompanied by the energy flow.

So, the total energy content in a volume ΔV at any time is $K = \left[\frac{1}{2} \epsilon_0 E^2 + \frac{1}{2}\frac{B^2}{\mu_0}\right] \Delta V$

$$\text{Now,} \quad B = \frac{E}{C} = E\sqrt{\mu_0 \epsilon_0}$$

$$\therefore \quad K = \left[\frac{1}{2} \epsilon_0 E^2 + \frac{1}{2}\frac{1}{\mu_0} E^2 \mu_0 \epsilon_0\right] = \frac{1}{2} \epsilon_0 E^2 + \frac{1}{2} \epsilon_0 E^2$$

It shows that total store of energy is equally shared by both the electric magnetic fields.

1.8. Transmission of EM Radiation through atmosphere

The earth is surrounded by a blanket of air known as atmosphere. The density of air decreases as we go up. The upper portion of the atmosphere is so much rarefind that under the action of solar radiations it is partly ionised. The composition and physical properties of the atmosphere are found to be different at different heights. The atmosphere is therefore, divided into a number of layers with no sharp boundaries.

Uppermost layer containing mainly positive ions and electrons is known as **ionosphere**. Between 50-80 km, the layer is called ozone layer because it contains high concentration of ozone. This ozone layer is largely responsible for absorption of ultraviolet rays coming from the sun. The ozone layer effectively protects us from the harmful UV and other radiations coming from the sun.

The atmosphere is quite transparent to visible light (0·4 μm – 0·8 μm) and also infrared radiation of the sun can pass through air. Our earth gets heated by absorbing these radiations.

The ionosphere plays an important role in propagation of electromagnetic waves which provide a means for transmission of information from one place to another without wires or use of any material medium.

●● Numerical Examples ●●

● **Example. 1.1. A parallel plate capacitor is made of two circular plates each of radius 5 cm. It is charged such that the electric field is increasing at the rate of 10 ^{12} Vm^{-1}s^{-1}. (i) Determine the induced magnetic field at R = 5 cm. (ii) What is the displacement current ?**

○ **Solution :** (i) Here R = 5 cm, $\dfrac{dE}{dt} = 10^2\, vm^{-1}s^{-1}$

In a parallel plate capacitor, the magnetic field is produced due to change of electric field. Therefore, from Maxwell's equation we write,

$$\oint B dl = \mu_0 \varepsilon_0 \frac{d\varphi_E}{dt} \quad \text{or,} \quad B.(2\pi R) = \mu_0\varepsilon_0 \frac{d\varphi_E}{dt} \quad [\text{ Here } r = \text{R}]$$

$$\therefore \quad 2\pi R B = \mu_0\varepsilon_0 \frac{d}{dt}(EA) = \mu_0\varepsilon_0 \frac{d}{dt}(E\pi R^2)$$

$$\therefore \quad B = \frac{1}{2}\mu_0\varepsilon_0 R\frac{dE}{dt} = \frac{1}{2}\times 4\pi\times 10^{-7}\times 8{\cdot}9\times 10^{-12}\times 5{\cdot}0\times 10^{-2}\times 10^{12} = \mathbf{2{\cdot}8\times 10^{-7}T}$$

(ii) Displacement current, $I_D = \varepsilon_0 \dfrac{d\varphi_E}{dt} = \varepsilon_0 \dfrac{d}{dt}(EA)$

$$\therefore \quad I_D = \varepsilon_0 (\pi R^2)\frac{dE}{dt}$$

$$= \varepsilon_0 (\pi R^2)\frac{dE}{dt} = 8{\cdot}9\times 10^{-12}\times 3{\cdot}14\times (5\times 10^{-2})^2\times 10^{12} = \mathbf{0{\cdot}07\,A}$$

● **Ex. 1.2. The electric field due to microwaves of wavelength 2·0 mm has an amplitude of 2 V*m*$^{-1}$. Determine (i) the frequency (ii) the amplitude of the magnetic field associated with the microwave.**

○ **Solution :** (a) Frequency $\upsilon = \dfrac{c}{\lambda} = \dfrac{3\times 10^8}{2\times 10^{-3}} = 1{\cdot}5\times 10^{11}\,\text{Hz}$

(b) $\mathbf{E}_m = c\,\mathbf{B}_m$

$$\therefore \quad B_m = \frac{E_m}{c} = \frac{2{\cdot}0}{3\times 10^8} = \mathbf{6{\cdot}67\times 10^{-9}\ wb / m^2}$$

● **Ex. 1.3. Electromagnetic waves travel in a medium with a speed of 2 × 10 ^{8} ms^{-1}. The relative permeability of the medium is 1. Find the relative permittivity.**

○ **Solution.** The speed of electromagnetic wave is a medium is given by

$$v = \frac{1}{\sqrt{\mu\varepsilon}} \quad [\ \mu = \text{absolute permeability;}\ \ \epsilon = \text{absolute permittivity}\ ]$$

Here $v = 2\times 10^8\ ms^{-1}$; $\mu = \mu_0\mu_r = \mu_0\ [\because \mu_r = 1]$; $\varepsilon = \varepsilon_0\varepsilon_r$

$$\therefore \quad v = \frac{1}{\sqrt{\mu_0\varepsilon_0\,\varepsilon_r}} = \frac{1}{\sqrt{\mu_0\varepsilon_0}}.\frac{1}{\sqrt{\varepsilon_r}}$$

$$\therefore \quad \varepsilon_r = \frac{c^2}{v^2} = \frac{(3\times 10^8)^2}{(2\times 10^8)^2} = 2{\cdot}25$$

● **Exa. 1.4. A capacitor is made of two circular plates each of radius 12 cm and separated by 5 mm. The capacitor is being charged by a constant charging current of 0.15 A.**

(i) Calculate the capacitance and rate of change of potential difference between the plates.

(ii) Obtain the displacement current across the plates.

○ **Solution :** Capacitance of the parallel plate capacitor

$$C = \frac{\varepsilon_0 A}{d} = \frac{8{\cdot}85\times 10^{-12}\times \pi\,(12\times 10^{-2})^2}{5\times 10^{-3}} = \mathbf{80{\cdot}1\times 10^{-12}\,F}$$

Now, $\quad Q = CV$

$$\therefore \quad \frac{dQ}{dt} = C\frac{dV}{dt} \qquad \therefore \quad \frac{dV}{dt} = \frac{1}{C}.\frac{dQ}{dt} = \frac{1}{C}.I$$

$$\therefore \quad \frac{dV}{dt} = \frac{0{\cdot}15}{80{\cdot}1\times 10^{-12}} = \mathbf{1{\cdot}87\times 10^9\ Vs^{-1}}$$

(ii) Displacement current, $I_d = \varepsilon_0\dfrac{d\varphi_E}{dt} = \varepsilon_0\dfrac{d}{dt}(EA)$

$$\therefore \quad I_d = \varepsilon_0 A\frac{dE}{dt} = \varepsilon_0 A\frac{d}{dt}\left(\frac{Q}{\varepsilon_0 A}\right) \qquad \left[\because\ E = \frac{Q}{\varepsilon_0 A}\right]$$

$$= \varepsilon_0 A.\frac{1}{\varepsilon_0 A}\frac{dQ}{dt} = \frac{dQ}{dt}$$

$$\therefore \quad \mathbf{I_d = 0{\cdot}15\ A}$$

● **Ex. 1.5. For vacuum absolute permittivity is $\varepsilon_0 = 8{\cdot}85\times 10^{-12} C^2N^{-1}m^{-2}$ and absolute permeability is $\mu_0 = 4\pi\times 10^{-7}\ NS^{-2}\ C^{-2}$. What is the velocity of electromagnetic wave ?**

○ **Solution :** The velocity of electromagnetic wave is vacuum is

$$C = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = \frac{1}{\sqrt{8{\cdot}85\times 10^{-12}\times 4\pi\times 10^{-7}}} = 3\times 10^8\ ms^{-1}$$

● **Ex. 1.6. The wavelength of electromagnetic wave travelling along X- axis is 6 mm. The electric vector acts along Y- axis and its peak value 33 Vm^{-1}. Obtain the equations for electric field and magnetic field.**

○ **Solution :** When the electromagnetic wave travels along X-axis, the electric field vector and magnetic field vector will be acting along Y and Z-axis respectively. So, the equation for electric field and magnetic will be as

$$E_y = E_0\sin(\omega t - kx) \quad \text{and} \quad B_z = B_0\sin(\omega t - kx) \qquad \left[k = \frac{2\pi}{\lambda}\right]$$

Here $\lambda = 6\,mm = 6\times10^{-3}\,m$; $E_0 = 33\ Vm^{-1}$

$$\omega = 2\pi\upsilon = \frac{2\pi c}{\lambda} = \frac{2\pi\times3\times10^{8}}{6\times10^{-3}} = \pi\times10^{11}\,\text{rad/s}$$

$$B_0 = \frac{E_0}{c} = \frac{33}{3\times10^{8}} = 1{\cdot}1\times10^{-7}\,T$$

$$\therefore \quad E_y = 33\sin\left(\pi\times10^{11}t - \frac{2\pi}{6\times10^{-3}}x\right) Vm^{-1}$$

$$B_z = 1{\cdot}1\times10^{7}\,\sin\left(\pi\times10^{11}t - \frac{2\pi}{6\times10^{-3}}x\right) T$$

●● Short Answer Type Questions (with answer) ●●

● **Question 1. What is Maxwell's displacement current ?**

○ **Ans.** The displacement current is produced when electric field between the plates of a parallel plate capacitor changes with time. Mathematically, $I_d = \varepsilon_0\dfrac{d\varphi_E}{dt}$

● **Q. 2. What are electromagnetic waves ?**

○ **Ans.** The transverse time varying electric and magnetic fields propagating in space in a direction perpendicular to the directions of both the electric and magnetic fields constitute electromagnetic waves.

● **Q. 3. State the principle of production of electromagnetic waves ?**

○ **Ans.** The accelerated charge produces electric and magnetic fields, which vary both in space and time. These varying electric and magnetic fields give rise to electromagnetic waves.

● **Q. 4. What does an electromagnetic wave consist of ? On what factors does it velocity in vacuum depend ?**

○ **Ans.** An electromagnetic wave consists of electric and magnetic fields varying both in space and time. These two fields are perpendicular to each other and to the direction of propagation of wave.

The velocity of propagation of the e. m. wave is vacuum depends upon the values of the absolute permeability (μ_0) and permittivity ($\in_0$).

● **Q. 5. What is electromagnetic spectrum ?**

○ **Ans.** The orderly distribution of electromagnetic waves in the form of distinct groups having widely differing properties is called electromagnetic spectrum.

● **Q. 6. When a capacitor is being charged, the instantaneous current in the connecting wires is 0.5 A. What is the displacement current at the instant in the region between the plates of the capacitor ?**

○ **Ans.** From the principle of continuity of current, the displacement must be equal to the conduction current. Hence the displacement current is 0.5 A.

● **Q. 7. A variable magnetic flux induces an electric field. Does a variable electric flux induce a magnetic field ?**

○ **Ans.** Yes. A variable electric flux produces a displacement current, which in turn creates a magnetic field.

● **Q. 8. What is dimensional formula for $\mu_0\varepsilon_0$?**

○ **Ans.** The velocity of electromagnetic wave is $e = \dfrac{1}{\sqrt{\mu_0\varepsilon_0}}$

$$\therefore \quad \mu_0\,\varepsilon_0 = \frac{1}{C^2} \qquad \therefore \quad [\mu_0\varepsilon_0] = [L^{-2}T^{2}]$$

EXERCISE

(A) Short answer type questions :

1. What is an electromagnetic wave ? Write what speed does it travel in free space ?
2. What is displacement current due to ? Does it satisfy the continuity requirement ?
3. Write down Ampere's law as generalized by Maxwell.
4. Which of the following has the shortest wavelength ?
   (a) Radio waves (b) X-rays (c) Green light (d) Ultraviolet light.
5. Which of the following has the greatest wavelength ?
   (a) VHF radio wave (b) γ- rays (c) violet rays (d) microwaves
6. Write down the expression for displacement current.
7. Write the expression for velocity of light in vacuum.
8. A wire is connected to an AC source. Is there any electromagnetic field in the surrounding ?

B. Essay type questions :

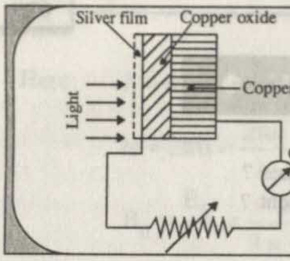
1. How Maxwell was led to the existance of a displacement current ?
2. How is displacement is produced ? Conduction and displacement currents individually are discontinuous, but their sum is continuous— Explain.
3. (a) State the principle of production of electromagnetic wave.
   (b) State any four properties of electromagnetic waves.
   (c) On which factors does the velocity of e. m. wave depend ?
4. (a) A plane progressive wave is moving with velocity *e*. $\vec{E}$ and $\vec{B}$ are mutually at right angles. Prove from Faraday's law, prove that $\vec{E} = \vec{B}\,C$.
   (b) From Ampere's law prove that, $C = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}}$

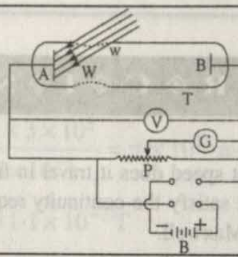
C. Numerical problems :

1. The plates of a parallel plate capacitor is circular having radius 12 cm. Separation between the plates is 5 mm. Its charging current's 0·15 A. Find (i) capacity of the capacitor and rate of change of potential of capacitor (ii) displacement current between the plates. [**Ans :** (i) 80·1 *p*F; $1{\cdot}87 \times 10^{9}\ \mathrm{V}s^{-1}$; (ii) 0·15 A]

   [**Hint :** $\dfrac{dv}{dt} = \dfrac{1}{c}\dfrac{dq}{dt} = \dfrac{\mathrm{I}}{\mathrm{C}}$]
2. The electric field of an electromagnetic wave is varying sinusoidally at frequency 2×10^{10} Hz. Amplitude of variation with time is 48 Vm^{-1}. (a) What is the wavelength of the wave ? (b) What is the intensity of the varying magnetic field ? [**Ans :** (a) $\lambda = 1{\cdot}5 \times 10^{-2}\,m$; (b) $\mathrm{B}_0 = \dfrac{\mathrm{E}_0}{\mathrm{C}} = 1{\cdot}6 \times 10^{-7}\,\mathrm{T}$]
3. The intensity of the electric field vector of a radio wave is $\mathrm{E} = 10^{-4}\ \mathrm{V\,m}^{-1}$. Calculate intensity of the magnetic field vector of the radio wave. [**Hint :** $\mathrm{B} = \dfrac{\mathrm{E}}{\mathrm{C}}$] [**Ans :** 3.3×10^{-13} T]
4. The energy of a photon is $E = h\upsilon$. What is the energy of a photon of an electromagnetic wave of wavelength 1 m ? Planck's constant, $h = 6.63 \times 10^{-34}$ J. S. [**Ans.** $\mathrm{E} = 1{\cdot}24 \times 10^{-6}$ eV]
5. The frequency of the electric vector of a plane electromagnetic wave is 10^{10} Hz and amplitude 50 $\mathrm{V}m^{-1}$. Find (i) wavelength (ii) amplitude of magnetic field. [**Ans :** $3 \times 10^{-2}\,m$ (ii) $1{\cdot}67 \times 10^{-7}$ T]
6. The radius of a plate of a parallel plate capacitor is 10 cm. During charging the electric field between the plates changes at the rate of $10^{10}\ \mathrm{V}m^{-1}s^{-1}$. (a) What is the electric field at a distance of 10 cm from the connecting wire ? (b) What is displacement current ? [**Ans :** (i) $5{\cdot}59 \times 10^{-9}$ T (ii) $2{\cdot}8 \times 10^{-3}$ A]

   [**Hints :** $\mathrm{B} = \dfrac{1}{2}\mu_0\varepsilon_0 \mathrm{R}\dfrac{d\mathrm{E}}{\mathrm{dt}}$, $\mathrm{I_D} = \varepsilon_0 \pi \mathrm{R}^2 \dfrac{d\mathrm{E}}{dt}$]
7. The separation between the plates of a parallel plate capacitor is 5·0 cm. During charging the electric field between the plates changes at the rate $10^{12}\ \mathrm{V}m^{-1}\ \mathrm{s}^{-1}$. calculate displacement current ? [**Ans :** 0·07 A]
8. The voltage between the parallel plates of a capacitor of capacity 1 μF change at 5 Vs^{-1}. Calculate displacement current. [**Ans :** 5·0 μA]





CHAPTER 2

PARTICLE NATURE OF RADIATION

• **TOPICS :** ○ Dual nature of Radiation; ○ Properties of photons; ○ Collision between photon and matter particle; ○ Photoelectric effect; ○ Experiment on photoelectric effect; ○ Failure of electromagnetic theory; ○ Particle nature of radiation : Quantum theory; ○ Einstein's photoelectric equation : Explanation of photoelectric phenomenon; ○ The photoelectric cells; ○ Short answer type questions (with answers); ○ Exercise.

2.1. Dual Nature of Radiation

A number of theories has been put forward at different times regarding the true nature of radiation. But not a particular theory is really complete and successful in explaining the behaviour and nature of light.

In the later half of the seventeenth century Newton proposed the **'Corpuscular theory'** of light, according to which, light consists of tiny particles or corpuscles which are given out by luminous object. These corpuscles travel with a tremendous velocity in straight lines in all directions. This theory satisfactorily explained rectilinear propagation of light, reflection, refraction etc. But other phenomena exhibited by light such as interference, diffraction and polarisation of light can not be explained on the basis of corpuscular theory of light.

In the year, 1678 a new theory called the **wave theory of light** was put forward by Christian Huygens. According to this theory a luminous object sends out disturbances in the form of waves. Huygens theory can explain reflection and refraction of light. Young and Fresnel applied the wave theory to explain the phenomena of interference and diffraction of light. Fresnel treated light wave as transverse to explain polarisation of light waves.

Huygen assumed the existence of a hypothetical medium ether, through which light wave could pass. Later, in the year 1886 Clerke Maxwell modified the concept by stating that light consists of electromagnetic waves which require no material medium for their propagation. According to the electromagnetic theory, light consists of oscillating electric field and magnetic field. These field are perpendicular to each other and also perpendicular to the direction of propagation of the waves. Maxwell's theory of light could explain rectilinear propagation, reflection, refraction etc, in addition to the phenomena like interference, diffraction and polarisation. In the year 1887, Hertz experimentally established electromagnetic theory of light.

Towards the end of nineteenth century when wave theory of light is firmly established and scientists were about to forget corpuscular theory, Max Planck proposed a new theory about the nature of light very similar to old corpuscular theory of light. This new theory was named **'Quantum theory'**.

The electromagnetic theory failed to explain several phenomena like black body radiation, origin of line spectrum, photoelectric effect, crompton effect etc. All these phenomena are related to the emission and absorption of light.

In 1900 Max Planck, while explaining the energy distribution in different wavelengths of black body radiation, suggested that radiant energy may be given out or taken in not continuously but in **small packets**. A packet of energy is called **quantum**. So by quantum theory light or radiation consists of energy packets or energy quanta. The amount of energy in one quantum is given by $\mathbf{E} = h\upsilon$ where h is known as Planck's constant and υ is frequency of light. The quantum of electromagnetic radiation is now called a **photon**. Einstein explained the photoelectric effect by considering light as the streams of photons instead of waves.

So, radiation appears to possess dual character. It some times behaves as waves and at other times as particles. For example, radiation is considered as a wave in propagation experiments like interference, diffraction and polarisation. These experiments firmly establish the wave nature of radiation.

On the other hand, the phenomenon like photoelectric effect, Crompton effect, Ramon effect etc show that radiation behaves as a particle— the photon. Thus radiation has a dual characteristic. However, radiation does not exhibit its wave and particle aspect simultaneously. It is found to behave as a wave in transmission and as a particle, when it interacts with matter.

2.2. Properties of Photons

Light quantum hypothesis or photon theory of light firmly establishes particle nature of radiation on theoretical basis. For any type of radiation photon is its only constituent. The photon has the following properties.

(i) Energy :

Photon is called energy quantum or energy packet. Evidently, photon is different from material particle. In fact photon is concentrated energy. The energy of photon depends on the frequency of radiation. So the energy of photon, $E = h\upsilon = \frac{hc}{\lambda}$ $\left[\, \upsilon = \frac{c}{\lambda},\ \lambda = \text{wavelength of light} \,\right]$

Also, $E = \frac{h}{2\pi} . 2\pi\upsilon = \hbar\omega$ [ω = angular velocity]

(ii) Mass and momentum of photon :

Now according to theory of relativity a mass m has an energy equivalent to mc^2.

So $E = mc^2 \quad \therefore \quad m = \frac{E}{c^2}$

$\therefore$ mass of the photon of energy $h\upsilon$ is $m = \frac{h\upsilon}{c^2}$

$\therefore$ momentum of photon, $p = \frac{h\upsilon}{c^2} . c = \frac{h\upsilon}{c} = \frac{h}{\lambda}$

(iii) Energy of photon does not depend on intensity of radiation :

Energy of photon depends on frequency of radiation. It does not depends on intensity of radiation. Intensity of radiation determines the number of photon present in the radiation. Higher the intensity, larger is the number of photons flowing normally through unit area.

(iv) Photon is electrically neutral :

So it is not deflected by electric or magnetic field.

- **A discussion :** (a) A wave is specified by its (i) frequency, υ (ii) wavelength λ, (iii) phase or wave velocity, u, (iv) amplitude and (v) intensity. More over, wave occupies a relatively large region of space. (b) A particle is specified by (i) mass m, (ii) velocity v, (iii) momentum, p and (iv) energy, E.

Moreover, a particle occupies a definite position in space.

2.3. Collision between Photon and matter particle

According to quantum mechanics, conservation of energy can be applied to the collision between matter and photon. Let during collision, a photon of frequency υ is created or anihilated [fig. 2.1]. Let the mass of a particle is m and v is the velocity.

Hence its increase or decrease of kinetic energy is $\frac{1}{2}mv^2$. Then by the principle of conservation of energy $h\upsilon = \frac{1}{2}mv^2$ . . . . (i)

here,

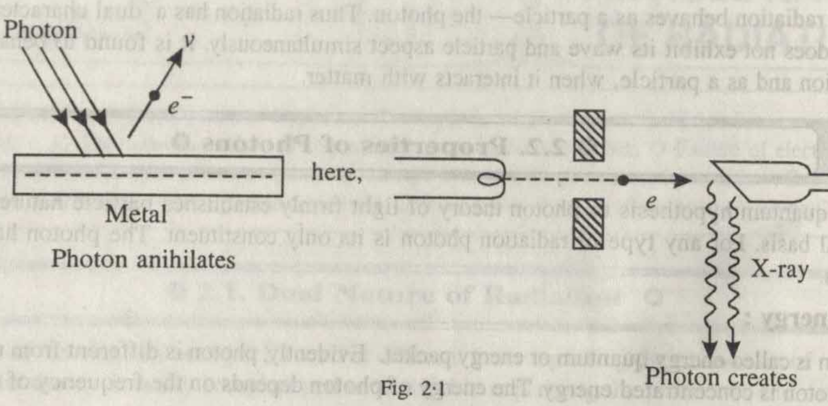


Fig. 2·1

If a particle having charge e is allowed to pass through a potential v, then $h\upsilon = e\text{V}$

$$\therefore \quad \frac{1}{2}mv^2 = h\upsilon = e\text{V}$$

$\therefore$ Velocity of the particle, $v = \sqrt{\frac{2e\text{V}}{m}}$

$$\therefore \quad v = \sqrt{2 \times 1\cdot76 \times 10^{11}\,\text{V}} = 5\cdot93 \times 10^5 \sqrt{\text{V}}\ ms^{-1}$$

When an accelerated charged particle strikes a metal surface, the frequency of the emitted photon is $\upsilon = \frac{e\text{V}}{h}$ and wavelength, $\lambda = \frac{hc}{e\text{V}}$

$$\therefore \quad \lambda = \frac{6\cdot62 \times 10^{-34} \times 3 \times 10^8}{1\cdot6 \times 10^{-19}\,\text{V}}\,m = \frac{12400}{\text{V}}\,\text{Å}$$

● **Example. 2.1. (a) How many photons be incident on a metal plate so that a force of 10^{-5} N is exerted on the plate ? Wavelength of radiation = 5×10^{-7} m.**

(b) Also calculate rise of temperature of the plate. Given mass of the plate is 1·5 g and specific heat 0·1 c. g. s. and $h = 6\cdot62 \times 10^{-4}$ J. s.

○ **Solution :** (a) Momentum of photon, $p = mc = \frac{h}{\lambda}$

If n electrons fall on the plate per sec, then change of momentum per sec or force on the plate,

$$\text{F} = np = \frac{nh}{\lambda} = 10^{-5}$$

$$\therefore \quad n = \frac{\lambda \times 10^{-5}}{h} = \frac{5 \times 10^{-7} \times 10^{-5}}{6\cdot62 \times 10^{-34}} = \mathbf{7\cdot54 \times 10^{21}}$$

(b) As n electrons hits the plate per sec, then energy absorbed by the plate per sec is $\text{E} = \frac{nhc}{\lambda}$

$$\text{E} = 7\cdot54 \times 10^{21} \times \frac{6\cdot62 \times 10^{-34} \times 3 \times 10^8}{5 \times 10^{-7}} = 3000\ \text{Js}^{-1} = \frac{3000}{4\cdot2}\ \text{cal / s}$$

If θ°C be the temperature rise per sec then, $ms\theta = \frac{3000}{4\cdot2}$ $\therefore\ \theta = \frac{3000}{4\cdot2\times ms}$

$$\therefore\ \theta = \frac{3000}{4\cdot2\times1\cdot5\times0\cdot1} = \mathbf{4\cdot75\times10^3\,{}^\circ C/s}$$

● **Ex. 2.2. Calculate the frequency and wavelength of a photon whose energy is 75 eV. $h = 6{\cdot}62 \times 10^{-34}$ J. s.**

○ **Solution.** Energy, $E = h\upsilon$

$$\therefore\ \text{Frequency, } \upsilon = \frac{E}{h} = \frac{75\times1\cdot6\times10^{-19}}{6\cdot62\times10^{-34}} = 18\cdot12\times10^{15}\,\text{Hz}$$

$$\text{Wave length, } \lambda = \frac{c}{\upsilon} = \frac{3\times10^8}{18\cdot12\times10^{15}} = 165\cdot5\times10^{-10}m = \mathbf{165\cdot5\text{Å}}$$

● **Ex. 2.3. Find the energy of a photon having momentum equal to the momentum of an electron of mass $9{\cdot}1 \times 10^{-31}$ kg and energy 2 eV.**

○ **Solution :** Energy of an electron, $\frac{1}{2}mv^2 = 2\times1\cdot6\times10^{-19}$ J

$$\therefore\ (mv)^2 = 2m\times2\times1\cdot6\times10^{-19}$$

$$\therefore\ \text{Momentum of an electron, } mv = p = \sqrt{4m\times1\cdot6\times10^{-19}}$$

$$\therefore\ p = \left[4\times9\cdot1\times10^{-31}\times1\cdot6\times10^{-19}\right]^{1/2} = 2\cdot4\times10^{-25}\ \text{kg.m/s}$$

$$\therefore\ \text{Momentum of the photon, } p = 2\cdot4\times10^{-25}\ \text{kg - m/s}$$

$$\text{Now, } p = \frac{h}{\lambda} = \frac{h\upsilon}{c} = \frac{E}{c}$$

$$\therefore\ E = pc = 3\times10^8\times2\cdot4\times10^{-25} = 7\cdot2\times10^{-17}\,\text{J}$$

✪ 2.4. Photoelectric effect ✪

When a light or any other radiation of suitable frequency falls on a metal surface, electrons are ejected from the metal. This phenomenon of electron emission from a metal surface due to light falling on it is called photoelectric effect.

Heinrich Hertz discovered photoelectric effect in 1887. After one year in 1888, Hallwach detected that a negatively charged zinc plate lost its charge quickly when ultraviolet light fall on it. But no such effect was observed with a positively charged plate. Hallwach showed that an illuminated metal emits negatively charged particles. In 1889 De Lenard measured the value of $\frac{e}{m}$ for the particles emitted from the illuminated metals and found that its value of $\frac{e}{m}$ is same as that for electrons.

Experiments with different metals shows that any metal can emit photoelectrons when it is illuminated by light of suitable frequency. The alkali metals like sodium, potassium and ceasium are sensitive even to rays from the visible part of the spectrum, whereas zinc, cadmium etc are sensitive to only ultra-violet light. Light of shorter wavelength or higher frequency has more energy and is thus more effective in producing photoelectrons.

✪ 2.5. Experiment on photoelectric effect ✪

Lenard in the year 1900 studies the photoelectric effect experimentally. The experimental arrangement to demonstrate the effect and obtain the important results is shown in fig 2.2.

It consists of an evacuated glass tube T fitted with electrodes A and B in the form of plates. The electrode A is photosensitive. A variable potential difference can be applied across the two electrodes. Through the window w light is allowed to fall on the electrode A. So A is the **emitting electrode** and B is the collecting electrode. By means of the battery E, the plate A can be maintained at higher potential than the plate B. With the help of commentator, the plate A can also be made negative with respect to B. The voltmeter V measures the potential differences across the plates A and B.

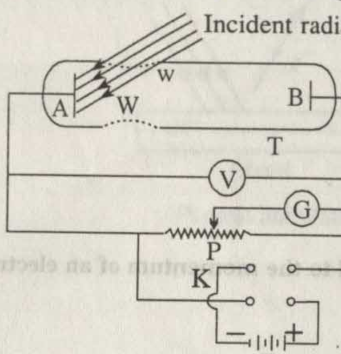


Fig. 2·2

Keeping the plate B at positive potential, when a light of suitable frequency is allowed to fall on plate A, electrons are emitted from A and are attracted by B. A current flows in the external circuit. The current is measured by the milliammeter.

From the experiment it is found that if the frequency of light incident of plate A, is equal to or greater than a minimum frequency, the electron emission is instantaneous, whatever be the intensity.

If the plate B is made negative with respect to A, the electrons are retarded and photoelectric current decreases. As the collecting electrode B is made more and more negative, the current gradually decreases and at a particular retarding potential V_s, no electron will be able to reach the plate B (now cathode). The current is now zero. This potential V_s is called **stopping potential**. Its value depends on nature of the electrode and on the frequency of incident light.

The work done by the stopping potential is equal to the maximum kinetic energy of the electrons, i.e

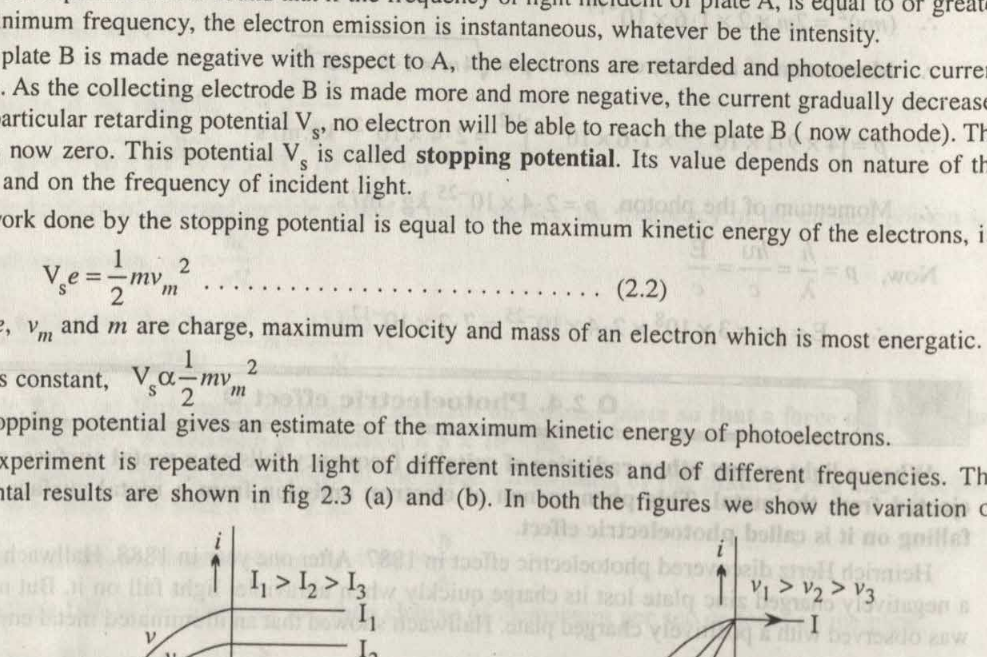
$$V_s e = \frac{1}{2} m v_m^2 \quad \dots\dots\dots\dots\dots\dots\dots\dots \quad (2.2)$$

Here e, v_m and m are charge, maximum velocity and mass of an electron which is most energatic.

As e is constant, $V_s \alpha \frac{1}{2} m v_m^2$

So, stopping potential gives an estimate of the maximum kinetic energy of photoelectrons.

The experiment is repeated with light of different intensities and of different frequencies. The experimental results are shown in fig 2.3 (a) and (b). In both the figures we show the variation of photoelectric current (i) with potential difference (V) between the plates A and B. From fig 2.3 (a) we see



(a) (b)

Fig. 2·3

that whatever be the intensity of light, the stopping potential does not change if the frequency (υ) of light is kept constant. With increase of intensity, the saturation current increases.

Fig 2.3 (b) shows the experimental results in the case of light of different frequencies and equal intensity. It is found that maximum or saturation current does not depend on frequency. But with higher frequency of incident light, the magnitude of stopping potential increases which means that **kinetic energy of photoelectrons increases with frequency of light.**

To show the effect of frequency on stopping potential a graph is plotted between the frequency (υ) of the incident light and the corresponding stopping potential (V_s). It is shown in fig 2.4. For any emitting surface, the graph is a straight line not passing through the origin. It shows that the maximum velocity of photoelectrons is proportional to the frequency of light.

The intercept on the frequency axis gives the minimum frequency υ_0 for which the stopping potential V_s or the maximum kinetic energy eV_s is zero. This minimum frequency is called **threshold frequency**.

So, **the threshold frequency for a given surface is defined as the minimum frequency of incident light below which no photoelectrons are emitted, whatever be the intensity of incident light**.

The slope of the graph is a constant, $m = h / e$. It is same for all emitting surfaces.

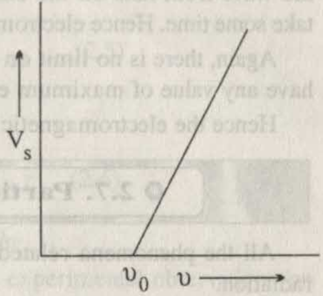


Fig. 2·4

➤ *2.5.1. Laws of photoelectric emission* :

On the basis of the experimental studies the following laws have been obtained for the photoemission of electrons.

(1) For all emitting surface there is a minimum frequency called the threshold frequency υ_0 below which electron emission does not take place no matter how much is the intensity of light.

(2) For a given frequency υ ($\upsilon > \upsilon_0$) the number of photoelectrons emitted per second for a given surface under a constant accelerating potential is directly proportional to the intensity of the incident light.

(3) If the frequency is higher than the threshold frequency, the process of photoelectric emission is almost instantaneous.

(4) The maximum kinetic energy and hence the maximum velocity of the ejected photoelectrons increases with the increase of frequency of the incident light provided it is higher than the threshold frequency.

(5) The maximum kinetic energy (or maximum velocity) is independent of the intensity of light.

(6) The stopping potential for a given metal surface is proportional to the frequency of incident light and does not depend on intensity of light.

✪ 2.6. Failure of electromagnetic theory ✪

The laws of photoelectric emission deduced from the observations from different experiments on photoelectric effect could not be explained on the basis of electromagnetic theory. According to this theory, light is a type of wave in which oscillating electric and magnetic field travel with velocity of light. Intensity and energy of light depends on the amplitude of electric and magnetic field comprising the wave. When light falls on an atom, its outermost valence electrons are influenced by the oscillating electric field. Higher the amplitude of the electric field, greater is the energy gained by the electron. So according to wave theory, energy of ejected electron should depend on intensity of light. But actually it does not happen.

Again according to electromagnetic theory, the kinetic energy of photoelectron should not depend on frequency of incident light. Whatever be the frequency of the incident light, the electron on the metal surface should absorb necessary amount of energy from the light wave and be emitted from the metal. In other words, there should not be any threshold frequency for a material. This is contrary to the observed facts.

Photoelectric effect is instantaneous. But according to electromagnetic theory there should be a time lag between the instant the light falls and that of emission of photoelectrons. Because, energy of a wave is spread over whole wave front. Now, electron is an extremely small particle and a very small portion of the wave front falls on the electron. To gain necessary amount of energy for emission the electron will take some time. Hence electromagnetic theory fails to explain why the photoelectric process is instantaneous.

Again, there is no limit on the maximum energy to be transferred to the electron i.e., the electron can have any value of maximum energy. This is contary to observed facts.

Hence the electromagnetic theory of light fails to explain the basic facts of photoelectric effect.

❂ 2.7. Particle nature of radiation : Quantum theory ❂

All the phenomena related to photoelectric effect experimentally establishes the quantum theory of radiation.

Previously, it was thought that light or any radiation is a wave. The quantum theory of radiation was proposed by Max Planck to explain the distribution of energy with wavelengths in black body radiation.

Planck had come to the conclusion that absorption or emission of thermal energy is not a continuous process but takes place in discrete amounts called **energy packets or quantum of energy**. Einstein called this energy quantum as **photon**. The energy carried by each photon is proportional to the frequency of radiation. So its energy is given by $E = h\upsilon$ [h = Planck's constant, υ = frequency of radiation] Exchange of energy between matter and radiation takes place as an integral multiple of minimum amount $E = h\upsilon$.

Hence, according to quantum theory, radiation consists of photons. Each photon carries fixed amount of energy which depends on frequency of radiation only. So, the energy content of a photon varies with frequency only. Intensity of light depends on number of photons it carries.

❂ 2.8. Einstein's photoelectric equation : Explanation of photoelectric phenomenon ❂

Einstein proposed an explanation of photoelectric phenomena as early as 1905 on the basis of quantum theory of radiation, according to which light consists of photons. He gave an equation now known as Einstein's **photoelectric equation**. For this he made the following assumptions.

(i) A light of frequency υ consists of quantum of energy or photons. Each photon has an energy $h\upsilon$ and travels with velocity of light.

(ii) In photoelectric effect one photon is completely absorbed by one electron of the metal surface.

- **Derivation of the equation :** According to Einstein, when a photon of light falls on a metal, it is absorbed and it imparts its energy $h\upsilon$ to a single electron. The photon energy is utilised in two ways :

(i) a part of energy is spent just to get the electron free from the atom and away from the metal surface. This energy is known as **photoelectric work function** of the metal. It is denoted by ω_0.

(ii) the rest of the photon energy is stored in the electron as kinetic energy whose measurement is $\frac{1}{2}mv^2$.

Then by the principle of conservation of energy we write, $h\upsilon = \omega_0 + \frac{1}{2}mv^2$ . . . . (2.3)

In case the photon energy is just sufficient to liberate the electron only, then kinetic energy of the electric is zero. Then the equation is $h\upsilon_0 = \omega_0$ . . . . (2.4)

here υ_0 is called **threshold frequency**. So, it is defined as the minimum frequency which can cause photoelectric emission.

When a radiation of frequency υ greater than threshold frequency is incident on the metal, the maximum velocity $v_{\max}$ with which the electron is emitted, is given by the relation.

$$\therefore \quad h\upsilon = h\upsilon_0 + \frac{1}{2}mv^2_{\max} \quad \ldots\ldots\ldots\ldots (2.5)$$

The equation (2.5) is known as **Einstein's photoelectric equation.**

$$\therefore \quad \frac{1}{2}mv^2_{\max} = h(\upsilon - \upsilon_0) = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right) \quad \ldots\ldots\ldots\ldots (2.6)$$

Where λ and λ_0 are wavelengths corresponding to frequencies υ and υ_0.

- **Explanation of the laws of photoelectric emission :** Different experimental observations on photoelectric effect can be adequately explained by Einstein's photoelectric equation.

(i) It is evident from the equation (2.6) that if $\upsilon < \upsilon_0$, the threshold frequency, the kinetic energy of emitted electron is **negative**, which is meaningless.

So, no electrons are emitted if the frequency of the incident light is less than the threshold frequency.

(ii) If $\upsilon = \upsilon_0$, the electron will be emitted with zero kinetic energy or zero velocity. Hence to start photoelectric effect, the frequency must be equal to or greater than threshold frequency.

(iii) For a frequency $\upsilon > \upsilon_0$, the kinetic energy of photoelectron is directly proportional to $(\upsilon - \upsilon_0)$.

As υ_0 is a constant, the kinetic energy of the electron increases with increase in frequency of the incident light.

Thus the maximum velocity of photoelectron depends on the frequency of the incident light and independent of intensity of light.

(iv) As photons are absorbed in single units, there is localisation of energy. The photons are immediately absorbed and there is practically no time delay in the emission of electron. Hence it is an instantaneous process.

(v) If the intensity of light is increased keeping frequency same, more photons each of same energy are incident on the metal surface. Hence more electrons are ejected. Each electron will have the same maximum energy and will be ejected with the same maximum velocity. Hence, **increase of intensity of light only increases the number of photoelectrons ejected and not their velocity**.

(vi) The stopping potential V_s is given by $V_s e = h\upsilon - h\upsilon_0 = h(\upsilon - \upsilon_0)$

As υ_0 is constant, the **stopping potential increases with the frequency of incident light.**

- **A discussion :** Light or any radiation is a wave. By wave theory of light different experimentally observed phenomena can be explained. But from Planck and Einstein's work on black body radiation and photoelectric effect, it is firmly established that radiation has particle property. Hence radiations possess dual properties wave and particle. On the basis of wave theory of light interference, diffraction and polarisation of light can be explained. But phenomena related to transference of energy to matter by light, particle property of light has to be considered.

○ 2.9. The photoelectric cells ○

A photoelectric cell is an arrangement which converts light energy into electrical energy. It can measure the intensity of light by measuring the photoelectric current when light falls on a photosensitive metal. We consider three types of photo cells :

(i) Photo-emissive cell, (ii) Photo-voltaic cell, (iii) Photo-conductive cell.

(i) **Photo-emissive cell :**

It action depends on the emission of electrons from metal cathode when it is exposed to light or other radiations.

● **Construction :** It consists of two metallic electrodes (cathode and anode) supported inside an evacuated glass or quarter bulb. The cathode is semi cylindrical in shape and is made of metal coated with photo sensitive material. The anode is in the form of a thin wire facing the cathode. In fig 2.5, K is the cathode and A is the anode. When visible light is used, the cathode is made of alkali metals.

The anode is connected to the positive terminal of a battery and the cathode to negative terminal, when light of frequency greater than threshold frequency is made to fall on the photo sensitive cathod surface, phtoelectrons are emitted. These electrons are attracted by the anode A. The current is measured by sensitive galvanometer. Since the number of electrons emitted depends upon the intensity of incident light, the photoelectric current measured by the galvanometer gives a measurement of intensity of light.

The photo emissive cell may be **vacuum type** and **gas filled type**. In vacuum type (described above) the photo electric current is very small of the order of µa. In the gas filled type of cell, the current is enhanced due to ionisation of the gas.

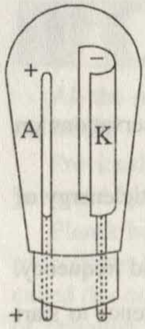


Fig. 2·5

● **Uses :** The function of a photo emissive cell is to convert in intensity of illumination into a corresponding change in electric current. This cell is used to control electron devices such as televisions, computers etc. The cell is also used to switch on and off the automatic switches of street lights.

(ii) **Photo-voltaic cell :**

A photo voltaic cell is itself a source of emf and does not require any external battery for its operation. So, it a self generating cell and is capable of producing appreciable current in a low resistance circuit even when ordinary light fall on it.

The ejected electrons themselves set up the necessary p.d. between the plates for driving current in the external circuit.

One of the common form of photo voltaic cell consists of a disc of copper coated with cuprous oxide. The oxide coated face is covered with an extremely thin film of silver or gold. The film is so thin than light can easily pass through it.

Hence, when light fall on the silver film, it penetrates and falls on the oxide coated surface and electrons are emitted. These electrons cannot go to the surrounding air but remain confirmed on the film of silver or gold. The oxide layer thus becomes positively charged and the film negatively. An emf is this set up. If an external circuit R and a sensitive galvanometer G are connected between them, a small current will flow through the circuit and the galvanometer will show a deflection.

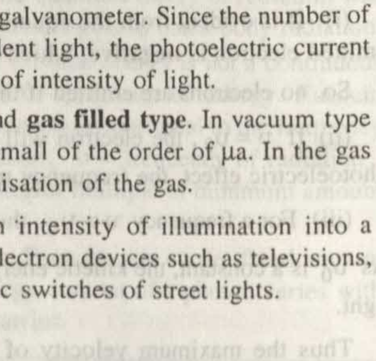


Fig. 2·6

The current available from the cell is proportional to the intensity of light. Hence it can be used to measure intensity of light.

● ● Short Answer Type Questions (with answers) ● ●

● **Q. 1. Production of X-rays and electron emission in photoelectric effect are opposite process.—Explain.**

○ **Ans.** When high speed electrons strike a metal surface X-rays are produced which are photons. On the other hand, when light of suitable frequency fall on a photo sensitive surface, electrons are emitted.

So, for the production of X-rays electrons produce photon. But in photoelectric phenomena photons produce electrons. Hence these two processes are opposite to each other.

● **Q. 2. Photoelectric effect opposes wave theory, but supports quantum theory of light. Explain.**

○ **Ans.** According to wave theory, if the intensity of light is large, the light can start photoelectric effect whatever be the frequency. But quantum theory or photon theory of light predicts that to start photoelectric effect, the frequency of light should be at least equal to a minimum frequency (threshold frequency) whatever be the intensity.

From different experiment it is found that photoelectric effect depends on frequency and not on intensity of incident light. Threshold frequency for a particular substance is fixed. Evidently, photoelectric effect supports photon concept of light.

● **Q. 3. A metal may show photoelectric effect with violet light, but may not show the effect with red light.**

○ **Ans.** It is known that for photoelectric emission, there is a minimum frequency of light called threshold frequency. If is frequency is less than this particular frequency there will be phtoelectric effect.

Now, frequency of violet rays is less than the frequency of red rays. In the present case, the frequency of violet light is higher than the minimum threshold frequency. Hence electron emission is possible with violet light. But the frequency of red light may be less than the required threshold frequency. In that case, photoelectric effect will not occur with red light.

● **Q. 4. On which factors maximum kinetic energy of photoelectrons depend ?**

○ **Ans.** From Einstein's photoelectric equation, $\frac{1}{2}mv^2_{\max} = h\upsilon - \omega_0$

As work function ω_0 is constant, the maximum kinetic energy $\frac{1}{2}mv^2_{\max}$ of photo electrons depends on frequency only.

● **Q. 5. How can the photoelectric current be increased ?**

○ **Ans.** The photoelectric current can be increased by increasing the intensity of incident light. **Reason :** With higher intensity, number of photons striking the surface per sec increases. Then rate of phtoelectric emission increases. Hence the photoelectric current also increases.

● **Q. 6. On which factor, the value of stopping potential depends ?**

○ **Ans.** Stopping potential is the potential which is just able stop photoelectric current, Now from Einstein's equation $eV_s = h\upsilon - \omega_0$

As ω_0 is constant for a metal, stopping potential increases with the increase of the frequency of the incident light.

● **Q. 7. If the intensity of incident light increases, what is the effect on the following factors ? (i) Threshold frequency (ii) Stopping potential (iii) Kinetic energy of photoelectron and (iv) photoelectric current.**

○ **Ans.** (i) As the energy of radiation does not depend on its intensity, increase in intensity does not affect the value of threshold frequency.

(ii) Stopping potential of photoelectrons which is the measure of kinetic energy of most energetic electron does not depend on intensity. So, the value of stopping potential will not change with increase of intensity.

(iii) Kinetic energy of photoelectron does not depend on intensity of incident light. So, with the increase of intensity of incident light, kinetic energy of photoelectron will not change.

(iv) With the increase of intensity of light, photoelectric current increase. **Reason :** If the intensity is increases, number of photons falling on the surface per sec also increases. The rate of emission of photo electron increases. Hence with the increase of intensity of light, photoelectric current increases.

● **Q. 8. Give examples of (i) electron emission by photon and (ii) production of photon by electron.**

○ **Ans.** (i) Photoelectric effect is the example of electron emission by photon.

(ii) Production of X-rays is an example of generation of photon by electrons.

● **Q. 9. The work function of two metals are 2 eV and 4 eV. For which metal threshold wavelength is smaller ?**

○ **Ans.** Work function, $\omega_0 = \frac{hc}{\lambda_0}$ $\quad\therefore\quad \lambda_0 \,\alpha\, \frac{1}{\omega_0}$

Evidently, for the second metal threshold wavelength will be smaller.

● **Q. 10. The work function of sodium is 2·3 eV. If light of wavelength 6800 Å falls on sodium, will photo emission occur ?**

○ **Ans.** Threshold frequency of sodium is $\lambda_0 = \frac{12400}{2\cdot 3}\text{Å} = 5391\text{Å}$

So, for the photo electric to occurs, wavelength should be equal to 5391 Å or less than this. But the given wavelength 6800 Å is higher than the threshold wavelength.

So, the photoelectric emission will not take place.

● ● Numerical Examples ● ●

● **Ex. 2.4. With what velocity must an electron travel so that its momentum is equal to that of photon of wavelength λ = 5200 Å**

○ **Solution :** Momentum of a photon of wavelength λ = 5200 Å is

$$P = \frac{h}{\lambda} = \frac{6\cdot 62 \times 10^{-34}}{5200 \times 10^{-10}} \text{kg. m/s} = \frac{6\cdot 62}{5\cdot 2} \times 10^{-27} \,\mathbf{kg.\,m/s}$$

Now, momentum of electron $= mv = 9\cdot 1 \times 10^{-31} v$

$$\therefore\quad 9\cdot 1 \times 10^{-31} v = \frac{6\cdot 62}{5\cdot 2} \times 10^{-27} \quad \therefore\quad \mathbf{v = 1400\ m/s}$$

● **Ex. 2.5. An X-ray tube operates at 20 kV. Calculate the maximum speed of the electron striking the anticathode. Given, $\frac{e}{m} = 1\cdot 76 \times 10^{11}\,\mathrm{C.kg^{-1}}$**

○ **Solution :** $v=\sqrt{\frac{2eV}{m}}=\sqrt{2\times1\cdot7\times10^{11}V}=5\cdot93\times10^{5}\sqrt{V}$

$$=5\cdot93\times10^{5}\times\sqrt{20,000}=\mathbf{8\cdot4\times10^{7}}\ m/s$$

● **Ex. 2.6. When light falls on sodium metal, the maximum velocity of photoelectron is 0.73 *e*V. If the work function of sodium be 1.82 *e*V, what is the energy of photon is *e*V unit. Also calculate wavelength of incident light ? $h = 6{\cdot}63 \times 10^{-27}$ erg-sec.** **[H. S. 2001]**

○ **Solution :** From Einstein's photoelectric equation $h\upsilon=\frac{1}{2}mv^{2}+\omega_0$

Here, $\frac{1}{2}mv^{2}=0\cdot73\,eV$ and $\omega_0=1\cdot82\,eV$

$\therefore$ Energy of photon, $h\upsilon=0\cdot73+1\cdot82=\mathbf{2\cdot55}\ eV$

2nd part : If λ be the wavelength incident light then $\frac{hc}{\lambda}=2\cdot55\times1\cdot6\times10^{-19}$

$$\therefore\quad \lambda=\frac{6\cdot63\times10^{-34}\times3\times10^{8}}{2\cdot55\times1\cdot6\times10^{-19}}=4\cdot875\times10^{-7}m=\mathbf{4875\ \text{Å}}$$

● **Ex. 2.7. An electron in hydrogen atom jumps from the first orbit to next orbit by absorbing 10·2 *e*V energy. When it returns to the original orbit, what will be the wavelength of the emitted photon.** **[H. S. 2000]**
$h = 6{\cdot}55 \times 10^{-27}$ erg-s.

○ **Solution :** The electrons absorbs 10·2 *e*V to jump from 1st orbit to the next orbit. So it will given the same energy when it returns to initial orbit.

$\therefore$ Energy of the emitted photon $\frac{hc}{\lambda}=10\cdot2\times1\cdot6\times10^{-12}$

$$\therefore\quad \lambda=\frac{6\cdot55\times10^{-27}\times3\cdot0\times10^{10}}{10\cdot2\times1\cdot6\times10^{-12}}=\mathbf{1\cdot204\times10^{-5}\ cm}$$

$\therefore$ Wavelength of the emitted photon = $\mathbf{1{\cdot}204 \times 10^{-5}}$ **cm**

● **Ex. 2.8. Work function of zinc is 3·6 eV. If threshold frequency for zinc be 9×10^{14} Hz, Calculate Planck's constant.** **[H. S. '96]**

○ **Solution :** Work function, $\omega_0=h\upsilon_0$

$$\therefore\quad h=\frac{\omega_0}{\upsilon_0}=\frac{3\cdot6\times1\cdot6\times10^{-12}}{9\times10^{14}}\quad\left[\upsilon_0=9\times10^{14}\,\text{Hz}\right]$$

$$=\mathbf{6{\cdot}4\times10^{-27}\ erg\text{-}sec}$$

$\therefore$ Value of Planck's constant = $\mathbf{6{\cdot}4 \times 10^{-27}}$ **erg-sec**

● **Ex. 2.9. Light of wavelength 6000 Å is allowed to fall on a metal surface. Energy required to liberate an electron from the metal is 1·77 *e*V. Calculate the kinetic energy of fastest photoelectron. What is threshold frequency, [$h = 6{\cdot}62 \times 10^{-27}$ erg-sec; 1 *e*V = $1{\cdot}6 \times 10^{-12}$ erg]** **[J. E. E. '91]**

○ **Solution :** Energy of incident photon, $E=\frac{hC}{\lambda}$

$$\therefore\quad E=\frac{6\cdot62\times10^{-27}\times3\times10^{10}}{6000\times10^{-8}}=3\cdot31\times10^{-12}\text{erg}=\mathbf{2\cdot07}\ eV.$$

Kinetic energy of fastest electron, $E_k=E-\omega_0=2\cdot07-1\cdot77=\mathbf{0\cdot30}\ eV$

$$\therefore\quad \text{Threshold frequency, }\upsilon_0=\frac{\omega_0}{h}=\frac{1\cdot77\times10^{-12}}{6\cdot62\times10^{-27}}=\mathbf{4\cdot28\times10^{14}}\,\text{Hz}$$

● **Ex. 2.10. Photoelectric threshold wavelength of a metal is 3800 Å. Ultraviolet rays of wavelength 2600Å is allowed to fall on the metal. Calculate maximum kinetic energy of electric and work function for the metal.** **[J E E '94]**

○ **Solution :** Work function for the metal, $\omega_0 = \dfrac{hC}{\lambda_0}$

$$\therefore \quad \omega_0 = \frac{6\cdot625\times10^{-34}\times3\times10^{8}}{3800\times10^{-10}} = \frac{6\cdot625\times3}{3\cdot8}\times10^{-19}\,\text{J} = \frac{6\cdot625\times3}{3\cdot8\times1\cdot6}\,eV = \mathbf{3\cdot27\,eV}$$

Energy of photon, $E = \dfrac{hC}{\lambda} = \dfrac{6\cdot625\times10^{-34}\times3\times10^{8}}{2600\times10^{-10}\times1\cdot6\times10^{-19}} = \mathbf{4\cdot78\,eV}$

∴ Maximum kinetic energy of emitted photon $= 4\cdot78 - 3\cdot27 = \mathbf{1\cdot51\,eV}$

● **Ex. 2.11. In fig 2.7 we show the variation of stopping potential (V) with frequency (υ) of the incident light. Find (i) threshold frequency (ii) work function (iii) value of Planck's constant.**

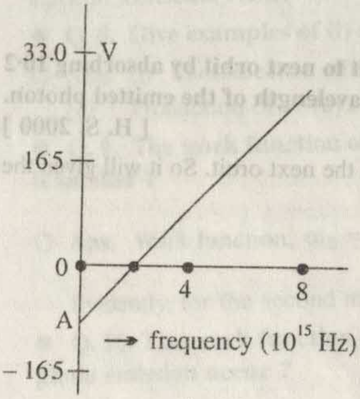


Fig. 2·7

○ **Solution :** From Einstein's equation : $h\upsilon = \dfrac{1}{2}mv^2{}_{\max} + \omega_0$

Now, $eV_s = \dfrac{1}{2}mv^2{}_{\max} = h\upsilon - \omega_0 \quad \therefore \quad V_s = \dfrac{h}{e}\upsilon - \dfrac{\omega_0}{e}$

It is apparent from the equatin that the slope of the $(V_s - \upsilon)$ graph is $\dfrac{h}{e}$ and intercept on the negative V_s- axis is $\dfrac{\omega_0}{e}$. Hence

(i) Threshold frequency, $\upsilon_0 = 2\cdot0\times10^{15}\,\text{Hz}$

(ii) Intercept on the negative V_s- axis, OA = – 8·25 volt

∴ Work function = 8·25 eV

(iii) Slope of the graph is $\dfrac{h}{e} = \dfrac{16\cdot5}{4\times10^{5}} = 4\cdot125\times10^{-15}$

$$\therefore \quad h = 4\cdot125\times10^{-15}\times1\cdot6\times10^{-19} = \mathbf{6\cdot8^{-34}\,J\text{-}s}$$

● **Ex. 2.12. The energy of photon is equal to rest energy of a electron. Calculate (i) frequency (ii) wavelength and momentum of that photon.**

○ **Solution :** Energy of an electron when it is at rest is $E = m_0c^2$

$$\therefore \quad E = 9\cdot1\times10^{-31}\times(3\times10^{8})^2 = 8\cdot19\times10^{-14}\,\text{J}$$

(i) Frequency of the photon, $\upsilon = \dfrac{E}{h} = \dfrac{8\cdot19\times10^{-14}}{6\cdot6\times10^{-34}} = \mathbf{1\cdot24\times10^{20}\,Hz}$

(ii) Wave length of photon, $\lambda = \dfrac{c}{\upsilon} = \dfrac{3\times10^{8}}{1\cdot24\times10^{20}} = \mathbf{2\cdot41\times10^{-12}\,m}$

(iii) Momentum of the photon, $p = \dfrac{E}{c} = \dfrac{8\cdot19\times10^{-14}}{3\times10^{8}} = \mathbf{2\cdot73\times10^{-22}\,kg\,m/s}$

● **Ex. 2.13. A transmitter of power 1 KW generates waves of frequency 880 Hz. How many photons will be emitted per sec ?**

○ **Solution :** Power of the transmitter = 10^3 W = 10^3 Js^{-1}

Energy of a single photon, $E_1 = h\upsilon = 6\cdot6\times10^{-34}\times880\times10^3 = \mathbf{58\cdot08\times10^{-29}\ J}$

$\therefore$ No. of photons emitted / sec $= \dfrac{10^3}{58\cdot08\times10^{-29}} = \mathbf{1\cdot73\times10^{30}}$

• Ex. 2.14. Light of wavelength 2000 Å falls on a photo sensitive material having work function 4.2 *e*V. What is the kinetic energy of the fastest and slowest photoelectron ? Also calculate the stopping potential. Given *h* = 6·62 × 10⁻³⁴ J. s., *e* = 1·6 × 10⁻¹⁹C.

○ **Solution :** Energy of photon, $E = \dfrac{hc}{\lambda} = \dfrac{6\cdot62\times10^{-34}\times3\cdot0\times10^{8}}{2000\times10^{-10}} = \mathbf{9\cdot93\times10^{-19}\ J}$

Work function, $\omega_0 = 4\cdot2\ eV = 4\cdot2\times1\cdot6\times10^{-19} = 6\cdot72\times10^{-19}$ J

$\therefore$ Kinetic energy of the fastest photoelectron is

$$\frac{1}{2}mv_m{}^2 = h\upsilon - \omega_0 = (9\cdot93 - 6\cdot72)\times10^{-19} = 3\cdot21\times10^{-19}\ \text{J} = \mathbf{2\ eV}$$

And the kinetic energy of the slowest electron = 0

Stopping potential : If V_s be the stopping potential, then $e.V_s = \frac{1}{2}mv^2{}_m = 2eV$

$\therefore$ $\mathbf{V_s = 2}$ **volt**

• Ex. 2.15. Ultra violet light of wavelength 350 nm and intensity 1 watt / m^2 is directed at a potassium surface. Find (i) The maximum kinetic energy of photoelectron (ii) If 0.5 % of incident photons produce photoelectrons, how many photoelectrons are emitted per sec if the surface of potassium is 1cm². Work function of potassium is 2.1 *e*V.

○ **Solution :** Frequency of incident light, $\upsilon = \dfrac{c}{\lambda}$

$$\therefore\ \upsilon = \frac{3\times10^{8}}{350\times10^{-9}} = \mathbf{8\cdot57\times10^{14}\ Hz}$$

$\therefore$ Work function, $\omega_0 = 2\cdot1\ eV = 3\cdot36\times10^{-19}$ J

(i) Energy incident per sec on 1 cm² surface of potassium = $\mathbf{10^{-4}\ J}$

Effective energy per sec on 1 cm² surface $= \dfrac{0\cdot5}{100}\times10^{-4} = \mathbf{5\times10^{-7}\ J}$

$\therefore$ Number of photon electrons emitted per sec from 1 cm² area of the surface of potassium

$$= \frac{5\times10^{-7}}{\omega_0} = \frac{5\times10^{-7}}{3\cdot36\times10^{-19}} = \mathbf{1\cdot5\times10^{12}}$$

• E ◇ X ◇ E ◇ R ◇ C ◇ I ◇ S ◇ E •

(A) Objective type questions :

• Fill up the blanks :

1. According to quantum theory radiation is made of ———— .
2. Energy of photon is ———— to frequency.
3. Momentum of photon ———— .
4. Photoelectric effect confirms ———— of radiation.
5. The minimum frequency to start photoelectric effect is called ———— ———— .

6. With increase of frequency stopping potential ———— .
7. Rest mass of photon ———— .
8. Velocity of photon in vacuum ———— .

◢ (B) Very short answer type questions :

1. What is quantum theory of radiations ?
2. What is quantum of energy ?
3. What is photon ? What is its rest mass ?
4. How is energy of photon measured ?
5. Does the energy of a photon depend on its intensity ?
6. What is threshold frequency ?
7. What is work function ? How it is measured ?
8. On which factor maximum energy of photoelectric depends ?
9. Give example where photon is emitted by electron.
10. If wavelength of light be λ, what is the momentum of photon ?
11. The wave length of electromagnetic radiation is doubled. What will happen to the energy of the photons ?
12. Two metals A and B have work functions 2 eV and 4 eV respectively. Which metal has a lower threshold wavelength for photoelectric effect ?
13. If the intensity of the incident radiation is increased, how does the stopping potential vary.
14. What is the energy associated with a photon of wavelength the 4000 Å. **[Ans.** $4{\cdot}98 \times 10^{-19}$ **J]**
15. Obtain the energy in joule acquired by an electron when accelerated through a potential difference of 2000 V ? **[Ans :** 3.2×10^{-16} **J]**
16. If the maximum kinetic energy of photoelectron be 5 eV, what is the stopping potential ? **[Ans : 5 V]**
17. The frequency υ of incident radiation is greater them the threshold frequency υ_0. How will the stopping potential vary if frequency υ is increased, keeping other factors constant ?

◢ (C) Short answer type questions :

1. Define work function and threshold frequency ? How are they related ?
2. Write down Einstein photoelectric equation ?
3. What is the difference between thermionic emission and photoelectric emission.
4. What is wave theory of light ? How does it differ from quantum theory ?
5. Why does wave theory fail to explain the observations of photoelectric effect ?
6. What is quantum theory of radiation ?
7. A photon of energy 3·0 eV is incident on a metal surface with work function 2·5 eV ? Will it necessarily eject photoelectron ?
8. Draw a graph to show the variation of stopping potential with frequency of radiation incident on the metal plate. How can the value of Plank's constant be determined from the graph ?
9. If we go on increasing the wavelength of light incident on a metal surface. What changes in the number of electrons and energy take place ?
10. Radiations of frequency 10^{15} Hz is incident on two photosensitive surfaces P and Q. Following observations are recorded

    (i) Surface P : No electron emission.

    (ii) Surface Q : Electrons are emitted with zero kinetic energy. Based on Einstein's photoelectric equation, explain the two observations.

◢ (D) Essay type questions :

1. What is photoelectric effect ? What is the importance of this effect ?
2. Describe an experiment to demonstrate photoelectric effect. Show graphically the variation of stopping potential with frequency of light.

3. What are the characteristics of photoelectric effect ? What is threshold frequency ?
4. Discuss electromagnetic wave theory and quantum theory of light.
5. Obtain an expression for the maximum kinetic energy of the electrons emitted from a metal surface in terms of frequency of the incident light and the threshold frequency.
6. Explain the laws of photoelectric emission on the basis of Einstein's photoelectric equation. Why the laws cannot be explained on the basis of wave nature of light ?
7. How does (i) photoelectric current, and (ii) kinetic energy of the photo electrons emitted in a photoelectric will vary if the intensity of the incident radiation is doubled.
8. Describe a photoemissive cell. How does it work ?

◢ (E) Simple numerical problems :

1. Threshold wavelength of a metal is 6525 Å. If a light of wavelength 4000Å falls on the metal, what will be its stopping potential ? **[Ans : 1.2 Volt]**
2. Wavelength of ultraviolet radiation is 3000 Å. What is the energy of photon ? **[Ans : 4·125 *e*V]**
3. Light of wavelength 400 nm is incident on the cathode of a photocell ? The stopping potential is 6 V. If the wavelength of incident light is increased to 600 nm, calculate new stopping potential. **[Ans : 4.97 volt]**
4. Work function of potassium is 2·2 eV. What is the threshold frequency for photoelectric emission ? **[Ans : 5·31 × 10^{14} Hz] [J E E '98]**
5. When a light of wavelength 5000 Å fall on a metal, maximum kinetic energy of photoelectric is 0·3 *e*V. Find the work function of the metal in *e*V units. $h = 6{\cdot}64 \times 10^{-27}$ erg.s; 1 *e*V = 1·6 × 10^{-12} erg. **[Ans : 2·19 *e*V] [H. S. '98]**
6. Work function of copper 4·4 *e*V. If visible light is allowed to fall on a piece of copper, will photoelectrons be emitted ? **[Ans : No, as λ_0 = 2820 Å < 4000 Å]**
7. Work function of ceasium 1·9 *e*V. Calculate : (a) threshold frequency, (b) if wavelength of incident light be 4000 Å, find the kinetic energy of photoelectrons (c) stopping potential. **[Ans : (a) 0·46 × 10^{15} Hz; (b) 1·203 *e*V (c) 1·203 V]**
8. Work function of metal zinc is 3·6 *e*V and its threshold frequency 9 × 10^{14} Hz, find the value of Planck's constant. **[Ans : 6·4 × 10^{-34} J. s.] [H. S. '96]**
9. For a metal threshold wavelength is 3000Å. If a light of wavelength 1000 Å falls on the metal, what will be the maximum kinetic energy of photoelectron ? **[Ans : 8·1 *e*V]**
10. The power of a source is 25 W. If it emits monochromatic light of wavelength 6000 Å, calculate the number of photons emitted from the source per sec. **[Ans : 7·57 × 10^{19}]**
11. Photo work function of sodium is 2·3 *e*V. If light of wavelength 6800 Å falls on sodium, show whether photoelectric effect will occur or not. **[Ans : No]**
12. Energy of photon is 75 *e*V. What are its frequency and momentum ? **[Ans : 18·18 × 10^{15} Hz; 4 × 10^{-26} kg-m / s]**
13. At what temperature kinetic energy of an ideal gas molecule be equal to the energy of a photon of wavelength 6000 Å ? Given, Boltzmann constant K = 1·38 × 10^{-23} J K^{-1} **[Ans : 1·6 × 10^4 K] [J E E '99]**
14. Calculate the number of photons of wavelength 6123 Å from an electric bulb of power 100 W. Given, 80% of power is utilised for emitting radiation and only 2% of the radiant energy emits 6123Å wavelength, $h = 6{\cdot}62 \times 10^{-34}$ J-s. **[Ans : 4·93 × 10^{18}]**
15. When ultraviolet lights of wavelengths 830Å and 700Å fall on hydrogen atom in its ground state, electrons of kinetic energy 1·8 *e*V and 4·0 *e*V are emitted. Calculate Planck's constant. **[Ans : 6·57 × 10^{-34} J.s.]**
16. The work function of tungstein and sodium are 4·4 *e*V and 2.3 *e*V. If threshold wavelength of sodium be 5460Å, calculate that of tungsten. **[Ans : 2854 Å]**
17. When light of wavelength 3000 Å fall on sodium, stopping potential is 1·85 V, but when wavelength 4000 Å fall on sodium stopping potential is 0·82 V. Calculate (i) Planck's constant (ii) work function of sodium and (iii) threshold wavelength of sodium. **[Ans : (i) $h = 6{\cdot}6 \times 10^{-34}$ J. s. (ii) W = 2·27 *e*V (iii) 5451 Å]**

18. Light of wavelength 5000 Å falls on metal surface. If the metal acquires an energy of 10^{-7} J, how many photons fall on the surface. **[Ans :** $2{\cdot}5 \times 10^{11}$ **]**

19. Threshold frequency of a metal is υ_0. When the frequency of light is $\upsilon = 2\upsilon_0$, the maximum velocity of photoelectron is 4×10^8 m / s. What will be the maximum velocity of photoelectron when $\upsilon = 5\,\upsilon_0$? **[Ans :** 8×10^8 m / s **]**

20. In an experiment with photoelectric effect the following data are obtained :

Frequency (υ) × 10^{14} Hz	3	4	5	6	7	8
Maximum K E of electron (in *e*V units)	0	0.5	0·9	1·3	1·7	2·1

Draw the graph. From the graph calculate (i) work function of the metal and (ii) Planck's constant.
[Ans : (i) 1·1 *eV*; (ii) $6{\cdot}4 \times 10^{-34}$ J-s **]**

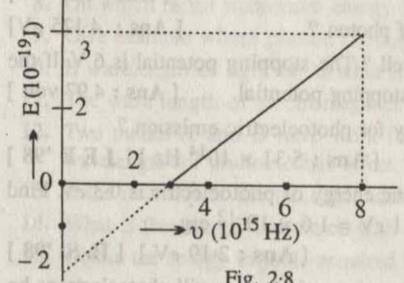


Fig. 2·8

21. In an experiment with photoelectric effect, the fig 2.8 shows the variation of kinetic energy (E) of electron with frequency (υ). Calculate :

(i) threshold frequency

(ii) work function

(iii) Planck's constant

(iv) Kinetic energy of electron at frequency 8×10^{14} Hz.

[Ans : (i) 3×10^{14} Hz (ii) 2×10^{-19} J = 1·25 *eV*, (iii) 6×10^{-34} J-s (iv) 3×10^{-19} J **]**

(F) Harder numerical problems :

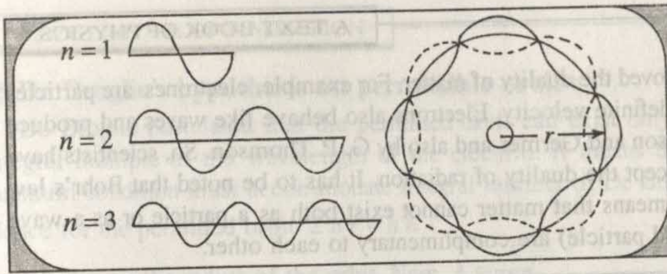
1. The power of incident radiation on earth from the sun is 1400 W / m². If the wavelength of rays coming from the sun is 6000 Å, how many photons falls on earth per sec on 1m² area ? Also calculate, the number of photons given at by the sun per sec. Given, average radius of the earth's orbit $1{\cdot}5 \times 10^{11}$m, $h = 6{\cdot}62 \times 10^{-34}$ J. s.
[Ans : $4{\cdot}23 \times 10^{21}$; $1{\cdot}195 \times 10^{45}$ **]**

2. The wavelength of a photon is 1.4 Å. Due to its collision with a stationary electron its wavelength becomes 2Å. What is the energy of the deflected electrion ? **[Ans :** $2{\cdot}66 \times 10^3$ *e*V **]**

3. When light of wavelength 4940Å falls on a metal, the stopping potential of fastest photoelectron is 0·6V. When a light of different wavelength is allowed to fall on the metal, the stopping potential becomes 1·1 V. Calculate (i) work function and (ii) wavelength of light in the second case. **[Ans.** (i) 1·9 *eV* (ii) 4125 Å **]**

4. Monochromatic light of wavelength $36{\cdot}5 \times 10^{-8}$ *m* and intensity 10^{-8} W / m^2 is allowed to fall on a surface, Absorption coefficient of the surface is 0·8 and work function 1·6 *e*V. Calculate (i) rate of emission of electron from 1m^2 area, (ii) energy absorbed by 1m^2 area and (iii) kinetic energy of photo electron.
[Ans : (i) $18{\cdot}38 \times 10^9$; (ii) $8{\cdot}0 \times 10^{-9}$ W / m^2; (iii) 1·8 *e*V **]**

5. In a hydrogen like atom (Z = 3) The electron jumps from fifth orbit to fourth orbit and then from fourth to third orbit. The two photons emitted in the two cases are allowed to fall on a metal and photoelectrons are emitted. The stopping potential for the shorter wavelength light is 3·95 volt. Calculate (i) work function and (ii) stopping potential for longer wavelength light. **[Ans :** (i) 2·0 *e*V (ii) 0·754 V **]**



CHAPTER 3

WAVE NATURE OF RADIATION

● **TOPICS :** ○ Dual nature of matter : De Broglie's wave; ○ De Broglie's wave equation; ○ Bohr's postulate of momentum of non-rediating stationary orbits; ○ De Broglie's wavelength of accelerated electron; ○ Properties of matter wave; ○ Short answer type questions (with answers); ○ Exercise.

❂ 3.1. Dual Nature of matter : De Broglie's wave ❂

It has been established that light has dual aspect behaving either as particles or as waves under suitable circumstances. For example, radiations (e.g. visible light, infrared, ultraviolet rays, X-rays etc) behave as waves in propagation experiments like interference, diffraction and polarisation. These experiments firmly establish wave nature of matter.

On the other hand, the phenomena like photoelectric effect, Crompton effect etc show that radiation behaves as a particle in intaction experiments. Here, radiation interact with matter in the form of photons or quanta.

Thus radiations possess a dual characteristic. However, radiation does not exhibit its wave and particle aspect simultaneously. Radiation behaves as a wave in transmission and as a particle when it interacts with matter.

A wave is characterised by its (i) frequency υ (ii) wavelength λ (iii) phase or wave velocity u (iv) amplitude a and (v) intensity I. Moreover, a wave spreads out and occupies a relatively large region of space.

A particle is characterised by its (i) mass m (ii) velocity v_p (iii) momentum p (iv) energy E.

Moreover, a particle occupies a definite position in space and hence is highly localised.

Evidently, the nature and properties of a wave and a particle are to a large extent conflicting. Because a wave is spread out in space and a particle is localised at a point in space. But these dual properties of light or radiation is experimentally confirmed.

Considering this dual nature of radiation (wave aspect and particle aspect) French theoretical physicist Louis De Broglie made a very bold and novel suggestion that like radiation, matter also possesses dual (i.e. particle like and wave like) characteristics. His suggestion was partly based on his intuitive feeling that nature is symmetrical. So, if radiation can show dual nature, then things like electrons which are considered particles should also act as waves at times. This is **dual nature of matter**. The waves associated with matter is called **matter wave** or **De Broglie wave**. According to his hypothesis, the mathematical relation representing dual nature of material particle can be obtained from the equation 3.1*. The relation between the wavelength λ associated with material particle moving with a momentum p is obtained from equation 3.1.

The wavelength associated with particle is $\lambda=\frac{h}{p}=\frac{h}{mv}$ . . . .(3.2)

Here m is the mass and v is the velocity of the particle. It must be noted that matter wave is not electromagnetic wave.

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* From the theory of Compton scalting, the momentum of light quanta or photon is

$p=\frac{h\upsilon}{c}=\frac{h}{\lambda}$ . . . . (3.1)

Experiment performed later actually proved the duality of matter. For example, electrones are particles having a discrete mass and moving with definite velocity. Electrons also behave like waves and produce diffraction patterns as was shown by Davison and Germer and also by G. P. Thomson. So, scientists have accepted this duality of matter, as they accept the duality of radiation. It has to be noted that Bohr's law of complimentarily hold good here. This means that matter cannot exist both as a particle or as a wave simultaneously. The two aspects (wave and particle) are complimentary to each other.

✪ 3.2. De Broglie's wave equation ✪

In his theory of relativity Einstein had proved the equivalence of mass and energy i.e., $E = mc^2$, where m is the mass and c is the velocity of light in vacuum.

Similarly, in accordance with quantum theory of radiation proposed by Max Planck a radiation of frequency υ consists of 'quanta' or 'photon' of energy $E = h\upsilon$, h = Planck's constant. Combining these two, we get $E = h\upsilon = mc^2$

But in the case of a wave $c = \upsilon\,\lambda$. Thus, for a quantum of light or photon we get,

$$E = \frac{hc}{\lambda} = mc^2 \qquad \therefore\ \lambda = \frac{h}{mc}$$

$$\therefore \quad \lambda = \frac{h}{p} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.3)$$

where $p = mc$ = momentum of photon.

Hence λ is the wavelength associated with quantum or particle having momentum p.

It means that a material particle of mass m moving with a velocity v, can be considered as a wave of wavelength λ given by the relation (3.3). This is **De Broglie wave equation.** This equation firmly establishes the concept of duality of matter, since the momentum p can be associated only with a particle while λ is associated only with a wave.

- **A discussion :** (i) $\lambda\ \alpha\ \frac{1}{v}$ i. e. if $v = 0$, $\lambda = \infty$ Thus, a material particle has associated matter wave only when it is motion.

(ii) $\lambda\ \alpha\ \frac{1}{m}$ if the moving particle has smaller mass, higher is the wavelength associated with it.

(iii) $\lambda\ \alpha\ \frac{1}{p}$. Greater the momentum of the particle, shorter is the associated wavelength.

(iv) Wave length of matter wave is independent of the charge of the particle.

✪ 3.3. Bohr's postulate of momentum of non-radiating stationary orbits ✪

According Bohr's theory related to stationary orbit of electron in atom, only those orbits are permissible in which the angular momentum of the electron is an integral multiple of $\frac{h}{2\pi}$. Mathematically, for a permitted stationary orbit, the condition to be satisfied is

$$mvr = \frac{nh}{2\pi} \quad [n = 1, 2, 3.......].$$

Here m is the mass of electron, v is the velocity of electron, r is the radius of the orbit. n is called principle quantum number. This condition is called **Bohr's quantum condition for stationary orbit**.

De Broglie's hypothesis on permissible orbit :

De Broglie postulated that the permitted orbit can exist only when orbit's circumference equals an integral multiple of the wavelength of the electron. It means that the electron orbit satisfying Bohr's quantum condition must accommodate integral number of De Broglie wave associated with the electron.

Hence for the permitted orbit $2\pi r = n\lambda$

Where r is the radius of the orbit. Now, $\lambda = \dfrac{h}{mv}$

$$\therefore \quad 2\pi r = n\frac{h}{mv} \quad \therefore \quad m\,v\,r = n\frac{h}{2\pi} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.4)$$

This is the same condition as given by Bohr's quantum postulate.

Evidently, the first Bohr orbit ($n = 1$) will accommodate only one full wave. Similarly the second and third orbit will accommodate two and three full waves associated with electron. In the diagram 3.1 we show the permitted stationary orbit for an electron where $n = 4$. Here the whole circumference of the orbit is equal to four full waves of wavelength λ given by de Broglie relation. So, we note that the concept of de Broglie waves is fully reconciled with the observed quantum condition of Bohr's model of the atom.

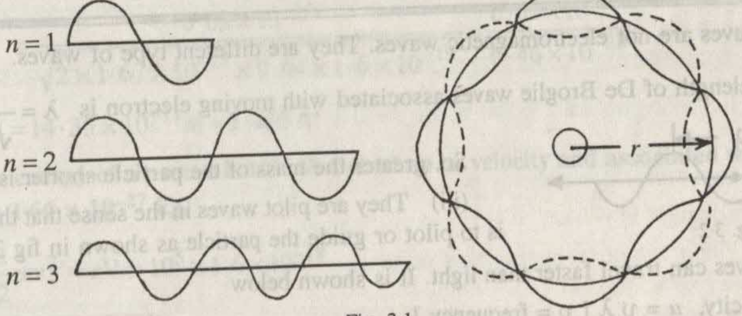


Fig. 3·1

3.4. De Broglie's wavelength of accelerated electron

The question of energy and momentum of an electron arises when it is treated as a particle. On the other hand, when we consider electron as a wave, the associated quantities are frequency and wavelength. Now if the electron is supposed to possess both particle and wave nature, then its energy, E = $h\upsilon$ and momentum, $p = \dfrac{h}{\lambda}$.

Now, let an electron is accelerated through a potential difference V. If its velocity be v, then by the principle of conservation of energy $\dfrac{1}{2}mv^2 = e\text{V} \quad \therefore \quad mv = \sqrt{2me\text{V}}$

$\therefore$ De Broglie wavelength associated with the moving electron is $\lambda = \dfrac{h}{mv} = \dfrac{h}{\sqrt{2me\text{V}}}$

Putting $h = 6\cdot62 \times 10^{-34}\,\text{J - s}$, $e = 1\cdot6 \times 10^{-19}\,c$ and $m = 9\cdot1 \times 10^{-31}\,\text{kg}$

$$\lambda = \frac{6\cdot62 \times 10^{-34}}{\sqrt{2 \times 9\cdot1 \times 10^{-31} \times 1\cdot6 \times 10^{-19} \times \text{V}}} = \frac{12\cdot25 \times 10^{-10}}{\sqrt{\text{V}}}\,m$$

$$\therefore \quad \lambda = \frac{12\cdot25}{\sqrt{\text{V}}}\,\text{Å} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.5)$$

(i) **Wave number, $\bar{\upsilon}$** : Number of waves that can be accommodated in unit length is called wave number.

$$\therefore \quad \bar{\upsilon} = \frac{1}{\lambda} = \frac{\sqrt{V}}{12\cdot25}\times10^{10}\,m^{-1}$$

(ii) **Wave velocity, u :** Energy equation E = $h\upsilon$

$$\therefore \quad \upsilon = \frac{E}{h} = \frac{1}{2}\frac{mu^2}{h} = \frac{m^2u^2}{2mh} = \frac{1}{2m}.h\frac{m^2u^2}{h^2} = \frac{h}{2\pi}\times\frac{2\,e\,m\,V}{h^2} = \frac{h}{2\pi}.\frac{1}{\lambda^2}$$

$$\therefore \quad \text{Wave velocity,} \quad u = \upsilon\lambda = \frac{h}{2\pi\lambda} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.6)$$

❂ 3.5. Properties of matter wave ❂

(i) Matter waves are not electromagnetic waves. They are different type of waves.

(ii) The wavelength of De Broglie waves associated with moving electron is $\lambda = \frac{h}{\sqrt{2m\,eV}}$

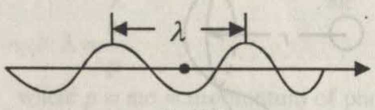


Fig. 3·2

So, greater the mass of the particle shorter is the wavelength.

(iii) They are pilot waves in the sense that their only function is to pilot or guide the particle as shown in fig 3.2.

(iv) These waves can travel faster than light. It is shown below

wave velocity, $u = \upsilon\,\lambda$ [υ = frequency] ……………… (i)

Now, $E = h\upsilon$ or, $\upsilon = \frac{E}{h}$ Also, $E = mc^2$

$$\therefore \quad \upsilon = \frac{mc^2}{h} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \text{(ii)}$$

$$\text{Now } \lambda = \frac{h}{mv} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \text{(iii)}$$

Hence, from the relations (i), (ii), and (iii) $u = \frac{mc^2}{h}\times\frac{h}{mv} = \frac{c^2}{v}$

Since, particle velocity $v < c,\ u > c$

So, the velocity of matter wave is greater than the velocity of light.

●● Numerical Examples ●●

● **Example. 3.1. The wave length associated with an electron is 2Å. What is its momentum ? Given, $h = 6{\cdot}626 \times 10^{-24}$ J. s.**

○ **Solution :** From De Broglie equation, $p = \frac{h}{\lambda}$

$$\therefore \quad p = \frac{6\cdot626\times10^{-34}}{2\times10^{-10}} = 3\cdot313\times10^{-24}\,kg\,ms^{-1}$$

It is the momentum of the electron.

- **Ex. 3.2. If an electron is allowed to pass through a potential difference of 150V, what is the wavelength of the wave associated with it ? $e = 1\cdot6 \times 10^{-19}$ C and $m = 9 \times 10^{-31}$ kg**

○ **Solution :** The required relation : $\lambda = \dfrac{h}{mv} = \dfrac{h}{\sqrt{2meV}}$

$$\therefore \text{ Wave length, } \lambda = \frac{6\cdot62\times10^{-34}}{\sqrt{2\times9\cdot1\times10^{-31}\times1\cdot6\times10^{-19}\,\mathrm{V}}} = \frac{12\cdot25}{\sqrt{\mathrm{V}}}\,\text{Å} = \frac{12\cdot25}{\sqrt{150}} = 1\,\textbf{Å}$$

- **Ex. 3.3. Mass of neutron is $1\cdot67 \times 10^{-27}$ kg and its kinetic energy 0·04 eV. Calculate the wavelength of the associated De Broglie wave of the neutron.**

○ **Solution :** $\lambda = \dfrac{h}{mv} = \dfrac{h}{\sqrt{2m\mathrm{E}}}$ [E = kinetic energy]

$$= \frac{6\cdot62\times10^{-34}}{\sqrt{2\times1\cdot67\times10^{-27}\times0\cdot04\times1\cdot6\times10^{-19}}} = \frac{6\cdot62\times10^{-34}}{0\cdot46\times10^{-23}}$$

$$\therefore \quad \lambda = 14\cdot35\times10^{-11}\,m = \mathbf{1\cdot435\ Å}$$

- **Ex. 3.4. The energy of a proton is 10^5 *e*V. Calculate its velocity and associated wavelength. Given, mass of proton $1\cdot66 \times 10^{-27}$ kg.**

○ **Solution :** $\dfrac{1}{2}mv^2 = e\mathrm{V} = 10^5\times1\cdot6\times10^{-19}$

$$v = \sqrt{\frac{2\times1\cdot6\times10^{-14}}{1\cdot66\times10^{-27}}} = 4\cdot47\times10^{6}\,ms^{-1} \text{ and De Broglie wavelength, } \lambda = \frac{h}{mv}$$

$$\therefore \quad \lambda = \frac{6\cdot62\times10^{-34}}{1\cdot66\times10^{-27}\times4\cdot47\times10^{6}} = \mathbf{8\cdot87\times10^{-14}}\,m$$

- **Ex. 3.5. Calculate the wavelength of thermal neutron at 27°C; given mass of neutron = $1\cdot67 \times 10^{-27}$ kg, Planck's constant, $h = 6\cdot6 \times 10^{-34}$ J-s, Boltzmann constant $k = 1\cdot376 \times 10^{-23}$ Jk^{-1}**

○ **Solution :** De Broglie wavelength, $\lambda = \dfrac{h}{\sqrt{2m\mathrm{E}}} = \dfrac{h}{\sqrt{3mk\mathrm{T}}} \left[\because \dfrac{1}{2}mv^2 = \mathrm{E} = \dfrac{3}{2}\mathrm{KT}\right]$

$$\therefore \quad \lambda = \frac{6\cdot6\times10^{-34}}{\left[3\times1\cdot67\times10^{-27}\times1\cdot376\times10^{-23}\times300\right]^{1/2}} = 1\cdot452\times10^{-10}\,m = \mathbf{1\cdot452\ Å}$$

- **Ex. 3.6. Find the de Broglie wavelength of an electron in the first Bohr orbit of hydrogen atom.**

○ **Solution :** Energy of electron in the *n* th Bohr orbit of hydrogen atom $\mathrm{E}_n = -\dfrac{13\cdot6}{n^2}\,e\mathrm{V}$

For the first Bohr orbit $n = 1$, $\mathrm{E}_1 = -13\cdot6\ e\mathrm{V} = -\ 13\cdot6 \times 1\cdot6 \times 10^{-19}$ J

$$\therefore \text{ De Broglie wavelength, } \lambda = \frac{h}{\sqrt{2m|\mathrm{E}_1|}} = \frac{6\cdot62\times10^{-34}}{\sqrt{2\times9\cdot1\times10^{-31}\times13\cdot6\times1\cdot6\times10^{-19}}}$$

$\therefore$ $\lambda = 3\cdot3 \times 10^{-10}$ m = **3·3 Å**

● **Ex. 3.7. The wavelength of yellow spectral line of sodium is 5890 Å. At what kinetic energy an electron will have the same de Broglie wavelength ?**

○ **Solution :** Let E be the kinetic energy of electron when its de Broglie wavelength will be equal to $\lambda = 5890$ Å $= 589 \times 10^{-9}$ m.

$$\text{Now,} \quad \lambda = \frac{h}{\sqrt{2m\text{E}}} \quad \text{or,} \quad \lambda^2 = \frac{h^2}{2m\text{E}}$$

$$\therefore \quad \text{E} = \frac{h^2}{2m\lambda^2} = \frac{(6\cdot62 \times 10^{-34})^2}{2 \times 9\cdot1 \times 10^{-31} \times (589)^2 \times 10^{-18}} = 6\cdot94 \times 10^{-25}\,\text{J}$$

$$= \frac{6\cdot94 \times 10^{-25}}{1\cdot6 \times 10^{-19}} = \mathbf{4\cdot34 \times 10^{-6}\,eV}$$

●● Short Answer Type Questions (with answers) ●●

● **Question 1. What is a photon ? What is the measurement of energy ?**

○ **Ans.** According quantum theory, radiation is made of energy packet or quanta whose energy depends on frequency υ of radiation. These energy packets are called photons.

Measurement of its energy, $\text{E} = h\,\upsilon$, h = Planck's constant.

● **Q. 2. From a radiation of frequency υ, n number of photons passes through a unit area per sec. What is the intensity of radiation ?**

○ **Ans.** Energy of photon, $\text{E}_1 = h\,\upsilon$

$\therefore$ Energy of n photons, $\text{E} = n\,\text{E}_1 = n\,h\,\upsilon$.

So, the energy that passes through unit area $= nh\upsilon$

Hence, the intensity of radiation, $\mathbf{I = nh\upsilon}$.

● **Q. 3. What is the momentum of a photon ?**

○ **Ans.** The momentum of a photon is, $\text{P} = \frac{\text{E}}{c} = \frac{h\upsilon}{c}$

$\therefore \quad \text{P} = \frac{h}{\lambda}$ here, E = Energy of photon.

● **Q. 4. The kinetic energy of a proton and an electron are equal. Which particle has longer De Broglie wavelength ?**

○ **Ans.** De Broglie wavelength, $\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2m\text{E}}}$

$\therefore \quad \lambda \propto \frac{1}{\sqrt{m}}$ as h and E are constant.

$\therefore \quad \frac{\lambda_e}{\lambda_p} = \sqrt{\frac{m_p}{m_e}}$ where m_p = mass of proton and m_e = mass of electron.

As $m_p > m_e$, $\lambda_e > \lambda_p$

Hence the de Broglie wavelength associated with electron is larger.

- **Q. 5. The de Broglie wavelengths associated with a proton and electron are equal. Whose kinetic energy is greater ?**

○ **Ans.** De Broglie wavelength, $\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2m\text{E}}}$

or, $\text{E} = \frac{h^2}{2m\lambda^2}$ $\quad \therefore\ \text{E} \propto \frac{1}{m}$ $\quad [\because\ h$ and λ are constant]

$$\therefore\ \frac{\text{E}_e}{\text{E}_p} = \sqrt{\frac{m_p}{m_e}} > 1 \quad \therefore\ \text{E}_e > \text{E}_p$$

∴ Energy of electron will be higher than that of proton.

- **Q. 6. The de Broglie wavelength both for a photon and an electron is 10^{-10} *m*. Whose kinetic energy is greater ?**

○ **Ans.** In the case of electron, $\text{E} = \frac{1}{2}mv^2 \quad \therefore\ v = \sqrt{\frac{2\text{E}}{m}}$

and $\lambda = \frac{h}{mv} = \frac{h}{m}\sqrt{\frac{m}{2\text{E}}} = \frac{h}{\sqrt{2m\text{E}}} \quad \therefore\ \text{E} = \frac{h^2}{2m\lambda^2}$

Next in the case of photon : $\text{E}' = \frac{hc}{\lambda}$

$$\therefore\ \frac{\text{E}'}{\text{E}} = \frac{hc}{\lambda} \times \frac{2m\lambda^2}{h^2} = \frac{2m\lambda c}{h} = \frac{2 \times 9{\cdot}1 \times 10^{-31} \times 10^{-10} \times 3 \times 10^8}{6{\cdot}62 \times 10^{-34}} = 82{\cdot}7$$

∴ $\text{E}' > \text{E}$. Energy of photon is greater.

- **Q. 7. A particle is moving with a kinetic energy K and its associated wavelength of de Broglie wave is λ. If the kinetic energy be $\frac{\text{K}}{4}$, what will be its de Broglie wavelength ?**

○ **Ans.** The expression for de Broglie wavelength is $\lambda = \frac{h}{\sqrt{2m\text{K}}}$ when the kinetic energy is K. If the kinetic energy be $\frac{\text{K}}{4}$, the corresponding wavelength is $\lambda' = \frac{h}{\sqrt{2m\frac{k}{4}}} \quad \therefore\ \lambda' = 2\lambda$

- **Q. 8. An electron is allowed to pass through a potential difference of 300 V. Calculate its de Broglie wavelength.**

○ **Ans.** The wavelength is given by, $\lambda = \frac{12{\cdot}3}{\sqrt{\text{V}}}\,\text{Å} = \frac{12{\cdot}3}{\sqrt{300}}\,\text{Å} = \mathbf{0{\cdot}71\ Å}$

- **Q. 9. What is the rest mass of photon ?**

○ **Ans.** The rest mass of photon is zero.

- **Q. 10. What are the maximum or minimum energy photons in visible light ?**

○ **Ans.** In visible light, the maximum energy of photon is 3·1 *e*V and minimum energy is 1·7 *e*V.

- **Q. 11. Is matter waves are electromagnetic waves ?**

○ **Ans.** Matter waves are not electromagnetic. Matter waves are the waves associated with moving particles. It does not depend on charge of the particle. But electromagnetic wave are produced only when electrons are accelerated.

● **Q. 12. What is the relation between energy and momentum of photon ?**

○ **Ans.** If P be the momentum and E be the energy, the relation between P and E is $P = \frac{E}{c}$

● **Q. 13. Show that the wavelength of electromagnetic wave is equal to the wavelength of photon associated with the wave.**

○ **Ans.** In the case of photon : $\lambda = \frac{h}{p} = \frac{h}{h\upsilon / c} = \frac{c}{v}$ and wavelength of the electromagnetic wave,

$$\lambda' = \frac{c}{v} \quad \therefore \lambda = \lambda'$$

● **Q. 14. How is the concept of stationary orbit in Bohr hydrogen atom is obtained from de Broglie hypothesis ?**

○ **Ans.** According Bohr's postulate of hydrogen atom, those orbits are permissible for the electron in the atom for which angular momentum is integral multiple of $\frac{h}{2\pi}$.

This concept of stationary orbit in an atom can also be obtained from the de Broglie hypothesis. According to the hypothesis, the de Broglie wave associated with a particle moving with momentum mv is $\lambda = \frac{h}{mv}$.

A stationary orbit must accommodate integral number of wavelength's. Hence if the radius of the orbit be *r*, then $2\pi r = n\lambda \quad \therefore \lambda = \frac{2\pi r}{n}$

or, $\frac{h}{mv} = \frac{2\pi r}{n}$ or, $mvr = n\frac{h}{2\pi}$. It is the angular momentum of electron in stationary orbit.

● EXERCISE ●

(A) Objective type questions :

1. During propagation, light behaves as ——— and during interaction with matter light behaves as ———.
2. The electron orbit described in Bohr's theory must accommodate ——— number of de Broglie wavelength.
3. ——— ——— is defined as number of waves per unit length.
4. If the mass of the particle is greater, the associated wavelength will be ———.
5. Momentum of photon of wave is p = ———

(B) Very short answer type questions :

1. Mention the characteristic quantities of waves ?
2. Mention the characteristic quantities of particles ?
3. What is the momentum of a photon of frequency υ ?
4. What are photons ? How do they differ from cathode rays ?
5. What is wave number ?

(C) Short answer type questions :

1. What is dual nature of light ?
2. Write the expression for the de Broglie wavelength.
3. What is the fundamental difference between photon and ordinary material particle ?

4. Why is the energy of a photon is measured ?

5. On which factors de Broglie wave depends ?

6. How does the wavelength of matter wave depends on energy of the particle ?

7. An electron is allowed to pass through a potential difference V. Write the expression for its wavelength.

8. What is the de Broglie hypothesis on the permitted stationary orbit ?

9. If υ be the frequency of photon what is its momentum ?

10. Show that the quantity $\frac{h}{mv}$ has the same unit as wavelength.

11. An electron and a photon has equal kinetic energy. Whose de Broglie wavelength is large ?

12. Show that the de Broglie wavelength of electron of mass m and moving with a kinetic energy E is $\lambda = \frac{h}{\sqrt{2mE}}$.

13. An electron and a proton possess equal kinetic energy. Whose wavelength is large ?

◢ (D) Essay type questions :

1. Discuss about the wave aspect of particle. Obtain an expression for the wavelength of de Broglie wave associated with a particle.

2. Obtain the expression for the de Broglie wavelength, when an electron is accelerated through a potential difference V.

3. Discuss the properties of matter waves.

4. Discuss the dual nature of particle.

◢ (E) Numerical problems :

1. Find the de Broglie wavelength of an electron which is moving with a velocity of 10^5 ms^{-1}. $h = 6{\cdot}62 \times 10^{-34}$ J-s mass of electron $9{\cdot}1 \times 10^{-31}$ kg [**Ans : 73.33 Å**]

2. If an electron is allowed to pass through a potential difference of 600 V, calculate de Broglie wavelength associated with the electron. $m = 9{\cdot}1 \times 10^{-31}$ kg, $e = 1{\cdot}6 \times 10^{-19}$ c [**Ans : 0·50 Å**]

3. The energy of a photon is 66.3 eV. Calculate its frequency. $h = 6{\cdot}63 \times 10^{-34}$ J-s [**Ans :** $1{\cdot}6 \times 10^{16}$ **Hz**]

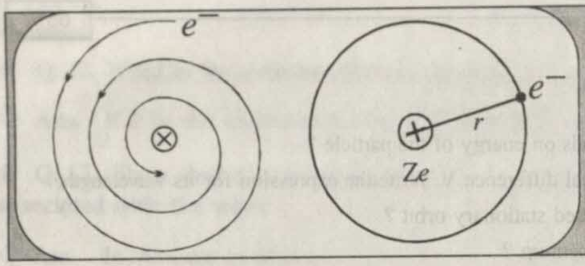
4. A particle of mass 1 gm is moving with a velocity of 2×10^{6} ms^{-1}. Calculate the wavelength of matter wave. [**Ans :** $3{\cdot}31 \times 10^{-76}$ **m**]

5. The energy of an electron is 400 eV. Calculate its de Broglie wavelength. $h = 6{\cdot}62 \times 10^{-34}$ J.s., $m = 9{\cdot}1 \times 10^{-31}$ kg [**Ans : 0·61 Å**]

6. The de Broglie wavelength of an electron is 2Å. What is its momentum ? [**Ans :** $3{\cdot}31 \times 10^{-24}$ **kg m.s^{-1}**]

7. Calculate the wavelength of a photon of energy 10^{-19} J [**Ans :** $1{\cdot}99 \times 10^{-6}$ **m**]

8. Find the de Broglie wavelength of an electron of speed 0·5 km / s. [**Ans :** $1{\cdot}5 \times 10^{-12}$ **m**]



CHAPTER
4

ELECTRONIC STRUCTURE OF ATOM

● **TOPICS :** ○ Introduction; ○ Thomson model of the atom; ○ Rutherford's experiment; ○ Bohr's atom model; ○ Origin of spectral lines; ○ Electron energy levels in hydrogen atom; ○ Limitations of Bohr's theory; ○ Shell and subshell; ○ Electron configuration in Atom; ○ Short answer type questions (with answers); ○ Exercise.

✪ 4.1. Introduction ✪

Any element is made of large number of atoms and for a particular element the atoms are identical. We know that electricity and radiation are atomic phenomena. The existence of atom can not be denied although we do not see atoms. In 1895 J. B. Perrin discovered that cathode ray consists of large number of negative charged particles called electrons. In the year 1897 J. J. Thomson determined the value of specific charge $\frac{e}{m}$ of electron. In 1906 Millikan by his famous **oil drop experiment** proved that electronic charge is quantised and also determined the electronic charge e.

After the discovery of positive ion in 1886, scientist were of the opinion that an atom consists of both positive and negative charges. Since an atom is electrically neutral, it is assumed that in an atom positive and negative charges are present in equal amount. Now, how the two opposite charges arrange inside the atom ? Two opposite charges always attract each other. So, due to attraction if the two opposite charges combine, they will anihilate each other. But this does not occur. The two types of charges exist inside the atom maintaining the separate entity.

Scientists tried to know the possible arrangement of two types of charges inside the atom so that the stability of atom is not destroyed. On the basis of the finding of spectral analysis of atom, different atomic models were proposed. Different atomic models that were proposed are :

(i) Thomson atom model
(ii) Rutherford nuclear atom model
(iii) Bohr's atom model
(iv) Somerfeld's model
(v) Wave mechanical model

✪ 4.2. Thomson model of the atom ✪

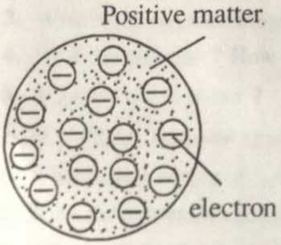


Fig. 4·1

In 1898, J. J. Thomson was the first to give an idea regarding structure of atom. The model is known as Thomson atom model.

Thomson atom model : According to the model, an atom consists of positivelly charged sphere of about 10^{-8} cm in diameter with the positive charges spread out uniformly throughout the sphere. The negatively charged electrons are arranged systematically within the sphere lying here and there like plums in a pudding [Fig 4.1]. The electrons get themselves arranged in such away that the force of attraction between positive charged sphere and electrons and that of repulsion among electrons balance each other. This model was applied to predict the frequencies of line spectrum emitted by an incandescent solid. The theoretical predictions did not agree with experimentally observed frequency, and it became clear that Thomson's picture of the atom was wrong.

✪ 4.3. Rutherford's experiment ✪

The first correct description of the distribution of positive and negative charges within the atom was given by Ernest Rutherford. His atom model was based on the epoch-making experiment of the scattering of α- particles from thin metal foils. An α-particle is a helium ion (He^{++}) emitted by radioactive substances. It charge is $q = +2e$ ($e = 1{\cdot}6 \times 10^{-19}$ C) and mass is equal to four times the mass of a proton ($m_p = 1{\cdot}67 \times 10^{-27}$ kg)

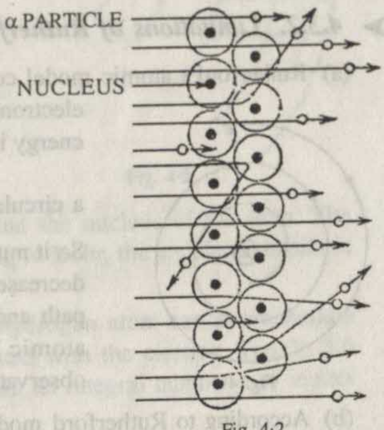


Fig. 4·2

In this experiment Rutherford was observing the course of fast α-particle directed towards the thin metal foil. He made the following observations.

(i) Most of the α-particles, either passed straight through the metal foil or suffered only small deflections. This could not be explained by Thomson atom model.

(ii) A few α-particles were deflected through angles which were less than or equal to 90°.

(iii) Very few particles were deflected through angles greater than 90°. Even a few of them were reflected back i.e., suffered a deflection of nearly 180°. In other words, a very few were sent back in the same direction from where it came (Fig 4.3). The large angle scattering came as a great surprise. It could not be explained by Thomson model.

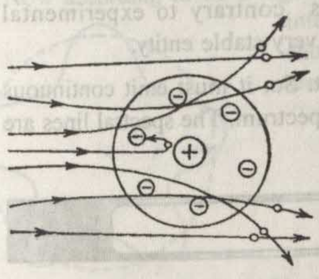


Fig. 4·3

● Conclusions from α-particle scattering :

(a) The fact that most of the α-particles passed undeviated led to the conclusion that an atom has a lot of empty space in it.

(b) As α-particles are heavy particles having large initial speeds, they could be deflected through large angles only by a strong electric force. This lead Rutherford to conclude that entire positive charge and nearly the entire mass of the atom were concentrated in a tiny central core. He named the core as **nucleus.**

Calculations show that, the α-particle should reach within 10^{-12} cm from the centre of the nucleus. Thus the radius of the nucleus is not greater than 10^{-12} cm. It is also determined that the diameter of the atom is of the order of 10^{-8} cm. Evidently, almost all the space occupied by the atom is empty. For this reason, in Rutherford's experiment majority of the α-particles went straight through the metal foil.

➤ *4.3.1. Rutherford's Atom model* :

On the basis of the conclusions drawn from Rutherford's experiment, a new atom model was proposed. This atom model known as Rutherford's atom model, had the following characteristics.

(a) An atom has a small central core where almost the entire mass and all the positive charge of the atom are concentrated. The core is called the **nucleus** of the atom. The size of nucleus is about 10^{-14} m.

(b) The remaining part of the atom contains electrons which surrounds the nucleus. The size of the atom is about 10^{-10} m.

(c) Since the atom as a whole is electrically neutral, the total negative charge of the electrons around the nucleus is equal to the total positive charge of the nucleus.

(d) In order to explain the stability of electron at a distance from the nucleus, it was proposed by Rutherford that the electrons revolve round the nucleus in circular orbits. The electrostatic force of attraction between the nucleus and the electron provide the necessary centripetal force to the electron to revolve in the orbit.

➤ *4.3.2. Limitations of Rutherford's model :*

(a) Rutherford's atomic model could not explain the stability at an atom. According to the laws of electromagnetic theory, a charged particle in accelerated motion must radiate energy in the form of electro-magnetic radiation. As the electron revolves in a circular orbit, it is constantly subjected to centripetal acceleration, $\frac{v^2}{r}$. So it must radiate energy continuously. As a result, there should be a gradual decrease of energy of electron and hence the electron should follow a spiral path and ultimately fall into the nucleus shown in fig 4.4. Thus the whole atomic structure should collapse. This is contrary to experimental observations which indicate that the atom is very stable entity.

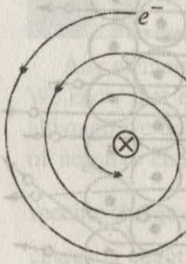


Fig. 4·4

(b) According to Rutherford model, electron can revolve in any orbit. So, it must emit continuous radiations of all frequencies. But a hydrogen atom is found to emit a line spectrum. The spectral lines are only of definite frequencies.

✪ 4.4. Bohr's atom model ✪

The drawbacks of Rutherford atom model were removed by Neil Bohr in 1913. Bohr proposed a bold and unprecedented theory of atomic structure. In contravention of the principle of classical mechanics and electromagnetic theory, he applied the quantum theory of radiation as developed by Max Planck and Einstein, to the Rutherford atom model. His model was based on the following **three postulates** concerning the motion of electrons within the atom. Bohr's postulates are—

(a) An atom consists of a very small positively charged nucleus. Nearly entire mass of the atom is concentrated in the nucleus.

(b) The electrons in an atom revolve around the nucleus in certain permitted non-radiating stationary circular orbits. The electrostatic force of attraction between the electron and the nucleus provides the necessary centripetal force.

Those orbits are permissible for which the angular momentum of the electron is an integral multiple of $\frac{h}{2\pi}$. Here h is Planck's constant.

If r be the radius of the permitted orbit, m is the mass of the electron and v is the velocity of the electron in that orbit, than $mvr = n\frac{h}{2\pi}$

Here n is an integer known as principal quantum number and has integral values 1, 2, 3, ...

This relation is regarded as a **quantisation condition on the angular momentum of the electron.**

(iii) When an electron jumps from higher energy orbit to lower energy orbit, electromagnetic radiation is emitted. Energy is absorbed when the electron moves from lower energy orbit to higher energy orbit.

Let n_1 and n_2 be the principal quantum number of two orbits. Let E_{n_1} and E_{n_2} is the energies associated with the respective orbits. Then the frequency of the emitted radiation is given by

$$h\upsilon = E_{n_2} - E_{n_1}$$

It is known as Bohr's **frequency condition.**

The concept is illustrated in fig 4.5 orbits having values $n = 1, 2, 3, \ldots\ldots$ are shown. When the electron jumps from higher to lower orbit, it emits a photon or quantum of energy. When the atom absorbs a quantum of energy, the electron jumps from a lower to higher orbit.

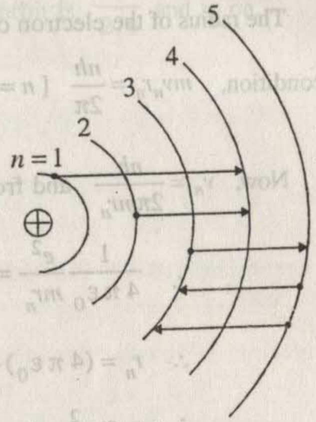


Fig. 4·5

➤ *4.4.1. Bohr's quantisation condition :*

Let an electron is moving in a circular orbit of radius r round the nucleus of the atom. The electron is confined to move in a circular path of length $2\pi r$. According to Bohr, the angular momentum of the electron is quantised i.e. it has only discrete values.

Now according to de Broglie, the permitted orbit of electron in hydrogen atom can accommodate integral number of matter wave associated with the electron. The fig 4.6 shows that the circumference $2\pi r$ contain an integral number n of matter wave.

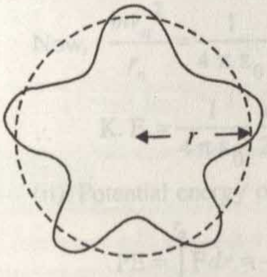


Fig. 4·6

$$2\pi r = n\lambda = n\frac{h}{p} \quad \left[\because \lambda = \frac{h}{p} \right] \quad p \text{ is the linear momentum.}$$

$$\therefore \quad \text{Angular momentum, } L = pr = \frac{nh}{2\pi} \qquad n = 1, 2, 3 \ldots\ldots\ldots \quad (4.1)$$

The equation 4.1 is called Bohr's quantisation condition for angular momentum.

➤ *4.4.2. Calculation concerning Bohr's atom model :*

The postulate concerning Bohr atom model can be utilised to calculate the radii of different electron orbits, the velocity and orbital frequency possessed by different electrons.

Let an electron of mass m and having charge $-e$ is revolving round the nucleus having charge $+Ze$ in a circular orbit of radius r_n. The atom is hydrogen or hydrogen like.

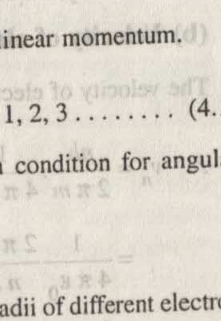


Fig. 4·7

Centripetal force required for the electron motion, $F_c = \dfrac{m{v_n}^2}{r_n}$

where v_n is the electron speed in n-th orbit.

Electrostatic force of attraction, $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Ze.e}{{r_n}^2}$

As electrostatic force provides the centripetal force we write

$$\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{{r_n}^2} = \frac{m{v_n}^2}{r_n} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4.2)$$

(a) Radii of Bohr orbits in hydrogen :

The radius of the electron orbit is governed not only by equation (4.2) but also by Bohr's quantisation condition, $mv_n r_n = \frac{nh}{2\pi}$ $[n = 1, 2, 3, \ldots\ldots]$ n has different values for different stationary orbits.

Now, $v_n = \frac{nh}{2\pi m r_n}$ and from equation 4.2 $v_n^{\,2} = \frac{1}{4\pi\varepsilon_0} = \frac{e^2}{mr_n}$ $[\;\because\; Z = 1$ for hydrogen $]$

$$\therefore \quad \frac{1}{4\pi\varepsilon_0}\frac{e^2}{mr_n} = \frac{n^2h^2}{4\pi^2m^2r_n^{\,2}}$$

$$\therefore \quad r_n = (4\pi\varepsilon_0)\frac{n^2h^2}{4\pi^2me^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.3)$$

$$\therefore \quad r_n \,\alpha\, n^2$$

So, the radius of the permitted orbit vary as the square of the principal quantum number (n) of the orbit.

$\therefore$ Radii of the permitted orbits are in the ratio $1 : 4 : 9 : \ldots\ldots\ldots\ldots$ etc

Again $r_n = n^2 r_1$

$\therefore$ Now, for hydrogen atom, $r_1 = 0{\cdot}53$ Å [Vide example 4.1]

and $r_2 = (2)^2 . r_1 = 4 \times 0{\cdot}53 = 2{\cdot}12$ Å

Evidently, the radius in between r_1 and r_2 **are not permitted.**

(b) Velocity of electron in *n*-the permitted orbit :

The velocity of electron in n th orbit, $v_n = \frac{nh}{2\pi m r_n}$

$$\therefore \quad v_n = \frac{nh}{2\pi m}\cdot\frac{1}{4\pi\varepsilon_0}\cdot\frac{4\pi^2me^2}{n^2h^2} \quad [\text{ by equation 4.3 }]$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{2\pi e^2}{nh} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.4)$$

$$\therefore \quad v_n \,\alpha\, \frac{1}{n}$$

So, the velocity of electron in any orbit is inversely proportional to the principal quantum number of the orbit. So orbital velocity of electron is maximum in the orbit nearest to the nucleus i.e. for $n = 1$

Again, $v_n = \frac{e^2}{2h\varepsilon_0 c}\left(\frac{c}{n}\right)$ [c = velocity of light in vacuum]

$$= \alpha . \frac{c}{n}$$

$\alpha = \frac{e^2}{2h\varepsilon_0 c}$ is called fine structure constant (a pure number) whose value is $\frac{1}{137}$

$$\therefore \quad v_n = \frac{1}{137}\frac{c}{n}$$

∴ velocity of the electron in first Bohr orbit is $\frac{c}{137}$ and in second orbit is $\frac{c}{274}$ and so on.

$$\therefore \quad v_n = \frac{3\times 10^8}{137}.\frac{1}{n} = \frac{2{\cdot}18\times 10^6}{n} \quad \ldots\ldots\ldots\ldots\ldots\ (4.5)$$

- **(c) Orbital frequency of electron :**

The orbital rotational frequency of the electron in n-th orbit.

$$\upsilon_n = \frac{v_n}{2\pi r_n} = \frac{1}{2\pi}\times\frac{1}{4\pi\varepsilon_0}\frac{2\pi e^2}{nh}\times\frac{1}{4\pi\varepsilon_0}\times\frac{4\pi^2 m e^2}{n^2h^2}$$

$$= \left(\frac{1}{4\pi\varepsilon_0}\right)^2\frac{4\pi^2 m e^4}{n^3h^3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ (4.6)$$

$$\therefore \quad \upsilon_n \,\alpha\, \frac{1}{n^3}$$

- **(d) Total energy of electron in n-th orbit :**

The revolving electron in an orbit round the nucleus has two types of energy— (i) kinetic energy and (ii) potential energy.

(i) Kinetic energy of the electron of mass m moving with a v_n in n-th orbit is $\mathrm{KE} = \frac{1}{2}mv_n{}^2$

$$\text{Now,} \quad \frac{mv_n{}^2}{r_n} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_n{}^2} \quad \therefore \quad mv_n{}^2 = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_n}$$

$$\therefore \quad \text{K. E.} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{2r_n} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ (\text{i})$$

(ii) Potential energy of the electron in the orbit is

$$\mathrm{PE} = \int_{\infty}^{r_n}\mathrm{F}dr = \frac{1}{4\pi\varepsilon_0}\int_{\infty}^{r_n}\frac{e^2}{r^2}dr = \frac{1}{4\pi\varepsilon_0}\frac{-e^2}{r_n}$$

Total energy of the electron in the n-th orbit is

$$\mathrm{E}_n = \mathrm{KE}+\mathrm{PE} = \frac{1}{4\pi\varepsilon_0}\left[\frac{e^2}{2r_n}-\frac{e^2}{r_n}\right] = \frac{1}{4\pi\varepsilon_0}\frac{1}{2}\frac{e^2}{r_n}$$

$$\therefore \quad \mathrm{E}_n = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{2}\times\frac{1}{4\pi\varepsilon_0}\frac{4\pi^2me^2}{n^2h^2} = \left(\frac{1}{4\pi\varepsilon_0}\right)^2.\frac{2\pi^2me^4}{n^2h^2} \quad \ldots\ldots\ldots\ldots\ldots\ (4.7)$$

Hence the total energy of the electron is negative and inversely proportional to n^2. Since the energy is negative, it follows that as n increases (i.e., for high orbits) energy increases. Hence in the orbit closed to the nucleus ($n = 1$) energy of electron is minimum.

- **Alternative form of E_n :** From equation (4.7) $\mathrm{E}_n = \frac{me^4}{8h^2\,\varepsilon_0{}^2}.\frac{1}{n^2}$

Substituting, $m = 9{\cdot}1 \times 10^{-31}$ kg, $e = 1{\cdot}6 \times 10^{-19}$ cm, $h = 6{\cdot}62 \times 10^{-34}$ J-s

and $\varepsilon_0 = 8.845 \times 10^{-12}\ e^2\ m^{-2}\ \mathrm{N}^{-1}$ and 1 eV = 1.6×10^{-19} J.

We get, $\mathrm{E}_n = -\frac{13{\cdot}6}{n^2}e\mathrm{V}$

Also, $$E_n = -\left(\frac{me^4}{8\,\varepsilon_0{}^2 ch^3}\right)\frac{ch}{n^2} = -(R\,ch).\frac{1}{n^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4.8)$$

Where R is known as Rydberg's constant $= \dfrac{8e^4}{8\,\varepsilon_0{}^2 ch^3} = 1\cdot097 \times 10^7 m^{-1}$

and R ch is Rydberg's energy = 2.17×10^{-18} J = 13.6 *e*V

It is the electron energy in first orbit of hydrogen atom.

(e) Frequency of the emitted radiation :

Let the electron jumps from the orbit n_2 to orbit n_1. If $n_2 > n_1$ and E_{n_2} and E_{n_1} be the energy of the electron in the two orbits, then $E_{n_2} - E_{n_1} = h\upsilon$

$$\therefore \quad \upsilon = \frac{E_{n_2} - E_{n_1}}{h} = Rc\left[\frac{1}{n_1} - \frac{1}{n_2{}^2}\right] \quad \text{[From equation 4.8]}$$

If c be the speed of light in vacuum and λ be the wavelength of emitted photon, then

$$\upsilon = \frac{c}{\lambda} = Rc\left[\frac{1}{n_1{}^2} - \frac{1}{n_2{}^2}\right]$$

$$\therefore \text{ Wave number } \quad \bar{\upsilon} = \frac{1}{\lambda} = R\left[\frac{1}{n_1{}^2} - \frac{1}{n_2{}^2}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4.9)$$

This relation holds for radiations emitted by hydrogen like atoms like He^+, Li^{++} and Be^{+++}.

✪ 4.5. Origin of spectral lines ✪

From Bohr's theory an equation for wave number emitted by hydrogen or hydrogen like atom can be obtained [eq. 4.9]. Long before the establishment of Bohr's theory, Balmer experimentally determined a similiar equation for wave number from the analysis of emission spectrum of hydrogen. He showed that the wave numbers of visible spectrum obtained from the radiation given out by hydrogen discharge tube can be expressed by the equation.

$$\bar{\upsilon} = A\left(\frac{1}{2^2} - \frac{1}{m^2}\right) \quad \text{here A = a constant.}$$

m is an integer greater than 2 ($m > 2$). If the wavelength of different spectral lines are measured by spectrometer, the value of the constant is found as $A = 10{\cdot}9678 \times 10^6\, m^{-1}$

Evidently, this value of the constant A is nearly equal to that of Rydberg's constant R obtained from Bohr's theory. This confirms the validity of Bohr's theory for atomic structure. The series consisting of the wavelength of the visible spectral lines of hydrogen is known as **Balmer series**. Evidently, the visible lines of Balmer series are obtained when electron jumps from orbits of quantum number 3, 4, 5, ... to the orbit of quantum number 2.

In the later times, a number of principal spectral series of hydrogen was discovered. They are—

(i) The spectral lines of **Lyman series**, discovered by Lyman are produced when the electron of the atom jumps from orbits of quantum number 2, 3, 4, ... etc to the orbit of quantum number 1. So the Lyman series is $\bar{\upsilon} = A\left(\dfrac{1}{1^2} - \dfrac{1}{m^2}\right)$ $m = 2, 3, 4, \ldots\ldots.$

(ii) The spectral lines of Paschen series are like wise emitted when electronic transition takes place from the orbits of quantum number 4, 5, 6, ... etc to the orbit of quantum number 3.

So, the Paschem series is represented by $\bar{\upsilon} = A\left(\frac{1}{3^2} - \frac{1}{m^2}\right)$ $m = 4, 5, 6, \ldots\ldots$

(iii) Similarly, Brackett series is given by $\bar{\upsilon} = A\left(\frac{1}{4^2} - \frac{1}{m^2}\right)$ $m = 5, 6, 7, \ldots\ldots$

Now, from Bohr's theory, one can understand the mechanism of the production as spectral lines. All the spectral series of hydrogen atom can be obtained from the Bohr's formula (4.9).

(a) If the electron in hydrogen atom makes a transition from the stationary orbits of quantum number $n_2 = 2, 3, 4, \ldots\ldots$ to the orbit of quantum number $n_1 = 1$, the wave number of the spectral lines is given by $\bar{\upsilon} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = R\left(\frac{1}{1^2} - \frac{1}{n_2^2}\right)$

Evidently, these spectral lines belong to Lyman series.

(b) Again, if the electron transition takes place from the orbits having $n_2 = 3, 4, 5, \ldots\ldots$ etc to the orbit having $n_1 = 2$, the spectral lines of Balmer series are obtained. So it is given as $\bar{\upsilon} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$

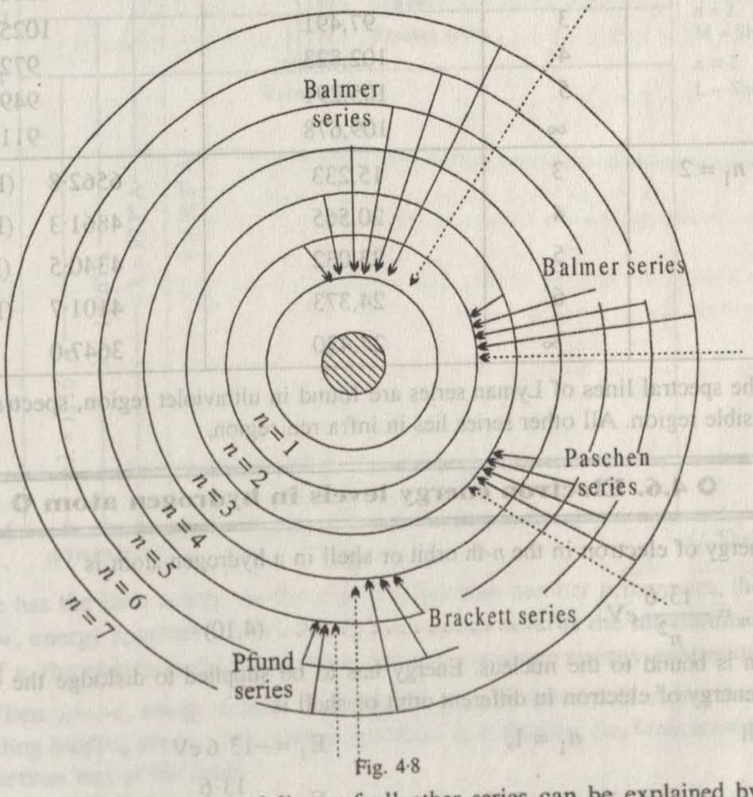


Fig. 4·8

Similarly, the production of spectral lines of all other series can be explained by Bohr's formula [eq. 4.9].

The production of spectral lines belonging to different series are shown in fig 4.8. Here transition of electron is shown by arrows.

The composite formula from which all spectra series can be derived is $\bar{\upsilon} = R\left(\frac{1}{N^2}-\frac{1}{n^2}\right)$

where N = 1, 2, 3, ... ∞ respectively for Lyman, Balmer and Paschen series etc. n is an integer greater than N.

In the table below we show the **different spectral series**

Value of n_1	Name of the series	Value of n_2
1	Lyman	2, 3, 4, ... ∞
2	Balmer	3, 4, 5, ... ∞
3	Paschen	4, 5, 6, ... ∞
4	Brackett	5, 6, 7, ... ∞
5	Pfund	6, 7, 8, ... ∞

Below we show wave numbers and wavelength of different spectral lines of different spectral series.

Spectral series	n_2	$\bar{\upsilon}$ (cm^{-1})	λ (Å)
Lyman $n_1 = 1$	2	82,258	1216·0
	3	97,491	1025·8
	4	102,823	972·5
	5	105,291	949·5
	∞	109,678	911·8
Balmer $n_1 = 2$	3	15,233	6562·8 (H_α)
	4	20,565	4861·3 (H_β)
	5	23,032	4340·5 (H_γ)
	6	24,373	4101·7 (H_δ)
	∞	27,420	3647·0

Evidently, the spectral lines of Lyman series are found in ultraviolet region, spectral lines of Balmer series lies in visible region. All other series lies in infra red region.

4.6. Electron energy levels in hydrogen atom

The total energy of electron in the n-th orbit or shell in a hydrogen atom is

$$E_n = -\frac{13 \cdot 6}{n^2} eV \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4.10)$$

The electron is bound to the nucleus. Energy has to be supplied to dislodge the electron from the atom. Binding energy of electron in different orbit or shell is

For K - shell $\quad n_1 = 1, \quad E_1 = -13 \cdot 6\, eV$

For L- shell $\quad n_1 = 2, \quad E_2 = -\frac{13 \cdot 6}{2^2} = -3 \cdot 4\, eV$

For M- shell $\quad n_1 = 3, \quad E_3 = -\frac{13 \cdot 6}{3^2} = -1 \cdot 51\, eV$

The total energy of the electron in different orbits or shells is shown by horizontal lines. These lines are known as **energy levels** and the diagram is called **'energy level diagram'** [Fig 4.9] In the table below we show the energy in eV units of different energy levels.

Energy Level (n)	**Energy (eV)**
1	− 13·6
2	− 3·4
3	− 1·51
4	− 0·85
5	− 0·54
∞	0

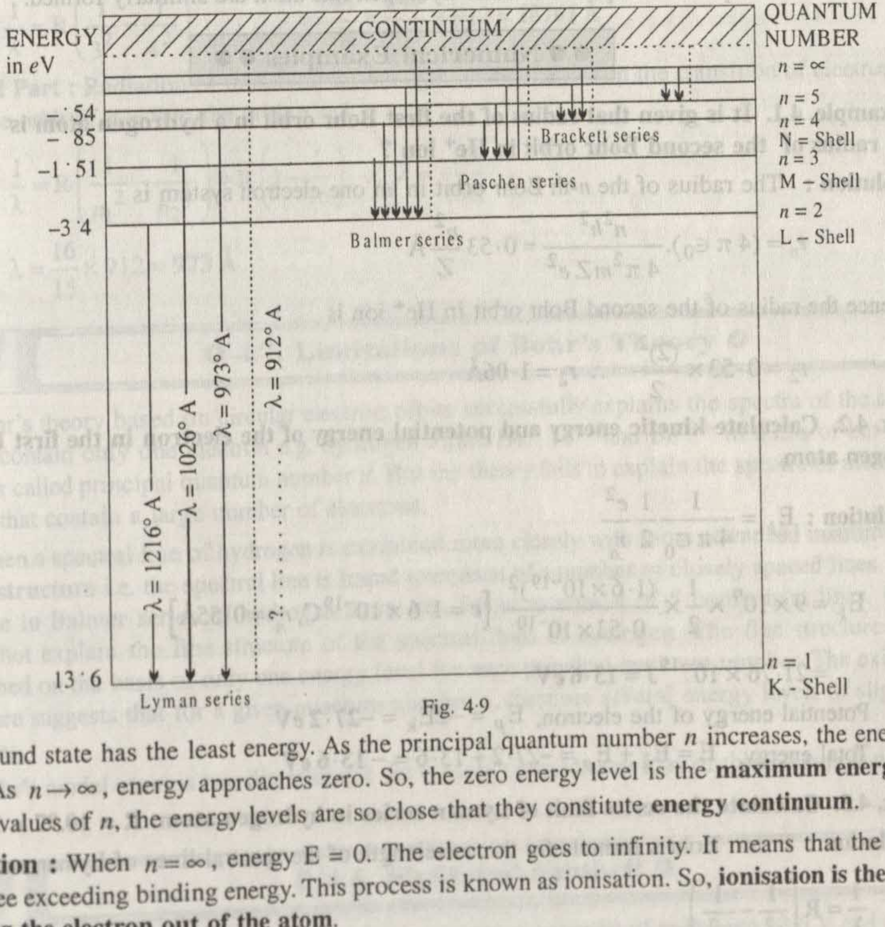


Fig. 4·9

The ground state has the least energy. As the principal quantum number n increases, the energy also increases. As $n \to \infty$, energy approaches zero. So, the zero energy level is the **maximum energy state**. For higher values of n, the energy levels are so close that they constitute **energy continuum.**

☐ **Ionisation :** When $n = \infty$, energy E = 0. The electron goes to infinity. It means that the electron becomes free exceeding binding energy. This process is known as ionisation. So, **ionisation is the process of knocking the electron out of the atom.**

☐ **Ionisation energy : Ionisation energy is the energy required to knock the electron completely out of the atom.**

It is the energy to excite the electric from $n = 1$ to $n = \infty$. Hence ionisation energy for hydrogen is $E_i = E_\infty - E_1 = [0 - (-13\cdot6)]$ eV = **13·6 *e*V**

- **A discussion :** At normal condition, the electron in a hydrogen atom remains in the inner most orbit or shell. It is called K-shell with principal quantum number $n = 1$. This state of the atom is called **normal state** or **ground state**. When it absorbs energy it goes to the higher energy states or permitted larger orbits. In the ground state, if the electron absorbs energy of 13·6 *e*V, it separates from the atom and the atom is said to be ionised. Now, if the electron in the ground state absorbs energy more than 13·6 *e*V, it will come out of the atom with a kinetic energy.

It is to be noted that when the electron is inside the atom its energy is quantised, i.e., it can have definite energy. But when it is free, it can have any energy. Its energy is no longer quantised. For this reason, the possible energy levels of free electrons are represented continuously above $n = \infty$. This region of energy levels is called continuum.

When hydrogen atom is given an energy, its electron absorbs the energy and goes to higher orbit. The atom is said to be excited. But the electron does not stay in that orbit. It returns to the lower energy orbits. During the transition photons of different frequencies are emitted. These radiations of different frequencies constitute emission spectrum. Spectra of other hydrogen like atom are similarly formed.

●● Numerical Examples ●●

- **Example. 4.1. It is given that radius of the first Bohr orbit in a hydrogen atom is 0·53Å. What is the radius of the second Bohr orbit in He⁺ ion ?**

○ **Solution :** The radius of the *n*-th Bohr orbit in an one electron system is

$$r_n = (4\pi\epsilon_0).\frac{n^2h^2}{4\pi^2 m Z e^2} = 0\cdot53\frac{n^2}{Z}\text{Å}$$

Hence the radius of the second Bohr orbit in He^+ ion is

$$r_2 = 0\cdot53\times\frac{(2)^2}{2} \quad \therefore\ \mathbf{r_2 = 1\cdot06Å}$$

- **Ex. 4.2. Calculate kinetic energy and potential energy of the electron in the first Bohr orbit in hydrogen atom.**

○ **Solution :** $E_k = \dfrac{1}{4\pi\epsilon_0}\dfrac{1}{2}\dfrac{e^2}{r_a}$

$$\therefore\ E_k = 9\times10^9\times\frac{1}{2}\times\frac{(1\cdot6\times10^{-19})^2}{0\cdot53\times10^{-10}}\ \left[e = 1\cdot6\times10^{-19}\text{C}, r_n = 0\cdot55\text{Å}\right]$$

$$= 21\cdot76\times10^{-19}\text{J} = \mathbf{13\cdot6\,eV}$$

$\therefore$ Potential energy of the electron, $E_p = -2E_k = \mathbf{-27\cdot2\,eV}$

$\therefore$ Total energy, $E = E_k + E_p = -27\cdot2 + 13\cdot6 = \mathbf{-13\cdot6\,eV}$

- **Ex. 4.3. Calculate the series limit of Lyman series in hydrogen atom. R = 10.97 × 10⁶m⁻¹ .**

○ **Solution :** From Bohr's formula for the wavelength of the spectral lines of Lyman series is,

$$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$$

The Lyman series limit or minimum wavelength is obtained by putting $n = \infty$

$$\therefore\ \frac{1}{\lambda} = R\ \therefore\ \lambda = \frac{1}{R} = \frac{1}{10\cdot97}\times10^{-6}m = \mathbf{912Å}$$

- **Ex. 4.4. Calculate the series limit of Balmer series in hydrogen atom.**

○ **Solution :** In the case of Balmer series, $\frac{1}{\lambda}=R\left(\frac{1}{2^2}-\frac{1}{n^2}\right)$

Minimum wavelength is obtained by putting $n=\infty$

$$\therefore \quad \frac{1}{\lambda}=\frac{R}{4} \qquad \therefore \quad \lambda=\frac{4}{R}=4\times 912=\mathbf{3648\ \text{Å}}$$

- **Ex. 4.5. Consider a hydrogen atom as proposed by Bohr. In this atom if the electron makes a transition from the orbit $n=4$ to the grand state, calculate the minimum and maximum wavelength of radiations emitted by the atom.**

○ **Solution :** Radiation of maximum wavelength is obtained when the electron jumps from the orbit of $n=4$ to the next inner orbit of $n=3$.

$$\frac{1}{\lambda}=R\left(\frac{1}{3^2}-\frac{1}{4^2}\right) \qquad \therefore \quad \lambda=\frac{144}{7R}=\frac{144}{7}\times 912=\mathbf{18761\ \text{Å}}$$

2 nd Part : Radiation of minimum wavelength is obtained when the transition of electron takes place from the orbit of $n=4$ to the orbit $n=1$

$$\therefore \quad \frac{1}{\lambda}=R\left(\frac{1}{{n_1}^2}-\frac{1}{{n_2}^2}\right)=R\left(1-\frac{1}{16}\right) \quad \therefore \lambda=\frac{16}{15R}$$

$$\therefore \quad \lambda=\frac{16}{15}\times 912=\mathbf{973\ \text{Å}}$$

4.7. Limitations of Bohr's Theory

Bohr's theory based on circular electron orbits successfully explains the spectra of the simpler atoms which contain only one electron e.g. hydrogen atom, He^+, Li^{++} and Be^{+++} in terms of but one quantum number called principal quantum number n. But the theory fails to explain the spectra of more complicated atoms that contain a large number of electrons.

When a spectral line of hydrogen is examined more closely with more advanced instruments it reveals a **fine structure** i.e. the spectral line is found to consist of a number of closely spaced lines. For example, H_α line in Balmer series of hydrogen atom was found to consist of 5 component lines. Bohr's theory could not explain the fine structure of the spectral lines of hydrogen. The fine structure could not be explained on the basis of only one energy level for each principal quantum number. The existence of fine structure suggests that for a given quantum number n, there are several energy levels of slightly different energies.

Bohr's model atom is two dimensional. So it cannot provide any idea of three dimensional structure of atom.

4.8. Shell and subshell

From different experiments it is found that ionisation potential of hydrogen 13·6 V and that of lithium 4·5 V. Ionisation potential is the energy required to remove an electron from the atom. It is found that ionisation potential of atom like He, Ne, A etc is high, but for the elements Li, Na, K etc ionisation potential is low. This suggests that all the electrons of an atom do not occupy same orbit, but they are distributed in so-called shells around the nucleus. For this reason, the atomic radius decreases from left to right in a period of the periodic table and in the same group atomic radius increases from top to bottom.

The shells around the nucleus in which the electrons are distributed, are numbered $n = 1, 2, 3, \ldots$. A shell can accommodate maximum $2n^2$ electrons. Thus only 2 electrons are allowed in shell $n = 1$, 8 electrons in $n = 2$ shell and $n = 3$ shell can accommodate maximum 18 electrons. The shell $n = 1, 2, 3, 4$ are designated by the symbols K, L, M, N.

A detailed spectroscopic analysis of the spectra emitted by atoms required that the shell be further subdivided into subshells. The first shell has no subshell, but the second has two subshells, while the third has three.

The shape of the subshell is determined by the orbital quantum number l. The subshells are designated s, p, d, f etc. l can take integral values from 0 to $(n-1)$ i. e. $l = 0, 1, 2, \ldots\ldots, (n-1)$

For different value of l, the electrons are designated as —

orbital quantum number l :	0	1	2	3	4	5
designation of electrons :	s	p	d	f	g	h

The electrons for which $l = 0, 1, 2, 3, 4, \ldots\ldots$ are said to be in $s, p, d, f, g \ldots\ldots$ subshells.

The electron having $n = 3$ and $l = 0$ is called $3s$ electron. Here the number denotes the value of n and the letter the value of l. Similarly when $n = 2$ and $l = 1$, the electron is called $2p$ electron. The electrons in the same subshell are equivalent. These electrons in the same subshell are denoted by index. For example, two $1s$ electrons are represented as $1s^2$, six $2p$ electrons by $2p^6$.

Few subshells are shown in the **table below :**

l	**Subshell**				
n	0	1	2	3	4
K 1	$1s$				
L 2	$2s$	$2p$			
M 3	$3s$	$3p$	$3d$		
N 4	$4s$	$4p$	$4d$	$4f$	
O 5	$5s$	$5p$	$5d$	$5f$	$5g$

Total number of electrons that can remain in a subshell is $2(2l+1)$. In the table below we show maximum possible electrons in shell upto $n = 4$ and corresponding subshells.

Subshell	K	L		M			N			
Principal quantum No. n	1	2		3			4			
Orbital quantum No. l	0	0	1	0	1	2	0	1	2	3
Subshell	$1s$	$2s$	$2p$	$3s$	$3p$	$3d$	$4s$	$4p$	$4d$	$4f$
maximum electron $2(2l+1)$	2	2	6	2	6	10	2	6	10	14
Total $2n^2$	2	8		18			32			

4.9. Electron configuration in Atom

The distribution of electrons in an atom is called electron configuration. The electron distribution in different shells and subshells obey the following three rules :

(a) The total number of electrons in a shell of quantum number n is $2n^2$.

(b) In the n th shell there are n subshells. Subshells are designated by orbital quantum number l such as 0, 1, 2, ... $(n-1)$

(c) Each shell can have maximum of $2(2l+1)$ electrons. Here we given electron configuration of a few elements.

(i) **Helium :** Its Z = 2. So it has two electrons. These electrons fill up the K-shell ($n = 1$) and both $l = 0$. The electron configuration of helium can be written as $\boxed{1s^2}$.

(ii) **Lithium :** It has Z = 3. It also has three electrons. Two electrons fill up the K – shell having $n = 1$ and $l = 0$. The third electron occupies the $l = 0$ subshell of L–shell ($n = 2$). So, the electron configuration of lithium atom is $\boxed{1s^2}\ 2s^1$.

(iii) **Beryllium :** Since its Z = 4, it has four electrons. Two are $1s$ electrons and the other two are $2s$ electrons. So, its electron configuration is $\boxed{1s^2}\ 2s^2$.

(iv) **Neon :** Z = 10. Its electron no. is 10. Its electron configuration is $\boxed{1s^2\ 2s^2\ 2p^6}$.

(v) **Sodium :** Z = 11. So, its electron no. is 11. So, its electron configuration is $\boxed{1s^2\ 2s^2\ 2p^6}\ 3s^1$

(vi) **Copper :** Z = 29. Its total number of electrons is 29.

Its electron configuration is $\boxed{1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^{10}}\ 4s^1$

●● Short Answer Type Questions (with answers) ●●

● **Question 1. Only one electron exists in a hydrogen atom. Then how can you explain the existence of many lines in its emission spectrum ?** **[H. S. 2002]**

○ **Ans.** According to Bohr's theory, an electron can revolve around the nucleus in different permitted orbits and energy is different in different orbits. In the orbit nearest to the nucleus ($n = 1$), the energy of electron is minimum. The energy of electron gradually increases in 2nd, 3rd, ... etc orbits. The energy increases with the increase of principle quantum number n. On absorbing energy the atom is excited and the electron goes to higher orbit. Then it returns to the lower orbits emitting radiation. The frequency of radiation is determined by the difference of energies of the two orbits. So, if this difference of energy be different, frequency of radiation will also be different.

There is one electron in hydrogen atom. When the atom is excited, the electron may go to any high energy orbit. Next the electron returns to lower orbit emitting a radiation of definite frequency. Due to transition of the electron in different lower orbits, radiations of different frequencies are produced.

● **Q. 2. What is the angular momentum of the electron revolving around the nucleus is 3rd orbit ? $h = 6{\cdot}60 \times 10^{-27}$ erg. sec.** **[H. S. 2002; J E E 2002]**

○ **Ans.** According Bohr's theory, the angular momentum of the electron in nth orbit is $L_n = \dfrac{nh}{2\pi}$

∴ Hence its angular momentum in third orbit is

$$L_3 = \frac{3h}{2\pi} = \frac{3 \times 6 \cdot 6 \times 10^{-27}}{2\pi} = \mathbf{3 \cdot 153 \times 10^{-27}\ erg.s}$$

● **Q. 3. The energy of the electron in the first Bohr orbit in hydrogen atom is – 13.6 eV. What will be its energy when it is in third orbit ?**

○ **Ans.** Energy of electron in n-th permitted orbit of the hydrogen atom is $E_n = -\dfrac{13 \cdot 6}{n^2}$ eV

∴ Energy of electron in 3rd orbit, $E_3 = -\dfrac{13 \cdot 6}{(3)^2} = \mathbf{-1 \cdot 51\ eV}$

● **Q. 4. In which element the wavelengths of spectral lines are four times smaller than the wavelengths of spectral lines of hydrogen ?**

○ **Ans.** From Bohr's theory we know that, if λ be the wavelength of a spectral line produced by an element of atomic number Z, then $\frac{1}{\lambda} \propto Z^2$ or, $\lambda \propto \frac{1}{Z^2}$

$$\therefore \quad \frac{\lambda_{He}}{\lambda_H} = \frac{{Z_H}^2}{Z^2{}_{He}} = \left(\frac{1}{2}\right)^2 = \frac{1}{4} \qquad \therefore \quad \boldsymbol{\lambda_{He} = \frac{1}{4}\lambda_H}$$

Hence the wavelength of the spectral line of helium is four times less than the wavelength of hydrogen.

● **Q. 5. The energy of an excited hydrogen atom is − 1.51 *e*V. What will be the angular momentum the electron according to Bohr's theory ?**

○ **Ans.** The energy of an electron in *n*th orbit, $\boxed{E_n = -\frac{13 \cdot 6}{n^2} eV}$

$$\therefore \quad -\frac{13 \cdot 6}{n^2} = -1 \cdot 51 \quad \text{or,} \quad n = 3$$

∴ The angular momentum of the electron is $L = \frac{nh}{2\pi}$

$$\therefore \quad L = \frac{3 \times 6 \cdot 62 \times 10^{-34}}{2 \times 3 \cdot 14} = \mathbf{3 \cdot 16 \times 10^{-34}\ J.s}$$

● **Q. 6. Calculate the number of revolutions executed by the electron per sec in the hydrogen atom. [J E E 2001]**

○ **Ans.** If the electron makes *n* revolutions in the first Bohr orbit $n_1 = \frac{v_1}{2\pi r_1}$

$$\text{Here} \quad v_1 = \frac{C}{137} = \frac{3 \times 10^8}{137} m/s, \quad r_1 = 0 \cdot 53 \times 10^{-10}\ m$$

$$\therefore \quad n_1 = \frac{3 \times 10^8}{137 \times 2\pi \times 0 \cdot 53 \times 10^{-10}} = \mathbf{0 \cdot 65 \times 10^{16}\ Hz}$$

● **Q. 7. How much greater is the radius of the fourth orbit than the radius of the second orbit in hydrogen atom. [J E E 2000]**

○ **Ans.** According to Bohr's theory the radius of any orbit is proportional to the square of the principle quantum number of the orbit.

$$\therefore \quad r \,\alpha\, n^2 \Rightarrow \frac{r_2}{r_1} = \left(\frac{4}{2}\right)^2 = 4 \qquad \therefore \ \boldsymbol{r_2 = 4\, r_1}$$

So, the radius of the fourth orbit is four times the radius of the second orbit in a hydrogen atom.

● **Q. 8. If the electron in an excited hydrogen atom goes to *n*th orbit, what maximum number of lines will be emitted ? [J E E 2000]**

○ **Ans.** No of transitions to $n = 1$ orbit from the orbits having $n > 1$ is $(n - 1)$ and No. of transitions to $n = 2$ orbit from the orbits having $n > 2$ is $(n - 2)$. Similarly we can calculate the number of transitions to orbits having $n = 3, 4, 5, \ldots\ldots$

∴ Total number of transitions are $N = (n - 1) + (n - 2) + \ldots\ldots\ldots + \{ n - (n - 1) \}$

$$\mathbf{N = \frac{1}{2} n\,(n - 1)}$$

●● Numerical Examples ●●

- **Example. 4.6. In a hydrogen atom, the frequency of the first Balmer line is 6563Å. What is the wavelength of the second line of the same series ?**

○ **Solution :** Balmer first line is obtained for the transition from $n = 3$ to $n = 2$ and the second line is obtained for the transition from $n = 4$ to $n = 2$.

Now, for the Balmer series : $\frac{1}{\lambda} = R\left(\frac{1}{4} - \frac{1}{n^2}\right)$ $n = 3, 4, 5, \ldots\ldots$

$\therefore$ for the first line : $\frac{1}{\lambda_1} = R\left(\frac{1}{4} - \frac{1}{9}\right) = \frac{5R}{36}$

and for the second line : $\frac{1}{\lambda_2} = R\left(\frac{1}{4} - \frac{1}{16}\right) = \frac{3R}{16}$

$$\therefore \frac{\lambda_2}{\lambda_1} = \frac{5R}{36} \times \frac{16}{3R} = \frac{20}{27} \quad \therefore \lambda_2 = 6563 \times \frac{20}{27} = \mathbf{4861\ \text{Å}}$$

- **Ex. 4.7. A hydrogen atom in a state having a binding energy (the energy required to remove an electron) of 0·85 *e*V makes a transition to a state with an excitation energy of 10·2 *e*V. Find the energy of emitted photon.**

○ **Solution :** We know, $E_n = -\frac{13\cdot 6}{n^2} = -0\cdot 85 \quad \therefore n = 4$

L-shell ($n = 2$) has an excitation energy of ($13\cdot6 - 3\cdot4$) = $10\cdot2$ *e*V with respect to the second state. Hence the state of the atom changes from $n = 4$ to $n = 2$.

$\therefore$ The energy of the emitted photon = $3\cdot4 - 0\cdot85$ = **2·55 *e*V**

- **Ex. 4.8. The electron in a hydrogen atom makes a transition from one orbit to another an absorbing an energy of 10·2 eV. When the electron returns to initial state, what will be the wavelength of the emitted photon ? $h = 6\cdot55 \times 10^{-27}$ erg. sec $C = 3 \times 10^{10}$ cm / s and 1 *e*V = $1\cdot6 \times 10^{-12}$ erg.** **[H. S. 2000]**

○ **Solution :** Evidently, when the electron returns from higher to lower initial orbit 10.2 *e*V energy will be emitted. Hence if λ be the wavelength of the photon then;

$$\frac{hc}{\lambda} = 10\cdot2 \times 1\cdot6 \times 10^{-12}$$

$$\therefore \lambda = \frac{6\cdot55 \times 10^{-27} \times 3 \times 10^{10}}{10\cdot2 \times 1\cdot6 \times 10^{-12}} = \frac{6\cdot55 \times 3}{10\cdot2 \times 1\cdot6} \times 10^{-5}\ cm = \mathbf{1204\ \text{Å}}$$

- **Ex. 4.9. The ionisation of hydrogen atom is 13.6 *e*V. Calculate the Rydberg constant for hydrogen.**

○ **Solution :** $R\,h\,c = 13\cdot6 \times 1\cdot6 \times 10^{-19}$

$$\therefore \quad R = \frac{13\cdot6 \times 1\cdot6 \times 10^{-19}}{6\cdot62 \times 10^{-34} \times 3 \times 10^{8}} = \mathbf{1\cdot096 \times 10^{7}\ m^{-1}}$$

$\therefore$ The value of Rydberg constant, R = $\mathbf{1\cdot096 \times 10^{7}\ m^{-1}}$

- **Ex. 4.10. Show that the velocity of hydrogen in the first Bohr orbit of hydrogen atom is $\frac{1}{137}$ of the velocity of light in vacuum.** $e = 1\cdot6 \times 10^{-9}$ **C,** $h = 6\cdot6 \times 10^{-34}$ **J. s.**

○ **Solution.** The velocity of the electron in the *n*th orbit in hydrogen atom is $v_n = \frac{2\cdot18 \times 10^{6}}{n}\ ms^{-1}$

So, the velocity of the electron in first Bohr orbit is $v_1 = 2{\cdot}18 \times 10^6$ ms^{-1}

Now, the velocity of light in vacuum is $c = 3 \times 10^8$ ms^{-1}

$$\therefore \quad \frac{c}{v} = \frac{3 \times 10^8}{2 \cdot 18 \times 10^6} = 137 \qquad \therefore \ v = \frac{c}{137}$$

• Ex. 4.11. Ionisation energy of hydrogen like atom in 4 rydberg. (i) If its electron returns from the first excited state of the ground state, what will be the frequency of the emitted radiation ? (ii) Calculate the radius of its first Bohr orbit. [1 rydberg = $2{\cdot}2 \times 10^{-18}$ J. The radius of the first Bohr orbit in hydrogen = 5×10^{-11} m] [I. I. T. '84]

○ **Solution. :** (i) Ionisation energy of the given atom = $13{\cdot}6 \times 4 = 54{\cdot}4$ *eV* = $87{\cdot}04 \times 10^{-19}$ J

If λ be the wavelength of the radiation given out when the electron jumps from $n = 2$ to $n = 1$ orbit,

$$\text{then } \frac{1}{\lambda} = \frac{87 \cdot 04 \times 10^{-19}}{ch}\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{87 \cdot 04 \times 10^{-19}}{ch} \times \frac{3}{4}$$

$$\therefore \quad \lambda = \frac{3 \times 10^8 \times 6 \cdot 62 \times 10^{-34} \times 4}{87 \cdot 04 \times 10^{-19} \times 3} = 3 \times 10^{-8} m = \mathbf{300\text{Å}}$$

(ii) The radius of the *n*-th orbit is $r_n = 0 \cdot 53 \times \frac{n^2}{Z}$ Å

$$\therefore \quad r_1 = 0 \cdot 53 \times \frac{1}{2} \times 10^{-10} m \ [\ Z = 2 \text{ because ionisation energy } \alpha\ Z^2\ ]$$

$= 0{\cdot}265 \times 10^{-10}$ m $=$ **0·265 Å**

• EXERCISE •

(A) Objective type questions :

• Fill in the blanks :

1. Millikan determined ———— of electron by oil drop experiment.
2. J. J. Thomson determined ———— ———— of electron.
3. According to Rutherford's nuclear atom model ———— exists at the centre and ———— revolve round it in different orbits.
4. The electron revolving in stationary orbit does not emit ———— .
5. When in the atom electron jumps from ———— to ———— orbit, a photon is emitted.
6. Ionisation potential of hydrogen atom is ———— volt.
7. Maximum number of electron in *n*th orbit is ———— .
8. Velocity of electron in the first Bohr orbit of hydrogen is ———— .
9. Energy of electron in *n*th orbit is ———— *eV*.
10. The value of Rydberg constant is ———— m^{-1} .
11. I rydberg energy ———— *eV*.
12. Ionisation energy of Lithium is ———— *eV*.

(B) Very short answer type questions :

1. What is meant by specific charge of electron ?
2. What is hydrogen like atom ?
3. State Bohr's quantisation law of orbit.

4. What is the angular momentum of electric in *n*th orbit ?
5. State Bohr's frequency condition.
6. What is the total energy of electron in *n*th orbit ?
7. When the spectral lines of Lyman series are emitted ?
8. When the spectral lines of Balmer series are emitted ?
9. What is meant by ionisation potential of an atom ?
10. Define excitation energy.

◢ (C) Short answer type questions :

1. What is nuclear atom model of Rutherford ?
2. State Bohr's postulates for hydrogen atom model.
3. Explain why the spectrum of hydrogen has many lines although a hydrogen atom has only one electron.
4. What is the frequency of electron in first Bohr orbit of hydrogen atom ?
5. Write down the expression for the angular momentum of electron in third Bohr orbit.
6. Why Rutherford model fails to explain emission of spectral lines of hydrogen atom ?
7. Write down the expression for the radius of the *n*th orbit in hydrogen atom.
8. How the radius of the orbit depends on atomic number of the atom ?
9. What is meant by energy levels in hydrogen atom ?
10. What is Rydberg constant ? What is its value ?
11. What is ionisation potential ? Mention its value for hydrogen atom.
12. Express one rydberg energy in Joule.
13. Radius of the first Bohr orbit in helium atom is smaller than the radius of the corresponding orbit in hydrogen. Why ?
14. What is orbital quantum number ? What is the number of subshells in M-shell ?
15. What is the number of subshells in *n*th shell ?
16. The spectral lines of which series lie in visible region ?
17. Calculate the maximum and minimum wavelengths in Lyman series.

◢ (D) Essay type questions :

1. (a) Why is the observation from the Rutherford α- particle scattering ?

   (b) Discuss Rutherford's muclear atom model ? What are the defects of the model ?
2. (a) State Bohr's postulates of hydrogen atom model ?

   (b) How these postulates help to explain the origin of the line spectra of hydrogen atom ?
3. Obtain the expression for the radius, velocity and total energy of the electron in the *n*th orbit.
4. How Bohr's theory explains the origin of all the spectral series like Lyman, Balmer and Paschem series.
5. Obtain an expression for ionisation potential of hydrogen.
6. Prove that when an electron jumps form n_2 to n_1 orbit $(n_2 > n_1)$ in a hydrogen like atom, the wavelength of the emitted radiation is given by $\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$.

◢ (E) Simple numerical problems :

1. Calculate the radius of the first Bohr orbit in hydrogen atom. Also find the velocity of the electron in that orbit.

   [**Ans :** 0·53Å; $V = 2{\cdot}18 \times 10^6$ ms^{-1}]
2. The wavelength of first Balmer line of hydrogen spectrum is 6553 Å. Calculate the Rydberg constant.

   [**Ans :** 109860 cm^{-1}]

3. The energy of electron in the first Bohr orbit of hydrogen atom is $-13{\cdot}6$ eV. If the electron transition taken from the second orbit to first orbit, what will the energy of the photon emitted ? **[Ans :** 10·2 *e*V**] [W. B. H. S.]**

4. In a discharge tube an electron of energy 12·09 eV collides with a hydrogen atom. Find the wave lengths of the photons that may be emitted from the hydrogen. **[Ans :** 1028 Å, 6581 Å, 1217 Å **]**

5. What is the velocity of electron in the second Bohr orbit in hydrogen atom ? **[Ans :** $1{\cdot}1 \times 10^6$ m/s **]**

6. In an excited hydrogen atom the energy of electron is $-0{\cdot}54$ *e*V. Find the angular momentum of the electron by Bohr's theory. **[Ans :** $5{\cdot}29 \times 10^{-34}$ J-s **]**

7. The energy required to ionise a hydrogen atom is $21{\cdot}7 \times 10^{-19}$ J. Calculate the wavelength of the radiation emitted when a proton and an electron combine to form a hydrogen atom. **[Ans :** 912·4 Å **]**

8. Find the longest and shortest wavelengths of the spectral lines of the Lyman series. $R = 1{\cdot}097 \times 10^7\ m^{-1}$ **[Ans :** 1216·0 Å, 911·8 Å **]**

9. Ultraviolet light of wavelength 800 Å is incident on hydrogen atom. Calculate the energy of the emitted electron in unit of *e*V. Given, ionisation potential of hydrogen is 13·85 *e*V. **[Ans :** 1·89 *e*V **]**

10. Find the wave number of the following wavelengths (a) 5 cm (b) 5890 Å. **[Ans :** (a) 20 m^{-1} (b) $1698 \times 10^3\ m^{-1}$ **]**

11. In an excited hydrogen atom, electron transition occurs from the third orbit to second orbit. What is the wavelength of emitted photon ? **[Ans :** 6565 Å belongs to Balmer series **]**

12. The atomic number of lithium is 3. In a Li^{++} ion, how much energy is required to raise the electron from the first orbit to third orbit ? How many lines can be obtained in the emission spectrum ? Given, ionisation energy of hydrogen = 13·6 *e*V. **[I I T] [Ans :** 113.7 Å **]**

13. Calculate the wavelengths of third and fifth spectral line of the Paschen series. **[Ans :** $\lambda_3 = 10930$ Å, $\lambda_5 = 9548$ Å **]**

14. The orbital speed of the electron in the ground state is *v*. What will be the orbital speed when it is excited to the state having energy $-\ 3{\cdot}4$ *e*V ? **[Ans :** v / 2 **]**

15. Find the ratio of the wavelengths of the longest wavelength lines in the Lyman and Balmer series of hydrogen spectrum. **[Ans :** 5 : 27 **]**

◢ (F) Harder numerical problems :

1. The electron in a hydrogen atom makes a transition $n_1 \rightarrow n_2$ are the principal quantum number of the two states. Assume that the Bohr model is valid. The time period of the electron in the initial state is 8 times in the final state. What are the possible values of n_1 and n_2 ? [Ans : $n_1 = 6$, $n_2 = 3$]

2. The radiation emitted when an electron jumps from $n = 3$ to $n = 2$ orbit of hydrogen atom falls on a metal to produce photoelectrons. The electron emitted with maximum kinetic energy are made to move perpendicular to a magnetic field of $\dfrac{1}{320}$ Tesla in a radius of 10^{-3} *m*. Find :
   (a) The kinetic energy of the electrons,
   (b) Work function of the metal
   (c) The wavelength of the radiation. $h = 6{\cdot}62 \times 10^{-34}$ J.s **[Ans :** (a) 0·88 *e*V (b) 1·03 *e*V (c) 6570 Å**]**

3. An electron in hydrogen like atom is in the excited state. It has total energy $-\ 3{\cdot}4$ *e*V. Calculate (a) the kinetic energy (b) the de Broglie wavelength of the electron $h = 6{\cdot}63 \times 10^{-34}$ J-s **[Ans :** (a) 3·4 *e*V, (b) 6·66 Å **]**

4. If the series limit of Balmer series for hydrogen is 3644 Å, find the atomic number of the element. Which gives X-ray wavelengths of 1Å. **[Ans :** 31, gallium **]**

5. An electron is revolving round the nucleus of charge Ze. The electron requires an energy of 47·2 *e*V to raise the electron from the second orbit to the third orbit. Find the following quantities :
   (a) Value of Z
   (b) Energy required to move the electron from the third to fourth orbit.

(c) Energy required to move the electron from the first orbit to infinity.

(d) Kinetic energy, Potential energy and total energy of the electron is the first Bohr orbit.

(e) The radius of the first Bohr orbit.

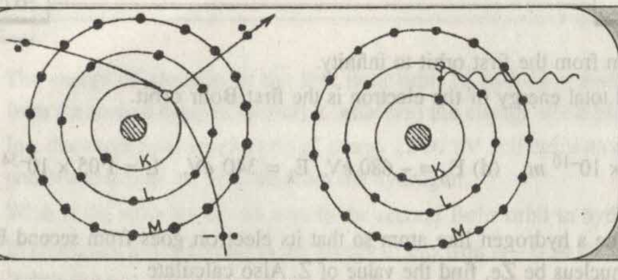
[**Ans :** (a) 5, (b) 16·53 *eV*, (c) $36{\cdot}5 \times 10^{-10}$ *m*, (d) $E_p = -680$ *eV*, $E_k = 340$ *eV*, $L = 1{\cdot}05 \times 10^{-34}$ J-s, and (e) $r_1 = 1{\cdot}06 \times 10^{-11}$ *m*]

6. The energy of 68·0 *eV* is required to exite a hydrogen like atom so that its electron goes from second Bohr orbit to third orbit. If the charge on the nucleus be Ze, find the value of Z. Also calculate :

(i) Kinetic energy of the electron in the first Bohr orbit.

(ii) The wavelength of the photon required to send electron from the first orbit to infinity.

[**Ans :** Z = 6; (i) 489·6 *eV*; (iii) 25·28Å]



CHAPTER 5

CATHODE RAYS AND X-RAYS

● **TOPICS :** ○ Introduction; ○ Electrical discharge through a gas : Cathode rays; ○ Nature of Cathode rays; ○ X-rays; ○ Variation of intensity with wavelength of X-rays : X-rays spectra; ○ Moseley's law; ○ Importance of Moseley's law; ○ Explanation of Moseley's law from Bohr's theory; ○ Comparison between X-rays and visible light rays; ○ Comparison betwen X-rays and Cathode rays; ○ Short answer type questions (with answers); ○ Exercise.

✪ 5.1. Introduction ✪

The last decade of nineteenth century is regarded as the beginning of the atomic physics, which is a part of modern physics. During this time all the discoveries was related to atom. Some eminent scientists while carrying out experiments on the electrical conductivity of gases discovered **electron**. This discovery finally brought about a radical change in the conception of the structure of atom.

✪ 5.2. Electrical discharge through a gas : Cathode rays ✪

Any dry gas including air at normal pressure is almost an insulator and does not conduct electricity. However, a gas can be made conducting in various ways—(a) by the passage of ultraviolet rays, X-rays, gamma rays, α-particle, β-particle etc., through the gas, (b) by heating the gas, the process is called thermal ionisation and (c) by passing electric spark through it.

From experiment it is observed that the above process render a gas conductive because they create ion in the gas. Neutral normal atoms and molecules become positive ions by losing one or more electrons which are negatively charged. This process is called ionisation.

The conduction of electricity through gases has been studied in a variety of experiments and has resulted in several important discoveries. In order to study electric discharge in gases at low pressure a **discharge tube** is taken. It consists of a long glass tube about 4 cm in diameter and 100 cm in length with a side tube at the middle which may be connected to vacuum pump. Two electrodes, cathode and anode are enclosed in the tube. A high voltage 10 KV to 15KV is applied between the electrodes.

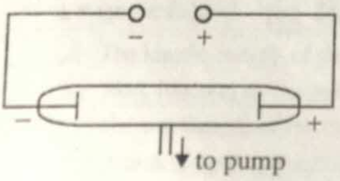


Fig. 5·1

As the pressure of the gas is gradually reduced, at a certain stage an electric discharge is produced. The discharge consists of long, thin lines which are bluish in colour.

When the gas pressure is further reduced the spark gets transformed into a general glow. If the pressure of the gas is further reduced to 10^{-2} mm of Hg the glow disappears. The tube becomes dark and a greenish glow appear on the wall of the tube near the anode. This glow was found to be due to the fluorescence of glass caused by the impact of certain invisible rays emitted from cathode.

- **The invisible rays coming from the cathode of the discharge tube are known as Cathode rays.**

Cathode rays have the following properties :

(i) They travel in straight line.

(ii) Cathode rays start normally from the surface of the cathode.

(iii) Cathode rays produce intense heating of the objects on which they impinge.

(iv) They can exert pressure and possess inertia.

(v) Cathode rays produce fluorescence.

(vi) These rays can affect photographic plate.

(vii) **Cathode rays are deflected by magnetic field**. The direction of deflection suggests that the cathode rays are constituted by negatively charged particles.

(viii) **Cathode rays are deflected by electric fields**. The direction of deflection clearly shows that the cathode rays contain negatively charged particles.

(ix) Cathode rays can ionise a gas.

- **These negative charged particles of cathode ray are now universally called electrons.**

5.3. Nature of Cathode rays

Cathode rays are deflected by both by electric field and by magnetic field. From the direction of deflection it is concluded that the cathode rays are made of negative charged particle. Perrin experimentally proved that cathode rays are invisible streams of negatively charged particle. These particles are called electrons. Hereby cathode rays are stream of negatively charged particles called electrons.

Electron is a fundamental particle :

J. J. Thomson tried to measure the specific charge i.e., charge to mass ratio of an electron. He found the value of the ratio $\frac{e}{m} = 1\cdot76\times10^{11}\ \text{C/Kg}$. From subsequent experiments, it is proved that the specific charge of electron is independent of the cathode plate and the nature of the gas inside the discharge tube. It is found that electrons available from the tubes are all identical, whatever gas taken in the discharge tube.

Electrons emitted from a surface of a metal by photoelectric process or by hot filament in vacuum are all identical. Beta ray emitted from a radioactive substance are also electrons. Electrons are, therefore, regarded as fundamental particles.

Charge, mass and energy of an electron :

In 1913, R. A. Millikan performed his famous oil drop experiment on the measurement of electronic charge. The experimentally measured charge of an electron is $e = 1\cdot6\times10^{-19}\text{C}$.

$\frac{e}{m}$ of the electron is $1\cdot76\times10^{11}\ \text{C. Kg}^{-1}$

$$\therefore \quad \text{mass of electron, } m = \frac{e}{e/m} = \frac{1\cdot6\times10^{-19}}{1\cdot76\times10^{11}} = 9\cdot1\times10^{-31}\ \text{Kg}$$

Now, mass of lightest element hydrogen is $\text{M} = 1\cdot67\times10^{-27}\ \text{Kg}$

$$\therefore \quad \frac{\text{M}}{m} = \frac{1\cdot67\times10^{-27}}{9\cdot1\times10^{-31}} = \mathbf{1836} \qquad \therefore \ \text{A hydrogen atom is 1836 times heavier than electron.}$$

Energy of electron :

Mass of an electron is extremely low. For this reason, its velocity can easily be increased. When an electron is passed through a potential difference, it gains kinetic energy which can be expressed in the unit of *e*V. Electron volt is a practical unit of energy.

❖ **Definition of 1 *e*V : It is the energy acquired by an electron when it passes through a potential difference of 1 volt.**

$\therefore$ 1 eV = 1 electric charge × 1 volt

$= 1{\cdot}6 \times 10^{-19}$ coulomb × 1 volt = $\mathbf{1{\cdot}6 \times 10^{-19}}$ **J**

Velocity gained by the electron :

Let an electron, initially being at rest, pass through a potential difference V and it gain a velocity v. Then

its final kinetic energy = work done on it

$$\therefore \quad \frac{1}{2}mv^2 = eV \qquad \therefore \quad v = \sqrt{\frac{2eV}{m}} \qquad [\ m = \text{mass of electron}\ ]$$

● **Example 5.1. An electron passes through a potential difference of 100V. Calculate : (a) energy gained by the electron and (ii) velocity of the electron.**

○ **Solution :** (i) Energy gained by the electron E = 100 eV = $\mathbf{1{\cdot}6 \times 10^{-17}}$ **J**

(ii) Its velocity, $v = \sqrt{\dfrac{2eV}{m}}$

$$\therefore \quad v = \sqrt{2\times1{\cdot}76\times10^{11}\times100} \qquad \left[\frac{e}{m} = 1{\cdot}76\times10^{11}\,\text{C/Kg}\right]$$

$$\therefore \quad v = \mathbf{5{\cdot}93\times10^{6}\ m/s}$$

● **Ex. 5.2. An electron is moving with a velocity 10^6 m/s. Express its energy in eV. Its mass $m = 9{\cdot}1 \times 10^{-31}$ Kg.**

○ **Solution :** Kinetic energy of electron $E = \dfrac{1}{2}mv^2$

$$\therefore \quad E = \frac{1}{2}\times9{\cdot}1\times10^{-31}\times10^{12} = 4{\cdot}55\times10^{19}\ \text{J} = \mathbf{2{\cdot}84}\boldsymbol{eV}$$

5.4. X-rays

5.4.1. *Production of X-rays :*

X-rays were quite accidentally discovered by Wilhelm Rontgen in 1895 while experimenting with a discharge tube. Now it is known that X-rays are produced whenever fast moving electrons are suddenly stopped by some hard materials.

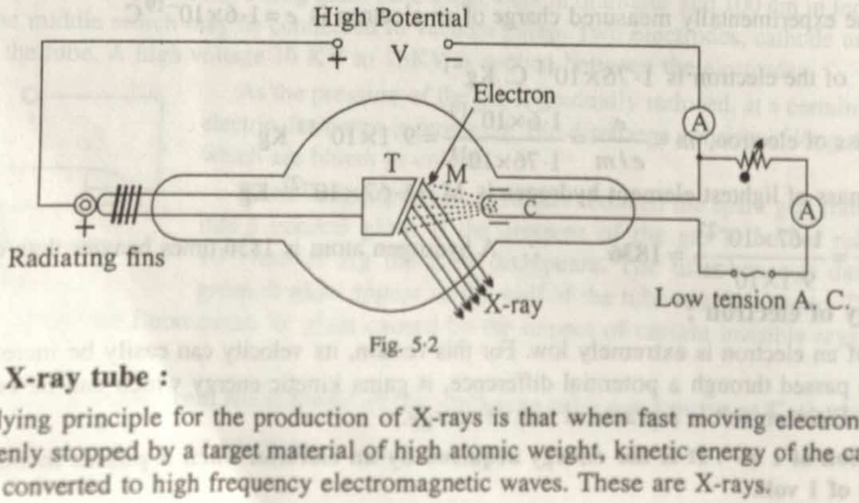


Fig. 5·2

Coolidge X-ray tube :

The underlying principle for the production of X-rays is that when fast moving electrons (Cathode rays) are suddenly stopped by a target material of high atomic weight, kinetic energy of the cathode rays fully or partly converted to high frequency electromagnetic waves. These are X-rays.

Modern form of X-ray tube is coolidge tube designed by Dr. William Coolidge. It is a special type of evacuated discharge tube. The cathode C is a thin filament of tungsten coated with BaO. The filament is heated by a current supplied by a battery. The electrons emitted by the cathode are focussed on the anti cathode or target metal with the help of molybdenum cylinder M surrounding the filament. M is maintained at negative potential.

The anti cathode is a tungsten block. A long, hollow handle made of copper is attached with it. It conducts away the heat produced due to impact of high speed electrons from the cathode.

The target is connected to the positive and cathode to the negative of a high tension power supply.

● **Working :** The electron emitted from the filament are focussed on the target. On account of high potential difference between the cathode and the target, the electrons arrive at the target with high kinetic energy. On striking the target, the electrons are stopped. A part of the energy is lost as heat and the rest of the energy appear in the form of X-rays.

Coolidge tube has the advantage of independent control of intensity and quality. The **intensity** of X-rays depends on the number of electrons emitted from the filament. So, the intensity is controlled by adjusting the filament current.

The quality of X-rays depends upon its energy which, in turn, depends upon the energy of the incident electrons. The energy of the incident electrons depends upon the potential difference between the target and the cathode. This potential difference is called accelerating potential. Thus, by controlling accelerating potential we can control the quality of X-rays.

The X-rays with low energy and low penetrating power are called **soft** X-rays. On the other hand, X-rays having high energy and high penetrating power are called **hard** X-rays.

➤ *5.4.2. Characteristics of X-rays :*

Some of the characteristics of X-rays are :

(i) X-rays are electromagnetic waves like ordinary light. But its wavelength is extremely small. The range of X-ray wavelengths is 1 Å to 0·01 Å.

(ii) X-rays are invisible and travels with velocity of light.

(iii) According quantum theory X-rays are made of uncharged particle called photons.

(iv) X-rays are not deflected by electric or magnetic field.

(v) X-rays can be reflected and refracted. They also show the phenomena of interference, diffraction and polarisation.

(vi) They affect photographic plates.

(vii) They can ionise a gas.

➤ *5.4.3. Shortest wavelength of X-rays :*

The minimum wavelength of X-rays produced in a X-ray tube depends on the potential difference applied between the two electrodes.

If the whole of kinetic energy of the electron falling on the target is converted to X-ray photon, then its energy or frequency will be maximum and so wavelength will be minimum.

If V be the potential difference between the two electrodes of the X-ray tube, then energy that the electron acquire is $E = eV$.

If ν_{max} be the frequency of the X-ray photon, then its energy $E = h\nu_{max}$

$$\therefore \; h\nu_{max} = Ve \qquad \therefore \; \nu_{max} = \frac{Ve}{h} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

Again, if λ_{min} be the minimum wavelength of X-ray produced then,

$$\nu_{max} = \frac{c}{\lambda_{min}} = \frac{Ve}{h} \qquad \therefore \; \lambda_{min} = \frac{hc}{Ve} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(5.1)}$$

Putting $h = 6{\cdot}62\times10^{-34}$ J, $C = 3\times10^{8}$ m/s and $e = 1{\cdot}6\times10^{-19}$ C

We get, $$\lambda_{\min} = \frac{6{\cdot}62\times10^{-34}\times3\times10^{8}}{V\times1{\cdot}6\times10^{-19}} = \frac{12{\cdot}42}{V}\times10^{-7}\,m \;\ldots\ldots\ldots\ldots\ldots \;(5.2)$$

$$\therefore\ \lambda_{\min}\ \alpha\ \frac{1}{V}$$

5.5. Variation of intensity with wavelength of X-rays : X-ray spectra

When a fast moving electron beam strike a target, X-rays are produced. X-rays produced in a X-ray tube are of different wavelength. Intensity of X-rays varies with wavelengths. The graph showing the variation of intensity (I) with wavelength (λ) of X-rays emitted from an X-ray tube are **two types**. It means that X-rays spectra are two types :

(a) Continuous spectrum, (b) Line spectrum or characteristic spectrum.

◻ (a) Continuous spectrum :

It consists of radiations of all possible wavelengths within a definite wavelengths range having a definite short wavelength limit. In this case intensity varies continuously with wavelengths of X-rays. It is shown in fig. 5.3 for different accelerating voltages. Some of the important features of the continuous X-rays spectrum are as follows :

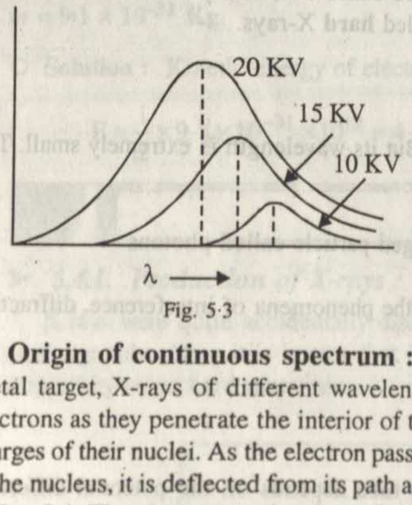


Fig. 5·3

(i) The intensity of X-rays increases at all wavelengths as the accelerating voltage is increased.

(ii) The shortest wavelength $\lambda_{\min}$ of X-rays emitted is sharply defined and it depends on voltage applied. Higher the applied voltage lower is the shortest wavelength.

(iii) As the accelerating voltage is increased the wavelength at which maximum emission occurs shifts towards the shorter wavelength side.

(iv) Intensity variation with wavelengths of X-rays is independent of the nature of the target material.

● **Origin of continuous spectrum :** In general, when stream of high velocity electrons falls of a metal target, X-rays of different wavelengths are produced. These are produced due to deceleration of electrons as they penetrate the interior of the atoms of the target material and are attracted by the positive charges of their nuclei. As the electron passes close to the nucleus, it is deflected from its path as shown in fig. 5.4. The electron is decelerated during its deflection in the strong field of the nucleus. The kinetic energy lost during this retardation is transformed into electromagnetic radiation in the form of X-rays of continuously varying wavelengths. These X-rays form continuous spectrum when analysed by Bragg spectrometer.

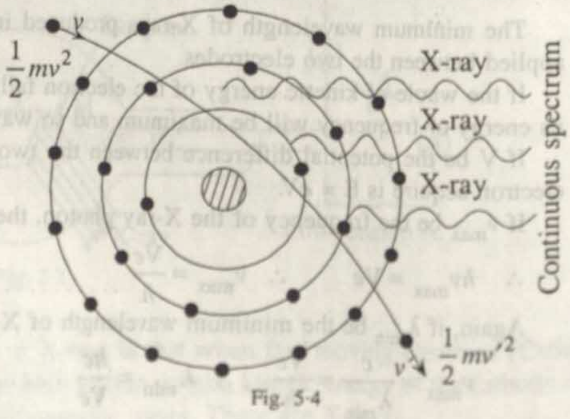


Fig. 5·4

Now, photon energy corresponding to the wavelength λ is

$$E = h\nu = \frac{hc}{\lambda}$$

Minimum limiting wavelength $\lambda_{\min}$ is obtained when entire kinetic energy of the bombarding electron is converted to X-ray energy.

If V be the accelerating potential, then

$$\frac{hc}{\lambda_{\min}} = eV \quad \therefore \; \lambda_{\min} = \frac{hc}{eV} \; \ldots\ldots\ldots\ldots\ldots\ldots\ldots \; (5.3)$$

This is known as **Duane Hunt** law.

- **Note :** Continuous spectrum is produced due to the radiation given out by decelerated beam of electrons. If, as shown in fig. 5.4, the striking electron has its velocity reduced from v to v' during its passage through the atom of the target material, then the loss of kinetic energy

$$\Delta E = \frac{1}{2} m (v^2 - v'^2)$$

It is equal to the energy of the X-ray photons emitted

$$\therefore \quad \frac{1}{2} m (v^2 - v'^2) = hv$$

For maximum value of frequency v, the velocity $v' = 0$.

$$\therefore \quad \frac{1}{2} mv^2 = hv_{\max} = h \frac{c}{\lambda_{\min}}$$

(b) Characteristic spectrum :

Its consists of radiations of definite wavelengths superposed on continuous spectrum. These lines are characteristic of the material used as the target. It is observed that on the continuous spectrum at certain wavelength large amounts of energy are radiated. The position of these lines do not depend on the voltage applied but depend only on the nature of the target used.

In fig. 5.5, graphs have been drawn between the wavelengths and the relative intensity for different values of accelerating potential in the case of molybdenum target. It is observed from the graph that as the accelerating potential is increased, relative intensity increases and the value of $\lambda_{\min}$ decreases. At lower potential difference only continuous spectra is observed. When the accelerating potential is 25 KV peaks are observed on the continuous spectrum. These are K_α and K_β characteristic lines.

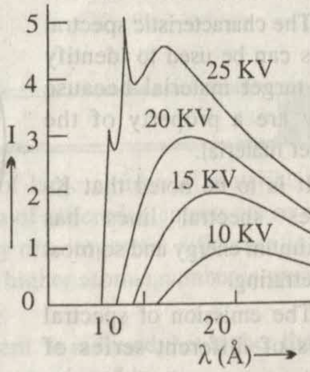
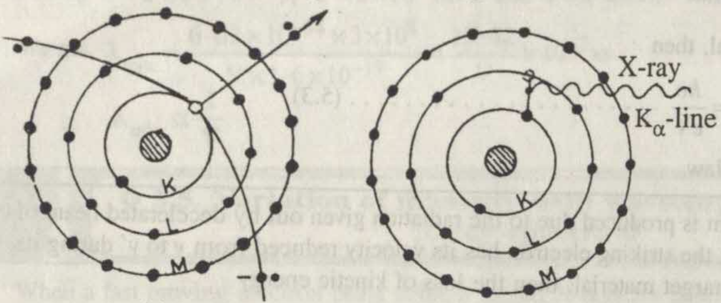


Fig. 5·5

- **Origin of characteristic spectrum :** The peaks on the continuous spectrum observed at high voltage are actually line spectrum or characteristic spectrum. The frequencies of these spectral lines depends on the nature of the target material.

The origin of the characteristic lines can be explained on the basis of Bohr theory of atom. According to the theory, around the nucleus of the atom a number of electrons are revolving in various shells K, L, M, N etc., as dictated by quantum conditions. When electron from higher orbits jumps to the lower orbit, a photon of definite energy and frequency is emitted. This energy is equal to the energy difference between the two orbits.

Sometimes, it may so happen that a high speed electrons while penetrating the interior of the atoms of the target material, knock off the tightly bound electrons from K or L-shell of the atoms [fig. 5.6(a)]. This creates a vacant space in the shell. Almost immediately, an electron from the outer shell jumps to fill up the vacancy so produced. The difference of energies of the two shells is emitted as electromagnetic radiation of definite frequency. This wavelength constitutes the line spectrum, which is the characteristic of the material of the target. As is shown in fig. 5.6(b), this vacancy in K shell is filled up by an electron from the L-shell. Then the frequency of the emitted X-ray is $E_K - E_L = h\nu$



(a) (b)

Fig. 5·6

This spectral line is called K_α line. Here E_K and E_L are the energies required to dislodge an electron from the K-shell and L-shell respectively.

If an electron from the M-shell jumps to K-shell to fill the vacancy, the frequency of the X-ray photon will be given by

$$h\nu' = E_M - E_K$$

Since $E_M - E_K > E_L - E_K$, $\nu' > \nu$. It is called K_β line.

In general, for a given target material, the wavelengths of different X-ray spectral lines are different. Spectral lines obtained due to the transition of electrons from L, M, N shells to K-shells are known as K-series of spectral lines. The respective X-ray lines are called K_α, K_β and K_γ as shown in fig. 5.7.

Similarly, L-series of characteristic spectral lines L_α, L_β and L_γ are obtained by the quantum jump of electrons from the outer M, N, O shells to the L-shell respectively. In the same manner, we can get other series M, N, O etc.

The characteristic spectral lines can be used to identify the target material because they are a property of the target material.

It is to be noted that K-series spectral lines has maximum energy and so most penetrating.

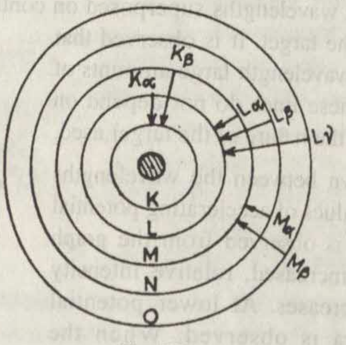


Fig. 5·7

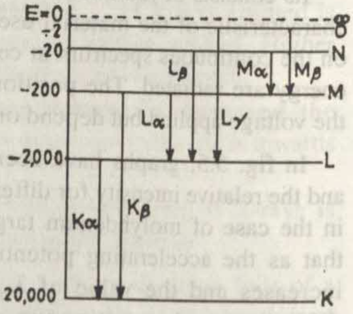


Fig. 5·8

The emission of spectral lines of different series of X-ray spectrum is shown in energy-level diagram of fig. 5.8.

Hence the X-rays obtained in X-ray tube consists of two parts.

- **(i) Continuous spectrum :** It consists of a series of uninterrupted wavelengths having a shortest wavelength limit λ_{min}.
- **(ii) Line spectrum :** It consists of a number distict and discrete wavelengths.

These two types of spectrum are shown in fig. 5.9.

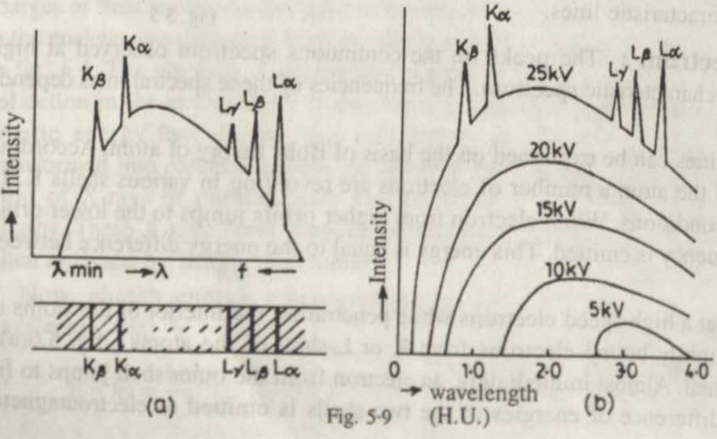


Fig. 5·9

- **Example 5.4. When the potential difference across an X-ray tube is 2 KV, the minimum wavelength of X-ray is 3Å. What will be the minimum potential if the applied potential difference be 10 KV.**

○ **Solution :** We know that, $\lambda_{\min} \alpha \dfrac{1}{V}$

$\therefore \quad V_1 \lambda_{\min} = \text{constant}$

$\therefore \quad 2\times3 = 10\times\lambda_{\min} \quad \therefore \ \lambda_{\min} = \mathbf{0\cdot6\,Å}$

- **Ex. 5.5. An X-ray tube operates at 5 KV and current of 2 *m*A. Calculate : (i) No. of electrons striking the target per sec. (ii) velocity of the incident electron. (iii) minimum wavelength of X-ray produced, (v) power of the tube.**

○ **Solution :** (i) If *n* number of electrons strike the target per sec, then

$$i = ne \qquad \therefore \ n = \frac{i}{e} = \frac{2\times10^{-3}}{1\cdot6\times10^{-19}} = \mathbf{1\cdot25\times10^{16}\ sec}$$

(ii) $\dfrac{1}{2}mv^2 = eV \qquad \therefore \ v = \sqrt{\dfrac{2eV}{m}}$

$$\therefore \quad v = \sqrt{2\times1\cdot76\times10^{11}\times5\times10^3} = \mathbf{4\cdot15\times10^7\ m/s}$$

(iii) Minimum wavelength of X-ray produced

$$\lambda_{\min} = \frac{12400}{V}\ Å = \frac{12400}{5000} = \mathbf{2\cdot48\,Å}$$

(iv) Power generated in the tube, P = V*i*

$$\therefore \quad P = 5000\times2\times10^{-3} = \mathbf{10\ watt}$$

5.6. Moseley's law

In 1913-14 Moseley examined the characteristic X-ray spectra of large number of materials used as targets in the X-ray tube. He found that the characteristic X-ray spctra of different elements are remarkably similar to each other and each consists of K-, L- and M-series. Only one important difference is that the frequency of spectral line of each series emitted from an element of higher atomic number is greater than that emitted by an element of lower atomic number.

Consider the K_α line of characteristic X-ray spectra of any element. It is found that higher the atomic number of the target material, higher is the frequency of the K_α line produced by it.

Moseley observed that if the square root of the frequency v of the most intense spectral line i.e., K_α line is plotted against the atomic number z, then a straight line graph is obtained [fig. 5.10] for each and every element. The graph is known as **Moseley diagram**. The exact mathematical relationship between frequency and atomic number is given by $v \alpha (z-b)^2$.

where '*b*' is the intercept on the atomic number axis.

or, $\sqrt{v} = a\,(z-b)$ ....(5.3)

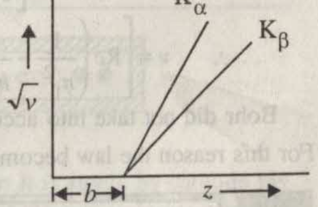


Fig. 5·10

a and *b* are constant for a particular series but vary from one series to another. Their values of K-series are different from those of L-series. The eq. (5.3) is known as Moseley law for the characteristic X-ray spectrum.

An exact form of Moseley's law is

$$\frac{1}{\lambda} = \mathrm{R}\,(z-b)^2\left(\frac{1}{n_1^{\,2}}-\frac{1}{n_2^{\,2}}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5.4)$$

Here n_1 and n_2 are principal quantum numbers.

5.7. Importance of Moseley's law

The most important significance of Moseley's law lies in the fact that it proves for the first time that it is the atomic number and not the atomic weight of an element which determines its characteristic properties. Both physical and chemical properties of an element depends on its atomic number only. He proved that elements must be arranged in the periodic table according to their atomic numbers and not their atomic weights.

Moseley's law has been used to place elements in their proper sequence in the periodic table in certain doubtful cases. For example, if we go by the atomic weight potassium $\left({}_{19}\mathrm{K}^{39}\right)$ should come before argon ${}_{18}\mathrm{A}^{40}$. Similarly, nickel $\left({}_{28}\mathrm{Ni}^{58\cdot7}\right)$ should precede cobalt $\left({}_{27}\mathrm{Co}^{58\cdot9}\right)$. But Moseley's law dictates that as per their atomic numbers, their order should be just opposite of the above. This fact is supported by the chemical properties of these elements.

Moseley's law also predicted the existence of new elements like hafnium (72), technitium (43) and rhenium (75) etc by the indication of gaps in Moseley diagram. This law also help to determine the atomic number of rare earth elements and predicted proper position of these elements.

5.8. Explanation of Moseley's law from Bohr's theory

If the atomic number of an element be z and the frequency of a particular spectral line be ν, then by Moseley's law

$$\sqrt{\nu} = a\,(z-b) \quad \text{or,} \quad \nu = a_1\,(z-b)^2 \quad [\ a,\ a_1 \text{ and } b \text{ are constants }]$$

It can be shown that Moseley's law can be obtained from Bohr's theory of spectral emission from atoms. We know that when an electron jumps from an orbit n_2 to the orbit n_1, the frequency of radiation emitted is

$$\nu = \frac{mz^2e^4}{8\,\varepsilon_0^{\,2}h^3}\left(\frac{1}{n_1^{\,2}}-\frac{1}{n_2^{\,2}}\right)$$

This may be put as $\nu = \left[\dfrac{me^4}{8\varepsilon_0^{\,2}h^3}\left(\dfrac{1}{n_1^{\,2}}-\dfrac{1}{n_2^{\,2}}\right)\right]z^2$

$$\therefore \quad \nu = \left[\mathrm{R}c\left(\frac{1}{n_1^{\,2}}-\frac{1}{n_2^{\,2}}\right)\right]z^2 \quad \therefore \quad \nu \ \alpha\ z^2 \quad \therefore \quad \sqrt{\nu}\ \ \alpha\ \ z$$

Bohr did not take into account the screening effect of electrons. But Moseley considered this effect. For this reason the law becomes $\sqrt{\nu}\ \ \alpha\ (z-b)$.

5.9. Comparison between X-rays and visible light rays

▲ A. Similarity :

1. Both X-rays and visible light are electromagnetic waves.

2. Both the rays show wave and particle nature.
3. Both the rays are not deflected by electric or magnetic field.
4. Both travel with equal velocity.
5. Both the rays are reflected, refracted. They can undergo interference, diffraction and polarisation.
6. Both the rays affect photographic plate.

▲ B. Dissimilarity :

1. Wavelength of X-rays ($10^{-9}\ m - 10^{-11}\ m$) is much smaller than the wavelength of visible light ($10^{-7}\ m$).
2. The frequency of X-rays is greater. So its energy $E = h\nu$ is more than the energy of visible light.
3. Penetrating power of X-rays is higher than visible light.
4. X-rays can ionise a gas. But visible light cannot.
5. Visible light helps to see object. But X-rays cannot make object visible.

❂ 5.10. Comparison between X-rays and Cathode rays ❂

▲ A. Similarity :

1. Both X-rays and Cathode rays can ionisation a gas.
2. Both the rays affect photographic plate.
3. Both the rays exhibit dual nature—Particle and Wave.

▲ B. Dissimilarity :

Cathode rays	X-rays
1. Cathode rays are stream of negatively charged electrons.	1. X-rays are electromagnetic wave of very short wavelengths.
2. They are deflected by electric and magnetic field.	2. They are not deflected by electric or magnetic field.
3. Velocity of cathode rays is much less than the velocity of light and its velocity can be controlled.	3. X-rays travel with velocity of light and can not be controlled.
4. When cathode rays fall on a metal, metal is heated.	4. When X-rays falls on a metal, it is not heated.

●● Short Answer Type Questions (with answers) ●●

● Question 1. Mention the difference between X-rays and γ-rays.

○ **Ans :** X-rays are produced from the extra nuclear part of the atom when it is struck by cathode ray particle. On the other hand, γ-rays are produced from the nucleus of the atom and the process is spontaneous. The frequency of γ-rays is higher than that of X-rays.

● Q. 2. Energy of X-rays is greater than visible light. Explain.

○ **Ans :** X-rays and visible light both are electromagnetic waves. But the frequency of X-rays is much higher than that of visible light. Now according to quantum theory of radiation, its energy is $E = h\nu$ where *h* is Planck's constant and *ν* is the frequency of radiation i.e., energy of radiation is proportional to its frequency. So, radiation of higher freqnency possesses higher energy. For this reason, energy of X-rays is greater than visible light.

● **Q. 3. Mention the difference between cathode rays and β-rays.**

○ **Ans :** Although both cathode rays and β-rays are stream of electrons, there is some difference between these two rays :

(i) Cathode rays are emitted from the extra nuclear part of the atom. But β-rays are emitted from the nucleus of a radioactive substance.

(ii) Emission of cathode rays does not affect the nucleus. But due to the emission of β-rays, number of neutrons decreases and number of protons increases.

$$ {}_0n^1 = {}_1p^1 + {}_0e^{-1} + \bar{\nu} $$

(iii) Emission of cathode rays depends on external condition. But emission of β-rays is a sponteneous process.

(iv) The velocity of β-rays is less than that of cathode rays. Velocity of cathode rays can be controlled. But the velocity of β-rays does not depend on external condition.

● **Q. 4. What are hard and soft X-rays ?**

○ **Ans :** X-rays of light frequency and high penetrating power are called hard X-rays. On the other hand, X-rays of low frequency and low penetrating power are called soft X-rays.

● **Q. 5. How can you control the intensity and quantity of X-rays obtained from an X-ray tube ?**

○ **Ans :** Intensity of X-rays depends on rate of emission of photon from the target and rate of photon emission depends on the number of electrons hitting the target per sec. This can be controlled by changing the cathode temperature. Hence intensity of X-rays emitted from an X-ray tube can be controlled by changing the cathode temperature.

Quality of X-rays is measured by the frequency and energy of X-ray photons. If the accelerating potential difference is large, the frequency of X-rays will be high and it is hard X-rays. But if the accelerating potential is low, frequency of X-rays will be low. These X-rays are soft X-rays. So quantity of X-rays can be controlled by changing the potential difference across the X-ray tube.

● **Q. 6. An X-ray tube operates at a potential difference V. What is the minimum wavelength of X-rays ?**

○ **Ans :** The minimum wavelength of X-rays produced in an X-ray tube due to potential difference. V is given by

$$ \frac{hc}{\lambda_{\min}} = eV \qquad \therefore \lambda_{\min} = \frac{hc}{eV} $$

● **Q. 7. Potential difference between two points is 100V. If an electron passes through this potential difference, how much energy, it will gain ?**

○ **Ans :** Energy gained by the electron is $E = 100eV = 100 \times 1{\cdot}6 \times 10^{-19}$ J $\therefore$ $E = 1{\cdot}6 \times 10^{-17}$ J

●● Numerical Examples ●●

● **Example 5.6. An electron initially at rest is sent through a potential difference of 220V. If the electron acquires a velocity of $4{\cdot}40 \times 10^6$ ms^{-1}, find its specific charge.**

○ **Solution :** The required relation, $\frac{1}{2}mv^2 = eV$

$$\therefore \quad \frac{e}{m}=\frac{v^2}{2V}=\frac{(4\cdot4\times10^6)^2}{2\times220} \qquad [\,v=4\cdot4\times10^6\ \text{m/s},\ V=220\text{V}\,]$$

$$=\mathbf{4\cdot4\times10^{10}\ C/Kg}$$

● **Ex. 5.7. An electron is projected perpendicular to a magnetic field of intensity 0·01 T with a velocity of 10^7 ms^{-1}. It describes a circle of radius 0·6 cm. Calculate its $\frac{e}{m}$ ratio.**

○ **Solution :** The required relation, $Bev=\frac{mv^2}{r}$

$$\therefore \quad \frac{e}{m}=\frac{v}{Br}=\frac{10^7}{0\cdot01\times6\times10^{-3}}$$

$$=\mathbf{1\cdot67\times10^{11}\ C/Kg}$$

$v = 10^7$ ms^{-1}

$B = 0\cdot01$ T

$r = 6\times10^{-3}\ m$

● **Ex. 5.8. A stream is electron is projected perpendicular to an electric field of intensity 1800 Vm^{-1} with a velocity of 3 × 10^7 m/s. After travelling a distance of 10 cm, it suffers a deflection of 2 mm. Calculate specific charge of electron.**

○ **Solution :** The acceleration of electron in the electric field is $a=\frac{Ee}{m}$.

The time taken by the electron to travel a distance l with a velocity v is $t=\frac{l}{v}$

$$\therefore \quad \text{Deflection of electron, } y=\frac{1}{2}at^2=\frac{1}{2}\frac{Ee}{m}\times\frac{l^2}{v^2}$$

$$\therefore \quad \frac{e}{m}=\frac{2yv^2}{El^2}=\frac{2\times2\times10^{-3}\times(3\times10^7)^2}{1800\times(0\cdot01)^2}$$

$$\therefore \quad \frac{e}{m}=\mathbf{2\times10^{11}\ C/Kg}$$

$y = 2\times10^{-3}\ m$

$E = 1800\ Vm^{-1}$

$v = 3\times10^7$ ms^{-1}

$l = 0\cdot1$ m.

● **Ex. 5.9. An X-ray tube operates at 30 KV and 20 *m*A. Calculate : (i) the number of electrons striking the target per sec, (ii) minimum wavelength of X-ray, (iii) rate of heat produced in the tube.**

○ **Solution :** (i) The current in the tube, $i=\frac{q}{t}=\frac{ne}{t}$

$$\therefore \quad n=\frac{it}{e}=\frac{20\times10^{-3}\times1}{1\cdot6\times10^{-13}}=\mathbf{1\cdot25\times10^{17}}$$

$$\text{(ii)}\quad \lambda_{\min}=\frac{1240}{V}\times10^{-9}\,m=\frac{1240}{30\times1000}\times10^{-9}\,m=\mathbf{0\cdot413\ \text{Å}}$$

(iii) Rate of heat production, $Q=Vi=30\times10^3\times20\times10^{-3}=\mathbf{600\ Js^{-1}}$

● **Ex. 5.10. The minimum wavelength X-rays produced by an X-ray tube operating at 60 KV is 0·162 Å. Find the value of Planck's constant.**

○ **Solution :** $\lambda_{\min}=\frac{hc}{Ve} \qquad \therefore\ h=\frac{Ve\lambda_{\min}}{c}$

$$\therefore \quad h=\frac{60\times10^3\times1\cdot6\times10^{-19}\times0\cdot162\times10^{-10}}{3\times10^8}=5\cdot184\times10^{-34}$$

$\therefore$ Planck's constant, $h=\mathbf{5\cdot184\times10^{-34}}$ **J.s.**

● **Ex. 5.11. The distance between the two deflecting plates in a cathode ray tube is 2 cm and a potential difference of 3000 V is applied across the plates. A magnetic field of intensity $2\cdot5 \times 10^{-3}$ T is also applied perpendicular to the electric field. The cathode rays after passing through a potential difference of 10 KV enter the cross fields. If the cathode rays go undeviated, calculate (i) velocity of the electron and (ii) $\frac{e}{m}$ of electron.**

○ **Solution :** (i) The electron passes through cross fields without any deviation.

∴ Magnetic force on the electric = electric field

$$\therefore \quad Bev = Ee = \frac{Ve}{d}$$

∴ Velocity of electron before entering the fields is $v = \frac{V}{B.d}$

$$\therefore \quad v = \frac{3000}{2\cdot5\times10^{-3}\times0\cdot02} = \mathbf{6\times10^{7}\ ms^{-1}}$$

(ii) Again, $eV_1 = \frac{1}{2}mv^2$ $\quad\therefore\quad \frac{e}{m} = \frac{v^2}{2\,V_1}$

$$\therefore \quad \frac{e}{m} = \frac{(6\times10^7)^2}{2\times10\times100} = \mathbf{1\cdot8\times10^{11}\ C\,Kg^{-1}}$$

● **Ex. 5.12. Two metal plates each of length 1·5 cm are kept horizontally at a separation of 1·0 cm and the potential difference between the plates is 1200 V. If an electron of energy 2 KeV enters into the electric field normally and midway between the plates. What is the deflection of the electron ?**

○ **Solution :** The equation to calculate the deflection inside an electric field is

$$y = \frac{1}{2}\,\frac{Ee}{m}\,\frac{l^2}{v^2} = \frac{1}{2}\,\frac{Vel^2}{d.m.v^2}$$

Kinetic energy of electron, $K = \frac{1}{2}mv^2$ $\quad\therefore\quad mv^2 = 2K = 2\times2\times10^3\times1\cdot6\times10^{-19}\,J$

$$\therefore \quad mv^2 = 6\cdot4\times10^{-16}\,J$$

$$\therefore \quad y = \frac{1}{2}\times\frac{1200\times1\cdot6\times10^{-19}\times(1\cdot5\times10^{-2})^2}{10^{-2}\times6\cdot4\times10^{-16}} \qquad [d = 10^{-2}\,m]$$

$$= \mathbf{3\cdot375\times10^{-3}\,m = 3\cdot375\ mm}$$

● **Ex. 5.13. A charged oil drop is floating in an electric field of intensity 3×10^4 Vm^{-1}. What is the charge on the oil drop ? mass of the drop = $4\cdot5 \times 10^{-15}$ Kg.**

○ **Solution :** Here the weight of the oil drop is balanced by the electric force

$$\therefore \quad Eq = mg \qquad \therefore \quad q = \frac{mg}{E}$$

$$\therefore \quad q = \frac{4\cdot5\times10^{-15}\times9\cdot8}{3\times10^4} = \mathbf{1\cdot47\times10^{-18}\,C}$$

● **Ex. 5.14. An X-ray tube operates at 20 KV. With what velocity the electron will strike the target ? If only 5% of the electron energy is converted to X-rays, calculate maximum frequency of X-rays ? Given, $h = 6\cdot62 \times 10^{-34}$ J.s.**

○ **Solution :** (i) The velocity of electron just before striking the target is

$$v = \sqrt{\frac{2eV}{m}} = \sqrt{2\times1\cdot76\times10^{11}\times20\times10^3} = \mathbf{8\cdot4\times10^{7}\ ms^{-1}}$$

(ii) If the maximum frequency of X-rays be $\nu_{\max}$, then

$$h\nu_{\max} = eV \times \frac{5}{100}$$

$$\therefore \quad \nu_{\max} = \frac{1}{20}\,\frac{eV}{h} = \frac{1 \cdot 6 \times 10^{-19} \times 20 \times 10^{3}}{20 \times 6 \cdot 62 \times 10^{-34}} = \mathbf{2 \cdot 4 \times 10^{17}\ Hz}$$

● E ◈ X ◈ E ◈ R ◈ C ◈ I ◈ S ◈ E ●

◢ A. Objective type questions :

● Fill in the blanks :

1. Cathode rays are fast moving ______ .
2. Cathode rays are deflected by both ______ and ______ .
3. Specific charge of electron ______ and of proton ______ .
4. 1 *e*V = ______ Joule.
5. According to Moseley's law frequency of X-ray line spectrum is ______ .
6. Physical and chemical properties of an element are determined by its ______ .
7. Frequency of X-rays is ______ than γ-rays.

◢ B. Very short answer type questions :

1. How can you change the intensity of X-rays is coolidge tube ?
2. What is meant by quantity of X-rays ?
3. What are hard and soft X-rays ?
4. What is meant by specific charge of electron ?
5. Define 1 *e*V.
6. State Moseley's law.
7. When are K_α and K_β lines produced ?

◢ C. Short answer type questions :

1. What is meant by electric discharge ? What is the difference between electric conduction and electric discharge ?
2. What are cathode rays ? How are they produced ?
3. An electron of charge *e* and mass *m* passes through a potential difference of V volt. What is the velocity gained of electron ?
4. How can you increase the velocity of cathode rays ?
5. What is the difference between cathode rays and β-rays ?
6. What are X-rays ? How are they produced ?
7. How can the intensity of X-rays be changed ?
8. What we mean by hard and soft X-rays ?
9. How can you control the hardness of X-rays ?
10. Why is energy of X-rays is greater than that of ordinary light ?
11. How can you increase the penetrating power of X-rays ? How it is related to frequency ?
12. What is the difference between X-rays and γ-rays ?
13. If the filament temperature and accelerating potential difference is increased, what is the effect on intensity and hardness of X-rays produced in a coolidge tube ?
14. Why we use X-rays for crystal structure analysis and not ordinary light ?
15. An electron is moving with a velocity of 10 m/s. Calculate its energy in electron volt.

◢ D. Essay type questions :

1. What are cathode rays ? Describe how cathode rays can be produced. Mention the properties of X-rays ?
2. Explain with the help of a diagram, a coolidge tube for the production of X-rays. On what factor does the penetrating power of X-rays depends ? Mention the properties of X-rays.

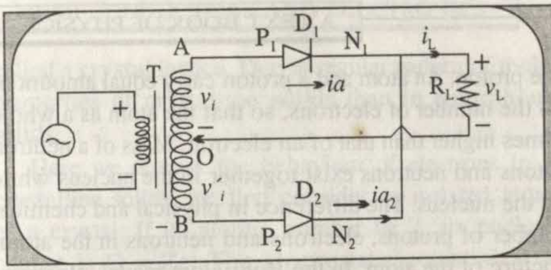
3. In a coolidge tube (i) the filament heating current is increased (ii) the p. d. between the anode and cathode is increased what changes in the nature of emitted X-rays will be observed in the two cases ?

E. Simple numerical problems :

1. An electron is accelerated through a potential difference of 1137·5 volt. What is the final energy (in eV) and velocity of the electron ? mass of electron = $9\cdot1 \times 10^{-28}$g
[**Ans :** (i) 1137·5 eV (ii) 2×10^{7} m/s] **[W. B. H. S. 1999]**
2. The velocity of electron is 10^{9} m/s. Calculate its energy in eV unit. [**Ans :** 284 eV]
3. From nest an electron passes through a potential difference of 60 KV. If the electron acquires a velocity of $1\cdot46 \times 10^{8}$ m/s, calculate its specific charge. [**Ans :** $1\cdot776 \times 10^{11}$ C / Kg] **[J. E. E. 1984]**
4. How much force an electron of charge $-1\cdot6 \times 10^{-19}$ C will experience in an electric field of intensity 2×10^{4} Vm^{-1} ?
[**Ans :** $-3\cdot2 \times 10^{-15}$ N]
5. An electron passes through a potential difference of 400 V. If it was initially at rest, calculate its velocity and energy. [**Ans :** (i) $11\cdot86 \times 10^{6}$ ms^{-1} (ii) 400 eV]
6. What minimum potential difference has to be applied across a coolidge tube so that minimum wavelength of X-rays emitted is 0·84 Å ? $h = 6\cdot62 \times 10^{-34}$ J. s. [**Ans :** 15·5 KV] **[J. E. E. 1984]**
7. An X-ray tube operates at 3 KV potential difference and 3 mA current. Calculate : (i) no. of electrons striking the target per sec. (ii) velocity with which the electron strike the target, (iii) Rate of heat production in the tube, (iv) minimum wavelength of X-ray. Given, mass of electron, $m = 9\cdot1 \times 10^{-31}$ Kg, charge, $e = 1\cdot6 \times 10^{-19}$ C and $h = 6\cdot62 \times 10^{-34}$ J. s. [**Ans :** (i) $1\cdot87 \times 10^{16}$, (ii) $3\cdot26 \times 10^{7}$ ms^{-1}, (iii) 9 Js^{-1}, (iv) 4·13 Å]
8. Wavelength of X-ray produced in a coolidge tube is 1 Å ; what is the energy of the photon ?
[**Ans :** $19\cdot86 \times 10^{-16}$ J]
9. A water drop of radius 10^{-5} cm is given a charge $1\cdot6 \times 10^{-19}$ C. If it floats in an electric field, what is the intensity of the electric field ? [**Ans :** E = 256·4 Vm^{-1}]
10. Calculate the maximum frequency of X-ray produced in an X-ray tube operating at 50 KV.
[**Ans :** $1\cdot21 \times 10^{19}$ Hz] **[J. E. E. 1990]**
11. A potential difference of 500V is applied between two parallel plates separated by a distance of 5 cm. An electron in the negative plate starts from rest toward the positive plate what will be its velocity after $6\cdot6 \times 10^{-9}\,s$? [**Ans :** $1\cdot16 \times 10^{7}$ m/s]
12. The charge of an oil drop is $-3e$. Its radius is $3\cdot2 \times 10^{-6}\,m$ and density 900 Kg / m^{3}. If it floats in an electric field, what is the intensity of the electric field ? [**Ans :** $2\cdot52 \times 10^{6}$ Vm^{-1}]
13. The wavelength associated with a photon is 4950Å, calculate its energy in eV unit. [**Ans :** 2·5 eV]
14. An electron enters normally into a magnetic field of intensity 3×10^{-2} T. If it describes a circle of radius 1·5 cm inside the field, calculate its velocity. [$e / m = 1\cdot7 \times 10^{11}$ C / Kg] [**Ans :** $7\cdot65 \times 10^{7}$ m/s]

F. Harder numerical problems :

1. An electron of charge $-1\cdot6 \times 10^{-19}$ C and mass $9\cdot1 \times 10^{-31}$ Kg is moving with a velocity of 10^{6} m/s along X-axis. Then it enters an electric field of intensity 10^{3} Vm^{-1} acting perpendicular to an electric field. The field acts over a distance of 2 cm along X-axis. What will be its deflection what it leaves the electric field ?
[**Ans :** 3·51 cm]
2. A stream of electron passes through a region where both electric and magnetic field is acting perpendicular to each other. Intensity of the magnetic field 2×10^{-3} Wb / m^{2} and that of the electric field is $3\cdot4 \times 10^{4}$ Vm^{-1}. If the electron passes through the region with no deflection, calculate the velocity of the electron. If the electric field is removed, find the radius of the path of the electron in the field.
[**Ans :** (i) $v = 1\cdot7 \times 10^{7}$ ms^{-1}, (ii) $r = 4\cdot78$ cm]
3. The separation between two parallel plates is 2·5 cm and a potential difference of 50 V is applied between the plates. An electron initially at rest is first allowed to pass through a potential difference of 1000V and then it enters the space between the plates. What will be the deflection of the electron after it travels 5 cm inside the electric field ?
[**Ans :** 1·25 mm]



CHAPTER 6

SEMI-CONDUCTORS AND ELECTRONICS

● **TOPICS :** ○ Introduction; ○ The electron; ○ Atom; ○ Energy bands in crystalline solids; ○ Valence and conduction band; ○ Conductors, insulators and semi-conductors; ○ Electrons and holes in semi-conductor; ○ Intrinsic semi-conductor; ○ Recombination of Electrons and holes; ○ Doping a semi-conductor; ○ Electrical resistivity of semi-conductor; ○ Current flow in a biased N-type semi-conductor; ○ Current flow in a biased P-type semi-conductor; ○ PN junction diode; ○ A biased PN junction; ○ Current voltage characteristics of a PN junction diode; ○ Zener diode or break down diode; ○ PN junction diode as rectifier; ○ Junction transistor; ○ Transistor action; ○ Different modes of using transistor; ○ Amplifying Action of transistor; ○ Transistor characteristics; ○ Transistor as an amplifier; ○ Current amplification factors; ○ Short answer type questions (with answers); ○ Exercise.

6.1. Introduction

We are living today in an age of electronics. Electronics is that branch of science and technology which deals with theory and application of a class of devices (such as vacuum and gas filled tubes, semi-conductors diodes and transistors etc.) in which transport of electrons occurs through vacuum, gas or semi-conductors. In such devices motion of electrons is controlled by electric field.

Electronics has found widespread application in many fields of science and engineering. It has became indispensiable in modern leaving.

6.2. The electron

The electron is one of the fundamental subatomic negatively charged particle. The charge of an electron has smallest possible value. Any charge is integral multiple of the electronic charge. The charge of an electron is $e = -1{\cdot}6 \times 10^{-19}$ coulomb. Its rest mass $m_0 = 9{\cdot}1 \times 10^{-31}$ Kg. If the electron moves with a velocity v, its moving mass is given by

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}} \quad \text{where } c = 3 \times 10^8\, ms^{-1}$$

The electron can be accelerated to high velocity by passing it through a high potential difference. If it is passed through a potential difference of V volts (initial velocity being zero), it will acquire a velocity v. It is obtained from

$$\frac{1}{2} mv^2 = e\text{V} \qquad \therefore\ v = \sqrt{\frac{2e\text{V}}{m}}$$

The energy of an electron or any other subatomic particle is expressed in **'electron volt'** (eV) unit. 1 eV is the energy acquired by an electron when it is passed through a potential difference of 1V. Hence 1 eV = 1 electronic charge × 1 volt = $1{\cdot}6 \times 10^{-19}$ J.

6.3. Atom

An atom is the smalled part of an element and it consists of electrons (negatively charged), protons (positively charged) and neutrons (uncharged). The only exception is the atom of hydrogen which does not contain neutron. It is made of one electron and one proton. An atom and a proton carry equal amount of charge. In an atom, the number of protons is equal to the number of electrons, so that the atom as a whole is electrically neutral. The mass of a proton is 1837 times higher than that of an electron. Mass of a neutron is slightly heavier than a proton. In an atom, the protons and neutrons exist together in the nucleus while the electrons revolve in some permitted orbits around the nucleus. The difference in physical and chemical properties of elements is due to the difference in number of protons, electrons and neutrons in the atom.

Different models have been proposed for the structure of the atom. In the Bohr atom model, electrons are assumed to revolve round the nucleus, without radiating energy, in certain discrete circular orbits. The orbital radius is such that the angular momentum of the electron is equal to integral multiple of $h/2\pi$.

So, $mvr = \frac{nh}{2\pi}$ $[n = 1, 2, 3, \cdots\cdots]$

The higher the value of n, the greater is the radius of the orbit.

As n is an integer, energy of orbital electron has difinite value. So, the electron energy is **quantised : Corresponding to different integral values of n, only certain values of energy are allowed.** The energy for different values of n are shown in the diagram by horizontal lines [fig. 6·1]. It is called energy level diagram.

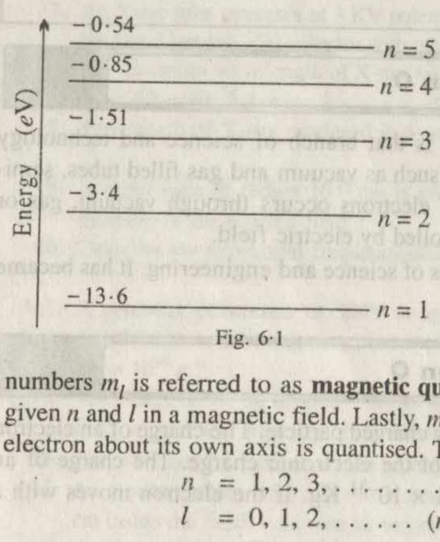


Fig. 6·1

Later to explain the fine structure of spectral lines of several elements some improvements of the simple Bohr atom model have been made. Four quantum numbers, denoted by n, l, m_l, m_s have been introduced. The energy of orbital electron is determined primarily by the quantum number n, called as **principal quantum number**. The other three quantum number have negligible effect on the total energy of orbital electron. The quantum number l determines the angular momentum of the electron and is known as **orbital angular momentum quantum number**. The third quantum numbers m_l is referred to as **magnetic quantum number**. It specify the splitting of the energies with a given n and l in a magnetic field. Lastly, m_s is called **spin quantum number**. It shows that the spin of the electron about its own axis is quantised. The values of the four quantum numbers are given as—

$$n = 1, 2, 3, \ldots\ldots\ldots$$
$$l = 0, 1, 2, \ldots\ldots (n-1)$$
$$m_l = 0 \pm 1 \pm 2 \ldots\ldots \pm l$$
$$\text{and } m_s = \pm \tfrac{1}{2}$$

The state of electron in an atom is uniquely determined by these four quantum numbers. The position of an electron in the atom is determined by **Pauli's exclusion principle**. According to this principle, no two electrons can occupy the same quantum state. It means that any two electrons in the atom can not have same four quantums n, l, m_l and m_s.

6.4. Energy bands in crystalline solids

A solid substance is rigid. It has difinite shape and volume. Solids can be divided into two groups—**Amorphous** and **Crystalline**. In amorphous solids, although the molecules are rigidly fixed, but they are not regularly arranged. Their molecular arrangement is very similar to that of liquids. For this reason, the amorphous substance are treated as super cooled liquids.

On the other hand in a crystal the atoms are arranged in a regular and repetitive geometrical pattern. The positions of atoms in a crystal are represented by an array of points in space. The array of points is called a **crystal lattice**. Due to regular pattern of molecular arrangement, it is easier to explain the physical properties of crystalline solids than in amorphous solids.

Here we discuss the behaviour of electrons in a crystalline solids, we first consider an isolated atom of a crystal. If its atomic number be Z, its nuclear charge is Q = Ze. The electrostatic potential at a distance r from the nucleus is

$$V(r) = \frac{1}{4\pi\epsilon_0}\frac{Ze}{r} \quad \ldots\ldots\ldots\ldots\ldots\ldots (6.1)$$

So, the potential energy of the electron at a distance r from the positive nucleus

$$E_p(r) = -eV(r) = \frac{-1}{4\pi\epsilon_0}\frac{Ze^2}{r} \quad \ldots\ldots (6.2)$$

The variation of V (r) with r and variation E_p (r) with r are shown in fig. 6.2(a) and (b) respectively. It is to be noted that V (r) is positive and E_p (r) is negative. Both potential and potential energy are zero at infinity.

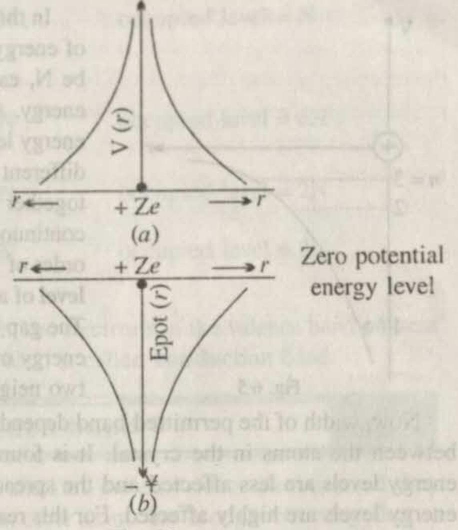


Fig. 6·2

Now we consider two similar atoms close together. The resultant potential energy of an electron will now be equal to the sum of the potential energies due to the two individual nuclei. The resultant potential energy in a region between the two nuclei will be smaller than the potential energy due to an isolated nucleus. It is shown in fig. 6.3.

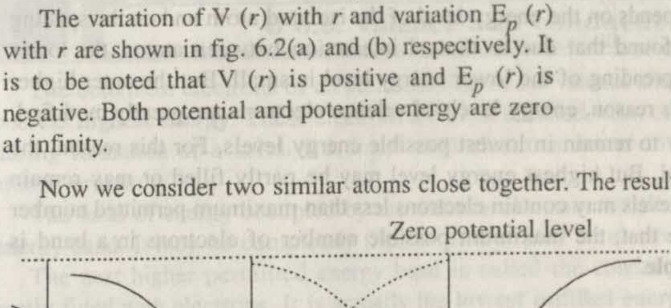


Fig. 6·3

The variation of potential energy of the electron due to the electrostatic attraction of the nucleus along a line through a row of nuclei of the atoms is shown in fig. 6.4. It is seen from the fig. that the distribution of potential energy between the nuclei is a series of humps. The potential energy increases at the boundary of the solid and becomes zero at infinity.

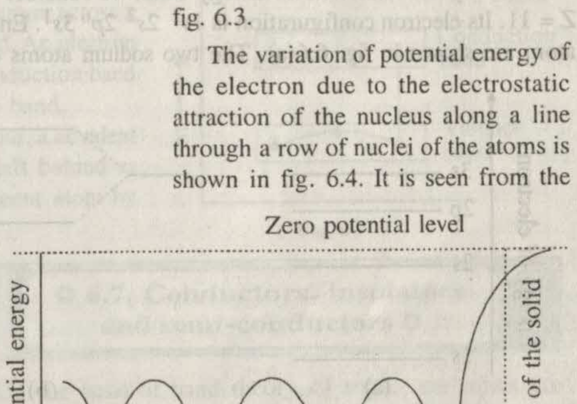


Fig. 6·4

From quantum mechanics it is known that when a particle exists in the potential well due to attractive force, the energy of the particle is discrete or quantised. For example an electron in hydrogen or hydrogen like atom exists in a potential well produced due to attraction of the nucleus [fig. 6.5]. By solving Schrodinger equation in wave mechanics, the energy levels of the electron are found to be discrete and quantised. These energy levels are identical with energy levels obtained from Bohr's theory.

Now, if we consider two neighbouring atoms, the number of energy levels will be double than due to a single atom. Here each energy level of single atom will split into two energy levels of very close energy.

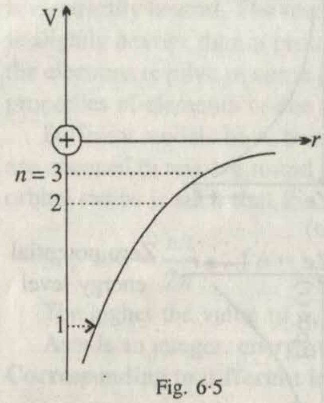


Fig. 6·5

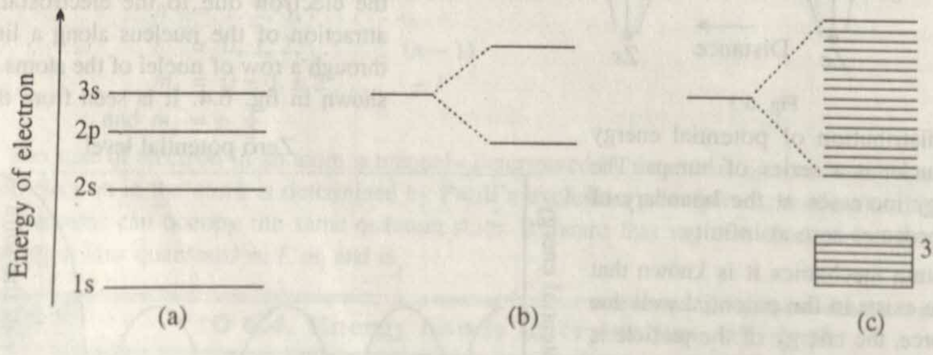
In this way as the number of atoms increases, the possible number of energy levels also increases. If the number of atoms in the crystal be N, each of the energy levels of the atom splits into N levels of energy. As these splited energy levels are very close, each original energy level becomes a **band** of very closely spaced levels of slightly different energy. The individual energies within the band are so close together that, for many purposes, the energy band may be considered continuous. As an example, if $N = 10^{18}$, then this separation is of the order of 10^{-18} eV for a band width of 1 eV. Thus a **permitted energy level** of a single atom is converted into a **permitted band** in the crystal. The gap between two consecutive bands is called **forbidden gap**. The energy of an electron in a crystal can not lie in the forbidden gap. So two neighbouring permitted bands are separated by forbidden gap.

Now, width of the permitted band depends on the energy level of the isolated atom and on the spacing between the atoms in the crystal. It is found that due to mutual interaction between atoms, the lower energy levels are less affected and the spreading of the lower energy level is small. But the outer higher energy levels are highly affected. For this reason, energy levels of valence electrons are greatly modified.

The electrons in an atoms always try to remain in lowest possible energy levels. For this reason, the lower energy levels are completely filled. But highest energy level may be partly filled or may remain completely empty i.e., the higher energy levels may contain electrons less than maximum permitted number of electrons. It must be mentioned here that, the maximum possible number of electrons in a band is determined by **Pauli's exclusion principle**.

➤ *6.5.1. Energy band in sodium :*

To study the formation of energy band, we consider sodium metal. Its atom has eleven electrons i.e., Z = 11. Its electron configuration is $1s^2\ 2s^2\ 2p^6\ 3s^1$. Energy levels of the electrons in an isolated sodium atom are shown in fig. 6.6(a). The two sodium atoms come very close to each other, the two valence electrons of both the atoms are affected by the electric field of Na^+ ions. As the energy level of 3s electron of the two atom system is divided into two close energy levels [fig. 6.6(b). As the number of atoms increases each energy level is divided into more and more number of energy levels. Inner level electrons are less affected. Now, in sodium, there are 10^{23} atoms /c.c. and each atom has one valence electron. So, the energy level of the valence electron is divided into large number of very closely spaced infinite number of energy levels as shown in fig. 6.6(c).



(a) (b) (c)

Fig. 6·6

In the fig. 6.7 below we show different energy bands in a sodium crystal containing N atoms.

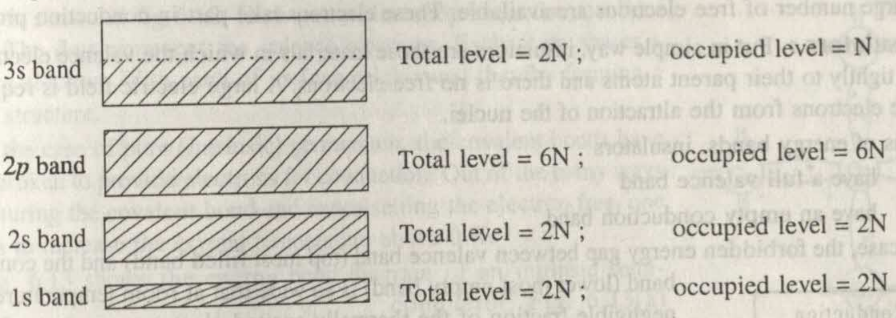

3s band
Total level = 2N ; occupied level = N
2p band
Total level = 6N ; occupied level = 6N
2s band
Total level = 2N ; occupied level = 2N
1s band
Total level = 2N ; occupied level = 2N

Fig. 6·7

The energy band of valence electrons is called valence band. The electrons in the valence band possess maximum energy. The vacant energy band above the valence band is called **conduction** band.

❂ 6.6. Valence and conduction band ❂

The outermost electrons of an atom existing in the valence shell are called valence electrons and they possess **highest energy**. These electrons are most affected when other atoms are brought close together as during formation of a solid. But inner shell electrons nearer to the nucleus are least affected by this atomic proximity.

The band of energy occupied by the valence electrons is called the **valence band** and is the highest energy band. It may be completely filed or partly filled by electrons.

The next higher permitted energy band is called the conduction band and may either be empty or partly fitted with electrons. It is actually the **lowest unfilled energy band**. The gap between the valence and conduction band is called forbidden energy gap. In conduction band the electrons can move freely and hence are known as **conduction electrons**.

If a valence electron gets sufficient energy, it jumps across a forbidden energy gap and enters the conduction band. An electron in the conduction band can jump to an adjacent conduction band more readily than it can jump back to the valence band.

When an electron is ejected from the valence band, a covalent band is broken and a positively charged hole is left behind as shown in fig. 6.8. The hole can travel to an adjacent atom by acquiring an electron from that atom.

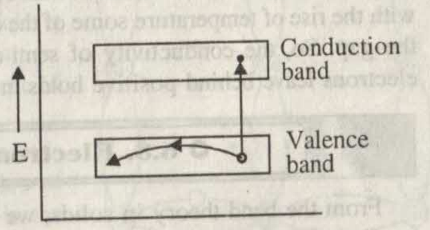


Fig. 6·7

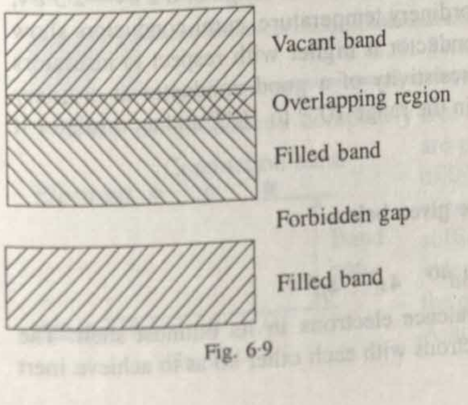


Fig. 6·9

❂ 6.7. Conductors, insulators and semi-conductors ❂

On the basis of band theory of solids, the solids are classified as—(i) conductors, (ii) insulators and (iii) semi-conductors.

- (i) **Conductors or metals :** Put in a simple way, conducting materials are those in which plenty of free electrons are available for electric conduction.

In terms of energy bands, it means that conductors are those which have overlapping valence and conduction bands as shown in fig. 6.9. So, in a conductor or metal, there is no physical distinction between the two bands. Hence a large number of free electrons are available. These electrons take part in conduction process.

➡ (ii) **Insulators :** Put in simple way, insulators are those materials in which the valence electrons are band very tightly to their parent atoms and there is no free electrons. A large electric field is required to remove the electrons from the altraction of the nuclei.

In terms of energy bands, insulators

(a) have a full valence band

(b) have an empty conduction band.

In this case, the forbidden energy gap between valence band (top most filled band) and the conduction band (lower most empty band) is so wide that at room temperature only a negligible fraction of the thermally excited electrons can jump from the valence band to the conduction band as shown in fig. 6.10.

Increase of temperature enables some electrons to go to the conduction band which accounts for the negative temperature coefficient of resistance of insulators.

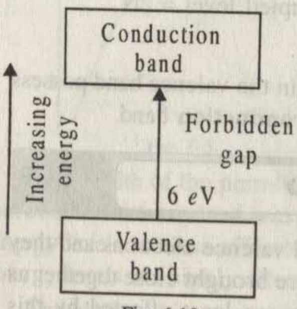


Fig. 6·10

➡ (iii) **Semi-conductors :** A semi-conductor is a material whose conductivity lie in between those of insulators and good conductors. Examples are : germanium and silicon.

In terms of energy band, semi-conductors are the materials which has almost an empty conduction band and almost filled valence band with a very narrow energy gap (of the order of 1 *e*V) separating the two [fig. 6.11]. In the case of germanium $E_G = 0\cdot 74$ *e*V and for silicon $E_G = 1\cdot 1$ *e*V.

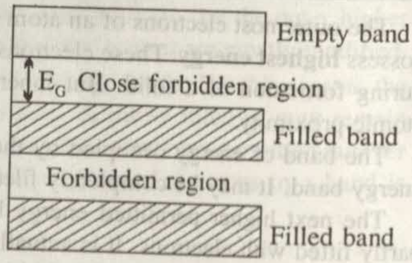


Fig. 6·11

At 0 K, there are no electron in the conduction band and the valence band is completely filled. However with the rise of temperature some of the electrons from the valence band go to conduction band overcoming the gap. So, the conductivity of semi-conductors increases with temperature. Moreover, the departing electrons leave behind positive holds in the valence band.

❂ 6.8. Electrons and holes in semi-conductor ❂

From the band theory in solids, we know that band structure in semi-conductor is similar to that in insulators. Only difference is that forbidden gap in semi-conductor is very narrow, but it is very wide in insulators, e.g., in the case of semi-conductor width of the energy gap lies in the range of 0·2 *e*V—2·5 *e*V, whereas in the case insulators it is about 9 *e*V. As a result, at ordinery temperature, semi-conductors show small electrical conductivity. In fact, conductivity of semi-conductor is higher with respect to insulators and lower with respect to good conductors. For example—resistivity of a good conductor at ordinary temperature is $10^{-6}\ \Omega$-cm, for a semi-conductor it value lies in the range 10^{-3} to 10^{+6} and an insulator it is about $10^{12}\ \Omega$-cm.

Widely used semi-conductors are germanium and silicon.

The electronic configuration of silicon and germanium are given below :

Silicon, Si_{14} : $1s^2\ \ 2s^2\ \ 2p^6\ \ 3s^2\ \ 3p^2$

Germanium, Ge_{32} : $1s^2\ \ 2s^2\ \ 2p^6\ \ 3s^2\ \ 3p^6\ \ 3d^{10}\ \ 4s^2\ \ 4p^2$

Both these materials are tetravalent i.e. each has four valence electrons in its outmost shell. The neighbouring atoms form **covalent** bonds by sharing four electrons with each other so as to achieve inert gas structure (i.e., 8 electrons in the outermost orbit). A two-dimensional view of the germanium crystal lattice is shown in fig 6.12. Each pair of lines represent the covallent bond. The dots represent the valence electrons. Each atom shares a valence electron. With each of its four neighbours, thereby forming a stable structure.

In the case of pure (intrinsic) germanium, the covalent bonds have to be broken to provide electrons for conduction. Out of the many ways of rapturing the covalent bond and hence setting the electron free, one way is to increase the crystal temperature above 0 K.

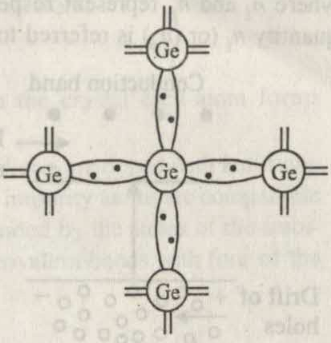


Fig. 6·12

Fig. 6.13 shows the energy bond diagram of an intrinsic semiconductor. Fig. 6.13(a) shows same charge carrier at absolute zero of temperature. Fig. 6.13(b) shows the situation at room temperature. At room temperature, some electrons are in conduction band leaving an equal number of holes (0) in the valence band. Here each horizontal line represents an energy level and in each energy level, there can be at most two electrons.

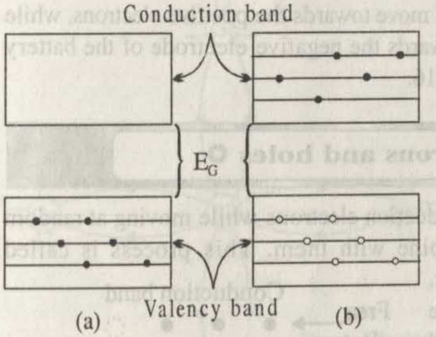


Fig. 6·13

The electrons which move from valence band to conduction band at higher temperature are called **intrinsic carriers** and the corresponding vacancies created in the valence bands are called **holes** which behave as positive charge carrier.

The creation of a hole can also be understood by referring to fig. 6.14. On receiving extra energy, an electron forming a covalent band breaks and is free to move in the crystal lattice leaving behind a hole in the covalent bond shown as a hollow circle. An electron in the neighbouring atom may jump into the vacant hole. A new hole is created at another place. The holes move randamly in a crystal lattice. It must be clearly understood that holes are due to movement of electrons in the valence band.

When an electric field is applied to the semi-conductor, current is produced not only due to the flow of electrons, but also due to flow of holes. The electrons remains in conduction bands and holes in the valence bands. Hence in a semi-conductor, both holes and electrons are treated as current carriers.

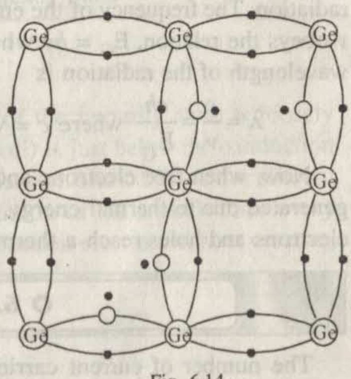


Fig. 6·14

✪ 6.9. Intrinsic semi-conductor ✪

It is a semi-conductor completely free from any impurity. Common example of such semi-conductors are pure germanium and silicon, which have forbidden energy gap of 0·74 *e*V and 1·1 *e*V respectively. The energy gap is so small that even at ordinary temperature, there are many electrons which possess sufficient energy to jump across the small energy gap from the valence band to the conduction band. For each electron transition to the conduction band, a hole is created in the valence band [fig 6.15]. A hole carries a positive charge equal in magnitude to the electron

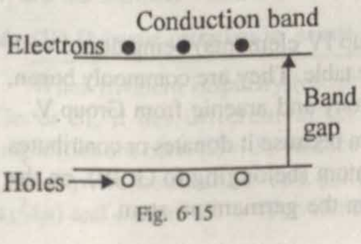


Fig. 6·15

charge. The number of thermally generated electrons always equals to the number of holes. So, $n_i = n_p$, where n_i and n_p represent respectively the concentrations thermaly generated electrons and holes. The quantity n_i (or (n_p) is referred to as the **intrinsic carrier concentration**.

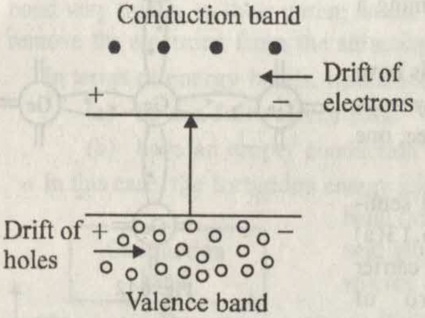


Fig. 6·16

The electrons in the conduction band and holes in the valence band move freely and randamly within the crystal due to their thermal energy. When a potential difference is applied to the semi-conductor a directed drift velocity is superposed on the random thermal motion of the electrons. This gives rise to a current flow. So, when a semi-conductor is connected to battery the current is contributed by the electrons in the conduction band and the holes in the valence band. The electrons move towards the positive electrons, while the holes move towards the negative electrode of the battery as shown in fig. 6.16.

6.10. Recombination of Electrons and holes

It is very likely that at every instant of time some of the conduction electrons while moving at random inside a semi-conductor crystal encounter holes and combine with them. This process is called **recombination**. When electrons and holes combine they are lost. When the electron jumps from the conduction band into the valence band and combine with a hole [fig. 6.17] energy equal to the band gap energy is released in the form of electromagnetic radiation. The frequency of the emitted electromagnetic radiation ν obeys the relation, $E_G = h\nu$ where h is Planck's constant. The wavelength of the radiation is

$$\lambda = \frac{c}{\nu} = \frac{ch}{E_G} \quad \text{where } c = \text{velocity of light in vacuum.}$$

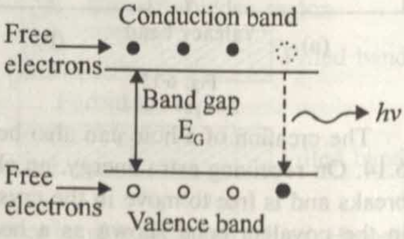


Fig. 6·17

Now, when free electrons and holes are lost by recombination new pairs of electrons and holes are generated due to thermal energy. At a fixed temperature for a pure semi-conductor, the concentration of electrons and holes reach a thermal equilibrium.

6.11. Doping a semi-conductor

The number of current carriers—electrons and holes in a semi-conductor crystal can be increased with the help of heat or light energy. But its conductivity remains rather low. The most convenient and efficient method of generating free electrons and holes is to deliberately add some amount (1 part in 10^8) of selected impurity inside the crystal. This process of adding impurity is called **doping** and the material added is termed **dopant**. The semi-conductor containing impurity atoms is known as extrinsic semi-conductor. The conductivity of the extrinsic semi-conductor is determined by the excess electrons or holes due to impurity atoms.

Usually extrinsic semi-conductors are made of Ge and Si (both Group IV elements) being doped with either elements of Group III or with elements of Group V of the periodic table. They are commonly boron, gallium, indium and aluminium from Group III and phosphorus, antimony and arsenic from Group V.

Penta valent doping atom (belonging to Gr V) is known as **donor** atom because it donates or contributes one electron to the conduction band of pure germanium. The trivalent atom (belonging to Gr III), on the other hand, is called acceptor atom because it accepts one electron from the germanium atom.

Depending upon the type of dopant present in the semi-conductor, the extrinsic semi-conductors are divided into two classes : (i) N-type semi-conductors, (ii) P-type semi-conductors.

▲ (i) N-type extrinsic semi-conductor :

Germanium and silicon are tetravalent elements having valency 4. In the crystal each atom forms covalent bonds with neighbouring four atoms.

To obtain N-type extrinsic semi-conductor crystal, pure germanium or silicon are doped with antimony (Z = 51, Valence shell $5s^2\ 5p^3$) or arsenic (Z = 33, valence shell $4s^2 4p^3$). The impurity atoms are comparable in size to the Ge or Si atom. In the crystal, an impurity atom will be surrounded by the atoms of the semi-conductor material [fig. 6.18(b). The four neighbouring atoms will form covalent bonds with four of the five valence electrons of the impurity atom. The fifth valence electrons of the impurity atom is loosely bouned to the nucleus and is almost free. Its energy level (called donor level) is just below the conduction band. Usually the donor level is 0·01 *eV* below conduction band for germanium and 0·054 *eV* for silicon. This small amount of energy is very easily provided by the thermal agitation of the crystal. Thus every antimony atom introduced into the germanium lattice contributes one conduction electron without creating a positive hole. Antimony is called donor impurity and makes the pure germanium an N-type extrinsic semi-conductor.

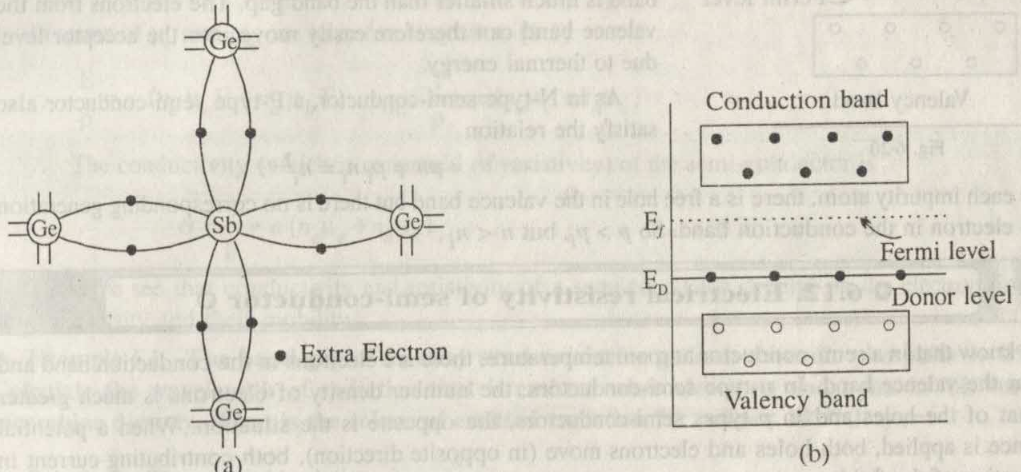


Fig. 6·18

Evidently the N-type semi-conductor, **electrons are the majority carriers** and thermally generated **holes are the minority carriers**. Hence, N-type semi-conductor conducts principally by electrons in the nearly empty conduction band.

If *n* and *p* represent respectively the electron and hole concentrations it can be shown that $np = n_i\, p_i = n_1^{\,2}$, n_i and p_i are the intrinsic values.

▲ (ii) P-type extrinsic semi-conductor :

When trivalent impurity (elements of Gr III) is added to Ge or Si, it has different effect. Some typical trivalent impurities are born (Z = 5, valence shell $2s^2\ 2p$), aluminium (Z = 13, valence shell $3s^2\ 3p$), galium (Z = 31, valence shell $4s^2 4p$) and indium (Z = 49, valence shell $5s^2\ 5p$).

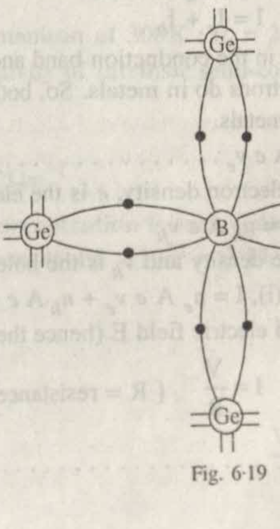


Fig. 6·19

In this case, the three valence electrons of impurity atom form covalent bands with four surrounding germanium atom, but one band is left incomplete and gives rise to a hole as shwon in fig. 6.19. Thus boron which is called acceptor impurity causes as many positive holes in a germanium crystal as there are boron atoms. So, a P-type extrinsic semi-conductor is produced.

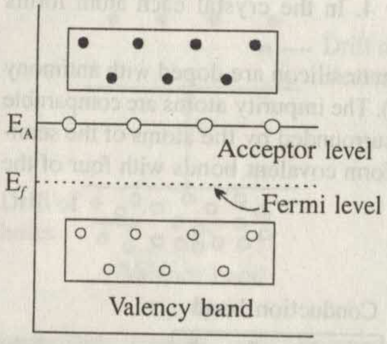


Fig. 6·20

In this type of semi-conductor, conduction is by means of holes in the valence band. Accordingly, holes form the majority carriers whereas electrons constitute minority carriers.

The acceptor impurity produces an energy level just above the valence band as shown in fig. 6.20. The energy difference between the acceptor energy level and the top of the valence band is much smaller than the band gap. The electrons from the valence band can therefore easily move into the acceptor level due to thermal energy.

As in N-type semi-conductor, a P-type semi-conductor also satisfy the relation

$$pn = p_i\, n_i = n_i^2$$

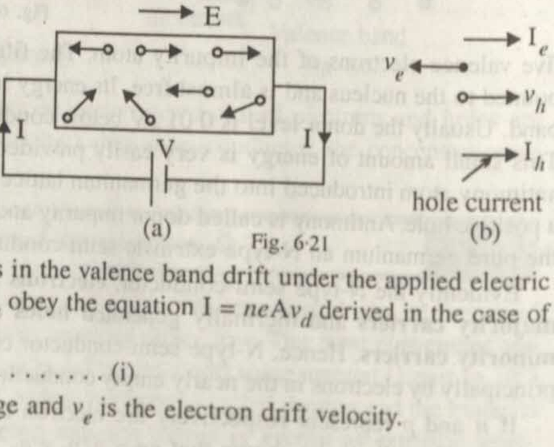
For each impurity atom, there is a free hole in the valence band but there is no corresponding generation of free electron in the conduction band. So $p > p_i$, but $n < n_i$.

6.12. Electrical resistivity of semi-conductor

We know that in a semi-conductor at room temperature, there are electrons in the conduction band and holes in the valence band. In *n*-type semi-conductors, the number density of electrons is much greater than that of the holes and in *p*-types semi-conductors, the opposite is the situation. When a potential difference is applied, both holes and electrons move (in opposite direction), both contributing current in the direction of the field.

Consider a block of semi-conductor of cross sectional area A and length *l*. A potential difference V is applied across the block [fig. 6.21]. Both electrons and holes contribute currents I_e and I_h in the same direction. Hence the total current I is given by [fig. 6.21(b).

$$I = I_e + I_h$$



(a) Fig. 6·21 (b)

The electron in the conduction band and the holes in the valence band drift under the applied electric field just as electrons do in metals. So, both I_e and I_h obey the equation $I = neAv_d$ derived in the case of current flow in metals.

$\therefore\ \ I_e = n_e A e v_e$ . . . . (i)

where n_e is electron density, *e* is the electric charge and v_e is the electron drift velocity.

Similarly, $I_h = n_h A e v_h$

n_h = hole density and v_h is the hole drift velocity.

Hence from (i), $I = n_e A e v_e + n_h A e v_h$ . . . . (ii)

If the applied electric field E (hence the applied voltage V) is low, semi-conductors obey ohm's law :

$$I = \frac{V}{R} \quad [\text{ R = resistance of the semi-conductor block }]$$

Also $|E| = \dfrac{V}{l}$ . . . . (iii)

And the resistivity ρ of the block is given by $\rho = \frac{RA}{l}$ . . . . (iv)

$$\therefore \quad \frac{V}{R} = Ae\,(n_e v_e + n_h v_h) \qquad \text{or,} \quad El \times \frac{A}{\rho l} = Ae\,(n_e v_e + n_h v_h)$$

$$\therefore \quad \frac{E}{\rho} = e\,(n_e v_e + n_h v_h) \; \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \; (6.1)$$

We now define a quantity called **mobility**. It is defined as the drift velocity per unit electric field.

$$\therefore \quad \text{mobility, } \mu = \frac{v}{E} \quad \therefore \quad \mu_e = \frac{v_e}{E} \quad \text{and} \quad \mu_h = \frac{v_h}{E}$$

In terms of μ_e and μ_h, we have from equation (6.1)

$$\therefore \quad \frac{E}{\rho} = e\,(n_e \mu_e E + n_h \mu_h E) \quad \text{or,} \quad \frac{1}{\rho} = e\,(n_e \mu_e + n_h \mu_h)$$

∴ The conductivity (which is reciprocal of resistivity) of the semi-conductor is

$$\sigma = \frac{1}{\rho} = e\,(n_e \mu_e + n_h \mu_h) \; \ldots\ldots\ldots\ldots\ldots\ldots \; (6.2)$$

Thus, we see that conductivity and resistivity of a semi-conductor depend on the electron and hole number density and their mobility.

● **Example 6.1. The band gap of an alloy semi-conductor gallium arsenide phosphide is 1·98 *e*V. Calculate the wavelength of radiation that is emitted when electrons and holes in this material recombine directly. What is the colour of emitted radiation ? $h = 6{\cdot}6 \times 10^{-34}$ J. s.**

○ **Solution :** The required relation, $\lambda = \frac{ch}{E_g}$

$$\therefore \quad \lambda = \frac{3 \times 10^8 \times 6{\cdot}6 \times 10^{-34}}{1{\cdot}98 \times 1{\cdot}6 \times 10^{-19}} \quad [\,E_g = 1{\cdot}98 \times 1{\cdot}6 \times 10^{-19}\,\text{J}\,]$$

$$= 6{\cdot}25 \times 10^{-7}\,m = \mathbf{6250\ \text{Å}}$$

The colour of emitted radiation is red.

● **Ex. 6.2. The following data are given for intrinsic germanium at 300K : $n_i = 2{\cdot}4 \times 10^{19}\ m^{-3}$, $\mu_n = 0{\cdot}39\ m^2V^{-1}s^{-1}$, $\mu_b = 0{\cdot}19$, $m^2\ V^{-1}s^{-1}$. Calculate conductivity of intrinsic semi-conductor.**

○ **Solution :** We know, $\sigma_i = n_i e\,(\mu_n + \mu_p)$

$$\therefore \quad \sigma_i = 2{\cdot}4 \times 10^{19} \times 1{\cdot}6 \times 10^{-19}\ (0{\cdot}39 + 0{\cdot}19) = 2{\cdot}22\,\Omega m^{-1}$$

● **Ex. 6.3. In pure silicon at 300K, the electron and hole concentration is each equal to $1{\cdot}5 \times 10^{16}\ m^{-3}$. When doped with indium the hole concentration increases to $4{\cdot}5 \times 10^{22}\ m^{-3}$. Find the electron concentration in the doped silicon.**

○ **Solution :** Given $n_i = 1{\cdot}5 \times 10^{16} m^{-3}$, $n_h = 4{\cdot}5 \times 10^{22} m^{-3}$

Now, $n_i^{\,2} = n_e n_h$

$$\therefore \quad n_e = \frac{n_i^{\,2}}{n_h} = \frac{(1{\cdot}5 \times 10^{16})^2}{4{\cdot}5 \times 10^{22}} = 5{\cdot}0 \times 10^9\ m^{-3}$$

6.13. Current flow in a biased N-type semi-conductor

In an N-type semi-conductor (i) electrons are majority carriers (ii) holes are minority carriers. Also there are large number of immobile positive ions.

Consider an *n*-type semi-conductor placed between a pair of metal electrodes across which a voltage is applied [fig. 6.22]. The electrons now drift towards the positive electrode. Through the positive electrode the electron goes towards the battery. Immediately the electron from the negative terminal of the battery enters the semi-conductor. Thus a continuous flow of electrons from one terminal of the voltage source to the other terminal via the semi-conductor takes place. The rate of flow of electrons or current in the circuit depends on the applied voltage and the conductivity of the semi-conductor.

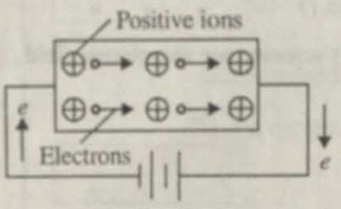


Fig. 6·22

6.14. Current flow in a biased P-type semi-conductor

In a P-type conductor (i) holes are majority carriers (ii) electrons are minority carriers. Also there are large number immobile negative ions in the crystal.

When a P-type semi-conductor is placed between a pair of electrodes across which a voltage is applied, the holes will drift towards the negative electrode. On reaching the negative electrode, they combine with electrons coming of the negative electrode of the battery and disappear. At the same time equal number of holes are generated at positive electrodes and released electrons go to the positive electrode of the battery. So, within the semi-conductor holes are moving and outside the electrons.

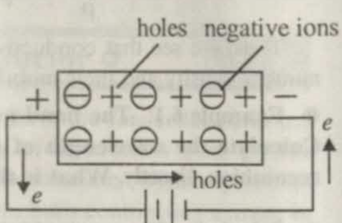


Fig. 6·23

6.15. PN junction diode

We know that the addition of impurity to a pure semi-conductor produces two types of charge carriers (electrons and holes). Depending on the nature of the impurity, N-type and P-type semi-conductors are obtained. Different semi-conductor devices are made by combining P-type and N-type crystals. For example—PN junction and transistors (N-P-N or P-N-P) are made of these extrinsic semi-conductor crystals.

Formation of P-N junction :

When two semi-conductors of P and N type are brought into contact as shown in fig. 6.24(a), they form what is called a PN junction. However, a PN junction cannot be formed merely by placing a piece of *n*-type material against a piece of *p*-type material. Two principal techniques of fabrication are : (i) grown junction and (ii) fused junction.

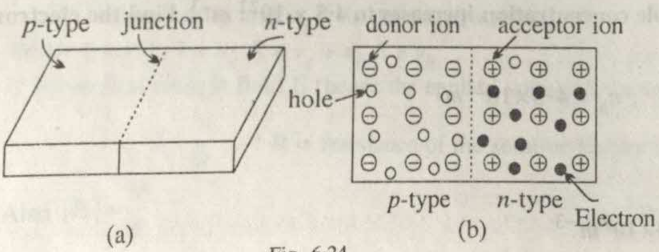


Fig. 6·24

In the fused junction technique, the *n*- and *p*-type silicon crystals are cut into thin slices called **wafers**. An aluminium film is placed on a wafer of *n*-type silicon and is placed in an oven maintained at a temperature of 600°C. Aluminium fuses to the surface of the semi-conductor and produces *p*-type semi-conductor immediately below the surface. Thus a PN junction is formed. The situation is shown clearly in fig. 6.24 (b). In P-region acceptor ion are shown by ⊖ symbol. In this case the impurity atom becomes negative ion by taking electron. In N-region donor ion is shown by ⊕ symbol. In this case the impurity atom becomes positive ion by donating electron.

In the N-region of the PN junction, number of electrons is higher than the number of holes and in the P-region the number of holes is higher than the number of electrons. When the PN junction is formed, even if no potential difference is applied, due to thermal vibration some electrons from the *n*-region will diffuse into the *p*-region. Conversely, some holes from the *p*-region will diffuse in the *n*-region. Thus electron hole combination occurs across the junction.

▭ Depletion region and Potential Barrier :

As a result of electron hole combination, the electrons in the *n*-region near the junction are neutralised by holes, so that in this region only positively charged donor atoms are left as shown in fig. 6.25. Similarly the holes in the P-region near the junction are neutralised by electrons, leaving only ionised acceptor atoms (negative charges) as shown in fig. The positive and negative ions in the small region around the junction are bound. This small region in the vicinity of the junction which has been depleted on free charge carriers (electrons and holes) and has only immobile ions is called **depletion region**.

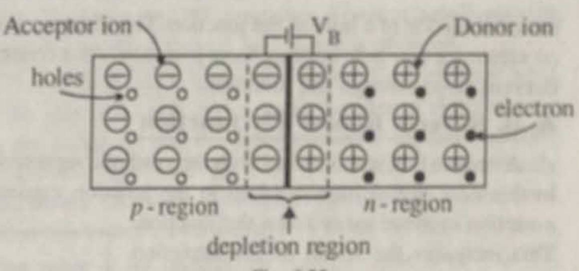


Fig. 6·25

The diffusion of holes and electrons across the junction continuous till a potential difference is set up in the depletion layer which then prevents further diffusion. This potential difference V_B is called **potential barrier**.

The electric field set up across the junction is called barrier field $\vec{E}_B$.

Due to the existance of the potential barrier V_B across the junction, an electron from the *n*-region can cause the junction only if energy $> e\ V_B$ is supplied to it from outside. An equal amount of energy is required to move a hole from the *p*-region to cross over to the *n*-region across the junction. The value of the barrier potential V_B for germanium is about 0·3 V and for silicon about 0·7 V. The width of the deplection region is about 10^{-4} cm to 10^{-6} cm.

When an external potential difference is applied barrier potential increases or decreases.

❂ 6.16. A Biased PN junction ❂

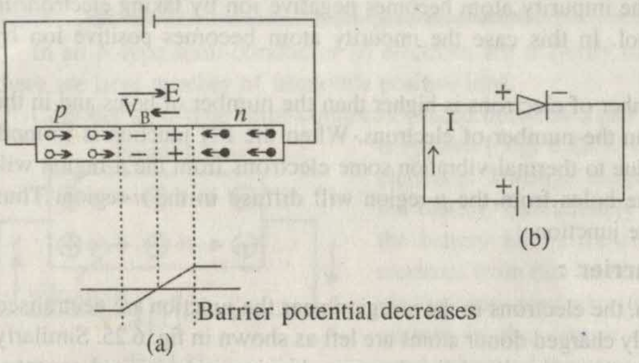
Let us consider the P-N junction connected to an external voltage source such as battery. When the positive terminal of a battery is connected to the *p*-side and negative terminal to the *n*-side, the junction is said to be **forward-biased**. At this bias the junction allows a large current to flow through it. If the polarity of the battery is reversed i.e., the positive terminal is connected to the *n*-side and negative terminal to the *p*-side, the junction is said to be **reverse-biased**. At this bias the junction allows a very small current to flow through it.

The above behaviour of a P-N junction makes it suitable for use as a rectifier. We now consider qualitatively the mechanism involved when the *p-n* junction is (i) forward biased and (ii) reverse-biased.

▲ A. Forward biased PN junction :

A forward biased P-N junction and its symbolic representation is shown in fig. 6.26(a) and (b) respectively. The forward biasing voltage V exerts a force on the holes of the *p*-side and on the electrons on the *n*-side. This force causes them move towards the junction. As a result the width of the depletion layer decreases and also the height of the barrier potential raduces. Just near the P-N junction, the electrons and holes combine and ceases to exist.

For each electron-hole combination that take place near the junction, a covalent bond breaks in the *p*-section near the positive pole of the battery. The electron moves towards the positive electrode of the battery and the hole moves towards the junction. At the same time an electron is released from the negative terminal of the battery enter into the *n*-section to replace the electron lost by the combination of a hole at the junction. This electron moves towards the junction and is again neutralised on meeting a new hole coming from *p*-section. As a consequence, a reletively large current, called **forward current** flows through the junction.



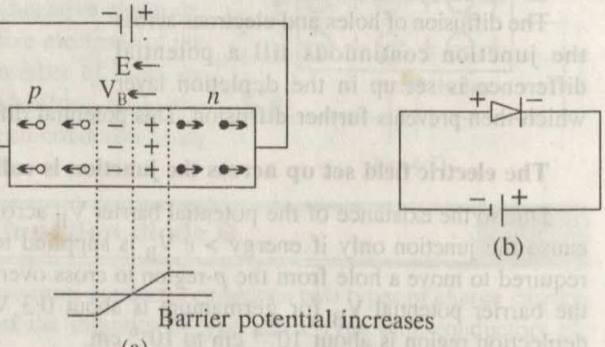
(a) (b)

Fig. 6·26

▲ B. Reverse Biased PN junction :

A reverse P-N junction and with the symbolic representation is shown in fig. 6.27(a) and (b) respectively. In this case, the voltage applied to the junction causes holes in the *p*-section and the electron in the *n*-section to move away from the junction. This increases the width of the depletion region and the height of the barrier potential. Hence electron from *n*-section and hole from the *p*-section cannot combine at the junction and majority current cannot flow.

However, the reverse bias can cause a small reverse current to flow which is caused by the presence of minority carriers. This current is called **reverse saturation current** (**I_S**). Its value depends as temperature of the diode.

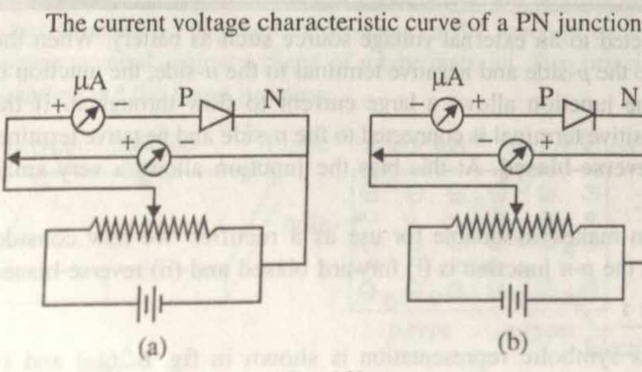


(a) (b)

Fig. 6·27

❂ 6.17. Current voltage characteristics of a PN junction diode ❂

The current voltage characteristic curve of a PN junction diode is a graph showing the variation of the current flowing through the junction with the change of voltage across the junction diode when it is forward biased [fig. 6.28(a)] and reversed biased [fig. 6.28(b)].



(a) (b)

Fig. 6·28

➡ Forward bias characteristic :

When the applied forward bias is low, practically no current flows through the junction diode. It is because the barrier potential (which is about 0·3 volt for Ge P-N junction and 0·7 volt for

Si P-N junction) opposes the applied voltage. Hence, a small forward current flows, till the applied forward bias does not became greater than barrier voltage V_B. It is represented by the portion OA of the graph between forward bias (F.B.) and forward current [fig. 6.29]. As soon the forward bias is greater than V_B, the forward current increases almost linearly.

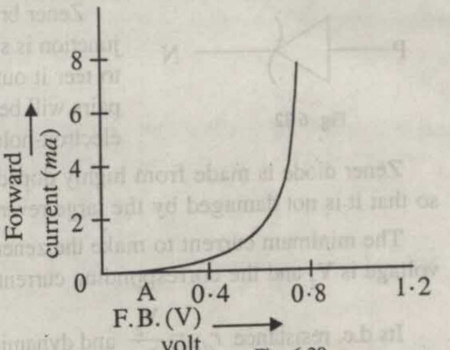


Fig. 6·29

Further, when the P-N junction is forward biased, the depletion layer becomes thin. As a result, the potential drop across the junction decreases. It leads to the **low resistances** of the junction diode during forward bias.

Reverse bias characteristic :

When the P-N junction is reverse biased, the current flows due to minority charge carriers. Hence during reverse bias current is measured by a micro ammeter. The reverse bias opposes majority carriers but makes the minority carriers to cross the PN junction. There a small reverse saturation current of about $I_s = 1{\cdot}2\ \mu A$ is obtained.

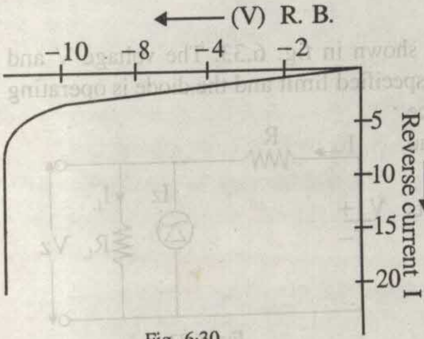


Fig. 6·30

If the reverse bias is made very high, all the covalent bond near the junction break and a large number of electron-hole pairs are produced and the reverse current increases rapidly to a relatively high value [fig. 6.30]. The material electrically **breaks down**. The reverse biased voltage at which this happens is called the **break down voltage** or **zener voltage.**

The value of the breakdown voltage varies from 1 or 2V to several hundred voltage depending on the number density of the dopants and the width of the depletion region.

Dynamic resistance :

The current-voltage (I-V) curve of fig. 6.29 and 6.30 show that I does not vary linearly with V, i.e., ohm's law is not obeyed. Therefore, the resistance offered by the junction diode depends upon the applied voltage. In such a case, it is useful to define a quantity called the dynamic resistance (or a. c. resistance) of the diode.

The dynamic resistance of the junction diode is defined as the ratio of small change in voltage ΔV to the corresponding small change in current ΔI. It is denoted by r_d.

$$\therefore \quad r_d = \frac{\Delta V}{\Delta I}$$

6.18. Zener diode or break down diode

It is a special type of junction diode. It operates in the reverse breakdown region of the reverse bias characteristic of the diode [fig. 6.31]. We know that when the junction diode is operated with reverse bias, at breakdown voltage or at zener voltage, the resistance of the junction becomes zero and the current increases rapidly. This property is utilised for zener diode construction. Due to breakdown of resistance at the junction, the difference of potential at the junction does not depend on current. For this reason, this diode is used as a voltage stabilizer.

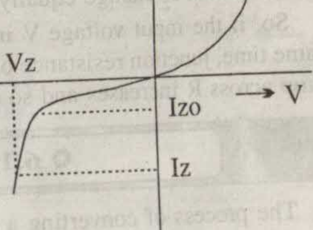


Fig. 6·31

A zener diode is represented by the symbol as shown in fig. 6.32.

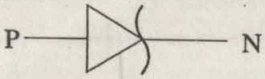


Fig. 6·32

Zener breakdown takes place when the reverse bias field across a P-N junction is such that the field may exert a strong force on a bound electron to teer it out from a covalent bond. Thus a large number of electron-hole pairs will be generated through a direct rupture of the covalent bonds. The electron-hole pairs so generated increase the reverse current considerably.

Zener diode is made from highly doped P and N type semi-conductors. Its power rating is kept high so that it is not damaged by the large reverse current.

The minimum current to make the zener diode operative is I_{zo} [fig. 6.31]. The Zener diode operating voltage is V_z and the corresponding current is I_z.

Its d.c. resistance $r_{dc} = \frac{V_z}{I_z}$ and dynamic resistance $r_z = \frac{\Delta V_z}{\Delta I_z}$, r_z is the reciprocal of the slope of the characteristic.

Use of the diode as voltage stabilizer :

The circuit diagram to use the diode as voltage stabilizer is shown in fig. 6.33. The voltage V and resistance R are determined so that the diode current is within the specified limit and the diode is operating in the breakdown region. The voltage V_0 across the load resistance R_L is a constant here although the supply voltage V and the load resistance R may vary.

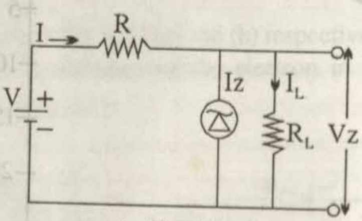


Fig. 6·33

If I be the current supplied by the source, I_z and I_L be the currents through the Zener diode and load resistance respectively then,

$$I = I_Z + I_L \quad \text{. . . . . (i)}$$

$$V_0 = V - IR \quad \text{. . . . . (ii)}$$

$$\text{and } V_0 = I_L R_L \quad \text{. . . . . (iii)}$$

Let the supply voltage V remains constant and load resistance R_L is varied. Since the Zener voltage V_0 tends to remain constant, equation (ii) gives $\Delta I = 0$. Then from equation (i),

$$\Delta I = \Delta I_z + \Delta I_L$$

$$0 = \Delta I_z + \Delta I_L \qquad \therefore \; \Delta I_z = -\Delta I_L$$

Thus when the load resistance is increased, but the supply voltage is fixed, the current I_L decreases and the current I_z increases by an equal amount so that total current remains constant.

Now suppose that load resistance R_L is fixed and the supply voltage V is varied. Since V_0 tends to remain constant, we from equation (ii),

$$\Delta V = R \cdot \Delta I$$

We obtain from equation (iii) $\Delta I_L = 0$ and from equation (i) $\Delta I = \Delta I_z$

That is, when the supply voltage is altered, but the load resistance is fixed, the total current I and the Zener current I_z change equally to keep the load current constant.

So, if the input voltage V increases then both Zener current I_z and load current I_L increases. At the same time, junction resistance of the Zener diode decreases. Zener current increases considerably. Voltage drop across R increases and so V_z remains constant.

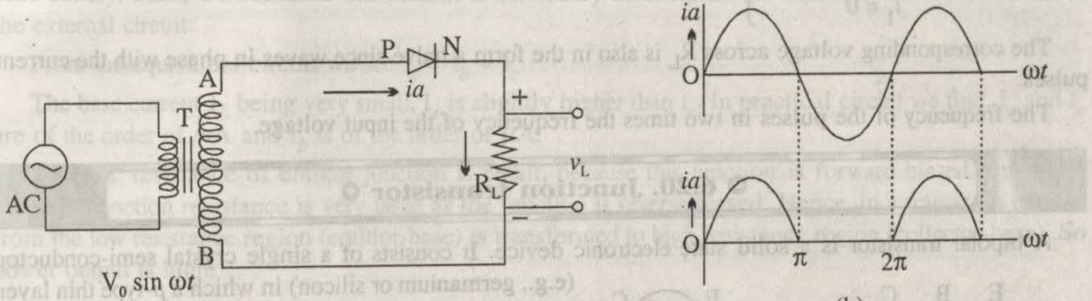
○ 6.19. PN junction diode as rectifier ○

The process of converting a simple harmonic a. c. voltage into a unidirectional voltage is known as **rectification**. The circuit required for this conversion is called **rectifier**.

- **Principle :** The junction diode offers a low resistance to current in one direction (when it is forward biased) and a high resistance in the opposite direction (when it is reverse biased). This unidirectional property of junction diode is utilised for rectification.

▲ A. Half-wave rectification :

A simple circuit for half wave rectification using a single diode is shown in fig. 6.34. The a. c. voltage to be rectified is applied to the input of the diode through a transformer T.



(a) Fig. 6·34 (b)

The a. c. is applied to the primary of the transformer T and the secondary (AB) is connected to the diode through the load resistance R_L. The output is taken across R_L.

- **Working of the rectifier :** Let the a. c. voltage across the secondary of the transformer, which is applied to the diode in series with R_L is given by

$$V_i = V_m \sin \omega t$$

V_m = maximum transformer secondary voltage. The variation of V_i with ωt is shown in fig. 8.34(b).

During positive half cycle of a. c. input, P-side of the diode is positive and N-side negative. So, the diode is forward biased. Hence a current flows through diode as well as through R_L. During the next half-cycle, P-side is negative and N-side positive. Now, the diode is reverse biased and hence no current flows through the circuit. Consequently, the currents through R_L is unidirectional and it flows in the form of half-sine wave as shown in fig. 8.34(b). The unidirectional current pulses are represented by

$$i = \frac{V_m \sin \omega t}{R_L + R_i} = I_m \sin \omega t \qquad [0 \le \omega t \le \pi]$$

$$i = 0 \qquad [\pi \le \omega t \le 2\pi]$$

when R_i is the dynamic forward resistance of the diode.

The corresponding voltage across R_L is also in the form of half sine waves in phase with the current pulses. The frequency of the pulses is the same as that of the input voltage.

▲ B. Full Wave Rectifier :

A full wave rectifier using two diods D_1 and D_2 are shown in fig. 6.35. Here also input is applied through the transformer. During the positive half cycle of the input a. c. voltage, the diode D_1 is forward biased and D_2 is reverse biased. So current flows only through D_1 and then through R_L.

During the next negative half cycle D_1 is reverse biased and D_2 is forward biased. So, the current flows through D_2 and then through R_L.

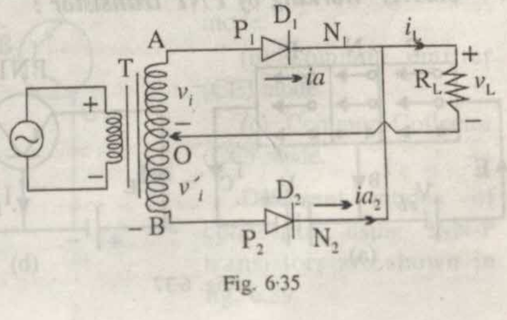


Fig. 6·35

Consequently, the current through R_L is unidirectional and flows in both halves of a. c. cycle. The current pulses are reperesented by

$$\left.\begin{aligned} i_1 &= I_m \sin \omega t \\ i_2 &= 0 \end{aligned}\right\} \text{ when } \; 0 \leq \omega t \leq \pi$$

$$\left.\begin{aligned} i_2 &= I_m \sin \omega t \\ i_1 &= 0 \end{aligned}\right\} \text{ when } \; \pi \leq \omega t \leq 2\pi$$

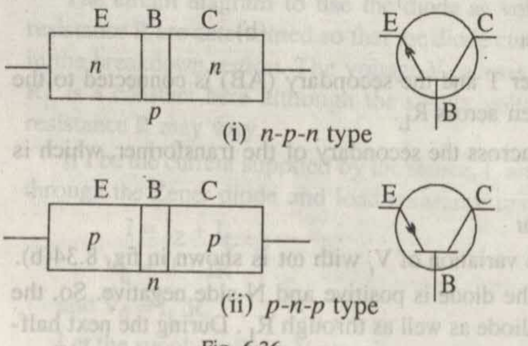
The corresponding voltage across R_L is also in the form a halve since waves in phase with the current pulses.

The frequency of the pulses in two times the frequency of the input voltage.

✪ 6.20. Junction transistor ✪

A bipolar transistor is a solid state electronic device. It consists of a single crystal semi-conductor (e.g., germanium or silicon) in which a *p*-type thin layer is sandwitched between two *n*-type layers. The resulting device is referred to as an ***n-p-n* transistor**. A transistor may also contain an *n*-type layer sandwitched between two *p*-type layers. It is named as ***p-n-p* transistor**. Both types of transistors with their circuit symbol are shown in fig. 6.36.

So, a transistor is a combination of three layers of semi-conductors. The thin middle portion is **base,** while the other two end portions are termed as **emitter** and **collector** as shown in fig. The constructional differences between emitter and collector are—



(i) *n-p-n* type

(ii) *p-n-p* type

Fig. 6·36

(i) Amount of dopant in emitter is higher than in collector.

(ii) The area of base-collector junction is larger than the area of base-emitter junction [fig. 6.36(a)].

In junction transistors both majority and minority carriers are involved. They are, therefore, called bipolar transistor.

In the normal operation of a transistor, the emitter-base junction is forward-biased while the collector-base junction is reverse-biased.

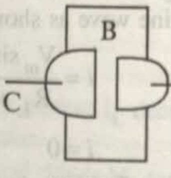
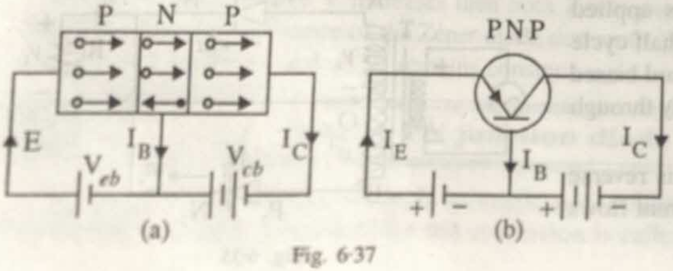


Fig. 6·36(a)

✪ 6.21. Transistor Action ✪

➤ *6.21.1. Working of PNP transistor :*



(a) (b)

Fig. 6·37

The working of PNP transistor can be studied with help of circuit shown in fig. 6.37 [(a) is the block diagram and (b) is the equivalent circuit in which the symbol of PNP transistor is used]. The emitter-base junction is forward biased with the voltage V_{eb}. The collector base junction is reversed bias with voltage V_{cb}.

Due to forward bias holes from the P-region (emitter) and electrons of the base move towards the junction. Since the base is thin and lightly doped, only a few holes (say, 1%) combine with electrons in the base. So, the base current I_B is small. Most of the holes (99%) rush through the base and are collected by the collector. For each hole reaching the collector, an electron is released from the negative terminal of battery of voltage V_{cb} to neutralise the hole. For each hole lost in the collector, a covalent bond near the emitter breaks down and an electron is released. This electron enters the positive terminal of the emitter-base battery. Thus, we conclude that current is carried by holes inside the transistor and by electrons in the external circuit.

From the equivalent circuit we see $I_e = I_b + I_c$.

The base current I_b being very small, I_e is slightly higher than I_c. In practical circuit we find, I_e and I_c are of the order of mA and I_b is of the order of μA.

Dynamic resistance of emittor junction is small, because this junction is forward biased. But base-collector junction resistance is very large as the junction is reverse biased. Hence, in a transistor, current from the low resistance region (emittor-base) is transformed to high resistance region (collector-base). So power output is high.

➤ *6.21.2. Working of NPN transistor :*

The necessary circuit diagram is shown in fig. 6.38. As usual the base-emitter junction is forward biased and base-collector junction is reversed-biased.

Due to forward bias of emitter junction, the electrons from emitter side and holes from the base move towards the junction. As the base is thin and lightly doped only a few (say, 1%) electrons combine of with the holes at the base. Most of the electrons enter the collector side.

The electrons reaching the collector move towards the positive terminal of the collector-base battery. Simultaneously electrons are released from the negative terminal of the emitter-base battery.

Hence, in this case current is carried by the electrons both in external as well as inside the transistor.

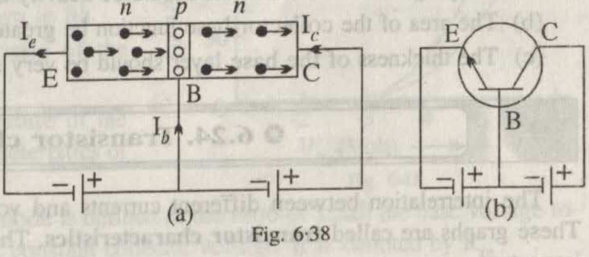


Fig. 6·38

The relation between the three currents is $I_e = I_b + I_c$

✪ 6.22. Different modes of using transistor ✪

A transistor is three terminal device. Depending upon the common terminal between the input and the output circuits of a transistor, it may be operated in any of the following three modes :

(a) Common base (CB) mode.

(b) Common emitter (CE) mode.

(c) Common Collector (CC) mode.

Different modes of connection using P-N-P transistors are shown in fig. 6.39.

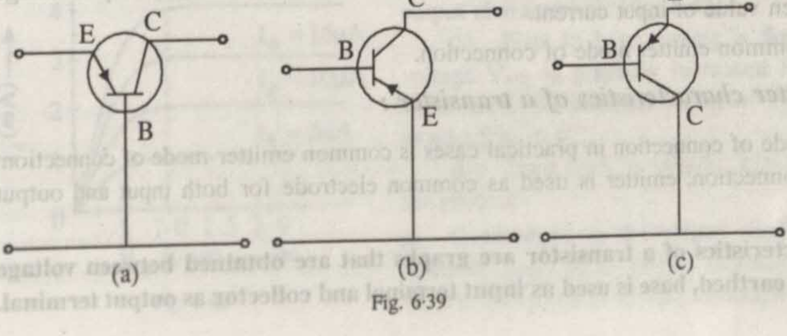


Fig. 6·39

In CB mode, the base terminal is common to both the input and the output circuit. Fig. 6.39(a) shows an *n-p-n* transistor connected in common-base mode.

In CE mode of operation, the emitter terminal is common to both the input and output circuits as shown in fig. 6.39(b).

In CC mode, the collector terminal is common to both input and output circuits as shown in fig. 6.39(c).

✪ 6.23. Amplifying action of transistor ✪

Since the emitter base junction is forward biased, the depletion region around this junction is much smaller than around the base-collector junction which is reverse biased. Consequently, the resistance of emitter-base junction (R_{EB}) is much smaller than the resistance of collector-base junction (R_{CB}).

The power dissipated in emitter-base junction is $P_{EB} = I_E^2 R_{EB}$
and the power dissipated in collector-base junction $P_{CB} = I_C^2 R_{CB}$

Now, $I_C \simeq I_E$ and $R_{CB} > R_{EB}$. Hence $P_{CB} > P_{EB}$

So, the power dissipated in the collector-base junction is much higher than that dissipated in the emitter-base junction. In other words, the output power is much greater than the input power. This is the amplifying action of the transistor.

For useful amplifying action, a transistor should have the following three features :

(a) The three elements of the transistor, namely the emitter, the base and the collector should have different doping levels—the emitter should be heavily doped, the collector should be less heavily doped.

(b) The area of the collector-base junction be greater than that of the emitter-base junction.

(c) The thickness of the base layer should be very small ($< 10^{-3}$ m).

✪ 6.24. Transistor characteristics ✪

The interrelation between different currents and voltages of a transistor can be shown graphically. These graphs are called **transistor characteristics**. The following two types of characteristics are very important.

(a) Common-emitter input and output characteristics.

(b) Common-base input and output characteristics.

In a particular mode of connection, the graph showing the variation of input current with input voltage for a constant output voltage, is called **input characteristic**.

On the other hand, the **output characteristic** is obtained by plotting the variation of output voltage with output current for a given value of input current.

Here we consider only common-emitter mode of connection.

➤ *6.24.1. Common emitter characteristics of a transistor :*

The most widely used mode of connection in practical cases is common emitter mode of connection. In this particular mode of connection, emitter is used as common electrode for both input and output circuit.

Common emitter characteristics of a transistor are graphs that are obtained between voltage and current when emitter is earthed, base is used as input terminal and collector as output terminal.

The circuit diagram required to study the common emitter (CE) characteristics of a *n-p-n* transistor is shown in fig 6.40. In CE mode of connection, input current and output voltage are treated as independent variables, while the input voltage and output current are dependent variables. So, the output current I_C and input voltage V_{BE} can be expressed as a function of input current I_B and output voltage V_{CE}.

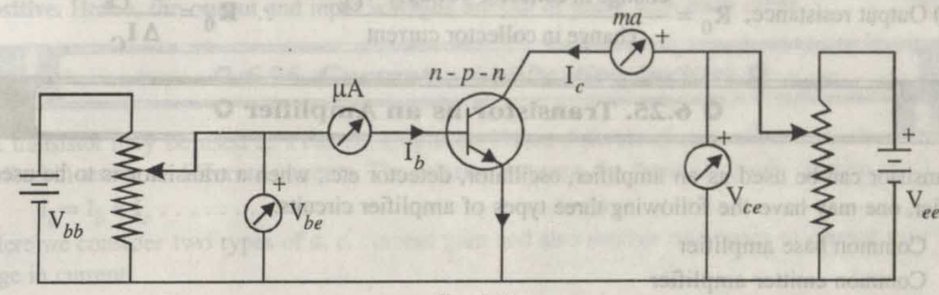


Fig. 6·40

(a) Input characteristics :

The graph of the base current I_B versus the base-emitter voltage V_{BE} keeping V_{CE} fixed is called the input characteristics [fig. 6.41].

(i) At first V_{CE} is fixed at value (say, 2V) then for different base-emitter voltages V_{BE}, the base currents I_B are noted. It is found that as long as V_{BE} is less than the barrier voltage, the current is small, as in the case of a forward-biased diode. But when V_{BE} is increased to a value greater than barrier voltage (0·3 volt), the current I_B increases sharply as shown in fig.

(ii) Similarly we can draw input characteristic of the transistor at V_{CE} = 3V and so on.

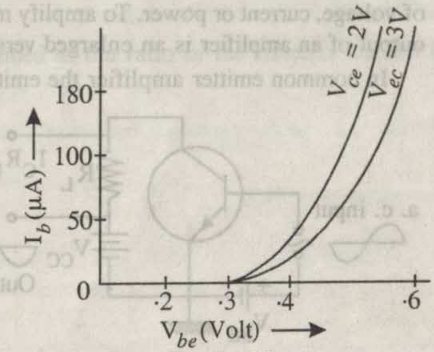


Fig. 6·41

Comments on the input graph : (i) The nature of the input characteristics is similar to the forward characteristics of a junction diode and it is forward biased.

(ii) a. c. input resistance of a transistor in CE mode is defined as the ratio of small the base voltage to the corresponding small change in base current at constant collector voltage. It is denoted by R_{in}

$$\therefore \quad R_{in} = \left(\frac{\Delta V_{BE}}{\Delta I_B}\right)_{V_{CE}}$$

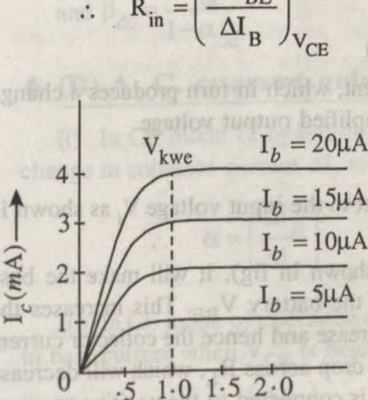


Fig. 6·42

(b) Output characteristics :

The graph of the collector current I_C versus the emitter-collector voltage V_{CE} keeping base current I_b fixed is called the output characteristic [fig. 6.42].

(i) First to base current is fixed at I_{b_1} = 5μA. Then the voltage V_{CE} is gradualy increased from zero. The readings are taken for I_C for different values of V_{CE}. Then ($I_C - V_{CE}$) graph is plotted [fig. 6.42].

(ii) Then for I_b = 10 μA, 15μA . . . . . . , ($I_C - V_{CE}$) graph are plotted.

Comments on the output characteristics : (i) For smaller value of V_{CE}– (0 to 1V) I_C increases rapidly. Then I_C becomes almost independent of V_{CE} although it is controlled by I_b.

(ii) For a given value of V_{CE}, I_C is larger for large values of I_b.

(iii) Output resistance, $R_0 = \frac{\text{change in collector voltage, } V_{CE}}{\text{change in collector current}}$ $\therefore\ R_0 = \frac{\Delta V_{CE}}{\Delta I_C}$

6.25. Transistor as an Amplifier

Transistor can be used as an amplifier, oscillator, detector etc., when a transistor is to be used as an amplifier, one may have the following three types of amplifier circuits.

1. Common base amplifier
2. Common emitter amplifier
3. Common collector amplifier

We shall study any common emitter amplifier.

➤ *6.25.1. Common emitter Amplifier :*

The transistor amplifier is a circuit consisting of at least one transistor, which is used for the amplification of voltage, current or power. To amplify means to increase the size (or to magnify) an input signal. So, the output of an amplifier is an enlarged version of the input signal.

In common emitter amplifier the emitter of the transistor is common to base and collector. Fig. 6·43 shows the common emitter amplifier circuit using a *n-p-n* transistor. The emitter is forward biased by using base bias battery V_{BE}. Due to forward bias, the resistance of the input circuit is low. The collector is reversed biased by using collector bias battery V_{CC}. The low input signal voltage is applied in base-emitor circuit and the amplified output is obtained across the collector and emitter. The direction of different currents are shown in fig.

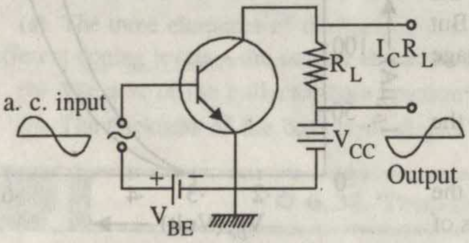


Fig. 6·43

The emitter current, base current and collector current are related to each other by the equation

$$I_C = I_b + I_e \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

Also corresponding to the emitter current I_e, the collector voltage i.e., the voltage across the collector and emitter will be

$$V_{CE} = V_{CC} - I_C R_L \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(ii)}$$

A change in the input voltage produces a change in emitter current, which in turn produces a change in collector current. This variation of collector voltage appear as amplified output voltage.

Phase relation between input and output signal :

In a common emitter amplifier, the output voltage V_O with respect to the input voltage V_i as shown in fig. The reason is :

If the first half cycle of the a. c. input voltage is positive (as shown in fig), it will make the base current more positive as it is connected to the positive terminal of the battery V_{BE}. This increases the negative forward bias of the emitter. The emitter current will also increase and hence the collector current [see equation (i)]. Increased collector current increases the potential drop across R_L, which will decrease the collector-base voltage V_{CE} [vide equation (ii)]. As the collector is connected to the positive terminal of the battery V_{CC}, the decrease in collector voltage means that the collector will become less positive. Thus, negative output voltage is obtained.

Hence, **when the input voltage is positive, the output voltage is negative.**

Similarly, **when the input voltage is negative (during the next half cycle of a.c.), the output voltage is positive.** Hence, the output and input voltages are out of phase with each other.

✪ 6.26. Current amplification factors ✪

A transistor may be used as a current amplifier. Hence different characteristics of a transistor can be described in terms of different currents. The relation between the three electrode currents is

$$I_e = I_b + I_c \qquad \text{.....(i)}$$

Here we consider two types of d. c. current gain and also another two types of current gain for small change in current.

▲ (A) D. C. current gain :

(i) In the CB mode of operation of a transistor, current gain α_{dc} is defined as ratio of the collector current I_C and emitter current I_e when V_{CB} is fixed.

$$\therefore \quad \alpha_{dc} = \frac{I_c}{I_c} \quad \text{[when } V_{CB} \text{ is constant]}$$

(ii) In the CE mode of operation, current gain β_{dc} is defined as the ratio of the collector correct to base current when V_{CE} is kept constant

$$\therefore \quad \beta_{dc} = \frac{I_c}{I_b} \quad \text{[when } V_{CE} \text{ is constant]}$$

● **Relation between α_{dc} and β_{dc} :**

$$I_e = I_b + I_c \quad \therefore \quad \frac{I_e}{I_c} = \frac{I_b}{I_c} + 1$$

$$\therefore \quad \frac{1}{\alpha_{dc}} = \frac{1}{\beta_{dc}} + 1 \quad \therefore \quad \alpha_{dc} = \frac{\beta_{dc}}{1 + \beta_{dc}}$$

and $\beta_{dc} = \dfrac{\alpha_{dc}}{1 - \alpha_{dc}}$

▲ (B) A. C. current gain :

(i) In CB mode of operation of a transistor, a. c. current gain α is defined as the ratio of the small change in collector current ΔI_c to the small change is emitter current when V_{CB} is held constant

$$\therefore \quad \alpha = \left(\frac{\Delta I_c}{\Delta I_e}\right)_{V_{CB}}$$

(ii) In CE mode of operation of a transistor, a. c. current gain β is defined as the ratio of small change in base current when V_{CE} is held constant

$$\therefore \quad \beta = \left(\frac{\Delta I_c}{\Delta I_b}\right)_{V_{CE}}$$

- **Relation between α and β : $I_e = I_b + I_c$**

$\therefore$ For a small change in current, $\Delta I_e = \Delta I_b + \Delta I_c$

$$\therefore \quad \frac{\Delta I_e}{\Delta I_c} = \frac{\Delta I_b}{\Delta I_c} + 1 \qquad \therefore \quad \frac{1}{\alpha} = \frac{1}{\beta} + 1$$

$$\therefore \quad \alpha = \frac{\beta}{1+\beta} \quad \text{and} \quad \beta = \frac{\alpha}{1-\alpha}$$

- **A discussion :**

(i) In general I_b is very small. So, I_e and I_c are very close. So, the value α_{dc} lies between 0·9 and 1·0.

(ii) Quality of a transistor is determined by β_{dc} and not by α_{dc}.

The value of β_{dc} may be of the order of 600.

●● Short Answer Type Questions (with answers) ●●

- **Example 6.1. In the common base connection of the transistor, $I_e = 1{\cdot}0\ mA$ and $I_c = 0{\cdot}95\ mA$. What is the value of base current ?**

○ **Solution :** The required relation : $I_e = I_b + I_c$

$\therefore \quad I_b = I_e - I_c = 1{\cdot}0 - 0{\cdot}95 = \mathbf{0{\cdot}05\ mA}$

- **Ex. 6.2. In the common base configuration $\alpha_{dc} = 0{\cdot}9$. If the emitter current be $1{\cdot}0\ mA$, calculate base current.**

○ **Solution :** Here $\alpha_{dc} = 0{\cdot}9$ and $I_e = 1{\cdot}0\ mA$

Now, $\alpha_{dc} = \dfrac{I_c}{I_e} \qquad \therefore \quad I_c = \alpha_{dc} \cdot I_e = 0{\cdot}9 \times 1{\cdot}0 = 0{\cdot}9\,mA$

also, $I_e = I_b + I_c \qquad \therefore \quad I_b = I_e - I_c = 1{\cdot}0 - 0{\cdot}9 = \mathbf{0{\cdot}1\,mA}$

$\therefore$ The required base current is **0·1 mA**

- **Ex. 6.3. For a transistor α = 0·9, calculate β.**

○ **Solution :** We know, $\beta = \dfrac{\alpha}{1-\alpha} \qquad \therefore \quad \beta = \dfrac{0{\cdot}9}{1-0{\cdot}9} = \mathbf{9}$

- **Ex. 6.4. In the circuit shown in fig 6.44, the difference of potential across the diode is 0·5 V and maximum power 100 mW. What is the maximum diode current. Calculate the diode resistance.** **[C. B. S. E. 1997]**

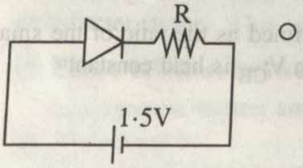


Fig. 6·44

○ **Solution :** In this case E = 1·5V, potential difference across the diode is

$$V_\alpha = 0{\cdot}5\ \text{V}$$

Its maximum power, P = 100 mW = 0·1 W

$$\therefore \quad \text{maximum diode current, } I = \frac{P}{V_\alpha} = \frac{0{\cdot}1}{0{\cdot}5} = 0{\cdot}2\,\text{A}$$

- **Ex. 6.5. A junction diode is used as a half wave rectifier. The peak value of the applied alternating voltage is 50 V. Load resistance is R_L. Calculate : (i) r. m. s. voltage across R_2, (ii) reading of the d. c. voltmeter connected across R_2.** **[C. B. S. E. 1997]**

○ **Solution :** $E_0 = 50\,\text{V} \qquad \therefore \quad E_{\text{r.m.s.}} = \dfrac{50}{\sqrt{2}}$ volt

(i) Since diode forward resistance is small. Hence, during first half cycle, the potential difference across R_2 is $\frac{50}{\sqrt{2}}$ volt .

$$V_{\text{r.m.s.}} = \sqrt{\frac{(50/\sqrt{2})^2 + 0^2}{2}} = \sqrt{\frac{1250+0}{2}} = \mathbf{25\ V}$$

(ii) DC voltmeter records average value of alternating voltage

$$\therefore \text{ Reading of voltmeter } = \frac{2E_0/\pi + 0}{2} = \frac{E_0}{\pi} = \mathbf{15 \cdot 92\ V}$$

● **Ex. 6.6. For a transistor α = 0·95 and its emitted is earth connected. If the change of base current be 0·2 *m*A, what is the change in collector current ?**

○ **Solution :** The required relation is $\beta = \frac{\alpha}{1-\alpha} = \frac{0 \cdot 98}{1 - 0 \cdot 98} = 49$

Also, $\beta = \frac{\Delta I_c}{\Delta I_b}$ $\therefore\ \Delta I_c = \beta \cdot \Delta I_b = 49 \times 0 \cdot 2 = \mathbf{9 \cdot 8\,mA}$

● **Ex. 6.7. For a transistor in common base connected d. c. current gain is 0·96. If the emitter current be 7·2 *m*A, calculate base current.**

○ **Solution :** $\alpha = 0 \cdot 96$ and $I_e = 7 \cdot 2\ mA$

Now, $\alpha = \frac{I_c}{I_e}$ $\therefore\ I_c = \alpha I_e = 0 \cdot 96 \times 7 \cdot 2 = \mathbf{6 \cdot 91\,mA}$

Again, $I_e = I_c + I_b$ $\therefore\ I_b = I_e - I_c = 7 \cdot 2 - 6 \cdot 91 = \mathbf{0 \cdot 29\,mA}$

∴ The required base current = **0·29 *m*A**

● **Ex. 6.8. In a transistor base current is 105 μA and collector current 2·05 *m*A. (a) Calculate β_{dc}, I_c and α_{dc}, (b) If the base current changes by 27 μA, collector current changes by 0·65 *m*A. What is the value of β_{ac}.**

○ **Solution :** Here $I_b = 105\mu A = 105 \times 10^{-6} A$, $I_c = 2 \cdot 05 \times 10^{-3} A$

(a) $\beta_{dc} = \frac{I_c}{I_b} = \frac{2 \cdot 05 \times 10^{-3}}{105 \times 10^{-6}} = 19 \cdot 5$

Again, $I_e = I_c + I_b = 2 \cdot 05 \times 10^{-3} + 105 \times 10^{-6} = 2 \cdot 155 \times 10^{-3} mA$

$\therefore\ \alpha_{dc} = \frac{I_c}{I_e} = \frac{2 \cdot 05 \times 10^{-3}}{2 \cdot 155 \times 10^{-3}} = \mathbf{0 \cdot 95}$

(b) Now, $\Delta I_b = 27\mu A = 27 \times 10^{-6} A$, $\Delta I_c = 0 \cdot 65 \times 10^{-3} A$

$\therefore\ \beta_{ac} = \frac{\Delta I_c}{\Delta I_b} = \frac{0 \cdot 65 \times 10^{-3}}{27 \times 10^{-6}} = \mathbf{24 \cdot 07}$

● E ◇ X ◇ E ◇ R ◇ C ◇ I ◇ S ◇ E ●

A. Objective type questions :

● **Fill in the blanks :**

1. Electron distribution in different energy levels in an atom is determined by Pauli's ______ ______ .

2. To obtain N-type semi-conductor dopant to germanium is ______ .

3. To obtain N-type semi-conductor dopant to germanium is ______.
4. In N-type crystal ______ is majority carrier and ______ minority carrier.
5. In a transistor emitter-base junction is biased and collector base junction is ______ biased.

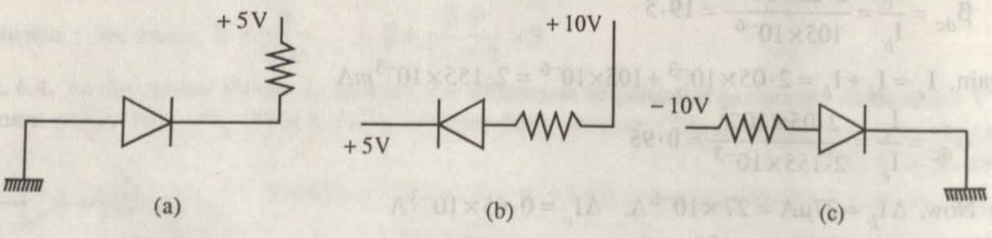
◢ B. Very short answer type questions :

1. Does the resistance increase or decrease with rise of temperature of a semi-conductor ?
2. How an N-type semi-conductor can be obtained ?
3. How a P-type semi-conductor can be obtained ?
4. What is forbidden energy gap ?
5. What is majority carrier in a semi-conductor ?
6. What is the principle of rectification ?
7. What is meant by forward bias ?
8. What is the relation between α and β ?

◢ C. Short answer type questions :

1. What are the four quantum number which determines the position of electron is an atom ?
2. Mention the difference between energy level and energy band.
3. What are valence band and conduction band ?
4. How conductor, insulator and semi-conductors are differentiated interms of energy bands ?
5. With rise of temperature the resistance of a conductor rises, but that of semi-conductor decreases. Explain.
6. How two types of carriers are produced in a semi-conductor ?
7. What we mean by intrinsic and extrinsic semi-conductor ?
8. How 'holes' are created in a semi-conductor ?
9. Why is temperature coefficient of resistance of a semi-conductor negative ?
10. What is the effect of mixing pentavalent impurity in pure germanium crystal ?
11. What is a semi-conductor diode ? What we mean by forward bias and reverse bias ?
12. How is depletion region forward in an unbiased *p-n* junction ?
13. What is the principle of rectification ?
14. How does a transistor works ?
15. What will happen if indium is mixed with silicon ?
16. Which diode of fig. 6.45 is forward biased and which are is reversed bias ?

[**Ans :** (a) and (c) reversed biased ; (b) Forward biased]



(a) (b) (c)

Fig. 6·45

17. In fig. 6.46, what is the nature of the bias of (i) emitter-base junction and (ii) collector-base junction ?

[**Ans :** (i) emitter-base junction reversed biased ; (ii) collector-base junction forward biased]

18. The resistance of semi-conductor decreases with rise of temperature. Explain.
19. You are given two bars of equal resistance—one is a conductor and the other is a semi-conductor. How would you proceed to identify them ?

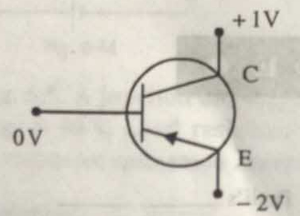


Fig. 6·46

◢ D. Essay type questions :

1. (a) How energy band and forbidden gap is formed in a crystal ? Mention the differences between conductor, insulator and semi-conductor on the basis of energy band and band gap.

(b) What we mean by N-type and P-type of semi-conductor ? **[W. B. H. S. 1994]**

2. How is a semi-conductor junction diode is formed ? Discuss forward bias and reverse bias of a junction. Also draw forward and reverse bias characteristic curve of the diode.

3. What is the principle of rectification by a junction diode. Describe the action of semi-conductor diodes as half-wave and as a full wave rectifier.

4. How is a transistor formed ? Describe its action.

5. Draw input and output characteristics of a transistor under CE made of operation. Also draw necessary circuit diagram.

6. Define α and β for a transistor. Obtain a relation between them.

7. How is a transistor used as an amplifier ?

◢ E. Numerical problems :

1. For a transistor $\alpha = 0{\cdot}9$, collector current is 1 *m*A. What is base current ? **[Ans : 0·1 *m*A]**

2. In the case of transistor $I_c = 0{\cdot}95$ *m*A, base current $I_b = 0{\cdot}05$ *m*A. Find the value of α. **[Ans : 0·95]**

3. In the circuit of fig. 6.47, for the transistor $\alpha = 0{\cdot}95$ and the potential difference across the 2 KΩ resistor is 2V. Find the values of collector current and base current. **[Ans : 1 *ma* ; 0·05 *ma*]**

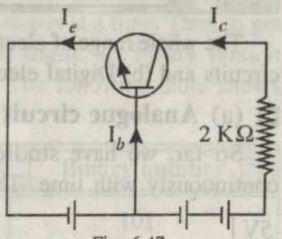


Fig. 6·47

4. Find β when (i) $\alpha = 0{\cdot}9$ and (ii) $\alpha = 0{\cdot}98$. **[Ans : (i) 9, (ii) 49]**

5. For a transistor $\alpha = 0{\cdot}98$. The transistor is connected in CE made with the emitter earthed. If the change in base current be 0·2 *m*A, what is the change in collector current ? **[Ans : $\beta = 49$; $\Delta I_c = 9{\cdot}8$ *ma*]**

6. For an N-P-N transistor $\alpha = 0{\cdot}98$. If the emitter current be 2 *m*A, calculate collector current and base current. **[Ans : $I_c = 1{\cdot}96$ *ma* ; $I_b = 0{\cdot}04$ *ma*]**

7. For a transistor base current is 75 μA and collector current 1·50 *m*A. (a) Calculate β, I_e and α. (b) If the change is base current be 25 μA, Collector current changes by 0·625 *m*A. What is β_{ac} ?

[Ans : (a) $\beta = 20$, $I_e = 1{\cdot}575$ *m*A, $\alpha = 0{\cdot}95$; (b) 25]

8. In CB mode connection of an N-P-N transistor $\alpha = 0{\cdot}98$. If the emitter current changes by 40 *m*A, calculate change in base current and collector current. **[Ans : 0·8 *m*A, 39·2 *m*A]**

9. Consider the circuit in fig. 6.48. Collector voltage $V_{cc} = 8$V. Potential difference across, R_c is 0·5 V. If $R_c = 800\Omega$, $\alpha = 0{\cdot}96$, Calculate (i) Collector-emitter voltage V_{CE} (ii) base current.

[Ans : (i) $V_{CE} = 7{\cdot}5$ V, (ii) $I_b = 0{\cdot}026$ *m*A]

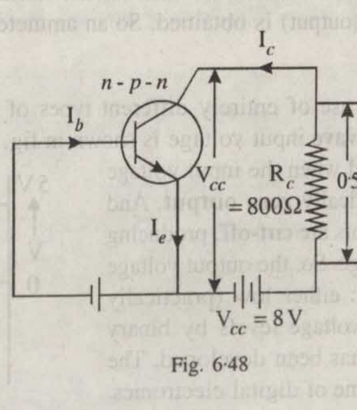
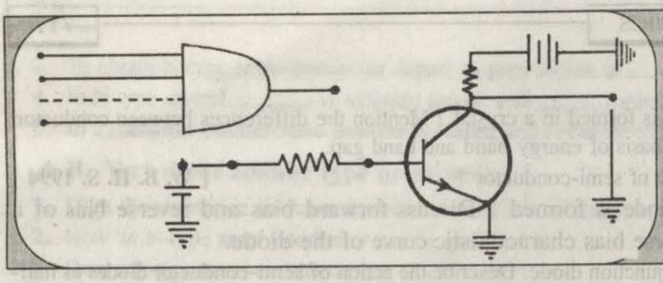


Fig. 6·48



CHAPTER 7

BINARY NUMBERS AND LOGIC GATES

• **TOPICS :** ○ Introduction; ○ Decimal and Binary number system; ○ Decimal to binary conversion; ○ Binary to decimal conversion; ○ Additon of Binary numbers; ○ Binary subtraction; ○ Binary multiplication and division; ○ Boolean operators; ○ Logic Gates; ○ OR Gate; ○ AND Gate; ○ NOT Gate; ○ Boolean algebra; ○ De Morgan's theorem; ○ Combination of gates; ○ NAND Gate as a building block of a digital circuit; ○ Short answer type questions (with answers); ○ Exercise.

❂ 7.1. Introduction ❂

The whole range of electronic circuits are broadly classified into two categories : (a) Analogue electronic circuits and (b) Digital electronic circuits.

(a) **Analogue circuit :**

So far, we have studied analogue electronic circuits. In such circuits, the voltage signals vary continuously with time. This type of signals is called analogue or continuous voltage signal. Fig. 7.1 shows a typical voltage signal varying sinusoidally between 0 to 5 V. The output voltage of an analogue circuit changes in accordance with the change in the input voltage. If the input voltage changes continuously, the output voltage changes accordingly. Examples of analogue instruments are : galvanometer, ammeter and voltmeter etc. What happens in the case of an ammeter is : when a current is passed through an ammeter, it supplies necessary information about current. The deflection of the pointer is the measure of current flowing through the circuit. Hence current and deflection are continuously varying quantity. Current is the input to the instrument. The current imitates some process inside the instrument and immediately a deflection (output) is obtained. So an ammeter is an analogue instrument.

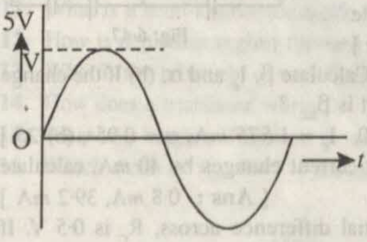


Fig. 7·1

(b) **Digital circuit :**

The digital electronic circuits make use of entirely different types of voltage signal. One way of operating a digital circuit with a **square-wave** input voltage is shown in fig. 7.2. If the peak value of the input is large, the transistors are **saturated** when the input voltage is equal to the peak value, giving a practically **zero output**. And when the input voltage is zero, the transistors are **cut-off**, producing **high output** equal to the d. c. supply voltage. So, the output voltage of a digital circuit has only two values : either low (practically zero) or high. By presenting these two voltage levels by binary numbers 0 and 1, the digital electronics has been developed. The calculators, computers etc., are the outcome of digital electronics.

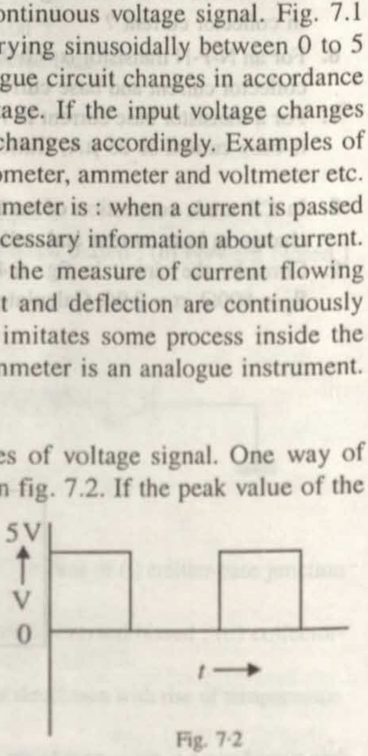


Fig. 7·2

We now first learn some elementary ideas about binary number system and then the logic gates. Logic gates are the basic building blocks of digital electronics.

7.2. Decimal and Binary number system

We are more familiar with the **decimal number system** with its ten digits : 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. In the **binary number system** only two digits, 0 and 1 are required to represent a number.

In the decimal system, to get a number greater than 9 (nine). We combine the decimal digits. For example, when we write ten, the next decimal number after 9, we use the second decimal digit 1 followed by the first digit 0. So ten is expressed as 10. Similarly to get 11, 12, 13, ... we use the second digit followed by the second, third, fourth, ... etc. This process goes upto 19 (nineteen). After this we use the third decimal digit 2 followed by first, second, third . . . . etc., digit to get 20, 21, 22, 23, ... etc. In this way we can get numbers upto 99. After 99 (ninety nine), to express a number we have to use three digits. Thus to express hundred, we use the second decimal digit followed by the two first digit. So, hundred is represented as 100.

In a binary system with its two digits 0 and 1, we proceed in the same manner as in decimal system to get a number greater than 1. Thus to represent 2 (two) we use the second binary digit 1 followed by the first 0. This gives 10 (read as one-zero). Similarly 3 (three) can be expressed as 11 (read as one-one). Evidently after representing 3, the two binary digits are exhausted. Now, we are to proceed in a manner similar to that in writing hundred (100) in the decimal system and to use three digits at a time. Thus to get 4 in the binary system we are to use the second digit followed by the two first digits. So, binary version of 4 is 100 (one-zero-zero). Similarly 5 is equivalent to 101 (one-zero-one). The following table shows thebinary equivalents of some decimal numbers.

Decimal number	Binary number	Decimal number	Binary number
0	0	5	101
1	1	6	110
2	10	7	111
3	11	8	1000
4	100	9	1001

Base :

The base of a number system refers to the total number of digits used in the system. Thus the decimal system has the base 10 (ten). A decimal system 1639 (one thousand six hundred thirty nine) can be expressed as

$$1639 = 1000 + 600 + 30 + 9 = 1 \times 10^3 + 6 \times 10^2 + 3 \times 10^1 + 9 \times 10^0$$

Thus the individual digits (1, 6, 3, 9) in the number (1639) are the coefficients of some power of 10 (ten). 10 is base in decimal system.

Similarly, $0{\cdot}2642 = 2 \times 10^{-1} + 6 \times 10^{-2} + 4 \times 10^{-3} + 2 \times 10^{-4}$

and $26{\cdot}42 = 2 \times 10^1 + 6 \times 10^0 + 4 \times 10^{-1} + 2 \times 10^{-2}$

In the binary system, the base is 2 (two) since only two digits 0 and 1 are used in the system. The individual digits in a binary number represent the coefficients of the power of two. For example, the binary number 1001 whose dicimal equivalent is 9, can be expressed as

$$1001 = 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 9$$

In digital work each of the binary digits is called a **bit** and a group of bits having a significance is called a **bite**. For example, the binary number 1010 has four binary digits or four bits. The number itself is a bite.

❂ 7.3. Decimal to binary conversion ❂

We have already learned that in decimal system we use ten digits from 0 to 9 and its base is 10 (ten). In binary system, only two 0 and 1 are used and its base is 2. Hence in binary system, any number has to be expressed by only two digits 0 and 1. It must be mentioned that the meaning of the digits 0 and 1 is same in both the system. But their positional value in a number in two systems are different.

Conversion of a decimal number into its binary equivalent consists in expressing the decimal number as the sum of ascending powers of 2 and collecting the coefficient of each term.

For example, to convert 7 into its binary equivalent, we write

$$7 = 4 + 2 + 1 = 2^2 + 2 + 1 = 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^6$$

So, the binary equivalent of 7 is 111 (one-one-one). Two other examples are :

$$11 \text{ (eleven)} = 8 + 2 + 1 = 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 \rightarrow 1011$$

$$23 \text{ (twenty three)} = 16 + 4 + 2 + 1 = 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 \rightarrow 10111$$

➡ Alternative method :

Another method of converting a decimal number into its equivalent binary form is to **divide** progressively the decimal number by 2 until a quotient of zero is obtained. So, the steps to be followed are—

(i) The decimal number is divided progressively be 2 until the quotient is zero.

(ii) The remainders after each division taken in reverse order gives the binary form of the number.

Here we consider two examples—

(a) To convert the decimal number 37 to binary form

$37 \div 2 = 18$ remainder $= 1$

$18 \div 2 = 9$ remainder $= 0$

$9 \div 2 = 4$ remainder $= 1$

$4 \div 2 = 2$ remainder $= 0$

$2 \div 2 = 1$ remainder $= 0$

$1 \div 2 = 0$ remainder $= 1$

∴ The required binary number = 100101 i.e., $(37)_{10} = (100101)_2$

(b) Conversion of decimal number 129 into binary form

$129 \div 2 = 64$ remainder $= 1$

$64 \div 2 = 32$ remainder $= 0$

$32 \div 2 = 16$ remainder $= 0$

$16 \div 2 = 8$ remainder $= 0$

$8 \div 2 = 4$ remainder $= 0$

$4 \div 2 = 2$ remainder $= 0$

$2 \div 2 = 1$ remainder $= 0$

$1 \div 2 = 0$ remainder $= 1$

$\therefore$ The binary equivalent of 129 is 10000001 ; $(129)_{10} = (10000001)_2$

If the decimal number be a fraction, the binary equivalent is obtained by multiplying the number continuously by 2, recording each time a carry in the integer position. The process is continued till the fractional part is zero or enough bit of binary equivalent is obtained. The carries in the forward order gives the required binary fraction. We consider two examples :

(a) Obtain the binary equivalent of 0·75.

$0{\cdot}75 \times 2 = 1{\cdot}50 = 0{\cdot}5$ with a carry of 1

$0{\cdot}5 \times 2 = 1{\cdot}00 = 0{\cdot}00$ with a carry of 1

$\therefore$ Binary equivalent of 0·75 is 0·11

(b) To obtain the binary equivalent of 37·625

The binary equivalent of 37 is 100101

Now, the binary equivalent of 0·625 is obtained as—

$0{\cdot}625 \times 2 = 1{\cdot}250 = 0{\cdot}250$ with a carry of 1

$0{\cdot}250 \times 2 = 0{\cdot}500 = 0{\cdot}500$ with a carry of 0

$0{\cdot}500 \times 2 = 1{\cdot}000 = 0{\cdot}000$ with a carry of 1

$\therefore$ $(0{\cdot}625)_{10} = (0{\cdot}101)_2$

$\therefore$ $(37{\cdot}0625)_{10} = (100101{\cdot}101)_2$

7.4. Binary to decimal conversion

The conversion of a binary number into its decimal equivalent can be carried out by the following procedure.

(i) The binary number is written.

(ii) Directly under the binary digits the corresponding decimal weights are written.

(iii) If zero occurs in a digit position of the binary number, the decimal weight for that position is crossed out.

(iv) The remaining weights are added to obtain the decimal equivalent.

- **First example :** We consider the decimal conversion of the binary number 10010

B. N.	→	1	0	0	1	0
Weights	→	16	$\cancel{8}$	$\cancel{4}$	2	$\cancel{1}$
Decimal	=	16			+ 2	
equivalent	=	18				

Hence $(10010)_2 = (18)_{10}$

- **Second example :** To convert the binary number 110101 into its binary equivalent.

B. N.	→	1	1	0	1	0	1
Weights	→	32	16	$\cancel{8}$	4	$\cancel{2}$	1

Decimal equivalent = 32 + 16 + 4 + 1

= **53**

∴ $(110101)_2 = (53)_{10}$

- **Third example :** To convert the binary fractions number 0·1101 to its decimal equivalent.

B. N.	→	0	·	1	1	0	1
Weights	→			$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$

Decimal equivalent $= \frac{1}{2} + \frac{1}{4} + \frac{1}{16} = 0{\cdot}5 + 0{\cdot}25 + 0{\cdot}0625$

= **0·8125**

∴ $(0{\cdot}1101)_2 = (0{\cdot}8125)_{10}$

- **Fourth example :** To convert the mixed binary number 10011·11 to its decimal equivalent.

B. N.	→	1	0	0	1	1	·	1	1
Weights	→	16	$\cancel{8}$	$\cancel{4}$	2	1	·	$\frac{1}{2}$	$\frac{1}{4}$

Decimal equivalent $= 16 + 2 + 1 + \frac{1}{2} + \frac{1}{4}$

= 19 + 0·5 + 0·25

= **19·75**

∴ $(10011{\cdot}11)_2 = (19{\cdot}75)_2$

7.5. Addition of Binary numbers

The binary addition can be made with following four rules :

(i) 0 + 0 = 0 i.e., zero added to zero gives zero.

(ii) 0 + 1 = 1 i.e., zero added to one gives one.

(iii) 1 + 0 = 1 i.e., one added to zero gives one.

(iv) 1 + 1 = 10 i.e., one added to one gives two.

Some example of binary addition are given below :

(a) decimal equivalent

$$\begin{array}{r} 1\,0\,1 \\ \text{L}\ 1\,1\,0 \\ \hline 1\,0\,1\,1 \end{array} \qquad \begin{array}{r} 5 \\ +6 \\ \hline 11 \end{array} \text{ (eleven)} \qquad 1011 = 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^2 + 1 \times 2^0 = 11$$

(b)

$$\begin{array}{r} 1\,0\,1\,1 \\ +\,1\,0\,0\,1 \\ \hline 1\,0\,1\,0\,0 \end{array} \qquad \begin{array}{r} 11 \\ +\,9 \\ \hline 20 \end{array} \qquad 10100 = 1 \times 2^4 + 1 \times 2^2 = \mathbf{20}$$

(c)

$$\begin{array}{r} 1\ 0\ 1\ 0 \\ +\ 1\ 1\ 0\ 1 \\ \hline 1\ 0\ 1\ 1\ 1 \end{array} \qquad \begin{array}{r} 10 \text{ (ten)} \\ +13 \\ \hline 23 \end{array} \qquad 10111 = 1 \times 2^4 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = \mathbf{23}$$

- **Example 7.1. Add the two binary numbers 10011 and 11010.**

○ **Solution :**

$$\begin{array}{r} 1\ 0\ 0\ 1\ 1 \\ +\ 1\ 1\ 0\ 1\ 0 \\ \hline 1\ 0\ 1\ 1\ 0\ 1 \end{array} \qquad \begin{array}{r} 1\ 0\ 0\ 1\ 1 = 19 \\ 1\ 1\ 0\ 1\ 0 = 26 \\ \hline 45 \end{array}$$

Now, $101101 = 1 \times 2^5 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^0$

$= 32 + 8 + 4 + 1 = \mathbf{45}$

- **Ex. 7.2. Add the binary numbers 101101 and 11111.**

○ **Solution :**

$$\begin{array}{r} 1\ 0\ 1\ 1\ 0\ 1 \\ +\ \ 1\ 1\ 1\ 1\ 1 \\ \hline 1\ 0\ 0\ 1\ 1\ 0\ 0 \end{array} \qquad \begin{array}{r} 1\ 0\ 1\ 1\ 0\ 1 = 45 \\ 1\ 1\ 1\ 1\ 1 = 31 \\ \hline 76 \end{array}$$

Now, $1001100 = 1 \times 2^6 + 1 \times 2^3 + 1 \times 2^2 = \mathbf{76}$

- **Ex. 7.3. Add the binary numbers 111·11 and 101·10**

○ **Solution :**

$$\begin{array}{r} 1\ 1\ 1 \cdot 1\ 1 \\ +\ 1\ 0\ 1 \cdot 1\ 0 \\ \hline 1\ 1\ 0\ 1 \cdot 0\ 1 \end{array} \qquad \begin{array}{r} 1\ 1\ 1 \cdot 1\ 1 = 7{\cdot}75 \\ 1\ 0\ 1 \cdot 1\ 0 = 5{\cdot}50 \\ \hline 13{\cdot}25 \end{array}$$

Now, $1101{\cdot}01 = 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^0 + 1 \times 2^{-2}$

$= 8 + 4 + 1 + 0{\cdot}25 = \mathbf{13{\cdot}25}$

○ 7.6. Binary subtraction ○

Binary subtraction obeys the following four rules.

(i) 0 – 0 = 0 (ii) 1 – 0 = 1

(iii) 1 – 1 = 0 (iv) 10 – 1 = 1

The last rule indicates that when 1 is subtracted from 2 (the decimal equivalent of 10 is 2) we get 1 (one). Now, we consider the following examples on subtractions.

$$\text{(i)}\ \begin{array}{r} 1\ 0\ 1 \\ -\ 1\ 0\ 0 \\ \hline 0\ 0\ 1 \end{array} \qquad \text{(ii)}\ \begin{array}{r} 1\ 0\ 0\ 1\ 1 \\ -\ 1\ 0\ 0\ 0\ 1 \\ \hline 0\ 0\ 0\ 1\ 0 \end{array} \qquad \text{(iii)}\ \begin{array}{r} 1\ 0\ 1\ 0\ 1 \\ -\ 1\ 0\ 1\ 1 \\ \hline 1\ 0\ 1\ 0 \end{array}$$

- **Example 7.4. Subtract 1010 from 1101.**

○ **Solution :**

$$\begin{array}{r} 1\ 1\ 0\ 1 \\ +\ 1\ 0\ 1\ 0 \\ \hline 0\ 0\ 1\ 1 \end{array} \qquad \begin{array}{r} 1\ 1\ 0\ 1 = \ \ 13 \\ -\ 1\ 0\ 0\ 1 = -10 \\ \hline 3 \end{array}$$

Also, $0011 = 1 \times 2 + 1 = 3$

Method : 1st column, 1 – 0 = 1 ; 2nd column 10 (borrowing one from preceeding column) – 1 = 1; 3rd column 0 (after borrow) – 0 = 0 and fourth column, 1 – 1 = 0.

∴ Result of subtraction : **0011 = 3**

- **Ex. 7.5. Subtract 1110 from 10001.**

○ **Solution :**

$$\begin{array}{r} 1\,0\,0\,0\,1 \\ -\quad 1\,1\,1\,0 \\ \hline 0\,0\,0\,1\,1 \\ \hline \end{array}$$

- **Ex. 7.6. Subtract 111 from 11001.**

○ **Solution :**

$$\begin{array}{r} 1\,1\,0\,0\,1 \\ -\quad 1\,1\,1 \\ \hline 1\,0\,0\,1\,0 \\ \hline \end{array} \qquad \begin{array}{r} 25 \\ -7 \\ \hline 18 \\ \hline \end{array}$$

Complement methods of subtraction :

There are two other methods of binary subtraction, known as **1's complement method** and **2's complement method**. Both the methods will be discussed here.

(i) The 1's complement of a binary number is a number which is obtained by replacing each 0 (zero) by 1 and each 1 by 0 (zero). Thus, 1's complement of the binary number 10011 is 01100.

(ii) 2's complement of a binary number is obtained is obtained by adding 1 to its 1's complement. Thus the 2's complement of the binary number 10011 is

01100 + 1 = 01101

Similarly, 1's complement of 11001 is 00110 and its 2's complement is 00110 + 1 = 00111.

The following table shows 1's and 2's complement of same binary numbers.

Binary number	1's Complement	2's Complement
1010	0101	0110
1011	0100	0101
11010	00101	00110
10100	01011	01100

1's complement method of subtraction :

In doing subtraction by 1's complement method, the following steps are taken :

(a) find 1's complements of the number to be subtracted.

(b) add it to the number from which the subtraction is required

(c) if there is a 1 carry in the last position of the result obtained of addition in (ii) remove this carry and add it to the result without the carry to get the final answer.

Following 1's, complement method **let us subtract 1011 from 1101.**

$$\begin{array}{lr} & 1\,1\,0\,1 \\ & +\ 0\,1\,0\,0 \text{ (1's complement of 1011)} \\ \hline \text{1 Carry} & \boxed{1}\,0\,0\,0\,1 \\ \text{1 Carry is removed} & 0\,0\,0\,1 \\ \text{1 Carry is added} & 1 \\ \hline & 0\,0\,1\,0 \end{array}$$

Conventional method

$$\begin{array}{r} 1\,1\,0\,1 \\ -\ 1\,0\,1\,1 \\ \hline 0\,0\,1\,0 \\ \hline \end{array}$$

(d) In step (b), if there is no 1 carry in the last position, then the answer is negative and is in its 1's complement form. Hence to get the final answer, 1's complement of the result of addition in (b) is determined and a minus sign is added before it.

Example : 1 0 0 1 – 1 0 1 1 = ?

1 0 0 1
+ 0 1 0 0 (1's complement of 1011)
No Carry 1 1 0 1

∴ Result is – 0010 (1's complement of 1101)

2's complement method of subtraction :

In 2's complement method of subtraction, the 2's complement of the subtracting number is **added** to the number from which the subtraction is required and the carry in the last position is disregarded. For example we subtract 10001 from 10011.

1 0 0 1 1
+ 0 1 1 1 1 (2's complement of 10001)
$\boxed{1}$ 0 0 0 1 0

disregrading 1 carry, the result is 00010.

If there is no 1 carry in the last position, the answer of subtraction is negative and remains as the 2's complement of the magnitude of the actual answer. Hence, in this case, the 2's complement of the result is determined. A minus sign is added before it to get the final answer.

As an example we subtract 10011 from 10001

1 0 0 0 1
+ 0 1 1 0 1 (2's complement of 10011)
No Carry 1 1 1 1 0

Now, 2's complement of 11110 = 00001 + 1 = 00010

Hence the answer is **– 00010**.

- **N. B.** Evidently subtraction of two binary number can be down in any of the three ways :
(i) Conventional method (ii) 1's complement method and (iii) 2's complement method.

7.7. Binary multiplication and division

The following four rules are obeyed in the binary subtraction :

(i) 0 × 0 = 0, (ii) 0 × 1 = 0, (iii) 1 × 0 = 0, (iv) 1 × 1 = 1.

Binary multiplication of binary numbers can be done in the same way as decimal multiplication. As an example, we consider the multiplication of 10101 by 101.

- (i) **First example :**

1 0 1 0 1
× 1 0 1
1 0 1 0 1
0 0 0 0 0
1 0 1 0 1
1 1 0 1 0 0 1

1 0 1 0 1 = 2 1
1 0 1 = × 5
1 0 5

Again, 1 1 0 1 0 0 1 = 105

- (ii) **Second example :** Multiplication of 1011·11 by 101·1.

$$\begin{array}{r} \times\ 1\,0\,1\,1\cdot 1\,1 \\ 1\,0\,1\cdot 1 \\ \hline 1\,0\,1\,1\,1\,1 \\ 1\,0\,1\,1\,1\,1 \\ 0\,0\,0\,0\,0\,0 \\ 1\,0\,1\,1\,1\,1 \\ \hline 1\,0\,0\,0\,0\,0\,0\,0\cdot 1\,0\,1 \end{array} \qquad \begin{array}{rcr} 1\,0\,1\,1\cdot 1\,1 & = & 1\,1\cdot 7\,5 \\ 1\,0\,1\cdot 1 & = & \times\,5\cdot\,5 \\ \cline{3-3} & & 6\,4\cdot 6\,2\,5 \end{array}$$

Also, 1000000·101 = **64·625**

Binary division can also be done in the same way as in decimal division. As an example, we consider the division of 1010 by 10.

$$\begin{array}{r} 10\,)\,1\,0\,1\,0\,(\,1\,0\,1 \\ 1\,0 \\ \hline 1\,0 \\ 1\,0 \\ \hline \end{array} \qquad \begin{array}{l} 1\,0\,1\,0 = 1\,0 \\ 1\,0 = 2 \\ \therefore\ 10 \div 2 = 5 \end{array}$$

Also **101 = 5**

- **Example 7.7. Carry out the division of 1111 by 11.**

○ **Solution :**

$$\begin{array}{r} 1\,1\,)\,1\,1\,1\,1\,(\,1\,0\,1 \\ 1\,1 \\ \hline 1\,1 \\ 1\,1 \\ \hline \end{array}$$

- **Ex. 7.8. Find the value of 110111 ÷ 0101**

○ **Solution :**

$$\begin{array}{r} 1\,0\,1\,)\,1\,1\,0\,1\,1\,1\,(\,1\,0\,1\,1 \\ 1\,0\,1 \\ \hline 0\,1\,1\,1 \\ 1\,0\,1 \\ \hline 1\,0\,1 \\ 1\,0\,1 \\ \hline \end{array}$$

✪ 7.8. Boolean operators ✪

The principle of action of electronic circuits of a computer system are based on the principles of Boolean algebra invented by George Boole. Boolean algebra deals with logical statements that have only two values—either a **true** value or a **false** value. In digital circuits following binary system recognises only two states 'ON' and 'OFF'. A boolean variable or a logical statement is denoted by 1 corresponding to its true value and by 0 corresponding to its false value.

In ordinary algebra (using the decimal system) we use various mathematical operators, e.g. addition, subtraction and multiplication. In boolean algebra, the three basic operators used are : (i) OR, (ii) AND, and (iii) NOT.

✪ 7.9. Logic Gates ✪

The logic gates are building blocks of digital electronics. They are used in digital electronics to change one voltage level (input voltage) into another (output voltage) in accordance with some logical relationship between them.

A logic gate may have one or more inputs, but it has only one output. The relation between the possible values of input and output voltages are expressed in the form of a table called **Truth table**. Hence a truth table of a logic gate is a table that shows all the input and output possibilities for the logic gate.

Logic circuits may be of different types. The three basic logic gates of digital electrons are (i) OR gate, (ii) AND gate and (iii) NOT gates. These three gates are also combined to give rise to other gates e.g. NAND, NOR and XOR. Every gate is represented by specific symbol. The three basic logic gates obey the following Boolean expressions.

(i) The addition sign (+) is referred to as **OR**. The boolean expression $y = A + B$ is read as y equals A OR B.

(ii) The multiplication sign (·) is referred to as AND in Boolean algebra. The Boolean expression $y = A \cdot B$ is read as y equals A AND B.

(iii) The bar sign (–) is referred to as NOT in Boolean algebra. The Boolean expression $y = \bar{A}$ is read as y equals NOT A.

✪ 7.10. OR Gate ✪

The OR gate has two or more inputs and a single output. The output of the OR gate assumes the state 1 if one or more of the inputs are in state 1. The working of OR gate can be understood by the simple electrical circuit shown in fig. 7.3. Three switches A, B and C are connected in parallel and this combination is connected in series with a bulb and a battery. Evidently the lamp will glow or output is obtained when any of the three switches (A or B or C) is closed. Again if any two or all the three are closed, the bulb will glow and output is obtained. But if all the three switches are kept open, the bulb will not glow or no output is obtained.

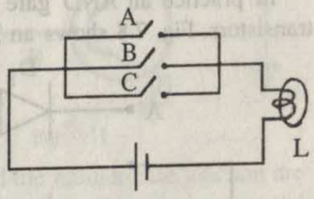


Fig. 7·3

The logic symbol for a 2 input OR gate is shown in fig. 7.4 A and B are the inputs and Y is the output.

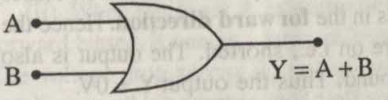


Fig. 7·4

In practice, the OR gate is realised with the help of electronic circuit using two diodes D_1 and D_2 as shown in fig. 7.5. Actually, the diodes acts as switches.

The input voltage to be applied at A and B is either 0 or 5V. The resistor R is the output load resistor. Suppose that the diodes are ideal. Three are four possibilities in which the input voltages may appear. There are :

(i) Both A = 0V and B 0V. Under this condition, none of the diodes conduct. Hence no voltage develops across resistor and accordingly output $y = 0$.

(ii) A = 0V, B = 5V. Under this condition, the diode D_2 is forward biased and hence it conducts. The whole of the input voltage appears at the output. Since the diode is forward biased, resistance is negligible. So, Y = 5V.

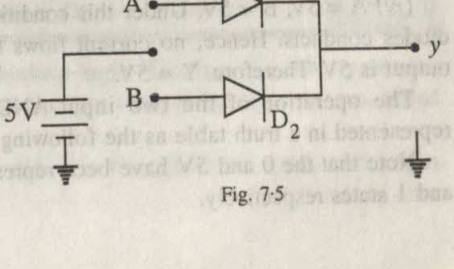


Fig. 7·5

(iii) A = 5V, B = 0V. Now, the diode D_1 conducts and the output Y = 5V.

(iv) Both A = 5V and B = 5V. Under this conditions, both the diode forward biased and they conduct. The output is Y = 5V and the voltages are in parallel.

The operation of the OR gate is represented in the **truth table** or in the **table of combination**. The truth table of the gate with two inputs is shown in the table. In the truth table, the 0 or 5V have been represented by the 0 and 1 states respectively.

Table–7·1

Input		Output
A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

❂ 7.11. AND Gate ❂

An AND gate is an electronic circuit which has two or more inputs but only one output. The output of an AND gate attains the state 1 if and only if all the inputs are in state 1. An AND gate is also called **coincidence circuit**. The working of AND gate can be easily understood by the simple electrical circuit as shown in fig. 7.6.

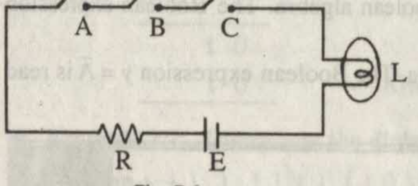


Fig. 7·6

It is evident that the bulb L will glow or output will be obtained only when all the three switches A, B and C are closed. The bulb L will not glow or output will not be obtained if any of the switches is kept open.

The AND gate is represented by the symbol is shown in fig. 7.7.

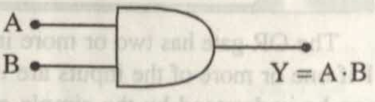


Fig. 7·7

In practice an AND gate is built by using either diodes or transistors. Fig. 7.8 shows an AND gate built by two diodes. It is supposed that the diodes are ideal and that the two input voltages are either 0 or 5V. The operation of AND gate can be discussed in the following four cases.

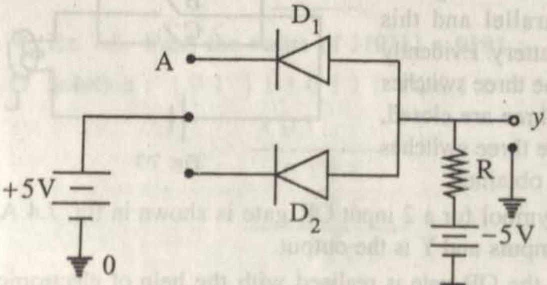


Fig. 7·8

(i) Both A = 0V and B = 0V. Under this condition, both the inputs are shorted to ground. The 5V battery in the output side, however, biases the two diodes in the **forward direction**. Hence the two diodes are on i.e., shorted. The output is also shorted to ground. Thus the output Y = 0V.

(ii) A = 5V, B = 0V. In this case the diode D_2 conducts and the output is short-circuited to ground through this diode. Thus Y = 0V.

(iii) A = 0V, B = 5V. Now, the diode D_1 will conduct, while D_2 will not conduct. Now, the output y is the voltage across D_1, which 0. So, Y = 0V.

(iv) A = 5V, B = 5V. Under this condition, none of the diodes conducts. Hence, no current flows through R. The output is 5V. Therefore, Y = 5V.

The operation of the two input AND gate may be represented in a truth table as the following table.

Note that the 0 and 5V have been represented by the 0 and 1 states respectively.

Table–7·2

Input		Output
A	B	Y = A · B
0	0	0
1	0	0
0	1	0
1	1	1

⚙ 7.12. NOT Gate ⚙

The NOT gate is a one input and one output logic gate. It combines the input A with the output y following the boolean expression $y = \bar{A}$.

The way the NOT gate gives the output, it is also called **invertor**. The output of this circuit assumes the state 1 if, and only if, the input does not assume the state 1 i.e., the output is high if the input is low and vice versa. Thus a NOT circuit inverts the sense of the output with respect to the input.

The possible input output combination of a NOT gate can be easily discussed with the help of electrical circuit as shown in fig. 7.9. Here the switch is connected in parallel to the lamp and the battery. It is found that

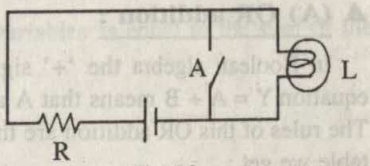


Fig. 7·9

(i) If the switch A is open (i.e. A = 0), the lamp will glow (i.e. $y = 1$).

(ii) If the switch A is closed (i.e. A = 1), the lamp will not glow (i.e. $y = 0$).

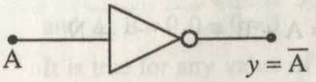


Fig. 7·10

Hence, in this circuit if the input is present, output is zero. Hence the electrical circuit can be called a NOT gate. Hence in a NOT gate, if the input is in OFF or 0 (zero) state, then output is in ON or 1 state. So a NOT gate inverts the output with respect to input.

The NOT gate is represented by the symbol as shown in fig. 7.10.

Realisation of NOT gate :

A NOT gate cannot be realised by diodes and we have to use a transistor. The required circuit diagram is shown in fig. 7.11.

The base (B) of the transistor is connected to A through a resistor R_b and the emitter is earthed. The collector is earthed through a resistor R_c and a 5V battery. The operation of the NOT gate can be discussed in following two ways :

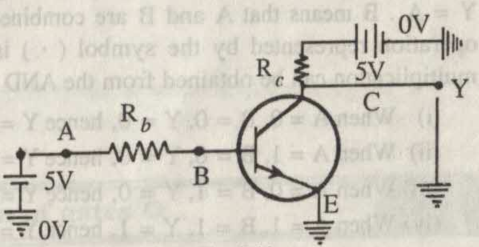


Fig. 7·11

(i) When the input A is connected to 0, the base-collector junction and the emitter-base junction are both reverse-biased. Hence the base current and the collector current are both zero. The transistor is said to be in **cut-off mode**. The voltage at Y is 5V, which corresponds to the state 1. So when A = 0, $y = 1$.

(ii) When the input A is connected to 1, the base-emitter junction is forward biased. It results a large collector current. The transistor **saturates.** The voltage drop across R_c is just equal to 5V and the output y is very nearly zero which corresponds to 0 state. So, when A = 1, $y = 0$.

The boolean expression in this case is $y = \bar{A}$. It is read as **'y equals NOT A'**.

The truth table of NOT gate is shown in the table.

Table–7·3

Input	Output
A	$Y = \bar{A}$
0	1
1	0

⚙ 7.13. Boolean algebra ⚙

The analysis and design of digital circuits can be done by **Boolean logic**. This logic is a binary or two-valued logic and resembles ordinary algebra in many respects. For this reason, the logic is also called **Boolean algebra.**

Boolean algebra is based on two values or states for a variable. The two permitted states of Boolean algebra are usually represented by 0 and 1.

The three operations that are employed on the variables in Boolean algebra are :

(i) the OR **addition** denoted by a plus (+) sign.

(ii) the AND **multiplication** denoted by a dot (.) sign and

(iii) the NOT **operation** represented by a bar over a variable.

The meaning of these Boolean operations are discussd here.

▲ (A) OR addition :

In Boolean algebra the '+' sign stands for addition. It represents the action of the OR gate. The equation $Y = A + B$ means that A and B are to be added in the same way as an OR gate combine them. The rules of this OR addition are the same as that given by the OR gate truth table (table 7.1). From this table we get :

(i) When $A = 0$, $B = 0$, $Y = 0$, hence $Y = A + B = 0 + 0 = 0$

(ii) When $A = 1$, $B = 0$, $Y = 1$, hence $Y = A + B = 1 + 0 = 1$

(iii) When $A = 0$, $B = 1$, $Y = 1$, hence $Y = A + B = 0 + 1 = 1$

(iv) When $A = 1$, $B = 1$, $Y = 1$, hence $Y = A + B = 1 + 1 = 1$

The Boolean expression for the OR gate with *n* inputs is written as $Y = A + B + \cdots\cdots + N$.

▲ (B) AND multiplication :

In Boolean algebra, the multiplication sign (·) has a new meaning. In this algebra the equation $Y = A \,.\, B$ means that A and B are combined in the same way as an AND gate combines them. The operation represented by the symbol (·) is referred to as AND multiplication. The rules of AND multiplication can be obtained from the AND gate truth table (table 7.2). From this table we get :

(i) When $A = 0$, $B = 0$, $Y = 0$, hence $Y = A \cdot B = 0 \cdot 0 = 0$

(ii) When $A = 1$, $B = 0$, $Y = 0$, hence $Y = A \cdot B = 1 \cdot 0 = 0$

(iii) When $A = 0$, $B = 1$, $Y = 0$, hence $Y = A \cdot B = 0 \cdot 1 = 0$

(iv) When $A = 1$, $B = 1$, $Y = 1$, hence $Y = 1 \,.\, 1 = 1$

Evidently, AND multiplication is the same as that of ordinary multiplication.

▲ (C) NOT operation :

The NOT operation is indicated by a bar over the variable to be operated and means that the variable is to be complemented. Now in Boolean algebra the variable can assume only two values 0 and 1. So, if $A = 0$, then $\bar{A} = 1$ and if $A = 1$, $\bar{A} = 0$. Alternatively if $\bar{A} = 0$, then $A = 1$ and if $\bar{A} = 1$, then $A = 0$.

In Boolean algebra, the equation $Y = \bar{A}$ means that the variable A is to be changed in the same way as a NOT circuit changes if so produce a Y output. The Boolean expression for the NOT circuit is thus

$$Y = \bar{A}$$

when A is the input to the NOT circuit and $\bar{A}$ is the output.

The rules for OR, AND and NOT operations of Boolean algebra are shown in the following table :

OR operation	AND operation	NOT operation
$0 + 0 = 0$	$0 \cdot 0 = 0$	$\bar{0} = 1$
$1 + 0 = 1$	$1 \cdot 0 = 0$	$\bar{1} = 1$
$0 + 1 = 1$	$0 \cdot 1 = 0$	
$1 + 1 = 1$	$1 \cdot 1 = 1$	

❂ 7.14. De Morgan's theorem ❂

- **Theorem 1 :** The complement of the sum of two or more variables is equal to the product of the complements of the variables.

If A and B are the variable, according to this theorem $\overline{A+B}=\overline{A}\cdot\overline{B}$.

- **Theorem 2 :** The complement of the product of two or more variables is equal to the sum of the complements of the variables.

$\therefore\ \overline{A\cdot B}=\overline{A}+\overline{B}$

(i) To prove $\overline{\mathbf{A}+\mathbf{B}}=\overline{\mathbf{A}}\cdot\overline{\mathbf{B}}$

Each variable can have a value either 0 or 1.

Let A = 0, B = 0.

$\therefore\ \overline{A+B}=\overline{0+0}=\overline{0}=1$

and $\overline{A}.\overline{B}=\overline{0}.\overline{0}=\overline{0}=1$ $\quad\therefore\ \overline{A+B}=\overline{A}\,.\,\overline{B}$

It is true for any value of A and B.

(ii) To prove $\overline{A.\,B}=\overline{A}+\overline{B}$.

Let A = 0, B = 0

Then, $\overline{A.\,B}=\overline{0.0}=\overline{0}=1$

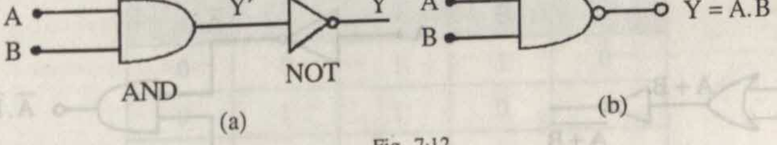
and $\overline{A}+\overline{B}=\overline{0}+\overline{0}=1+1=1$ $\quad\therefore\ \overline{A.B}=\overline{A}+\overline{B}$.

❂ 7.15. Combination of gates ❂

In the previous articles we have discussed operations of three basic logic gates—OR, AND and NOT. Different digital circuits are designed by the combination of these gates. Various combinations of three basic gates give rise to complicated digital circuits. Here we discuss a few combinations of these basic gates, e.g., NAND and NOR gates. These two gates are called universal gates.

▲ (A) NAND Gate :

It is a logic gate in which AND gate is followed by a NOT gate. Here the output Y′ of the AND gate is connected to the input of the NOT gate [fig. 7.12(a)]. The logic symbol of NAND gate is also shown in fig. 7.12(b).



(a) (b)

Fig. 7·12

In this case $Y' = A.B,\ Y=\overline{Y'}=\overline{A.B}$

The truth table of NAND gate is obtained by combining the truth tables of AND and NOT gates. It is shown in the following table.

A	B	$Y' = A . B$ (A AND B)	$Y = \overline{A.B}$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

The Boolean expression for the NAND gate is $Y = \overline{A.B} = \overline{A} + \overline{B}$

- **A discussion :** From the second De Morgan's theorem (i.e., $\overline{A.B} = \overline{A} + \overline{B}$) it is apparent that a logic circuit obtained by combining an AND and a NOT gate [fig. 7.13(a)] is equivalent to the logic circuit obtained by connecting the inputs of two NOR gates into an OR gate [fig. 7.13(b)].

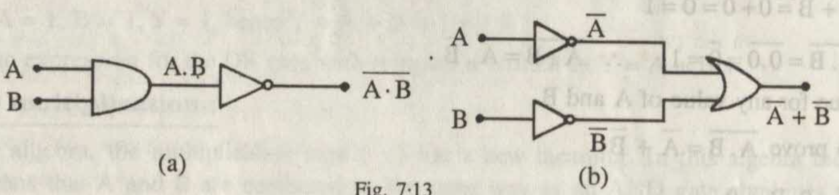


Fig. 7·13

▲ (B) NOR Gate :

It is a logic circuit OR gate is followed by a NOT gate. Here the output of an OR gate is connected to the input of a NOT gate [fig. 7.14(a)]. The logic symbol of a NOR gate is shown in fig. 7.14(b).

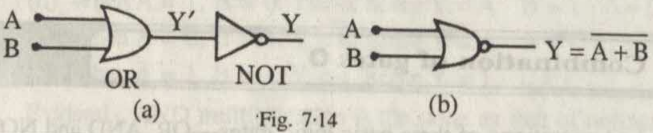


Fig. 7·14

The truth table of NOR gate can be obtained by combining the truth tables of OR and NOT gates. The combined truth table is given in table below.

A	B	$Y' = A + B$	$Y = \overline{A+B}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

The Boolean expression for NOR gate is : $Y = \overline{A+B}$

- **A discussion :** From De Morgan's first theorem (i.e., $\overline{A+B} = \overline{A}.\overline{B}$) it is evident that the logic gate

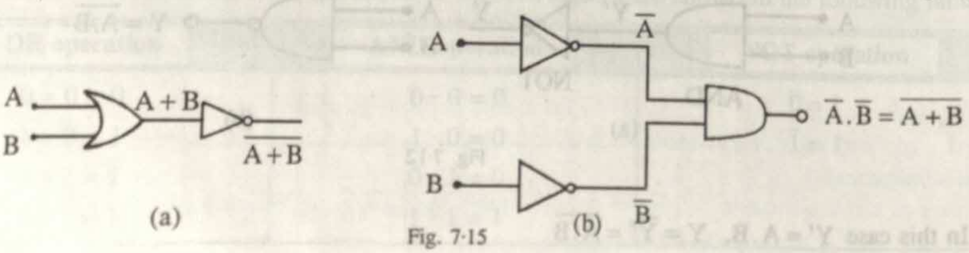


Fig. 7·15

obtainied by combining one OR gate with a NOT gate [fig. 7.15(a)] is equivalent to a logic gate when the outputs of two NOT gates is connected to the input of an AND gate [fig. 7.15(b)].

7.16. NAND Gate as a building block of a digital circuit

We have seen that different logic gates can be obtained by repeated use of the three basic gates—OR, AND and NOT. Conversely, repeated use of NAND gates can produce the three logic gates. Here we discuss how NAND gates can be connected to obtain OR, AND and NOT gates.

(a) To produce NOT gate from NAND gates : Fig. 7.16 shows how to obtain NOT gate from NAND gate. Here the two inputs of a NAND gate are joined so that it has only one input. So in this case A = B. The truth table is shown in the table. It is the truth table of a NOT gate. So NAND gate can act as a NOT gate when the two inputs of the NAND gate are joined.

A	B = A	Y
0	0	1
1	1	0

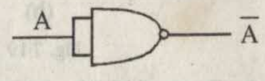


Fig. 7·16

(b) To produce AND gate from NAND : In fig. 7.17, we show how AND gate can be obtained from NAND gates. Here the output of the NAND gate is connected to the NOT gate (made from NAND gate by joining the two inputs). The combination acts as AND gate. The combined truth table is shown below :

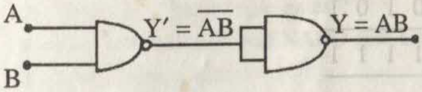
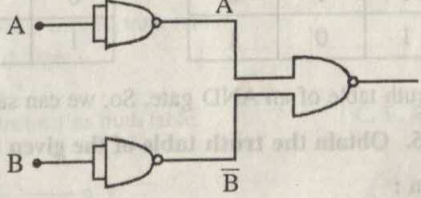


Fig. 7·17

A	B	Y′	Y
0	0	1	0
1	0	1	0
0	1	1	0
1	1	0	1

(c) To produce the OR gate by using NAND gates : By joining the input terminals of NAND gate we get a NOT gate. Now, the outputs of two such NOT gates be given to a NAND as shown in fig. 7.17.

The truth table shows that this arrangement works as a OR gate.



$$\begin{aligned} Y &= \overline{\overline{A}.\overline{B}} \\ &= \overline{A}.\overline{B} \\ &= A+B \end{aligned}$$

Fig. 7·18

A	B	$\overline{A}$	$\overline{B}$	Y
0	0	1	1	0
0	1	1	0	1
1	0	0	1	1
1	1	0	0	1

(d) XOR Gate : XOR gate can be produced by the combination of OR, AND and NOT gates as shown in fig. 7.19(a). It is also exclusive OR gate. Its logic symbol is shown in fig. 7.19(b).

The truth table for XOR gate is shown below :

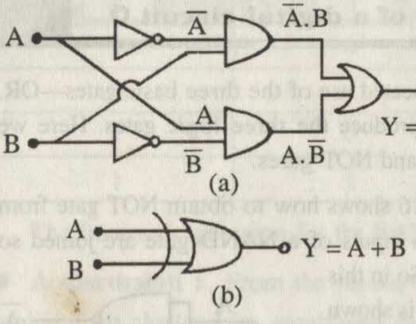


Fig. 7·19

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

- **Example 7.13. Add the following binary numbers.**

(i) $(110010)_2$ and $(111101)_2$ (ii) $(101010)_2$ and $(010101)_2$

○ **Solution :** (i)

$$\begin{array}{r} 1\,1\,0\,0\,1\,0 \\ 1\,1\,1\,1\,0\,1 \\ \hline 1\,1\,0\,1\,1\,1\,1 \end{array}$$

(ii)

$$\begin{array}{r} 1\,0\,1\,0\,1\,0 \\ 0\,1\,0\,1\,0\,1 \\ \hline 1\,1\,1\,1\,1\,1 \\ \hline \end{array}$$

- **Ex. 7.14. A combination of two NAND gates are shown in fig. 7.20. Show that the combination behaves as an AND gate.**

○ **Solution :** Truth table of the combination is shown in fig.

A	B	Y′	Y
0	0	1	0
1	0	1	0
0	1	1	0
1	1	0	1

≡

A	B	Y
0	0	0
1	0	0
0	1	0
1	1	1

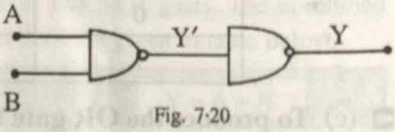


Fig. 7·20

It is a truth table of an AND gate. So, we can say that the combination behaves as an AND gate.

- **Ex. 7.15. Obtain the truth table of the given combination of gates.**

○ **Solution :**

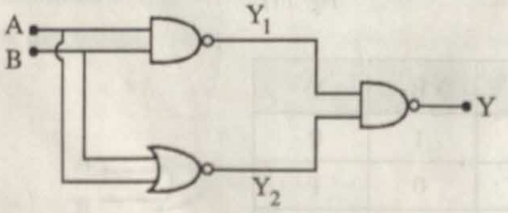


Fig. 7·21

A	B	Y_1	Y_2	Y
0	0	1	1	0
1	0	1	0	1
0	1	1	0	1
1	1	0	0	1

≡

A	B	Y
0	0	0
1	0	1
0	1	1
1	1	1

So the combination behaves as an OR gate.

• E ◈ X ◈ E ◈ R ◈ C ◈ I ◈ S ◈ E •

A. Objective type questions :

Fill in the blanks :

1. In binary number system only two digit ______ and ______ are used.
2. In binary system of numbers base is ______ .
3. Decimal equivalent of binary number (1111) is ______ .
4. 1's complement of the binary number 11001 is ______ .
5. 2's complement of the binary number 11001 is ______ .
6. For an OR gate 1 + 1 = ______ .
7. For an NOR gate 1 + 1 = ______ .
8. NAND gate and NOR gate is called ______ gate.
9. NAND gate is obtained by combining an ______ gate and a ______ gate.
10. NOR gate is obtained by combining an ______ gate and a ______ gate.

B. Very short answer type questions :

1. What is the base in decimal system ?
2. What is the base in binary system ?
3. What are the basic logic gates ?
4. How is a NOR gate and NAND gate are obtained ?
5. What is a logic gate ?

C. Short answer type questions :

1. What is an Analogue circuit ? Give examples.
2. What is an digital circuit ? Give example.
3. What we mean by decimal system and binary system ?
4. What is meant by the base of a number system ? What is the base for decimal system and binary system ?
5. Why is binary number is used in digital system ?
6. What is a truth table ?
7. Give symbolic representation of OR, AND and NOT gate.
8. Write the truth table for OR, AND and NOT gate.
9. How is a NAND gate formed. What is its logic symbol ? Give its truth table.
10. How is an XOR gate formed. Give is symbol and truth table.
11. Why are NAND and NOR gate called universal gates ?
12. What is the name of the gates shown in fig. 7.22. Construct its truth table. **[C. B. S. E. 1996]**

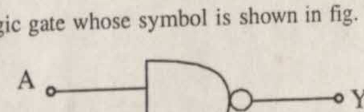


Fig. 7·22

13. What is the name of the logic gate whose symbol is shown in fig. 7.23.

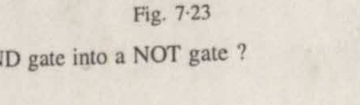


Fig. 7·23

14. How can you convert a NAND gate into a NOT gate ?

15. Which logic gate has the following truth table ?

A	B	Y
1	1	0
1	0	1
0	1	1
0	0	1

16. A combination of logic gates is shown in fig. 7.24. For all possible values of A and B, determine Y and obtain the truth table of the combination. **[Ans :** NAND Gate]

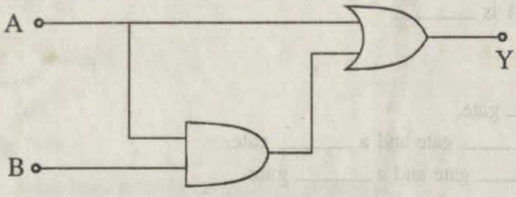


Fig. 7·24

A	B	Y
0	0	0
0	1	0
1	0	1
1	1	1

D. Essay type questions :

1. (a) Discuss the difference between analogue and digital circuit with examples.
(b) What are the difference between decimal system and binary system ?

2. What is an OR gate ? Explain its working with circuit diagram. Obtain its truth table.

3. How is an AND gate. How does it operates ? Explain with diagram. Also obtain its truth table.

4. What is a NOT gate. Explain its working with circuit diagram. Also obtain its truth table.

5. How a NAND gate and a NOR gate are obtained. Give their logic symbol. Also obtain truth table.

6. How can you obtain (i) NOT gate (ii) AND gate and (iii) OR gate by using NAND gates.

7. How is XOR gate is formed by using OR, AND and NOT gate ? What is its symbol ? Obtain its truth table.

8. The output of an AND gate is applied to the input of a NOT gate. If A and B be the inputs of the AND gate and output of the NOT gate be C. Obtain the Boolean expression for the output.

E. Numerical problems :

1. Convert the following decimal numbers to binary numbers.

(i) 13 (ii) 37 (iii) 58 (iv) 25 (v) 0·75 (vi) 15·625

[Ans : $(1101)_2$ (ii) $(100101)_2$ (iii) $(111010)_2$ (iv) $(11001)_2$ (v) $(0·11)_2$ (vi) $(1111·101)_2$]

2. Convert the following binary numbers to decimal numbers :

(i) $(100100)_2$ (ii) $(1011)_2$ (iii) $(1110)_2$ (iv) $(110·101)_2$ (v) $(10001·1)_2$ (vi) $(0·111)_2$ (vii) $(10011·0101)_2$

[Ans : $(36)_{10}$ (ii) $(11)_{10}$ (iii) $(14)_{10}$ (iv) $(6·625)_{10}$ (v) $(17·5)_{10}$ (vi) $(0·875)_{10}$ (vii) $(19·3175)_{10}$]

3. Add : $(1101)_2 + (1010)_2$ **[Ans :** $(10111)_2$]

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